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Models for charged relativistic spheres via hyper-geometric equations

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Abstract. Exact solutions to Einstein–Maxwell systems play an important role in relativistic astrophysics. In this paper, a new technique to generate exact solutions to the Einstein–Maxwell system is proposed. Corresponding to a spherically symmetric charged fluid sphere, by specifying the electric field and for a particular form of the metric potential g_{rr}, a new solution is obtained in terms of hypergeometric functions. Subsequently, for specific choice of model parameters, many closed-form solutions are developed. In the process, it is possible to regain a number of well-known stellar models which had been developed earlier with or without the presence of charge following the Vaidya and Tikekar ansatz for compact stars (*J. Astrophys. Astron*. **3**:325 (1982); *J. Math. Phys.* **31**:2454 (1990)). It is shown that the new class of solutions can be used as viable models for compact stars for a wide range of values of the model parameters. Physical behaviour of the resultant stellar configurations are studied.

Keywords. Einstein–Maxwell space–time; relativistic star; hypergeometry.

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1. Introduction

In gravitational theory, the Einstein field equations govern the space–time response to mass, just like Maxwell's equations govern how the electric and magnetic fields respond to charges and currents. An exact solution to the Einstein field equations was obtained by Schwarzschild which describes the exterior space–time of static spherically symmetric distribution of matter. Schwarzschild [1,2] also obtained an interior solution to the Einstein field equations for the static spherically symmetric homogeneously distributed matter. The presence of charge in the matter distribution makes it an Einstein–Maxwell system whose exterior space–time is uniquely described by the Reissner–Nordström (R–N) metric. A large class of well-behaved and physically interesting exact solutions to the Einstein–Maxwell system are available in the literature [3]. In his review, Ivanov [3] presented a classification scheme for the static charged spherically symmetric perfect fluid solutions.

Consideration of the Einstein–Maxwell system is well-motivated in astrophysics as strong electric repulsion balances the gravitation pull, thereby preventing the system from collapsing to a singularity [4]. Consequently, a uniformly distributed charged system is expected to have greater stability than an uncharged system [5]. By the enhancement of charge, the critical radius for instability can be decreased. Different techniques are available in the literature for obtaining solutions to the Einstein–Maxwell system which include the works of Sharma *et al* [6], Mukherjee [7], Tikekar and Singh [8] and Patel *et al* [9], to name a few. Of particular interest is the technique adopted by Vaidya and Tikekar [10] who developed a model for a superdense star like neutron star by noting the t = constant hypersurface of a 3-sphere to be spheroidal and by utilising the subsequent space–time geometry to obtain a closed-form solution. The Vaidya and Tikekar ansatz since then has been widely used to develop viable models for describing charged fluid spheres [6,11–13]. For an electrically charged bare strange star, the energy density associated with the electric fields has been shown to be of the same order of magnitude as the energy density of the fluid matter itself [14]. In a recent work, Sharma *et al* [15] utilised the Vaidya and Tikekar ansatz to develop a charged generalisation of the Buchdahl bound for

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compact stars. The effect of anisotropy over a charged stellar object with linear matter distribution was studied by Thirukkanesh and Maharaj [16,17]. Solutions for a linear as well as non-linear matter distribution in the presence of anisotropy as well as charge was explored by Varela *et al* [18].

While a broad class of solution to the charged static spherically symmetric stellar systems are available in the literature, this paper aims to enrich the field by proposing a new technique to obtain closed-form solutions of the Einstein–Maxwell system. The interior solution is then matched with the R–N metric at the stellar boundary where the isotropic pressure vanishes ($p = 0$). The matching conditions are utilised for fixing the constants which facilitate their physical analysis.

The paper is organised as follows. In §2, the Einstein–Maxwell equations for a static, spherically symmetric, charged stellar object, under a suitable transformation, are laid down. By choosing a particular form of metric potential and electric charge, the system is solved in §3. In §4, the interior space–time is matched over the boundary with the R–N exterior space–time. The solution obtained in hypergeometric functional form in §3 is expressed in terms of elementary functions in §5 for some specific cases. In §6 the physical acceptance of the model is examined. Physical features of the model are discussed in §7.

2. The field equations

We describe the gravitational field of a static spherically symmetric relativistic star by the metric

$$ds_-^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

in Schwarzschild coordinates $(x^a) = (t, r, \theta, \phi)$. The quantities $\nu(r)$ and $\lambda(r)$ are unknown gravitational potentials. The signature of the space–time is assumed to be $(-, +, +, +)$. We construct the Einstein field equations

$$G^{ab} = R^{ab} - \frac{1}{2}Rg^{ab} = 8\pi(M^{ab} + E^{ab}), \tag{2}$$

for the above space–time where respective expressions for the the energy–momentum tensor of the associated fluid distribution M^{ab} and the electromagnetic field E^{ab} are given by

$$M^{ab} = (\rho + p)u^a u^b + pg^{ab}, \tag{3a}$$

$$E_{ab} = \frac{1}{4\pi}\left(F_{ac}F_b{}^c - \frac{1}{4}g_{ab}F_{cd}F^{cd}\right), \tag{3b}$$

where ρ, p and $u^a = e^{-\nu}\delta_0^a$ denote the energy density, isotropic pressure and co-moving fluid four-velocity, respectively. The electromagnetic field tensor can be defined as

$$F_{ab} = A_{b;a} - A_{a;b}, \tag{4}$$

where $A_a = (\phi(r), 0, 0, 0)$ is the four potential. F_{ab} satisfies the covariant Maxwell's equations

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0, \tag{5a}$$

$$F^{ab}{}_{;b} = J^a, \tag{5b}$$

where J is the four-current density defined by

$$J^a = \sigma u^a, \tag{6}$$

and σ is the proper charge density.

The total energy–momentum tensor T^{ab} is the sum of M^{ab} and E^{ab}:

$$T^{ab} = M^{ab} + E^{ab}. \tag{7}$$

For the line element (1) and the matter distribution (7), the Einstein–Maxwell system of field equations are obtained as

$$8\pi\left(\rho + \frac{E^2}{8\pi}\right) = \frac{1}{r^2}[r(1 - e^{-2\lambda})]', \tag{8a}$$

$$8\pi\left(p - \frac{E^2}{8\pi}\right) = -\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\nu'}{r}e^{-2\lambda}, \tag{8b}$$

$$8\pi\left(p + \frac{E^2}{8\pi}\right) = e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right), \tag{8c}$$

$$4\pi\sigma = \frac{1}{r^2}e^{-\lambda}(r^2E)', \tag{8d}$$

where we have set the gravitational constant $G = 1$ and the speed of light $c = 1$. To make the system of equations (8) tractable, we use the transformation of Durgapal and Banerjee [19]

$$x = Cr^2,\ Z(x) = e^{-2\lambda(r)},\ A^2y^2(x) = e^{2\nu(r)}, \tag{9}$$

where A and C are arbitrary constants. Under the transformation (9), the Einstein–Maxwell system (8) takes the form

$$8\pi\left(\frac{\rho}{C} + \frac{E^2}{8\pi C}\right) = \frac{1 - Z}{x} - 2\dot{Z}, \tag{10a}$$

$$8\pi\left(\frac{p}{C} - \frac{E^2}{8\pi C}\right) = 4Z\frac{\dot{y}}{y} + \frac{Z - 1}{x}, \tag{10b}$$

$$4Zx^2\ddot{y} + 2\dot{Z}x^2\dot{y} + \left(\dot{Z}x - Z + 1 - \frac{2E^2x}{C}\right)y = 0, \tag{10c}$$

$$\frac{2\pi^2\sigma^2}{C} = \frac{Z}{x}(x\dot{E} + E)^2, \tag{10d}$$

where dots denote differentiation with respect to the new variable x.

3. Choosing potential

The Einstein–Maxwell system (10) contains two more variables than the number of independent equations. Therefore, we have the freedom to choose any two variables to integrate the system. In our approach, we choose Z and E in the form

$$Z = \frac{1-x}{1+ax}, \quad E^2 = \frac{Ckax}{2(1+ax)^2}, \tag{11}$$

where a is a real constant and k is a positive real constant.

It is to be stressed here that the number of variables involved in the EM-system in our case is more than the number of independent equations and hence, in principle, it is possible to generate an infinite class of solutions by suitably choosing some of the variables. Since the system has two degrees of freedom, we choose one of the metric potentials and the radial profile of the electric field. The motivation for choosing the specific form of the metric potential as well as the electric field is that these choices make the system integrable. The subsequent solution remains singularity-free. It turns out that the metric potential and the electric field remain well-behaved throughout the stellar configuration and regular at the centre $r = 0$. Making use of the appropriate boundary conditions, the constants that appear in these choices namely, a and C can be fixed. Thus, even though such choices make it a somewhat restricted model, it is noteworthy that the parameter k remains free in this construction. For a wide range of values of the parameter k, it is possible to develop different classes of closed-form stellar solutions which include many well-known solutions as well. In the following sections, we show that for particular choices of the parameter k, one can obtain closed-form solutions which fulfil all the necessary requirements of a realistic compact star.

Substituting (11) into (10c) we obtain

$$4(1-x)(1+ax)\ddot{y} - 2(1+a)\dot{y} + [a(a+1) - ka]y = 0. \tag{12}$$

To integrate (12), we introduce the following transformation:

$$X = \frac{1+ax}{1+a}, \quad Y(X) = y(x), \quad a \neq -1 \tag{13}$$

which yields

$$X(X-1)\frac{\mathrm{d}^2Y}{\mathrm{d}X^2} + \frac{1}{2}\frac{\mathrm{d}Y}{\mathrm{d}X} - \frac{1}{4}(a+1-k)Y = 0. \tag{14}$$

Equation (14) is a Gaussian-type hypergeometric equation. In general, the solution to eq. (14) can be given in terms of hypergeometric functions:

$$Y = c_1 F\left(\alpha, -\alpha-1, -\frac{1}{2}; X\right) + c_2 X^{\frac{3}{2}} F\left(\alpha + \frac{3}{2}, -\alpha + \frac{1}{2}, \frac{5}{2}; X\right), \tag{15}$$

where c_1, c_2 are arbitrary constants and $\alpha = (-1 \pm \sqrt{2+a-k})/2$.

Substitution of eq. (11) in eqs (10a) and (10d) yields:

$$\rho = \frac{6(1+a)C + aC[2(1+a) - k]x}{2(1+ax)^2}, \tag{16a}$$

$$\sigma = \frac{C(3+ax)\sqrt{2ak(1-x)}}{(1+ax)^{5/2}}. \tag{16b}$$

Subsequently, the energy density at the centre ($r = 0$) is obtained as

$$\rho_c = 3C(1+a), \tag{17}$$

which shows that the constants C and a are linked to the density of the star. Obviously, the central density will remain positive for $a > -1$ and $C > 0$. Using eq. (16b), we can calculate the central charge density at $r = 0$ as

$$\sigma_c = 3C\sqrt{2ak}. \tag{18}$$

The total mass within a sphere of radius r is defined as

$$m(r) = \frac{1}{2}\int_0^r \rho \tilde{r}^2 \mathrm{d}\tilde{r}, \tag{19}$$

which on integration gives

$$m(r) = \frac{1}{8a}\left[\frac{3k\tan^{-1}\sqrt{acr}}{\sqrt{ac}} - \frac{3kr - 2aC(2+2a-k)r^3}{(1+aCr^2)}\right]. \tag{20}$$

Clearly, $m(r = 0) = 0$. Similarly, the charge within a sphere of radius r is defined as

$$q(r) = \frac{1}{2}\int_0^r \sigma \tilde{r}^2 \mathrm{e}^{\lambda} \mathrm{d}\tilde{r}, \tag{21}$$

which on integration yields

$$q(r) = \frac{Cr^3\sqrt{2ak}}{2(1+aCr^2)}. \tag{22}$$

We note that the charge also vanishes at the centre.

4. Matching conditions

The exterior space–time of the static, spherically symmetric, charged object is described by the Reissner–Nordström metric

$$\mathrm{d}s_+^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2$$

$$+\frac{1}{\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)}\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2), \tag{23}$$

where M and Q are the total mass and charge of the object, respectively. Using eqs (20) and (22), we can express the total mass (M) and charge (Q) as

$$m(R)=M=\frac{1}{8a}\left[\frac{3k\tan^{-1}\sqrt{ac}R}{\sqrt{ac}}-\frac{3kR-2aC(2+2a-k)R^3}{(1+aCR^2)}\right], \tag{24a}$$

$$q(R)=Q=\frac{CR^3\sqrt{2ak}}{(1+aCR^2)}. \tag{24b}$$

The matching conditions over the boundary ($r=R$) are the continuity of $\mathrm{e}^{2\nu}$ and $\mathrm{e}^{2\lambda}$. The junction conditions yield

$$A^2y^2|_{r=R}=\left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right), \tag{25a}$$

$$\frac{1-CR^2}{1+aCR^2}=\left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right). \tag{25b}$$

Another boundary condition to be satisfied for a compact star is that the pressure should vanish at the surface which leads to

$$\left.\frac{\dot{y}}{y}\right|_{r=R}=\frac{(a+2)+aCR^2(a+2-k)}{8(1-CR^2)(1+aCR^2)}. \tag{26}$$

From eq. (25b), we determine the constant

$$C=\frac{a(4M-R)-R\pm\sqrt{16akMR+(1+a)^2R^2}}{2aR^2[R(1+a+2k)-2Ma]} \tag{27}$$

in terms of mass, radius and assumed values of either a or k. Note that the relation between a and k is guided by the term α in eq. (15). So, to fix the values of all the constants, we must have to set a proper relationship between a and k. We shall take up this issue in the following section.

5. Closed-form solutions

It is well known that in some special cases, the hypergeometric function can be expressed in terms of elementary functions [20]. We utilise this to obtain new closed-form solutions in our approach.

5.1 *Solution I*

If we set $k=a-2$, eq. (15) yields

$$y=d_1(1+ax)^{\frac{3}{2}}+d_2\sqrt{1-x}\,(a+3+2ax)\,, \tag{28}$$

where d_1 and d_2 are arbitrary constants. The physical quantities can then be expressed as

$$\mathrm{e}^{2\nu}=[d_3(1+ax)^{\frac{3}{2}}+d_4\sqrt{1-x}(a+3+2ax)]^2, \tag{29a}$$

$$\mathrm{e}^{2\lambda}=\frac{1+ax}{1-x}, \tag{29b}$$

$$\rho=\frac{C[6+a\,(6+4x+ax)]}{2(1+ax)^2}, \tag{29c}$$

$$p=\frac{C[d_1(1+ax)^{3/2}+d_2\sqrt{1-x}(a+3+2ax)]^{-1}}{2(1+ax)^2}\times[d_1(1+ax)^{3/2}(a^2x-14ax+10a-2)-d_2\sqrt{1-x}\times\{a^2x(1+a)(1+2ax)-34a^2x^2-52ax-18\}], \tag{29d}$$

$$\sigma=\frac{C(3+ax)\sqrt{2a(a-2)(1-x)}}{(1+ax)^{5/2}}, \tag{29e}$$

$$E^2=\frac{Ca(a-2)x}{2(1+ax)^2}, \tag{29f}$$

where $d_3=Ad_1$ and $d_4=Ad_2$.

In this solution, we have four unknowns C, d_1, d_2 and a. If we choose any one these four unknowns (in our case, we choose a), then all the remaining constants can be calculated by using eqs (25a), (26) and (27) provided the mass M and radius R of the star are known.

It is interesting to note that when $k=0$, $C=1/R^2$ and $\tilde{x}=\sqrt{1-x}=\sqrt{1-\frac{r^2}{R^2}}$, the general solution (28) reduces to

$$y=c_3\tilde{x}\left(1-\frac{4}{9}\tilde{x}^2\right)+c_4\left(1-\frac{2}{3}\tilde{x}^2\right)^{\frac{3}{2}}, \tag{30}$$

where $c_3=d_2/9$ and $c_4=d_1/3\sqrt{3}$. This solution is the Vaidya and Tikekar solution [10] for a superdense neutron star with spheroidal parameter $a=2$. Note that the electric field as well as charge density in this case vanish and the physical quantities take the form

$$\mathrm{e}^{2\nu}=\left[c_3\tilde{x}\left(1-\frac{4}{9}\tilde{x}^2\right)+c_4\left(1-\frac{2}{3}\tilde{x}^2\right)^{\frac{3}{2}}\right]^2, \tag{31}$$

$$\mathrm{e}^{2\lambda}=\frac{1+2x}{1-x}, \tag{32}$$

$$\rho=\frac{3C(3+2x)}{(1+2x)^2}, \tag{33}$$

$$p=\frac{9C(1-2x)}{(1+2x)^2}. \tag{34}$$

5.2 *Solution II*

If we set $k=a+\frac{7}{4}$, then (15) becomes

$$y=2^{\frac{3}{2}}d_1[\sqrt{1+a}+\sqrt{a(1-x)}]^{\frac{3}{2}} +d_2\left[\frac{1+ax}{\sqrt{1+a}+\sqrt{a(1-x)}}\right]^{\frac{3}{2}}, \tag{35}$$

where d_1 and d_2 are arbitrary constants. The physical quantities for this particular solution take the form

$$e^{2\nu}=\frac{[2^{\frac{3}{2}}d_3(\sqrt{1+a}+\sqrt{a(1-x)})^3+d_4(1+ax)^{\frac{3}{2}}]^2}{[\sqrt{1+a}+\sqrt{a(1-x)}]^3}, \tag{36a}$$

$$\rho=\frac{C[24+a\,(24+x+4ax)]}{8(1+ax)^2}, \tag{36b}$$

$$p=\frac{C}{(1+ax)}\left[\frac{3a(1-x)}{(2\sqrt{a(1+a)(1-x)}+a(1-x))}\left\{8d_2\sqrt{(1+ax)}(1+4\sqrt{a(1+a)(1-x)}+a(2-x))\right\}-d_1(2\sqrt{1+a}+\sqrt{a(1-x)})^3-\frac{[ax(1+4a)+8(1+a)]}{8(1+ax)}\right], \tag{36c}$$

$$\sigma=\frac{C(3+ax)\sqrt{a(4a+7)(1-x)}}{2(1+ax)^{\frac{5}{2}}}, \tag{36d}$$

$$E^2=\frac{aCx(4a+7)}{8(1+ax)^2}. \tag{36e}$$

5.3 *Solution III*

If we set $k=a+2$, then (15) becomes

$$y=d_1\sqrt{1-x}-d_2\left[\sqrt{1+ax}-\sqrt{a(x-1)}\times\ln\left(\frac{\sqrt{a(x-1)}+\sqrt{1+ax}}{\sqrt{-1-a}}\right)\right], \tag{37}$$

where d_1 and d_2 are arbitrary constants. The physical quantities for this solution take the form

$$e^{2\nu}=\left\{d_3\sqrt{1-x}-d_4\left[\sqrt{1+ax}-\sqrt{a(x-1)}\times\ln\left(\frac{\sqrt{a(x-1)}+\sqrt{1+ax}}{\sqrt{-1-a}}\right)\right]\right\}^2, \tag{38a}$$

$$\rho=\frac{C(6+6a+a^2x)}{2(1+ax)^2}, \tag{38b}$$

$$p=C\left[2(1+ax)^2\left[d_1\sqrt{1-x}-d_2\left\{\sqrt{1+ax}-\sqrt{a(x-1)}\times\ln\left(\frac{\sqrt{a(x-1)}+\sqrt{1+ax}}{\sqrt{-1-a}}\right)\right\}\right]\right]^{-1}\times\left[d_2\sqrt{a(x-1)}\times\left\{(a^2x+2+2a)\sqrt{\frac{1+ax}{ax-a}}-(a^2x+4ax+2a+6)\times\ln\left(\frac{\sqrt{a(x-1)}+\sqrt{1+ax}}{\sqrt{-1-a}}\right)\right\}-d_1\sqrt{1-x}(a^2x+4ax+2a+6)\right], \tag{38c}$$

$$\sigma=\frac{C(3+ax)\sqrt{2a(a+2)(1-x)}}{(1+ax)^{\frac{5}{2}}}, \tag{38d}$$

$$E^2=\frac{Ca(a+2)x}{2(1+ax)^2}. \tag{38e}$$

5.4 *Solution IV*

If we set $k=a-7$, then (15) becomes

$$y=d_1\,(1+ax)^{\frac{3}{2}}\sqrt{1-x}+d_2((a+1)^2+4(1+ax)(a-1-2ax)), \tag{39}$$

where d_1 and d_2 are arbitrary constants. The physical quantities for this solution take the form

$$e^{2\nu}=[d_3\,(1+ax)^{\frac{3}{2}}\sqrt{1-x}+d_4\left((a+1)^2+4(1+ax)(a-1-2ax)\right)]^2, \tag{40a}$$

$$\rho=\frac{C[6+6a+9ax+a^2x]}{2(1+ax)^2}, \tag{40b}$$

$$p=C\left[2(1+ax)^2\left[d_1\sqrt{1-x}(1+ax)^{\frac{3}{2}}+d_2\,\{4ax(a-3-2x)+a(a+6)-3\}\right]\right]^{-1}\left[d_2\,\{4ax\,[32ax^2-10x(4a-3)+(9a-43)]+a(a+38)-3\}-d_1\sqrt{1-x}(1+ax)^{\frac{3}{2}}\times\left(a^2x+13ax-a+6\right)\right], \tag{40c}$$

$$\sigma = \frac{C(3+ax)\sqrt{2a(a-7)(1-x)}}{(1+ax)^{5/2}}, \tag{40d}$$

$$E^2 = \frac{Ca(a-7)x}{2(1+ax)^2}. \tag{40e}$$

It is interesting to note that if we set $k = 0$, $C = 1/R^2$ and use the transformation

$$\tilde{x} = \sqrt{1-x} = \sqrt{1 - \frac{r^2}{R^2}},$$

solution (39) reduces to

$$y = c_3\tilde{x}\left(1 - \frac{7}{8}\tilde{x}^2\right)^{\frac{3}{2}} + c_4\left(1 - \frac{7}{2}\tilde{x}^2 + \frac{49}{24}\tilde{x}^4\right), \tag{41}$$

where $c_3 = d_1/8$ and $c_4 = -d_2/192$. This particular solution was obtained by Tikekar [21] for a superdense neutron star having spheroidal parameter $a = 7$.

In the following section, we shall examine the physical acceptability of the solutions.

6. Physical analysis

Any physically acceptable stellar interior solution should have the following features [22]:

1. The density and pressure should be positive throughout the interior of the star, i.e., $\rho, p > 0$;
2. the density and the pressure should be monotonically decreasing from the centre to the surface of the star;
3. the pressure should vanish at some finite radial distance, i.e., $p(r = R) = 0$;
4. the causality condition should be satisfied throughout the star, i.e., $0 \leq \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho}} \leq 1$; and
5. the strong energy condition should also be satisfied throughout the interior, i.e., $\rho - 3p > 0$.

To examine the fulfillment of the above requirements as well as to study the features of the new solutions, we consider a typical compact star of mass $M = 1.5M_{\odot}$ and radius $R = 10$ km. For different values of the spheroidal parameter a, values of the model parameters for the new set of solutions are given in table 1.

For a given set of values, physical features of our solutions are outlined as follows:

- Figure 1 depicts the density profile at all interior points of the star. The density decreases radially outward as expected. We also note that the parameter denoting departure from spherical geometry (a) increases the energy density.
- Figure 2 represents the pressure profile at all interior points of the star. The isotropic pressure decreases outward as expected and becomes zero at the boundary. Interestingly, the pressure falls off more rapidly with increasing spheroidal parameter a.
- Variation of the mass function for all types of solutions is shown in figure 3. The figure shows that the mass of the star increases more rapidly to its maximum value for greater values of a.
- Figure 4 represents variation of q^2 which depends on k.
- Figure 5 depicts the variation of charge density with radial distance. We note that as the distance from the stellar core increases, the charge density gradually decreases for each solution.
- The square of the electric field is plotted against the radial distance of the star in figure 6. The plot shows that the electric field is zero at the centre and increases monotonically.
- Figure 7 shows that the square of the sound velocity (v^2) lies between 0 and 1 for each solution as expected.
- Figure 8 shows that the strong energy condition is satisfied throughout the interior of the stellar configuration for each solution.
- Although a star is expected to be charge neutral, at certain epoch of its evolution, the star might be charged. The gravitational pull in such a star is counterbalanced by the Coulomb repulsion, which prevents the stellar configuration from collapsing to a point singularity. When the gravitational pull is counterbalanced by the combined effects of fermionic degenerate pressure as well as Coulomb repulsion, the star attains stability. Now, for a stable stellar configuration, we should have $\Gamma > \frac{4}{3}$ [23,24]. The relativistic adiabatic index is given by

  $$\Gamma = \frac{\rho + p}{p}\frac{\mathrm{d}p}{\mathrm{d}\rho}. \tag{42}$$

  Figure 9 shows that this condition is satisfied for each solution.

6.1 *Equation of state (EOS)*

The standard technique to develop a stellar model is to assume an equation of state (EOS) for the matter composition. For a given EOS, the stellar configuration and its physical properties are studied by numerically solving the Tolmann–Oppenheimer–Volkoff equations. In our method, we do not prescribe any EOS. Rather, we find the radial dependence of energy density and pressure by solving the field equations. Subsequently, a parametric plot of energy density and pressure provides a functional relationship $p = p(\rho)$ which in our case turns out to be

Table 1. Values of different model parameters for different choices of a for the assumed values of $R = 10$ km, $M = 1.5M_{\odot}$ and $A = 1.5$.

Solution No.	a	k	C	d_1	d_2
I	3.00	1.00	0.00156	0.08803	0.05471
II	0.50	2.25	0.00326	0.03236	0.17709
III	0.35	2.35	0.00336	0.14366	−0.78452
IV	8.00	1.00	0.00078	0.07014	0.00306

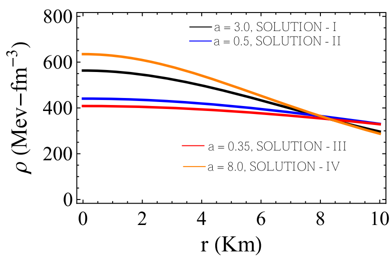

Figure 1. Energy density ρ plotted against the radial distance r.

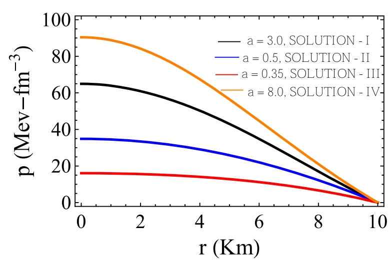

Figure 2. Isotropic pressure p plotted against the radial distance r.

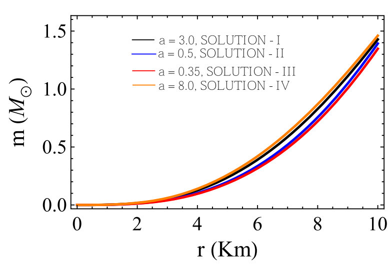

Figure 3. Mass function $m(r)$ plotted against the radial distance r.

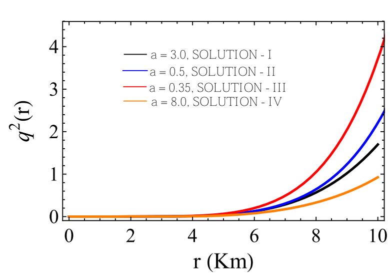

Figure 4. Square of charge q^2 plotted against the radial distance r.

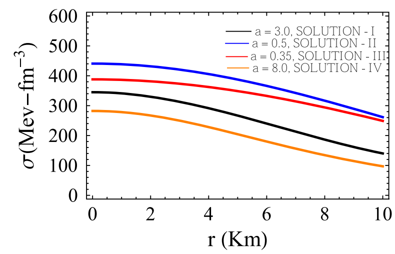

Figure 5. Charge density σ plotted against the radial distance r.

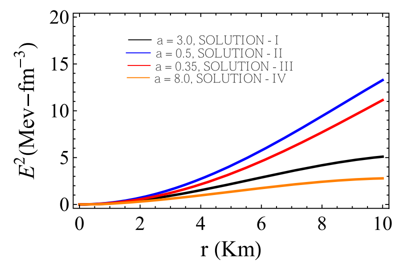

Figure 6. Square of electric field intensity E^2 plotted against the radial distance r.

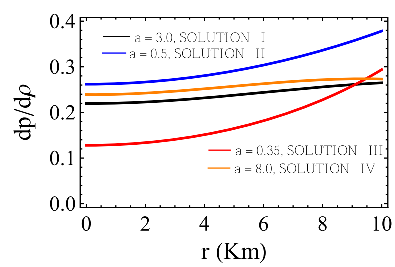

Figure 7. Square of sound velocity v^2 plotted against the radial distance r.

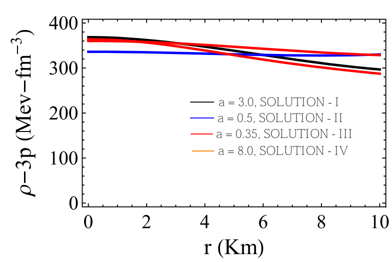

Figure 8. Fulfillment of strong energy condition.

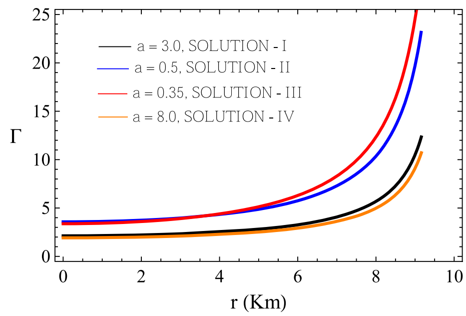

Figure 9. Γ plotted against the radial distance r.

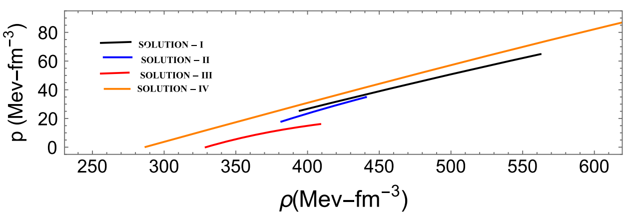

Figure 10. Equation of state (EOS) for different values of a.

linear for different classes of solutions as can be seen in figure 10. It is to be stressed that in the high-density regime of compact stars, a linear EOS is not uncommon. For example, the Bag model EOS for a quark star provides a linear relationship between energy density and pressure.

It should be pointed out here that in the Vaidya and Tikekar model for compact stars [10], the parameter a has a geometric interpretation and denotes a departure from homogeneous distribution of matter. Figure 10 shows that for a given density, the isotropic pressure increases with the increase in inhomogeneity specified by the parameter a.

7. Discussion

In this paper, we have developed a technique to generate exact solutions for describing a spherically symmetric charged fluid sphere for a specific choice of the electric field and one of the gravitational potentials. Even though the solution is obtained in terms of hypergeometric functions, an interesting feature is that it generates a wide range of closed-form solutions for some specific choices of the model parameters. The technique helps us to regain some of the earlier solutions, including the well-studied solutions of Vaidya and Tikekar. In fact, our approach permits solutions for a wide range of values of the geometric parameter a, which describes a departure from spherical geometric of the associated 3-space. The solutions can be used as viable models for compact stars of $\sim$ 1–2$M_{\odot}$ mass and $\sim$ 10 km radius.

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