Pramana – J. Phys. (2022) 96:181
https://doi.org/10.1007/s12043-022-02429-5

Imprint of dynamic localisation in frequency-dependent conductivity in a paradigmatic quantum system

SUSHANTA DATTAGUPTA

Bose Institute, Kolkata 700 054, India
E-mail: sushantad@gmail.com

MS received 24 March 2022; revised 29 April 2022; accepted 19 May 2022

Abstract. We consider a representative quantum system, namely a tight-binding chain, in which an electron can tunnel to nearest-neighbour sites with equal probability. When an AC drive is applied, a fascinating phenomenon called ‘dynamic localisation occurs – the electron keeps coming back to its starting site for specific values of amplitude and frequency of the drive. While this is a zero-temperature coherent effect, we enquire whether the oscillatory field has its influence on finite temperature, incoherent, Drude-like transport and quantum diffusive motion. Our treatment of dissipative dynamics is based on the adaptation of the well-known spin-boson model under Ohmic dissipation.

Keywords. Mesoscopic transport; tight-binding; dynamic localisation; bosonic bath; Ohmic dissipation.

PACS Nos **73.23.-h; 62.23.Hj; 72.20.Ee; 72.15.Nj; 66.30.Pa**

1. Introduction

A nanowire, which is of considerable interest in the contemporary topic of mesoscopic physics, can be conveniently modelled as a tight-binding (TB) chain. Our focus here, as in [1] and [2], is such a system under the influence of a bosonic heat bath, with a linear position-dependent coupling of strength g_l, as is familiar in the topic of dissipative quantum systems [3]. An additional input here is the incorporation of a driving oscillatory field of magnitude F_{o} and frequency ω_{o} with the express interest of studying the AC transport behaviour.

As illustration of dissipative quantum transport, our aim in the first instance would be to investigate the electrical current say, when the field F_{o} equals $-eE$, $-e$ being the electronic charge and E is the electric field of arbitrary strength. The specific question is: Under what limits can the familiar Drude–Kubo formula for classical transport [4] be retrieved and what new physics ensues for nonlinear effects for an arbitrary dependence on the frequency?

Having analysed the current, we shall turn our attention to the more general attribute of transport, viz., the long-time limit of the probability of finding the particle at an arbitrary site from its journey at the origin, from which its first moment or the current can also be calculated independently. Again, the results are illustrated by focussing on the second moment of the probability which yields the mean-squared displacement (MSD) in the diffusive limit.

The explicitly time-dependent Hamiltonian, relevant for the present discussion, can be written as

$$\begin{aligned}
\mathbf{H}_t &= \mathrm{d}F_{\text{o}}N\cos(\omega_{\text{o}}t) + \Delta/2(\mathbf{K}+\mathbf{K}^{\dagger}) \\
&\quad + N\Sigma g_l(\mathbf{b}_l + \mathbf{b}^{\dagger}{}_l) + \Sigma\omega_l(\mathbf{b}_l^{\dagger}\mathbf{b}_l), \\
\mathbf{N} &= \sum_{-\infty}^{\infty} |n\rangle|n\rangle\langle n|, \\
\mathbf{K} &= \sum_{-\infty}^{\infty} |n\rangle\langle n+1|, \\
\mathbf{K}^{\dagger} &= \sum_{-\infty}^{\infty} |n+1\rangle\langle n|,
\end{aligned} \tag{1}$$

where d is the lattice spacing and Δ is the tunnelling (over nearest neighbours) frequency. The subscript t under $\mathbf{H}$ signifies that the underlying time dependence is merely a parametric one. (We shall take the Planck constant to be unity throughout.) The last term is the free bosonic Hamiltonian (with frequency ω_l) that can be compactly denoted as

Published online: 20 September 2022

$$\mathbf{H}_b = \sum \omega_l \mathbf{b}_l^\dagger \mathbf{b}_l. \tag{2}$$

A remark is in order concerning another motivation behind the present analysis apart from the issue of studying the electrical transport anew. When the dissipative coupling is absent, the model in eq. (1) is exactly solvable for quantum coherent transport; for $F_o = 0$, the particle spreads like a wave packet with an MSD going like t^2 – much like a free particle with a 'quantal velocity' $v_{qu} = d\Delta$; for $F_o \neq 0$, but $\omega_o = 0$ (static field) the MSD shows Bloch oscillations with frequency dF_o. However, for an oscillatory field, there is a fascinating effect of what is called dynamic localisation when the itinerant particle (the electron, in this case) gets localised where it started from, for certain ratios of dF_o/ω_o [5,6]. Dynamic localisation may be viewed as the quantum analogue of the Kapitza effect in a rapidly driven classical pendulum [6,7]. While the quantum phenomenon is a zero-temperature purely coherent effect, we intend to look for some signature of such localisation in finite temperature dissipative transport when the latter is totally incoherent. It is pertinent to emphasise however that there is another source of zero-temperature quantum localisation – not of interest in the present paper – that occurs in the absence of the driving field – purely due to Ohmic dissipation in the TB chain that occurs in the strong damping regime [1]. Our focus here is on incoherent transport under the frequency-dependent driving, predominantly for weak damping, but at temperatures much larger than the characteristic bath frequencies (in suitable energy units).

With these preliminary comments, the paper is structured as follows: In §2 we introduce one of the objects of our primary emphasis, viz., the particle (or charge) current which is proportional to the time derivative of the average position weighted by an underlying density operator $\boldsymbol{\rho}(t)$. The latter is made to transform to a convenient form with the aid of two distinct unitary transformations which allow for the application of the standard master equation methods [8]. Specialising to Ohmic dissipation, the current in the limiting case of the so-called 'high temperatures' ($k_B T \gg \omega_c$, k_B being the Boltzmann constant and ω_c the cut-off frequency of the bosonic excitations) is explicitly worked out here. In §3 we connect with the Drude–Kubo formula in the limiting case of linear response to a static field, with an appropriate identification of the Drude scattering rate in terms of the bath parameters and the temperature T. Following this identification, we return to the case of arbitrary amplitude and frequency of the driving field and explore the possible occurrence of dynamic localisation in incoherent transport. This section also contains numerical results for the current for different values of F_o and ω_o, and other bath parameters. In §4, we consider the probability propagator and from it, derive the current independently, as well as provide an expression for the mean-squared displacement (MSD) in the long-time approximation. Our concluding remarks are presented in §5.

2. Master equation for $\rho(t)$, Ohmic dissipation and high temperature current

Our first attempt in this section is to eliminate the linear coupling term with the bath in eq. (1) in preference to a new coupling term in which the old coupling constant g_l is shifted to the exponent, thus allowing for a treatment to all orders in g_l albeit the 'effective' tunnelling is calculated to second order. This is achieved via a unitary 'polaronic' transformation

$$\mathbf{S} = \exp \sum [-g_l/\omega_l (\mathbf{b}_l - \mathbf{b}_l^\dagger)\mathbf{N}]. \tag{3}$$

The transformed Hamiltonian reads as

$$\tilde{\mathbf{H}}_t = \mathbf{S}\mathbf{H}_t\mathbf{S}^{-1} = \mathbf{H}_t^s + \mathbf{V} + \mathbf{H}_b, \tag{4}$$

where the superscripted $\mathbf{H}_\mathbf{t}^\mathbf{s}$ designates the 'system' Hamiltonian given by

$$\mathbf{H}_t^s = dF_o \mathbf{N} \cos(\omega_o t), \tag{5}$$

while the new coupling term is

$$\mathbf{V} = \Delta/2(\mathbf{B}_-\mathbf{K}^\dagger + \mathbf{B}_+\mathbf{K}), \tag{6}$$

where

$$\boldsymbol{B}_+ = \exp \sum [g_l/\omega_l (\mathbf{b}_l - \mathbf{b}_l^\dagger)], \quad \mathbf{B}_- = \mathbf{B}_+^\dagger. \tag{7}$$

Evidently, the polaronic transformation is such that even if the coupling Hamiltonian $\mathbf{V}$ is treated perturbatively, g_l remains in the exponent and therefore, an expansion of the exponential ensures that both weak and strong dissipation can be treated [3,8].

As mentioned in the Introduction, one of the objectives of this work is to compute the statistical average of the displacement (the time derivative of which yields the current), defined by

$$\langle \mathbf{N}(t) \rangle = \mathrm{Tr}(\boldsymbol{\rho}(t)\mathbf{N}) = \mathrm{Tr}(\tilde{\boldsymbol{\rho}}(t)\mathbf{N}), \tag{8}$$

where

$$\tilde{\boldsymbol{\rho}}(t) = \mathbf{S}\boldsymbol{\rho}(t)\mathbf{S}^{-1}. \tag{9}$$

In writing eq. (8) we have used the cyclic invariance of the trace and the fact that $\mathbf{N}$ and $\mathbf{S}$ commute.

Next, the onus of the time dependence of the system Hamiltonian can be transferred to the coupling term by going to the interaction picture via the operator

$$\mathbf{U}(t) = \exp[i(\eta(t)\mathbf{N} + \mathbf{H}_B t)], \tag{10}$$

where

$$\eta(t) = dF_{\rm o}\int_{\rm o}^{t} dt' \cos(\omega_{\rm o}t') = (dF_{\rm o}/\omega_{\rm o})\sin(\omega_{\rm o}t). \tag{11}$$

Consequently,

$$\tilde{\tilde{\mathbf{H}}}(t) = \mathbf{U}(t)\tilde{\mathbf{H}}_t\mathbf{U}^{-1} = \Delta/2[\mathbf{B}_-(t)\mathbf{K}^\dagger(t) + \mathbf{B}_+(t)\mathbf{K}(t)], \tag{12}$$

$$\boldsymbol{B}_+(t) = \exp(i\mathbf{H}_Bt)\mathbf{B}_+\exp(-i\mathbf{H}_Bt),\ \boldsymbol{B}_-(t) = \mathbf{B}_+(t)^\dagger, \tag{13}$$

$$\mathbf{K}(t) = \exp[i\eta(t)\mathbf{N}]\mathbf{K}\exp[-i\eta(t)\mathbf{N}] = \exp[-i\eta(t)]\mathbf{K}. \tag{14}$$

In writing eq. (14) (and correspondingly, its Hermitian adjoint (h.a.)) we have employed the commutation relations

$$[\mathbf{N}, \mathbf{K}] = -\mathbf{K},\ [\mathbf{N}, \mathbf{K}^\dagger] = \mathbf{K}^\dagger,\ [\mathbf{K}, \mathbf{K}^\dagger] = 0. \tag{15}$$

Once again, the invariance property of the trace and the fact that $[\mathbf{U(t)}, \mathbf{N}] = 0$ ($[\mathbf{U(t)}, \mathbf{S}] \neq 0$, however) yield

$$\langle\mathbf{N(t)}\rangle = \mathrm{Tr}[\tilde{\tilde{\boldsymbol{\rho}}}(t)\mathbf{N}],\quad \tilde{\tilde{\boldsymbol{\rho}}}(t) = \mathbf{U}(t)\tilde{\boldsymbol{\rho}}(t)\mathbf{U}^\dagger(t), \tag{16}$$

where $\tilde{\boldsymbol{\rho}}(t)$ is given by eq. (9).

We have thus reduced the calculation of $\langle\mathbf{N}(t)\rangle$ to an analysis of the transformed density operator $\tilde{\tilde{\boldsymbol{\rho}}}(t)$. The latter, upon partial averaging over the heat bath and in the so-called Born–Markov treatment of $\tilde{\tilde{\mathbf{H}}}_t$, satisfies the 'time-convolution-less' master equation (see eqs (I.A.19), (I.A.22) and (I.A.25) of [8])

$$\frac{\partial\tilde{\tilde{\boldsymbol{\rho}}}(t)}{\partial t} = -\int_0^t d\tau \mathrm{Tr}_B([\tilde{\tilde{\mathbf{H}}}(t)(\tau), [\tilde{\tilde{\mathbf{H}}}(t)(o), \boldsymbol{\rho}_B\tilde{\tilde{\boldsymbol{\rho}}}_R(t)]]), \tag{17}$$

where the 'reduced' density operator is defined by

$$\tilde{\tilde{\boldsymbol{\rho}}}_R(t) = \mathrm{Tr}_B(\tilde{\tilde{\boldsymbol{\rho}}}(t)). \tag{18}$$

The advantages of using the time convolution-less form for the master equation over the time convolution form have been spelt out in [9] and [10]. The statistically averaged electric current is then given by

$$I(t) = -e(d/dt)\langle\mathbf{N}(t)\rangle = -e\mathrm{Tr}_S[\partial\tilde{\tilde{\boldsymbol{\rho}}}_R(t)\mathbf{N}/\partial t], \tag{19}$$

where Tr_S $(\cdots)$ denotes a sum over the states of the subsystem. Before concluding this section, it may be stressed that the same procedure can also be adopted to compute all moments higher than $\langle\mathbf{N}(t)\rangle$, as discussed in §4.

The next task is to work on eqs (17)–(19) to arrive at a closed-form expression for the current. For this, the first step is to disentangle the double commutator in eq. (17), separate out the operators $\mathbf{B}_+$ and $\mathbf{B}_-$ for eventual averaging over the bath states and finally, sum over the quantum states of the subsystem. After some algebra (further elaborated upon in §3), we arrive at the compact expression

$$I(t) = -e\Delta^2\int_0^t d\tau\,\mathrm{Im}[\phi(\tau)]\sin[\eta(\tau)]. \tag{20}$$

Equation (20) encompasses a general result valid for arbitrary values of the amplitude and the frequency of the drive.

In eq. (20), $\phi(t)$ is the bath correlation function

$$\phi(t) = \mathrm{Tr}_B(\boldsymbol{\rho}_B\mathbf{B}_+(0)\mathbf{B}_-(t)) = \mathrm{Tr}_B(\boldsymbol{\rho}_B\mathbf{B}_-(0)\mathbf{B}_+(t)),\quad \phi^*(t) = \phi(-t). \tag{21}$$

In arriving at eq. (20) we have employed the fact that

$$\mathrm{Tr}_B(\boldsymbol{\rho}_B\mathbf{B}_+(\mathbf{t})\mathbf{B}_+(0)) = \mathrm{Tr}_B(\boldsymbol{\rho}_B\mathbf{B}_-(t)\mathbf{B}_-(0)) = 0,\quad \mathbf{K}\mathbf{K}^\dagger = \mathbf{K}^\dagger\mathbf{K} = I. \tag{22}$$

From eq. (21) we can show [8]

$$\phi(t) = \exp\left\{-\int_0^\infty d\omega\xi(\omega)[\coth(\beta\omega/2)) \times (1-\cos\omega\tau) + i\sin\omega\tau]/\omega^2\right\}, \tag{23}$$

β being $1/k_{\rm B}T$, T is the temperature. Here, $\xi(\omega)$ is the so-called spectral density

$$\xi(\omega) = 2\sum_q g_q^2\delta(\omega-\omega_q). \tag{24}$$

It may be easily checked that in the case of a static field, $\omega_0 = 0$, $\eta(t) = dF_{\rm o}t$, and eq. (20) reduces to

$$I_{\rm o}(t) = -e\Delta^2\int_0^t d\tau\{\mathrm{Im}[\phi(\tau)\sin[dF_{\rm o}(\tau)]\}. \tag{25}$$

The zero-frequency result of eq. (25) can be systematically improved for the slowly driven case by expanding $\sin[\eta(\tau)]$ in eq. (20) as a power series in $\omega_{\rm o}\tau$. The result (25) matches with the one obtained by Aslangul *et al* and Bandyopadhyay and Dattagupta [1,2] (who had used the same 'polaronic' transformation as implemented here) and also with Fisher and Zwerger who had however employed the 'dilute bounce gas' approximation (DBGA) in a path integral formulation of the problem [11], popularised by Leggett *et al* [12,13]. The diluteness of the bounces is reflected in the fact that the underlying results are valid to the second order in Δ, i.e., the effective tunnelling frequency is small. With this proviso, the expression of the current in eq. (20) or (25) is applicable to all temperatures and arbitrary strengths of the heat bath coupling, albeit within the Ohmic form of the spectral density [1].

Having said this, we now specialise in the incoherent transport that is obtained at higher temperatures and for the case in which the spectral density has a linear dependence on the frequency for small frequencies, i.e., it has the Ohmic form, given by

$$\xi(\omega) = 2\alpha\omega[\omega_c^2/(\omega^2 + \omega_c^2)], \tag{26}$$

where α is a dimensionless constant. As it turns out, the Lorentzian cut-off at a frequency ω_c (in contrast to an exponential form), as encapsulated in eq. (26), works well in the high-temperature domain [1]. It is time now to clarify the meaning of 'high' temperature – it merely implies that $1 > \beta\omega_c$. The actual temperature is however much lower than classical temperatures wherein transport would occur via Kramers activation over a barrier associated with a spatially periodic potential, an abstraction of which leads to the tight-binding Hamiltonian in eq. (1) [5].

Replacing then the cotangent function in eq. (23) by the inverse of its argument and doing the integral over ω with the aid of the spectral function in eq. (26), we obtain [1,2]

$$\mathrm{Im}\,\phi(t) = \sin[A_1(t)]\exp[-A_2(t)], \tag{27}$$

where, in the notation of Aslangul *et al*, the high-temperature expressions for $A_1(t)$ and $A_2(t)$ are [1]

$$A_1(t) = \alpha\pi[1 - \exp(-\omega_c t)],$$
$$A_2(t) = K[\omega_c t - 1 + \exp(-\omega_c t)]. \tag{28}$$

Here, the dimensionless parameter K, as defined by Fisher and Zwerger [11] is

$$K = (2\alpha\pi/\beta\omega_c). \tag{29}$$

Evidently, in the high-temperature domain of $1 \gg \beta\omega_c$,

$$K \gg \alpha, \tag{30}$$

and it is a measure of the strength of the coupling with the heat bath. Substituting eqs (27)–(29), we obtain the current at high temperatures as

$$I(t) = -e\Delta^2 \int_0^t d\tau \sin\{\alpha\pi[1 - \exp(-\omega_c\tau]\} \times \exp\{-K[\omega_c\tau - 1 + \exp(-\omega_c\tau)]\}\sin[\eta(\tau)]. \tag{31}$$

While eq. (31) will be numerically computed and graphically illustrated in the next section, we argue *a la* Aslangul *et al* [1] that for times $t \gg (\omega_c^{-1})$, we may drop the terms proportional to $\exp(-\omega_c\tau)$ in the integrand of eq. (31) and approximately write

$$I(t) = -e\Delta^2 \sin(\alpha\pi) \int_0^t d\tau \exp(-K\omega_c\tau)\sin[\eta(\tau)]. \tag{32}$$

The fact that the current vanishes in the absence of dissipative coupling [2] is reflected in the pre-factor in eq.(32).

3. Drude–Kubo limit, dynamic localisation and graphical plots

In this section, we first consider linear response to a weak static field to connect with the well-known expression for the electrical current in the Drude–Kubo theory, then return to our main focus on the oscillatory drive and obtain new results on how incoherent transport is influenced by a phenomenon called dynamic localisation, hitherto studied only in the pure quantum limit of coherent tunnelling [5,6]. We also present representative plots to check the realm of validity of our approximations and to further elucidate the analytical expressions.

In the asymptotic stationary domain, the upper time-limit in eq. (20) can be extended to ∞. From eq. (25) then, the stationary state current for a weak static field can be written as

$$I_o = -eF_o d\Delta_{\text{eff}}^2 \int_0^\infty \mathrm{d}\tau\,\tau \exp(-K\omega_c\tau)$$
$$= -edF_o(\Delta_{\text{eff}}/K\omega_c)^2. \tag{33}$$

Recalling that $F_o = -eE$, E being the static electric field and $d\Delta_{\text{eff}}$ ($\Delta_{\text{eff}}^2 \equiv \Delta^2 \sin\alpha\pi$) has the interpretation of a 'quantal speed' for the tight-binding model [6] which then scales as $(\beta m)^{-1/2}$ in the classical equipartition limit, and substituting for K from eq. (29), we may rewrite eq. (33) as

$$I_o = E(e^2/dm\gamma), \tag{34}$$

where

$$\gamma = (2\alpha\pi)^2/\beta \tag{35}$$

has the interpretation of the Drude relaxation rate [4]. Here m is the mass of the electron. The advantage of the present analysis is that the expression for the rate has an explicit microscopic structure in terms of the dimensionless parameter α which is a measure of the coupling of the tight-binding chain with the bosonic heat bath.

With this identification of the Drude rate, we return to our primary interest, viz., the case of oscillatory response. It is important to point out at the outset that the structure of $\eta(t)$ (see eq. (11)) is such that no straightforward linearisation in F_o is meaningful even in the linear response regime because F_o occurs in the pre-factor in the form of (dF_o/ω_o), thus invalidating any expansion scheme, especially for low frequencies. Therefore, in eq. (20) we are led to consider the following Bessel function $(J_p\mu))$ of the first kind [14]:

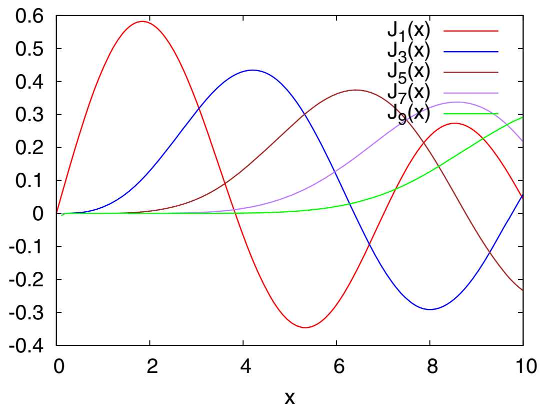

Figure 1. The odd-integer Bessel functions plotted along the ordinate as a function of their argument x (which in the present context equals dF_o/ω_o) along the abscissa.

$$\exp[i\eta(\tau)] = \sum_{(p=-\infty)}^{\infty} \exp(i\omega_o p\tau) J_p(x),$$
$$x \equiv dF_o/\omega_o. \tag{36}$$

The cosine of $\eta(\tau)$ and the sin of $\eta(\tau)$ are obtained from the real and imaginary parts of eq. (36) respectively. Substituting $t = \infty$ in eq. (32), the current is given by

$$I = -e\Delta_{\rm eff}^2 \sum_p [(K\omega_c)^2 + (p\omega_o)^2]^{-1}(p\omega_o) \times J_p(dF_o/\omega_o). \tag{37}$$

It is evident that the $p = 0$ term does not contribute to eq. (37) which in a way, is expected because the current must vanish for $F_o = 0$ (recalling that $J_o(x = 0) = 1$). Having recognised this, the negative p-terms can be rewritten in terms of positive p's by noting that $J_{-p}(x) = (-1)^p J_p(x)$. Hence, all $|p|$ = even terms vanish, leaving behind

$$I = -2e\Delta_{\rm eff}^2 \sum_{p=1,3,5...}^{\infty} [(K\omega_c)^2 + (p\omega_o)^2]^{-1})(p\omega_o) J_p(dF_o/\omega_o)]. \tag{38}$$

Our main result of eq. (38) for the generalised Drude formula – in a quantum dissipative nanowire – has the remarkable attribute of 'dynamic localisation'. As figure 1 suggests, the odd-integer Bessel functions go through zero, for certain values of the argument (dF_o/ω_o) depending on the value of p, though $J_9(x)$ is effectively zero for $x\ (= dF_o/\omega_o) > 6.5$. Thus, the mean current in the incoherent regime oscillates from one value to another – a pure quantum effect sans any classical counterpart. To study the oscillatory behaviour of the mean current, we re- express eq. (38) in terms of certain dimensionless quantities defined below.

$$j = -I\omega_o/2e\Delta_{\rm eff}^2, \quad \mu = \omega_c/dF_o,$$
$$r = \beta\omega_c, \tag{39}$$

in terms of which

$$j(x) = \sum_{p=1,3,5,...}^{\infty} [(pJ_p(x)]/[(2\pi\alpha\mu r x)^2 + p^2]. \tag{40}$$

In evaluating eq. (40) we take cognizance of the fact that as long as $x\ (= dF_o/\omega_o)$ is restricted to be below 6.5 (see figure 1), we can curtail the summation beyond $p = 7$. However, the chosen value of $x = 6.5$ does not come in the way of dynamic localisation because the latter occurs for $x < 2.5$ [5]. In addition, we set $\mu = 1.0$ assuming that the bath cut-off energy is comparable to the amplitude energy of the driving field [1]. Thus, we may write

$$j(x) \approx \{J_1(x)/[1 + (2\pi\alpha r x)^2] + 3J^3(x)/[9 + (2\pi\alpha r x)^2] + 5J_5(x)/[25 + (2\pi\alpha r x)^2] + 7J_7(x)/[49 + (2\pi\alpha r x)^2]\},$$
$$0 \le x < 6.5. \tag{41}$$

In figure 2a, we plot the L.H.S. of eq. (41) vs. x in the range $0 \le x < 6.5$, by fixing $\alpha = 0.01$ (weak damping), and for different values of $r = 5, 10, 15$. This computation is repeated in figure 2b for $\alpha = 0.001$. Evidently, the oscillations in the current are more pronounced when the value of α is smaller or decoherence is weaker.

To check the validity of the long-time approximation implemented in this section, we return to the general expression given in eq. (31) and rewrite the integral in terms of a dimensionless variable τ (upon scaling by the frequency ω_o). Following then the notations employed in eq. (39) and putting $\mu = 1.0$ at the outset, as in eq. (41), we derive

$$j(x, \nu = \omega_o t) = \int_0^\nu d\tau \sin\{\alpha\pi[1-\exp(-x\tau)]\}/[2\sin(\alpha\pi)] \cdot \exp\{-2\pi\alpha r[x\tau - 1 + \exp(-x\tau)]\} \sin[x\sin(\tau)], \tag{42}$$

where we have divided by the factor $2\sin(\alpha\pi)$ so that we recover eq. (41) in the appropriate long-time limit. We plot eq. (42) in figure 3a for $\alpha = 0.01$, as in figure 2a, and for $\nu = 100.0$, and the same r-values used earlier in figure 2a and x in the range $0 \le x < 6.5$. We repeat the plot in figure 3b for $\alpha = 0.001$. Comparison of figure 3 with figure 2 suggests that the approximations employed in arriving at the closed-form expression in eq. (41) are quite reasonable.

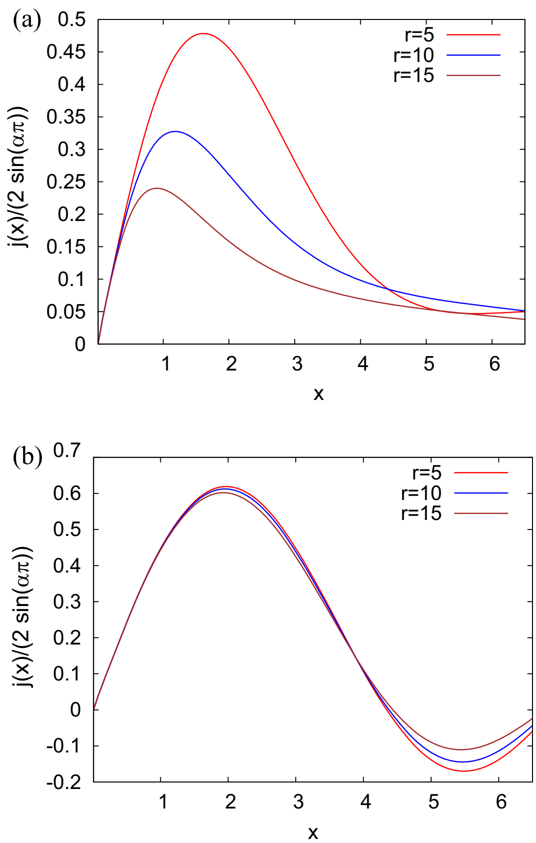

Figure 2. (**a**) Along the y-axis we plot the dimensionless current scaled as $j(x) = -I\omega_0/2e\Delta_{\text{eff}}^2$ vs. x, defined in the caption to figure 1, for $\alpha = 0.01$. The plots are given for three different values of $r = \omega_c$. (**b**) Same as in figure 2a, except that the quantum damping parameter α is weaker – an order of magnitude smaller, i.e., $\alpha = 0.001$.

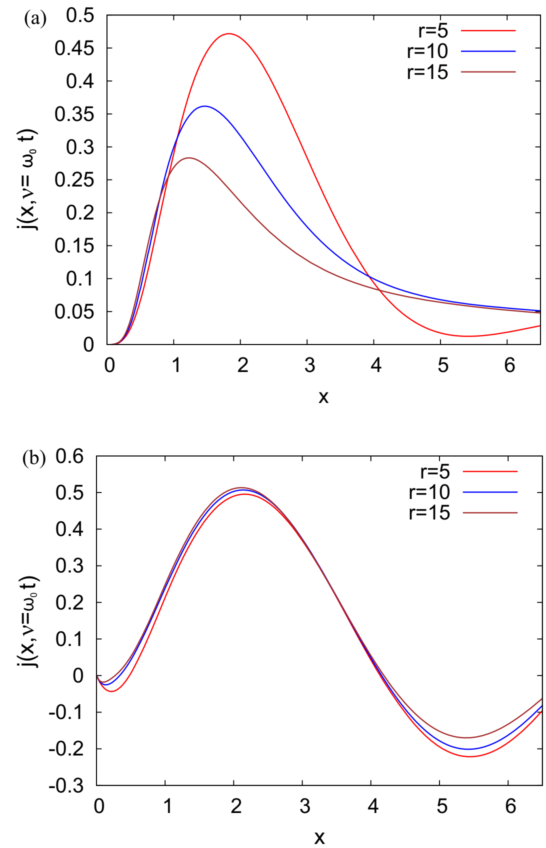

Figure 3. (**a**) The long-time limit is checked by plotting along the ordinate the left-hand side of eq. (42) for $\nu = \omega_0 t = 100.0$ vs. x for the same parameter values as in figure 2a. (**b**) The same as in figure 3a except that α is taken as 0.001, as in figure 2b.

4. Occupational probability and its moments

In this section, we investigate the full density operator $\boldsymbol{\rho}(t)$ and using this, We calculate the probability $P_m(t)$ of finding the particle at an arbitrary site m, given that it was at the origin $m = 0$ site, defined by

$$P_m(t) = \langle m|\boldsymbol{\rho}_R(t)|m\rangle = \langle m|\tilde{\boldsymbol{\rho}}_R(t)|m\rangle = \langle m|\tilde{\tilde{\boldsymbol{\rho}}}_R(t)|m\rangle, \tag{43}$$

where the reduced density operator has been defined in eq. (18). Here, we have used the definitions in eqs (9) and (16) and the cyclic invariance of the trace, facilitated by the fact that in eq. (43) we are interested only in the diagonal matrix elements in the representation of **N**. From the master equation of eq. (17) and writing out the double-commutator therein, we obtain

$$\begin{aligned}\partial\tilde{\tilde{\boldsymbol{\rho}}}_R(t)/\partial t &= -\Delta^2/4\int_0^t d\tau\{\phi(\tau)[\tilde{\tilde{\boldsymbol{\rho}}}_R(t)(\mathbf{K}^\dagger(o)\mathbf{K}(\tau) + \mathbf{K}(o)\mathbf{K}^\dagger(\tau)) - (\mathbf{K}^\dagger(\tau)\tilde{\tilde{\boldsymbol{\rho}}}_R(t)\mathbf{K}(o) \\ &\quad + \mathbf{K}(\tau)\tilde{\tilde{\boldsymbol{\rho}}}_R(t)\mathbf{K}^\dagger(o))] + \text{h.a.}\} \\ &= -\Delta^2\int_0^t d\tau\{\text{Re}[\phi(\tau)]\cos[\eta(t)] \\ &\quad \cdot[\tilde{\tilde{\boldsymbol{\rho}}}_R(t) - (\mathbf{K}^\dagger\tilde{\tilde{\boldsymbol{\rho}}}_R(t)\mathbf{K} \\ &\quad + \mathbf{K}\tilde{\tilde{\boldsymbol{\rho}}}_R(t)\mathbf{K}^\dagger)/2] + \text{Im}[\phi(\tau)\sin[\eta(t)] \\ &\quad \cdot(\mathbf{K}\tilde{\tilde{\boldsymbol{\rho}}}_R(t)\mathbf{K} - \mathbf{K}\tilde{\tilde{\boldsymbol{\rho}}}_R(t)\mathbf{K}^\dagger]/2\}.\end{aligned} \tag{44}$$

In deriving eq. (44), we have made use of eqs (12)–(14) and eqs (21) and (22) of the text.

From eq. (44) and the definition given in eq. (19) we can easily arrive at eq. (20) of the text for the first moment, wherein we make use of the commutation properties stated in eq. (15). However, we can calculate the entire probability $P_m(t)$ defined in eq. (43), using the steps below. Thus, we obtain

$$\begin{aligned}dP_m(t)/dt = X(t)[&P_{m+1}(t) \\ &+ P_{m-1}(t) - 2P_m(t)]/2 \\ &+ Y(t)[P_{m+1}(t) - P_{m-1}(t)]/2,\end{aligned} \tag{45}$$

where

$$X(t) = \Delta^2 \int_0^t d\tau \mathrm{Re}[\phi(\tau)] \cos[\eta(\tau)],$$
$$Y(t) = \Delta^2 \int_0^t d\tau \mathrm{Im}[\phi(\tau)] \sin[\eta(\tau)]. \quad (46)$$

In arriving at eq. (46), we have used the relations

$$\mathbf{K}^\dagger|m\rangle = |m+1\rangle, \quad \boldsymbol{K}|m\rangle = |m-1\rangle,$$
$$\langle m+1|\boldsymbol{\rho}(t)|m+1\rangle = P_{m+1}(t), \quad (47)$$

etc. Interestingly, eq. (45) has the same structure as in the classical random walk on a chain in which the left and right steps are asymmetric [15]. We can solve eq. (45) by discrete Fourier transforms:

$$|k\rangle = 1/\sqrt{2\pi} \sum_n \mathrm{e}^{-i\eta\chi}|n\rangle,$$
$$|n\rangle = 1/\sqrt{2\pi} \int_{-\pi}^{\pi} d\chi \mathrm{e}^{i\eta\chi}|k\rangle, \quad (48)$$

where χ is a dimensionless variable: $\chi = \mathrm{d}|\mathbf{k}|$, and introduce

$$P_m(t) = 1/\sqrt{2\pi} \int_{-\pi}^{\pi} d\chi \mathrm{e}^{in\chi} P(\chi, t), \quad (49)$$

and the corresponding inverse Fourier transform. Using these relations, we derive

$$dP(\chi,t)/dt = -[X(t)(1-\cos\chi) - iY(t)\sin\chi]P(\chi,t). \quad (50)$$

As we are interested in the high-temperature limit, we can replace $X(t)$ and $Y(t)$ by their $t = \infty$ values. In that case, the solution of eq. (49) reads as

$$P(\chi,t) = \exp\{-t[X(t=\infty)(1-\cos\chi) - iY(t=\infty)\sin\chi]\}, \quad (51)$$

where we have used the initial condition: $P(\chi, t = 0) = 1$. The probability $P_m(t)$ can then be evaluated from the inverse Fourier transform of eq. (50) Without its full knowledge, however, its moments can be calculated from

$$\langle m(t)\rangle^r = (-i)^r[dr/d\chi^r P(\chi,t)]_{\chi=0}. \quad (52)$$

For instance, the first moment, the time derivative of which is proportional to the current, is given by

$$\langle m(t)\rangle = -i[(d/d\chi)P(\chi,t)]_{\chi=0}, \quad (53)$$

where, from eq. (50),

$$dP(\chi,t)/d\chi = \{-t[X(t=\infty)\sin\chi - iY(t=\infty)\cos\chi]\}P(\chi,t). \quad (54)$$

Hence, from eq. (52),

$$\langle m(t)\rangle = tY(t=\infty) = t\Delta^2 \int_0^\infty d\tau \mathrm{Im}[\phi(\tau)]\sin[\eta(\tau)]. \quad (55)$$

The time derivative of this matches with the asymptotic current, as given by eq. (20). The second moment, which yields the diffusion coefficient in the long-time limit, is obtained from

$$\langle m^2(t)\rangle = -[(d^2/d\chi^2)P(\chi,t)]_{\chi=0}. \quad (56)$$

Note, from eq. (53) that

$$(d/d\chi)dP(\chi,t)/d\chi = \{-t[X(t=\infty)\cos\chi + iY(t=\infty)\sin\chi]\}P(\chi,t) + \{-t[X(t=\infty)\sin\chi - iY(t=\infty)\cos\chi]\}dP(\chi,t)/d\chi. \quad (57)$$

Therefore,

$$\langle m^2(t)\rangle = tX(t=\infty) + t^2[Y(t=\infty)]^2. \quad (58)$$

Hence,

$$\langle m^2(t)\rangle - \langle m(t)\rangle^2 = tX(t=\infty), \quad (59)$$

where, from eq. (46),

$$X(t=\infty) = \Delta^2 \int_0^\infty d\tau \mathrm{Re}[\phi(\tau)]\cos[\eta(\tau)]. \quad (60)$$

The variance in eq. (56) has the usual signature of diffusive motion with the diffusion coefficient given by

$$D = (\Delta^2/2)\int_0^\infty d\tau \mathrm{Re}[\phi(\tau)]\cos[\eta(\tau)], \quad (61)$$

which, like the mean current, is also expected to exhibit dynamic localisation.

5. Conclusions

A combination of an oscillatory driving field and a quantum field due to bosonic excitations, occasioned by a finite-temperature quantum bath characterised by Ohmic dissipation, as in the well-known spin-boson model of dissipative quantum systems [3,11], is incorporated here for studying the transport in a tight-binding chain. If the coupling to the bath is switched-off, the problem reduces to the coherent zero-temperature dynamics of an AC-driven tight-binding model, in which there is a well-investigated but intriguing effect of dynamic localisation in that the particle keeps returning to its original site for certain special ratios of the amplitude to the frequency of the drive. We find that even at moderate temperatures and in the presence of an interacting bosonic bath, wherein transport is expected to be incoherent, the oscillatory field leaves its signature in the mean current and the mean-squared displacement in creating dynamic localisation-type effect. By tuning the parameters of the oscillatory drive, one may then be able to see the interplay of coherence and decoherence.

As was indicated in [6] and discussed in detail in [16], dynamic localisation in a TB chain is akin to the Kapitza effect [7] when a rapidly and periodically-driven system behaves as though it is under a static potential which is however renormalised by a term proportional to the ratio of the amplitude and the frequency of the drive. In the present instance of the TB chain, what ensues is a renormalised tunnelling frequency $\Delta J_0(F_0/\omega_o)$, J_0 being the Bessel function of the first kind of order zero [6]. Indeed, dynamic localisation occurs as $J_0(F_0/\omega_o)$ oscillates and goes through a series of zeros for certain specific values of (F_0/ω_o) [14]. As argued earlier, while dynamic localisation is a coherent quantum phenomenon, incoherent effects due to the coupling with a quantum dissipative bath yield a different kind of renormalisation in that Δ is replaced by (see eq. (33)) $\Delta_{\text{eff}} = \Delta \sin 1/2(\pi\alpha)$, α being a measure of dissipation.

Acknowledgements

The author would like to thank the Indian National Science Academy for its support through their Senior Scientist scheme. He is also grateful to Shamik Gupta for a critical reading of the manuscript and for help with numerical computations.

References

[1] C Aslangul, N Pottier and D Saint-James, *J. Phys. (Paris)* **47**, 1671 (1986)
[2] M Bandyopadhyay and S Dattagupta, *Phys. Rev. B* **104**, 125401 (2021)
[3] U Weiss, *Quantum dissipative systems* (World Scientific, Singapore, 1993)
[4] N Ashcroft and D N Mermin, *Solid state physics* (Holt, Reinhart and Winston, New York, 1976)
[5] D H Dunlap and V M Kenkre, *Phys. Rev. B* **34**, 3625 (1986)
[6] S Dattagupta, *Resonance* **27**, 543 (2022)
[7] P L Kapitza, *Zh. Eksp. Teor. Fiz.* **21**, 588 (1951); *Collected papers of P. L. Kapitza* edited by D ter Haar (Pergamon Press, Oxford, 1965); L D Landau and E M Lifshitz, *Mechanics* (Pergamon Press, Oxford, 1960) p. 93
[8] S Dattagupta and S Puri, *Dissipative phenomena in condensed matter* (Springer-Verlag, Berlin, 2004)
[9] N G van Kampen, *Physica* **74**, 215 (1974); *ibid.* **74**, 239 (1974)
[10] N Hashitsume, F Shibata and M Shingu, *J. Stat. Phys.* **17**, 155 (1977); *ibid.* **17**, 171 (1977)
[11] M P A Fisher and W Zwerger, *Phys. Rev.* **32**, 6190 (1985)
[12] A J Leggett, S Chakravarty, A T Dorsey, M P A Fisher, A Garg and W Zwerger, *Rev. Mod. Phys.* **59**, 1 (1987)
[13] S Chakravarty and A J Leggett, *Phys. Rev. Lett.* **52**, 5 (1984)
[14] M Abramowitz and I A Stegun, *Handbook of mathematical functions* (Dover, New York, 1965)
[15] S Dattagupta, *Diffusion – Formalism and applications* (Taylor and Francis, London, 2013) Chap. 1
[16] M Bandyopadhyay and S Dattagupta, *Parmana – J. Phys.* **76**, 381 (2008)