



Pulsar PSR B0943+10 as an isotropic Vaidya–Tikekar-type compact star

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Abstract. In this paper, we have constructed a well-behaved, realistic and stable model for relativistic compact star in the presence of isotropic charged fluid by solving the Einstein–Maxwell field equations. To put the coupled differential equations into a closed system, we have employed the Vaidya and Tikekar (*J. Astrophys. Astron.* 3:325, 1982) form of the metric potential g_{rr} and assumed a completely new form of metric potential g_{tt} . The resulting energy–momentum components, i.e., energy density and pressure, contain six constants; two of these are determined through the boundary conditions. The remaining constants are constrained by physical requirements of a realistic compact star. The physical acceptability of the model is tested using the observational data of pulsar PSR B0943+10. Using graphical analysis, we have shown that this theoretical model obeys all the physical requirements and approximates observations of pulsar PSR B0943+10 to an excellent degree of accuracy. The stability of this model is evaluated using the Tolman–Oppenheimer–Volkoff equation, the adiabatic index and the Harrison–Zeldovich–Novikov criterion and it has passed the evaluation.

Keywords. Field equations; relativistic compact stars; charged fluid; Schwarzschild’s canonical coordinate system; isotropy.

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1. Introduction

In the general theory of relativity, the Einstein’s field equations establish a relation between the geometry of space–time and the distribution of matter in it. It has been a compelling field for both mathematicians and physicists to discover new analytical solutions of these equations. The very first exact solution of Einstein’s field equations representing a bounded matter distribution was provided by Schwarzschild [1]. This encouraged the researchers to search for physically applicable solutions for the same. For researchers, this is still an interesting topic. The exact solution plays a crucial role for the development of varied areas of gravitational field like black hole solution, gravitational collapse, solar system test, modelling of pulsars like PSR B0943+10 and so on.

Pulsar PSR B0943+10 [2] (discovered at Pushchino in December 1968), is 2000 light-years from Earth. The pulsar is about 5 million years old, which is relatively

old for a pulsar. PSR B0943+10 is one of the brightest pulsars at low frequency [3]. It has an interesting characteristic that it emits both radio waves and X-rays. In the radio band, PSR B0943+10 is one of the most studied pulsars showing the mode-switching phenomenon. In fact, at irregular intervals, every few hours or less, PSR B0943+10 switches between a radio bright mode with highly organised pulsations and a quieter mode with a rather chaotic temporal structure [4]. Its 0.1–10 GHz spectrum is very steep [5]. PSR B0943+10 exhibits a very interesting behaviour in the X-ray band as well. It was the first rotation-powered pulsar exhibiting variations in its X-ray emission [6], contradicting the common view that rotation-powered neutron stars are sources of constant X-ray emission.

Due to the robust nonlinearity of Einstein’s field equations and thus the shortage of a comprehensive rule to get all solutions, it becomes difficult to locate any new exact solution. Thousands of exact solutions of the field equations describing an outsized number of stellar

objects varying between perfect fluids, charged bodies, anisotropic matter distributions, higher-dimensional stars, exotic matter configurations, etc. are present so far. But most of them are physically irrelevant within the relativistic structure of compact stellar objects. For obtaining exact solutions describing static compact objects, some impositions like space–time dimensionality, symmetry requirements, an equation of state relating the pressure and energy density of the stellar fluid, the behaviour of the pressure anisotropy or isotropy, vanishing of the Weyl stresses are made to make the problem mathematically more identifiable [7].

Though there is a good range of stellar solutions exhibiting deviation from sphericity, spherical symmetry is the closest natural assumption to elucidate stellar objects. A collection of static, spherically symmetric solutions which provides useful guide can be seen in [8,9]. In this regard, the primary model had been proposed by Tolman [10], which was followed by some generalisations made by Wyman [11], Leibovitz [12] and Whitman [13]. Bayin [14] then used the strategy of quadratures and gave new astrophysical solutions for the static fluid spheres. The studies of Sharma *et al* [15] and Ivanov [16] show that the presence of an electrical field affects the values of surface red-shifts, luminosities and maximum mass of compact objects. Ray *et al* [17] performed the charged generalisation of Bayin [14]. Mak and Harko [18] and Komathiraj and Maharaj [19,20] highlighted the particular incontrovertible fact that the electromagnetic field features an important role in describing the gravitational behaviour of stars composed of quark matter. Models constructed in this manner are proven to be useful in describing the physical properties of compact relativistic objects with different matter distributions. There are several investigations on the Einstein–Maxwell system of equations for static charged spherically symmetric gravitational fields [21–23].

Exact solutions of the field equations for various ‘neutral as well as charged static spherically symmetric configurations’ for anisotropic pressure compatible to compact stellar modelling have been obtained in numerous works [24–37]. In recent times, various models of relativistic stars have been found with anisotropic pressures [38–43]. However, in addition to this, it is necessary to keep in mind the compact stars with isotropic pressure, as at times this may be typically thought to be the equilibrium state of gravitating matter. Physical analysis indicates that isotropic models may even be accustomed to describe compact charged spheres. Some samples of isotropic stars with an electromagnetic field are often seen in [44–47]. Various comprehensive investigations of charged isotropic spheres can be seen in [16,21–23,48,49].

There is not enough information regarding the equation of state of matter contained within the interior of compact stars. So it is difficult to apply analytic solutions to the equation of relativistic stellar structure to understand it [50,51]. Oppenheimer and Volkof [52] technique and Tolman [10] method are two customary methodologies which are generally followed to acquire a realistic stellar model. In the first approach, we start with an explicit equation of state. The integration starts at the centre of the star with a prescribed central pressure and iterated till the surface of the star has been reached, i.e., where pressure diminishes to zero. Normally, such input equations of state does not yield closed-form solutions. In the second approach, Einstein’s gravitational field equations need to be solved. For a static isotropic perfect fluid case, the field equations can be reduced to a set of three coupled ordinary differential equations in four unknowns. After getting exact solutions, one can solve the field equations by considering one of the metric functions or the energy density *a priori*. Consequently, the equation of state can be computed from the resulting metric. Since non-physical pressure–density configurations are found more frequently than physical ones, a new solution that ought to be regular, well behaved and can reasonably model a compact astrophysical stellar object is always appreciated [53]. We are going to follow Tolman’s methodology in this paper and specify one of the gravitational potentials as the Vaidya and Tikekar [62] potential which has been shown to model superdense stars in several papers.

The presence of five unknown functions and only three basic field equations permits one to specify the metrics and solve for the fluid attributes [65]. In 1959, Buchdahl [54] proposed a wonderful scheme for finding a viable spherically symmetric perfect fluid solution, which possesses monotonically decreasing density towards the boundary, and obtained a particular solution of Einstein’s field equations employing this scheme. Considering specific charge intensity, the charged generalisation of this solution has been done [55–59]. The proposed scheme is used for anisotropic modelling as well [60,61]. The Buchdahl scheme was further modified by Vaidya and Tikekar [62], Durgapal and Bannerji [63] and Wils [64]. Vaidya and Tikekar [62] proposed a static spherically symmetric model of a superdense star based on the exact solution of Einstein’s equations by prescribing an ansatz (Vaidya–Tikekar ansatz) for the metric functions. It was for the geometry of ‘ $t = \text{constant}$ ’ hypersurface and the physical 3-space of the star was spheroidal. Using the Vaidya–Tikekar ansatz, several studies have been performed. Gupta and Kumar [66] observed a particular form of electric field intensity, having positive gradient. He used Vaidya–Tikekar ansatz to generate exact solutions of the field equation in

charge analogue. Later, this form of electric field intensity was used by Sharma *et al* [15]. Komathiraj and Maharaj [20] additionally accepted a similar articulation to show another kind of Vaidya–Tikekar-type star. Bijalwan and Gupta [67,68] obtained a charged perfect fluid model of Vaidya–Tikekar-type stars with more generalised electric intensity. Additionally, some of the other researches on Vaidya–Tikekar stars can be found in [69–72]. Recently, Kumar *et al* [48] used the Vaidya–Tikekar metric potential to explore a class of charged compact objects filled with self-gravitating, charged, isotropic fluids.

The aforementioned literature survey motivates us to perform this research on the subsequent line of action. In this research paper, our objective is to get an exact solution of the field equations for a static spherically symmetric fluid sphere. The matter distribution is charged with isotropic pressures.

This paper is organised as follows: Following a quick introduction in §1, we have introduced the Einstein–Maxwell field equations for the static charged fluid spheres in general relativity in §2. In §3, we have proposed a new model to solve the system of equations analytically. For this, we have used Vaidya and Tikekar [62] ansatz for the metric potential and acquired the expression for density and pressure. In §4, we have discussed the requirements for a well-behaved solution. Boundary conditions are discussed in §5. In §6, we have shown that our model is compatible with the observational data of pulsar PSR B0943 + 10. We have done the stability analysis of the obtained model in this section. Finally, §7 is dedicated to concluding remarks.

2. Field equations in Schwarzschild canonical coordinates

Let us consider the metric in Schwarzschild coordinates $(x^i) = (t, r, \theta, \phi)$

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where the metric potentials $\nu(r)$ and $\lambda(r)$ are arbitrary functions of the radial coordinate r . This metric describes the interior of the static and spherically symmetric stellar system and the metric potentials uniquely determine the surface red-shift and gravitational mass function respectively. The signature of the space–time taken here is $(+, -, -, -)$.

The Einstein–Maxwell field equations for obtaining the hydrostatic stellar structure of the charged sphere can be written as

$$-\kappa(T_j^i + E_j^i) = R_j^i - \frac{1}{2}R\delta_j^i = G_j^i, \quad (2)$$

where $\kappa = 8\pi G/c^4$. G here stands for gravitational constant and c is the speed of light, R_j^i and R represent Ricci tensor and Ricci scalar respectively. Since we have considered the matter within the compact stars to be isotropic, the radial and transverse pressures will be equal at each interior point of the stellar configuration. Thus, the corresponding energy–momentum tensor T_j^i and electromagnetic field tensor E_j^i will be

$$T_j^i = (\rho + p)v^i v_j - p\delta_j^i \quad (3)$$

and

$$E_j^i = \frac{1}{4\pi} \left(-F^{im} F_{jm} + \frac{1}{4} F^{mn} F_{mn} \right), \quad (4)$$

where $\rho(r)$ is the energy density, $p(r)$ is the isotropic pressure and F_{ij} is the antisymmetric electromagnetic field strength tensor defined as

$$F_{ij} = \frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j}$$

which satisfies Maxwells equations,

$$F_{ik,j} + F_{kj,i} + F_{ji,k} = 0$$

and

$$[\sqrt{-g}F^{ik}]_{,k} = 4\pi J^i \sqrt{-g}.$$

Here $A_j = (\phi(r), 0, 0, 0)$ is the potential and J^i is the electromagnetic current vector defined as

$$J^i = \frac{\sigma}{\sqrt{g_{44}}} \frac{dx^i}{dx^4} = \sigma v^i,$$

where $\sigma = e^{(\nu/2)} J^0$ represents the charge density, g is the determinant of the metric g_{ij} which is defined as $g = -e^{\nu+\lambda} r^4 \sin^2\theta$ and J^0 is the only non-vanishing component of the electromagnetic current J^i for the static spherically symmetric stellar system. Since the field is static, we have $v = (0, 0, 0, 1/\sqrt{g_{44}})$.

Also, the total charge within a sphere of radius r is given by

$$q(r) = r^2 E(r) = 4\pi \int_0^r J^0 r^2 e^{(\nu+\lambda)/2} dr, \quad (5)$$

where $E(r)$ is the intensity of the electric field.

Thus, for the spherically symmetric metric (1), the Einstein’s field equation (2) provides the following relationship:

$$\frac{\lambda'}{r} e^{-\lambda} + \frac{1 - e^{-\lambda}}{r^2} = c^2 \kappa \rho + \frac{q^2}{r^4} \quad (6)$$

$$\frac{\nu'}{r} e^{-\lambda} - \frac{1 - e^{-\lambda}}{r^2} = \kappa p - \frac{q^2}{r^4} \quad (7)$$

$$\left(\frac{\nu''}{2} - \frac{\lambda' \nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) e^{-\lambda} = \kappa p + \frac{q^2}{r^4}. \quad (8)$$

Here prime denotes differentiation with respect to r . Using eqs (7) and (8), we can obtain the condition of pressure isotropy as

$$\left(\frac{v''}{2} - \frac{\lambda'v'}{4} + \frac{v'^2}{4} - \frac{v'+\lambda'}{2r} - \frac{1}{r^2}\right)e^{-\lambda} + \frac{1}{r^2} = \frac{2q^2}{r^4}. \tag{9}$$

We can get the definition of charged density σ by substituting this value in eq. (5).

For our convenience, we use geometrised units, and thus we have taken $G = c = 1$ throughout the discussion.

3. Proposed exact solution of the field equations

We have seen in the previous section that a charged isotropic fluid and its gravitation are described by five functions: λ, ν, ρ, p and q , which are dependent on the radius. We have to solve Einstein–Maxwell field equations for these unknowns. The three coupled differential equations will act as generating equations for any three of the fluid characteristics, provided the remaining two fluid characteristics are known. Thus, to get an analytical closed form solution, we have to specify two variables *a priori* and solve the field equations for the remaining three variables. As we have $\binom{5}{2}$ ways to choose a pair of unknowns *a priori*, we can find various solutions depending on the given pair of functions. Here it is important to note that the obtained solutions might not be realistic. We have full control over the functions which we choose, and thus we can choose simple as well as regular functions. But, we completely lose control over the generated functions, and as a result, they could be complex or physically unrealistic. In this paper, we make a prior choice for λ and ν in such a way that the behaviour of pressure, density and charge is according to the requirements of a realistic compact star.

Let us consider the widely used [62] metric potential

$$e^\lambda = \frac{K(1 + Cr^2)}{K + Cr^2}, \tag{10}$$

where C and K are two parameters with $0 < K < 1$. We can verify that e^λ is a regular and monotonically increasing function of r . It is positive, free from singularity at the centre with $\lambda'(0) = 0$, i.e., λ is as per the requirement of physically acceptable model as suggested by Delgaty and Lake [8]. Also, assume $\nu = 2 \log Z(r)$, where, as per the criteria of Delgaty and Lake [8], Z will further be chosen regular and monotonically increasing.

The above-mentioned substitutions, followed by some computations, lead us to an equivalent form of field

equations, which might be helpful to find the exact solutions more efficiently. Using (6), (7) and (9), we have

$$c^2\kappa\rho + \frac{q^2}{r^4} = \frac{C(K - 1)(3 + Cr^2)}{K(1 + Cr^2)^2} \tag{11}$$

$$\kappa p - \frac{q^2}{r^4} = \frac{K + Cr^2}{K(1 + Cr^2)} \frac{2Z'}{rZ} + \frac{C(1 - K)}{K(1 + Cr^2)} \tag{12}$$

$$\frac{2q^2}{r^4} = \frac{K + Cr^2}{K(1 + Cr^2)} \left[\frac{Z''}{Z} - \frac{Z'}{rZ} + \frac{Cr(K - 1)}{(K + Cr^2)(1 + Cr^2)} \left(Cr - \frac{Z'}{Z} \right) \right]. \tag{13}$$

We now find a suitable expression for Z . Introducing

$$X = \sqrt{\frac{K + Cr^2}{1 - K}}, \tag{14}$$

eq. (13) reduces to

$$\frac{d^2Z}{dX^2} - \frac{X}{1 + X^2} \frac{dZ}{dX} - (1 - K) \times \left[\frac{1}{1 + X^2} + \frac{2K(1 + Cr^2)q^2}{C^2r^6} \right] Z = 0. \tag{15}$$

Let us use the transformation

$$Z = (1 + X^2)^{1/4} Y \tag{16}$$

which converts eq. (15) into the normal form

$$\frac{d^2Y}{dX^2} + \psi Y = 0, \tag{17}$$

where

$$\psi = -\frac{1}{1 + X^2} \left[1 - K + \frac{2Kq^2(1 + Cr^2)^2}{C^2r^6} + \frac{3X^2 - 2}{4(1 + X^2)} \right]. \tag{18}$$

To solve the differential equation (17), we have assumed that ψ can take the following form:

$$\psi = -\frac{2a}{X^2(a + bX)} \tag{19}$$

for suitable choice of real constants $a (\neq 0)$ and b . We shall use the hit and trial method to find the values of these constants so that the numeric value of ψ obtained through eqs (18) and (19) will be the same.

If we put the value of ψ from eq. (19) to eq. (17), the resulting differential equation will be

$$X^2(a + bX) \frac{d^2Y}{dX^2} - 2aY = 0 \tag{20}$$

which can be easily solved for Y to get

$$Y = \frac{a + bX}{X} \left[A \frac{a}{b^3} H(X) + B \right], \tag{21}$$

where A and B are constants of integration and

$$H(X) = \frac{\sec^2 \left(\tan^{-1} \sqrt{\frac{bX}{a}} \right)}{2} - \frac{\cos^2 \left(\tan^{-1} \sqrt{\frac{bX}{a}} \right)}{2} + 2 \log \left| \cos \left(\tan^{-1} \sqrt{\frac{bX}{a}} \right) \right|. \tag{22}$$

Together, eqs (16) and (21) yield

$$Z = (1 + X^2)^{1/4} \frac{a + bX}{X} \left[A \frac{a}{b^3} H(X) + B \right]. \tag{23}$$

Also, comparison of both expressions of ψ from eqs (18) and (19) provides the following definition of electric field intensity:

$$E^2 = \frac{q^2}{r^4} = \frac{C^2 r^2}{2K(1 + Cr^2)^2} \left[\frac{5}{4} \frac{(1 - K)}{(1 + Cr^2)} + \frac{2a}{X^2(a + bX)} \frac{1 + Cr^2}{(1 - K)} + K - \frac{7}{4} \right]. \tag{24}$$

To achieve the expression for energy density and pressure, let us put eqs (23) and (24) into eqs (11) and (12), respectively. Hereby, we obtain the following expressions:

$$c^2 \kappa \rho = \frac{C(K - 1)(3 + Cr^2)}{K(1 + Cr^2)^2} - \frac{C^2 r^2}{2K(1 + Cr^2)^2} \times \left[\frac{5}{4} \frac{(1 - K)}{(1 + Cr^2)} + \frac{2a}{X^2(a + bX)} \frac{1 + Cr^2}{(1 - K)} + K - \frac{7}{4} \right] \tag{25}$$

$$\kappa p = \frac{C(K + Cr^2)}{K(1 + Cr^2)} \left[\frac{f_1 f_2 + f_3 f_4}{f_2 f_5} \right] + \frac{C^2 r^2}{2K(1 + Cr^2)^2} \times \left[\frac{5}{4} \frac{(1 - K)}{(1 + Cr^2)} + \frac{2a}{X^2(a + bX)} \frac{1 + Cr^2}{(1 - K)} + K - \frac{7}{4} \right] + \frac{C(1 - K)}{K(1 + Cr^2)}. \tag{26}$$

On differentiating eqs (25) and (26) with respect to r , we get the gradient of density and pressure respectively as

$$c^2 \kappa \frac{d\rho}{dr} = C^2 r [g_6 - g_7 + g_8] \tag{27}$$

$$\kappa \frac{dp}{dr} = C^2 r \left[\frac{K + Cr^2}{K(1 + Cr^2)} \frac{g(r)}{f_2 f_5} + \frac{2(1 - K)}{K(1 + Cr^2)^2} \times \left(\frac{f_1 f_2 + f_3 f_4}{f_2 f_5} - 1 \right) + g_7 - g_8 \right]. \tag{28}$$

See Appendix A for f_i ($i = 1, 2, \dots, 5$), N and g_j ($j = 1, 2, \dots, 8$).

4. Physical acceptability conditions for well-behaved solution

For a well-behaved nature of the solution, the pre-requisites are

1. The solution should be free from physical and geometrical singularities, i.e., values of central pressure (p) and central density (ρ) must be finite and positive, and e^λ and e^ν must have a non-zero positive value.
2. The solution should have positive and monotonically decreasing expressions for energy density and pressure when radius r increases. Mathematically,

$$\rho \geq 0, p \geq 0, \frac{d\rho}{dr} \leq 0 \quad \text{and} \quad \frac{dp}{dr} \leq 0.$$

At the stellar boundary ($r = R$) the radial pressure p should vanish, i.e., $p(R) = 0$.

3. The casualty condition should be obeyed, i.e. velocity of sound should be less than that of light throughout the model. Also, it should be decreasing towards the surface. Besides this, at the centre, $d\rho/dr$ and dp/dr must be zero and $d^2\rho/dr^2$ and d^2p/dr^2 must have negative values at the centre so that the gradient of density and pressure shall be negative within the radius. The condition

$$\frac{p}{\rho} < \frac{dp}{d\rho}$$

should be valid throughout within the sphere.

4. The red-shift z should be positive, finite and monotonically decreasing in nature with the increase of r .
5. The adiabatic constant γ should increase from its lowest value $\frac{4}{3}$ at the centre to infinity as we move outwards, for a stable model.
6. The solution must satisfy the Tolman–Oppenheimer–Volkoff (TOV) equation.
7. The solution is required to fulfill all the energy conditions simultaneously.
8. The interior metric functions should match smoothly with the exterior Schwarzschild space–time metric at the boundary.

5. Boundary conditions

The unique exterior metric for a spherically symmetric charged distribution of matter is the Reissner–Nördstrom solution. To explore the boundary conditions, we are going to use the principle that, the metric coefficients and their first derivatives in the interior and exterior solutions are continuous up to and on the boundary.

Consider $r = R$ as the outer boundary of the fluid sphere. For $Q = q(R)$, the gravitational field in the exterior region ($r \geq R$) is described by the Reissner–Nördstrom metric

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{29}$$

where $M = m(R)$, the total gravitational mass, is actually a constant, and defined by $M = \xi(R) + \zeta(R)$, with

$$\zeta(R) = \frac{\kappa}{2} \int_0^R \rho r^2 dr \text{ and } \xi(R) = \frac{\kappa}{2} \int_0^R r \sigma q e^{\lambda/2} dr.$$

Here, $\xi(R)$ is the mass and $\zeta(R)$ is the mass equivalence of the electromagnetic energy of distribution [74]. Applicable boundary conditions are

1. The interior metric (1) should join smoothly at the surface of spheres ($r = R$) to the exterior metric (29).
2. Pressure $p(r)$ should vanish at $r = R$.

Arbitrary constants A and B can be obtained using the boundary conditions. The continuity of e^ν , e^λ and Q when $r = R$ implies that

$$\begin{aligned} e^{\nu(R)} &= Z^2(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2}, \\ e^{-\lambda(R)} &= 1 - \frac{2M}{R} + \frac{Q^2}{R^2}, \\ Q &= q(R), \quad p(R) = 0. \end{aligned} \tag{30}$$

Using $P(R) = 0$, we can easily obtain B/A . Then, along with condition $e^{\nu(R)} = e^{-\lambda(R)}$ it will give us A and B as

$$\begin{aligned} A &= -\frac{1}{\sqrt{K}(1 + X_1^2)^{3/4}} \left[\frac{n_{11}X_1^2 + (J + 1)n_{51}}{(a + bX_1)n_{31}n_{41}} \right] \\ B &= \frac{1}{\sqrt{K}(1 + X_1^2)^{3/4}} \left[\frac{X_1^2}{a + bX_1} + \frac{a n_{11}X_1^2 + (J + 1)n_{51}}{b^3 (a + bX_1)n_{31}n_{41}} H(X_1) \right] \end{aligned}$$

with $n_{i1} = f_i(R)$ for $i = 1, 3, 4$ and 5 ,

$$\begin{aligned} n_{21} &= f_2(R) - \frac{B}{A}, \\ J &= \frac{CR^2}{2(1 - K)(1 + CR^2)} \\ &\times \left[\frac{5}{4(1 + X_1^2)} + \frac{2a(1 + X_1^2)}{X_1^2(a + bX_1)} + K - \frac{7}{4} \right] \end{aligned}$$

and

$$X_1 = X(R).$$

6. Compatibility of the model with the realistic compact star

The solutions discussed in this paper can be used to model a relativistic star. In this section, we have critically verified our model by performing mathematical analysis and plotting several graphs indicating that the result overcomes all the barriers of physical tests.

The expression for the energy density in eq. (25) implies that at $r = 0$,

$$\rho_0 = \frac{3C(K - 1)}{\kappa K}. \tag{31}$$

As $0 < K < 1$, the central density ρ_0 will be positive iff we take $C < 0$.

It is also necessary to restrict C so that the transformation (14) remains physically acceptable throughout the configuration. For this, we require $|C| \leq K/R^2$. However, the function we have chosen for ψ in eq. (19) requires that $C \neq -K/R^2$. Thus, in our model, we have to consider $|C| < K/R^2$. Also, we can say that K and R characterise the geometry of the star.

The physical acceptability of the model has been examined by plugging the mass and radius of the observed pulsar as input parameters. To validate our model, we have considered the pulsar PSR B0943+10, a low-mass bare quark star of radius $r \sim 2.6$ km and mass $M \sim 0.02M_\odot$ [2]. Using these values of mass and radius as input parameters, the required physical conditions have been utilised to determine the constants as $C = -4.01893 \times 10^{-5} \text{ km}^{-2}$, $K = 0.0119$, $a = 0.001$ and $b = 0.029$. Furthermore, using boundary conditions, values of arbitrary constants used in eq. (23) are obtained as $A = -0.177729$ and $B = 29.657254$.

Here we have used $G = 6.674 \times 10^{-11} \text{ N/ms}^2$, $c = 3 \times 10^8 \text{ m/s}$ and $1 M_\odot = 1.475 \text{ km}$, to obtain numerical values of physical quantities and have multiplied charge by 1.1659×10^{20} to convert it from relativistic unit (km) to coulomb.

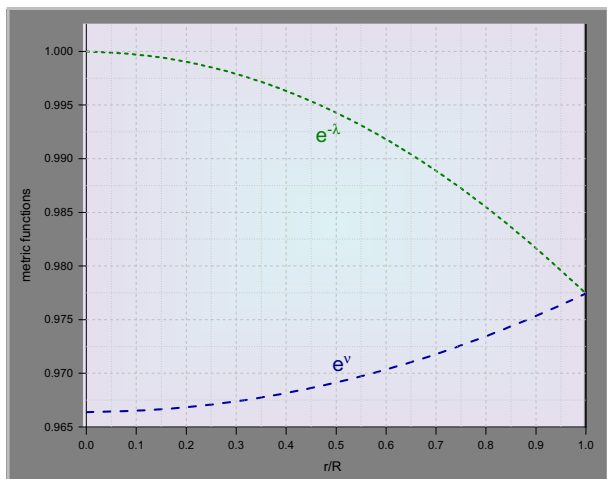


Figure 1. Variation of $e^{-\lambda}$ (green) and e^{ν} (blue) with respect to the fractional radius (r/R). For plotting the figure, we have used the values of free parameters as: $C = -4.01893 \times 10^{-5} \text{ km}^{-2}$, $K = 0.0119$, $a = 0.001$ and $b = 0.029$.

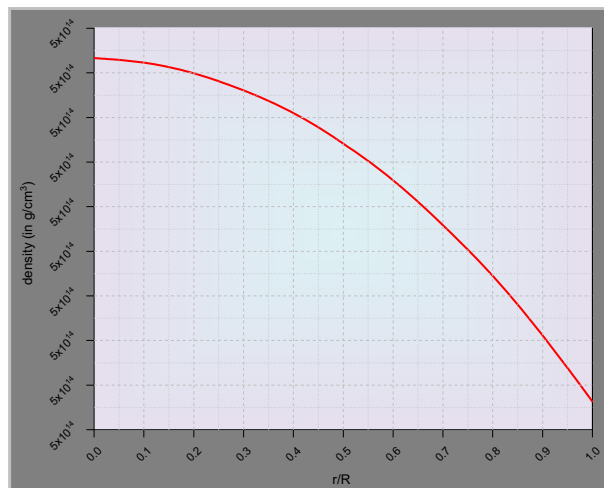


Figure 2. Variation of density with respect to the fractional radius (r/R). For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

6.1 Physical analysis of the model

6.1.1 Regularity and reality conditions.

- (i) The profiles of $e^{-\lambda}$ and e^{ν} for PSR B0943+10 in figure 1 show that the metric potentials are free from physical and geometrical singularities, regular at the centre (i.e., the metric potentials are positive and finite at the centre). e^{λ} and e^{ν} are monotonically increasing with the radius inside the star. Also, both $e^{-\lambda}$ and e^{ν} coincide at the surface. In short, the behaviour of metric functions is consistent with the requirements for a physically acceptable model.
- (ii) Figure 2 indicates that the energy density is positive with a maximum value at the centre and decreasing in nature throughout the star. Also, we can see in figure 3 that the pressure is monotonically decreasing towards the surface. At the centre, it is finite and vanishes at the boundary of the star.

Note: If we remove the electric intensity (i.e., when $q(r) = 0$) then we shall have the expression for density ρ_A and its gradient as

$$c^2 \kappa \rho_A = \frac{C(K - 1)(3 + Cr^2)}{K(1 + Cr^2)^2}$$

and

$$c^2 \kappa \frac{d\rho_A}{dr} = C^2 r \left[\frac{2(1 - K)(5 + Cr^2)}{K(1 + Cr^2)^3} \right],$$

respectively. It can be verified that for $0 < K < 1$, density (ρ_A) is positive throughout the structure and

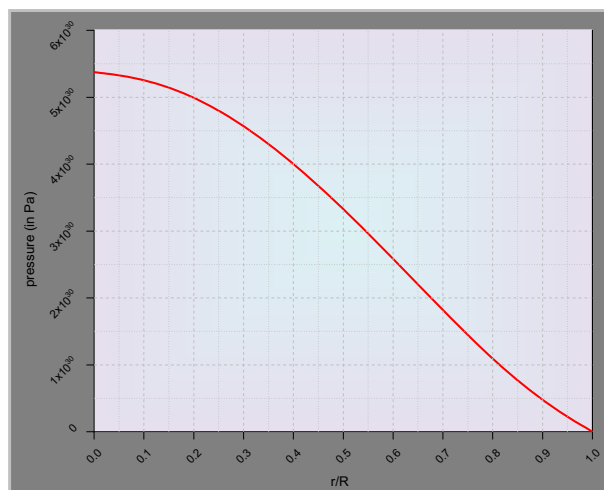


Figure 3. Variation of pressure with respect to the fractional radius (r/R). For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

$(d\rho_A/dr) > 0$, i.e., ρ_A is increasing towards the boundary. This shows that for the parameter K , $0 < K < 1$ only a charged isotropic model is valid.

6.1.2 Causality condition. For a physically acceptable isotropic model, the square in sound speed v_s must be less than 1 in the star's interior, i.e., $0 \leq v_s^2 = (dp/d\rho) \leq 1$ [73]. This condition is known as the causality condition. Figure 6 shows that, for our charged isotropic model, the velocity of sound remains less than the speed of light inside the star and decreases with an increase of r (figure 4).

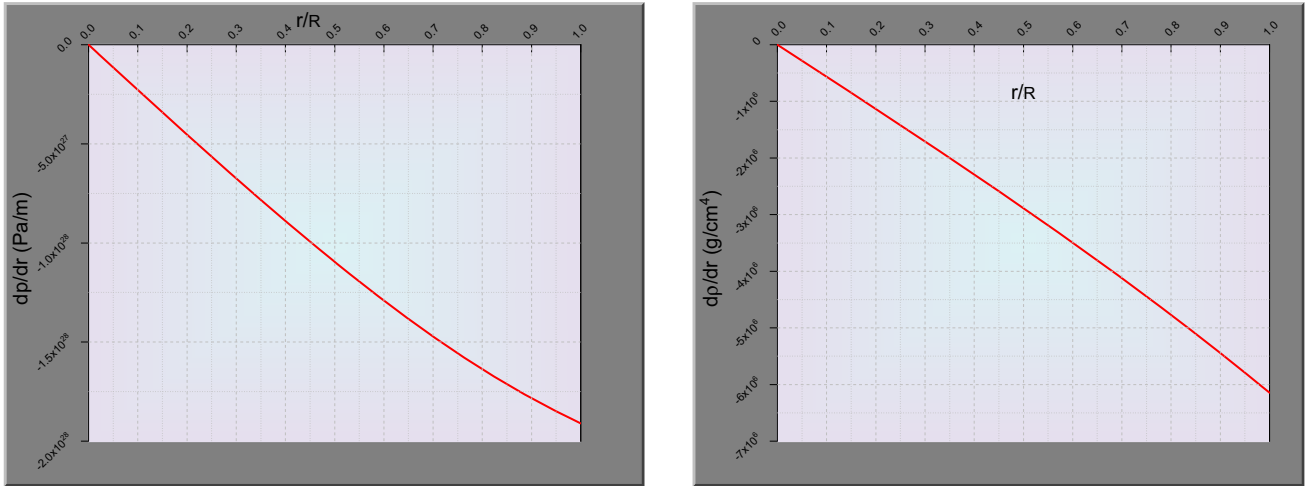


Figure 4. Variation of pressure gradient (left) and density gradient (right) with respect to fractional radius (r/R). For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

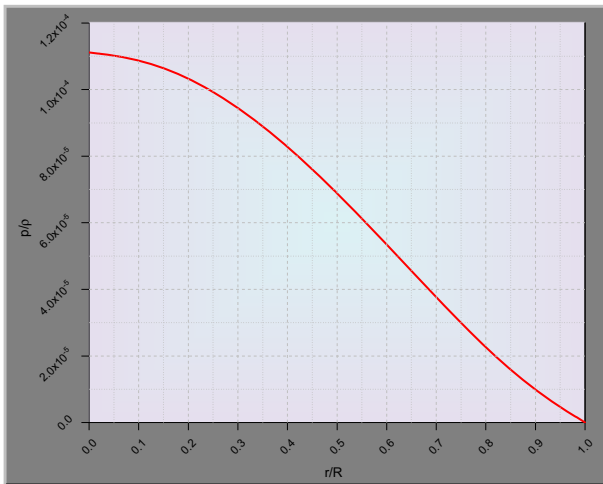


Figure 5. Variation of ratio of pressure to that of density with increase in radius (r/R). For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

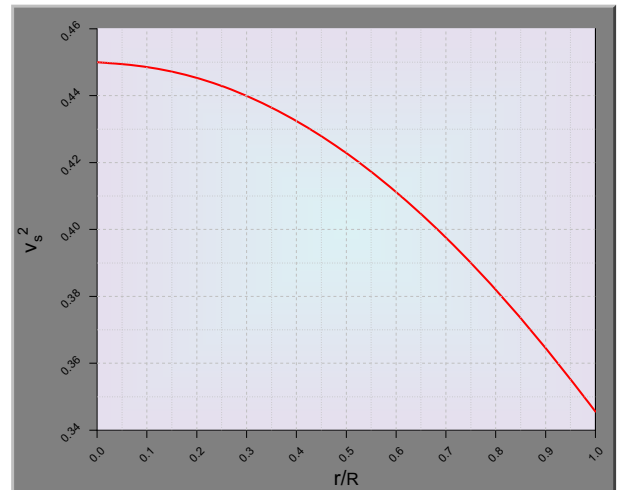


Figure 6. Variation of square of velocity of sound with respect to the fractional radius (r/R). For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

It is very clear from figures 5 and 6 that the ratio p/ρ is less than $dp/d\rho$ throughout the stellar model. One can also verify this using table 1. As we can see in figure 4, gradient of pressure and density is zero at the centre and has negative values at every other point in the region (table 2).

6.1.3 Energy conditions. In the study of stellar configurations describing charged isotropic matter distributions, it is necessary to check whether the energy–momentum tensor is well behaved, i.e. positive defined everywhere within the star. For this, the fulfillment of the following energy conditions are required:

1. Null energy condition (NEC):

$$c^2\rho + \frac{q^2}{\kappa r^4} \geq 0.$$

2. Weak dominating energy condition (WDEC):

$$c^2\rho - p + \frac{2q^2}{\kappa r^4} \geq 0.$$

3. Strong dominating energy condition (SDEC):

$$c^2\rho - 3p + \frac{2q^2}{\kappa r^4} \geq 0.$$

Table 1. Structural properties of PSR B0943+10 within its radius.

r/R	q (km)	ρ (km ⁻²)	p (km ⁻²)	p/ρ	$dp/c^2d\rho$	z	γ
0.0	0.0	3.98537×10^{-4}	4.42866×10^{-8}	0.000111	0.449988	0.017248	4047.839101
0.2	0.0002314	3.98513×10^{-4}	4.13532×10^{-8}	0.000104	0.445668	0.017019	4293.081396
0.4	0.0018512	3.98448×10^{-4}	3.31219×10^{-8}	0.000083	0.432779	0.016329	5203.995995
0.6	0.006266	3.98337×10^{-4}	2.13164×10^{-8}	0.000052	0.411541	0.015178	7686.893767
0.8	0.0149214	3.98178×10^{-4}	8.86689×10^{-9}	0.000022	0.382306	0.013562	17159.507244
1.0	0.029315	3.97966×10^{-4}	0.0	0.0	0.345547	0.011477	Inf.

Table 2. The obtained numerical values for charge at the surface, central density, surface density, central pressure and mass–radius ratio of the compact star PSR B0943+10.

q_s (C)	ρ_0 (g/cm ³)	ρ_s (g/cm ³)	p_0 (Pa)	M/R	R (km) (predicted)
3.41784×10^{18}	5.37433×10^{14}	5.36663×10^{14}	5.3749×10^{30}	0.011346	2.6

Table 3. The value of energy conditions throughout the interior of PSR B0943+10.

r/R	NEC (MeV/fm ³)	WDEC (MeV/fm ³)	SDEC (MeV/fm ³)	WEC (MeV/fm ³)	SEC (MeV/fm ³)
0.0	301.740646	301.707099	301.640004	301.774194	301.841289
0.2	302.120074	302.484687	302.422036	302.547338	302.609988
0.4	304.847908	307.996459	307.946279	308.046639	308.09682
0.6	312.335777	323.065401	323.033106	323.097696	323.12999
0.8	327.057366	352.638418	352.624985	352.651852	352.665285
1.0	351.582277	401.854051	401.854051	401.854051	401.854051

4. Weak energy condition (WEC):

$$c^2\rho + p + \frac{2q^2}{\kappa r^4} \geq 0.$$

5. Strong energy condition (SEC):

$$c^2\rho + 3p + \frac{2q^2}{\kappa r^4} \geq 0.$$

The behaviour of these energy conditions for our particular model PSR B0943+10 are shown in table 3. This table clearly indicates that all the energy conditions in our model are satisfied throughout the interior region of the spherical distribution.

6.1.4 *Mass–radius ratio.* For physically viable models, the ratio of the mass to that of the radius of a compact star cannot be arbitrarily large. According to Buchdahl [54], the ratio of mass to the radius for a perfect fluid compact star should satisfy the inequality $\frac{M}{R} < \frac{4}{9}$. This maximum limit of the mass–radius ratio was calculated in the framework of perfect fluid having decreasing energy density towards the boundary. However, the presence of an electric charge in the solution further modifies this limit of Buchdahl. Böhmer and Harko [76] have given the generalised expression of lower bound for a charged compact object as follows:

$$\frac{Q^4 + 18R^2Q^2}{12R^4 + R^2Q^2} \leq \frac{2M}{R}. \tag{32}$$

Subsequently, Andreasson [77] showed that, for a charged sphere, the model must satisfy the following inequality:

$$\sqrt{M} \leq \frac{\sqrt{R}}{3} + \sqrt{\frac{R^2 + 3Q^2}{9R}}. \tag{33}$$

We, therefore, conclude from the above two conditions that M/R must satisfy the following inequality:

$$\frac{Q^2}{2R^2} \left(\frac{Q^2 + 18R^2}{12R^2 + Q^2} \right) \leq \frac{M}{R} \leq \frac{1}{R} \left[\frac{\sqrt{R}}{3} + \sqrt{\frac{R^2 + 3Q^2}{9R}} \right]^2, \tag{34}$$

where

$$M = m(r) = \frac{\kappa}{2} \int_0^R c^2 \rho r^2 dr + \frac{1}{2} \int_0^R \frac{q^2}{r^2} dr + \frac{Q^2}{2R} = \frac{R}{2} \left(1 - \frac{M_0}{R} + \frac{Q^2}{R^2} \right)$$

is the total mass of the compact object for the charged isotropic fluid matter distribution. It is important to note

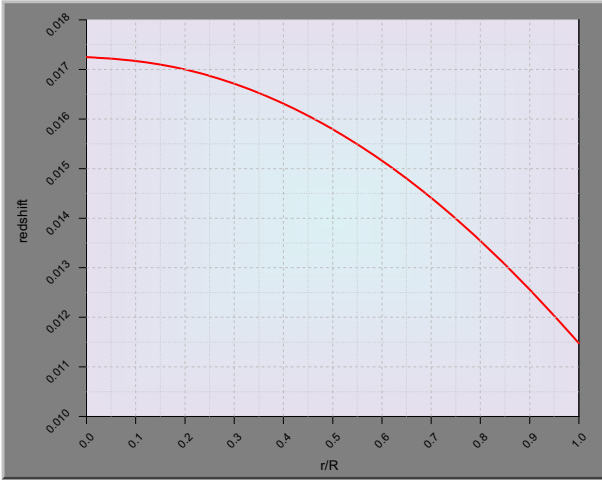


Figure 7. Variation of red-shift with respect to fractional radius (r/R).

that the total mass M for the charged stellar object will be more than the total mass M_0 of the compact object corresponding to the perfect fluid matter distribution [78,79].

6.1.5 Red-shift. The gravitational red-shift z within the static line element is given by

$$z = |e^{\nu(r)}|^{-1/2} - 1 = \frac{1}{|Z(r)|} - 1. \tag{35}$$

Note that surface red-shift also stabilises the following relationship:

$$z_s = |e^{\nu(R)}|^{-1/2} - 1 = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)^{-1/2} - 1. \tag{36}$$

We have central red-shift $z_0 = 0.017248$ and surface red-shift $z_s = 0.011477$ and as can be seen in figure 7, the red-shift function gradually decreases inside the star. Clearly, we have $z_s < 2$, which is required for spherically symmetric isotropic fluid spheres as suggested in [54,75,76].

6.2 Stability analysis of the model

Now, we shall examine the stability of our isotropic, charged fluid configuration in the Einstein–Maxwell space–time in the following subsections.

6.2.1 Electric charge in the sphere. Any fluid sphere with net charge, contains fluid elements with unbounded proper charged density located at the fluid–vacuum interface. This net charge can be huge (10^{19} C) [80]. Ray *et al* [17] have analysed the impact of charge in compact stars by considering the limit of the very extreme measure of the charges. They have demonstrated that

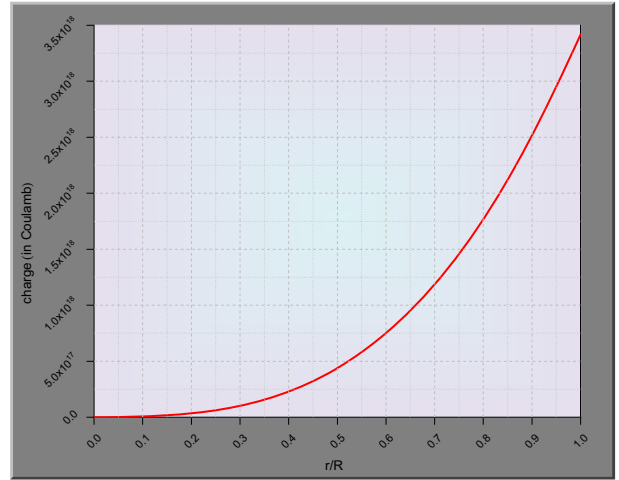


Figure 8. Behaviour of charge within the configuration of PSR B0946+10. For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

the global balance of the forces allows a huge charge (10^{20} C) to be available in a compact star.

In figure 8, we can observe that the electric field given by eq. (24) is positive and an increasing function with increasing radius. The charge starts from zero at the centre and acquires the maximum value at the boundary. In this model, the charge on the boundary is 3.41784×10^{18} C. Thus, we can say that, in this model the net charge is effective to balance the mechanism of the force.

6.2.2 Equilibrium analysis through TOV equation.

A star remains in static equilibrium under gravitational force (F_g), hydrostatic force (F_h) and electric force (F_e). This condition is formulated mathematically as the TOV equation by Tolman, Oppenheimer and Volkof [10,52]. In the presence of a charge, the same takes the following form [82]:

$$\frac{-M_G(\rho + p)}{r^2} e^{(\lambda-\nu)/2} - \frac{dp}{dr} + \sigma \frac{q}{r^2} e^{\lambda/2} = 0, \tag{37}$$

where $M_G(r)$ is the gravitational mass of the star within radius r and is defined by

$$M_G(r) = \frac{1}{2} r^2 \nu' e^{(\nu-\lambda)/2}. \tag{38}$$

Substituting the value of $M_G(r)$ in eq. (37), we obtain

$$-\frac{\nu'}{2}(\rho + p) - \frac{dp}{dr} + \sigma \frac{q}{r^2} e^{\lambda/2} = 0 \tag{39}$$

which is equivalent to

$$F_g + F_h + F_e = 0, \tag{40}$$

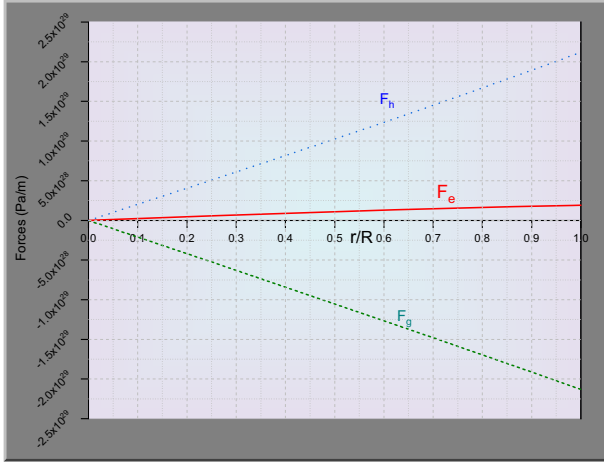


Figure 9. Variations of gravitational force (F_g), hydrostatic force (F_h) and electric force (F_e) acting on the system with respect to the fractional radius (r/R). For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

where

$$\begin{aligned}
 F_g &= -\frac{v'}{2}(\rho + p) = -\frac{Z'}{Z}(\rho + p) \\
 &= -\frac{C^2 r}{2\kappa} \left[\frac{f_1 f_2 + f_3 f_4}{f_2 f_5} \right] \left[\frac{2(K-1)}{K(1+Cr^2)^2} \right. \\
 &\quad \left. + \frac{K+Cr^2}{K(1+Cr^2)} \frac{f_1 f_2 + f_3 f_4}{f_2 f_5} \right] \\
 F_h &= -\frac{dp}{dr} = -\frac{C^2 r}{\kappa} \left[\frac{K+Cr^2}{K(1+Cr^2)} \frac{g(r)}{f_2 f_5} \right. \\
 &\quad \left. + \frac{2(1-K)}{K(1+Cr^2)^2} \left(\frac{f_1 f_2 + f_3 f_4}{f_2 f_5} - 1 \right) \right. \\
 &\quad \left. + g_7 - g_8 \right] \\
 F_e &= \sigma \frac{q}{r^2} e^{\lambda/2} = \frac{1}{\kappa r^4} \frac{dq^2}{dr} \\
 &= \frac{C^2 r}{8\kappa} \left[\frac{3+Cr^2}{K(1+Cr^2)^3} \left\{ \frac{5(1-K)}{4(1+Cr^2)} \right. \right. \\
 &\quad \left. \left. + \frac{2a}{X^2(a+bX)} \frac{1+Cr^2}{(1-K)} + K - \frac{7}{4} \right\} - g_8 \right].
 \end{aligned}$$

Figure 9 shows that F_h and F_e are positive and are nullified by F_g , which is negative to keep the system in static equilibrium.

6.2.3 Relativistic adiabatic index. The adiabatic index, defined as

$$\gamma = \left(\frac{c^2 \rho + p}{p} \right) \left(\frac{dp}{c^2 d\rho} \right) \tag{41}$$

is related to the stability of an isotropic stellar configuration.

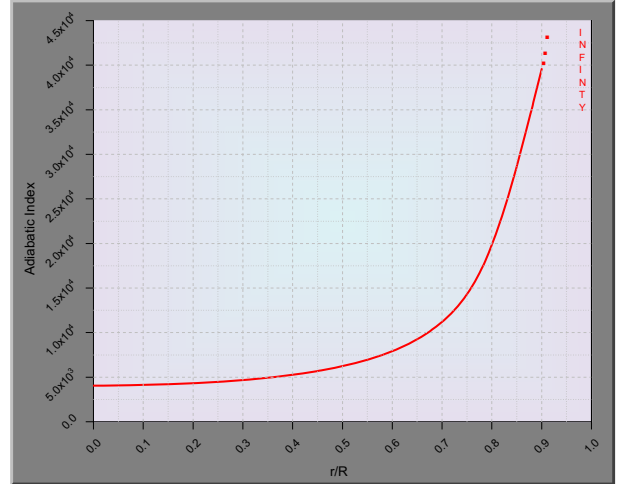


Figure 10. Nature of adiabatic index with increase in radius. For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

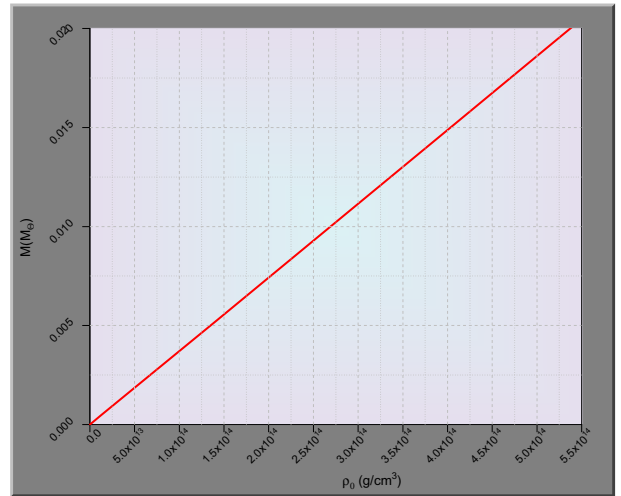


Figure 11. Behaviour of mass M vs. central density ρ_0 for PSR B0943+10. For plotting this figure, we have taken the same numerical values for the free parameters C , K , a and b as in figure 1.

We have demonstrated the behaviour of adiabatic index γ in figure 10, which shows the desirable features. The value of γ at the centre is 4047.839101, and the graph clearly indicates that with an increase in radius, γ increases drastically.

If we consider a Newtonian sphere to be in stable equilibrium, γ must have values strictly greater than $4/3$ throughout the region and $\gamma = 4/3$ is the condition for a neutral equilibrium [83]. This condition changes for a relativistic isotropic sphere due to the regenerative effect of pressure, which makes the sphere more unstable. In the post-Newtonian approximation, the condition for stability takes the following finer form [84]:

$$\Gamma > \frac{4}{3} + \max\left(\frac{\kappa r \rho p}{3 |p'|}\right). \quad (42)$$

This expression shows that the unstable range of Γ increases due to the relativistic effect of pressure. However, relativistic correction to the adiabatic index could introduce some instabilities inside the star [85,86]. To overcome this issue, a more strict condition on the adiabatic index was proposed by Moustakidis [87]. He suggested that the critical value of adiabatic index (Γ_{critical}) depends on the compactness and the amplitude of Lagrangian displacement from equilibrium, and obtained this value as

$$\Gamma_{\text{critical}} = \frac{4}{3} + \frac{19 M}{21 R}, \quad (43)$$

so that the stability condition becomes $\Gamma \geq \Gamma_{\text{critical}}$ [88,89]. The critical value of adiabatic index for the proposed model is calculated as 1.343598762, and it is clear from figure 10 that the obtained adiabatic index of Pulsar PSR B0943+10 surpasses this critical value throughout the stellar interior. Hence, the proposed model is stable.

6.2.4 Harrison–Zeldovich–Novikov stability criterion. According to Harrison, Zeldovich and Novikov [90,91] criterion, for a compact star to be stable, its mass should increase with rise in central density. Mathematically, $(dM/d\rho_0) > 0$ throughout the stellar configuration.

Using eqs (30) and (31) we have

$$M = \kappa R^3 \frac{\rho_0}{2M_1} \left[K - 1 + \kappa K R^2 \rho_0 \frac{M_2}{2M_1} \right], \quad (44)$$

where

$$M_1 = 3(K - 1) + \kappa K \rho_0 R^2$$

and

$$M_2 = \left[-\frac{15(1-K)^2}{4M_1} - \frac{2aM_1}{3(1-K)^2 X_1^2 (a + bX_1)} + K - \frac{7}{4} \right].$$

We observed in figure 11 that mass of the star is positive and increasing with increase in central density. Thus, we can conclude that the presented model satisfies the Harrison–Zeldovich–Novikov criterion. Hence, the model is stable.

7. Discussion and concluding remarks

In this paper, an isotropic charged fluid model has been developed by using the well-known Vaidya–Tikekar metric potential. The solution of this system involves six constants, two of them have been fixed using the junction

condition as well as the property of vanishing pressure at the boundary, leaving the remaining constants to be determined by studying a real compact star. The presented solution satisfies all the physical criteria of a relativistic compact object. A thorough physical analysis has been accomplished for the star PSR B0943+10. All the physical quantities are regular and well-behaved throughout the stellar interior. Energy density and pressure are decreasing functions as we move towards the surface of the star from the centre. This model satisfies causality conditions, energy conditions and stability conditions.

The main features of this study can be summarised as follows:

1. To demonstrate that our solution is compatible with a real compact star, we have used the pulsar PSR B0943+10, which has the mass of $\sim 0.02M_{\odot}$ and radius of ~ 2.6 km [91]. The use of mass and radius as input parameters helped us to fix four of the constants as $C = -4.01893 \times 10^{-5} \text{ km}^{-2}$, $K = 0.0119$, $a = 0.001$ and $b = 0.029$. Furthermore, the use of boundary conditions yields $A = -0.177729$ and $B = 29.657254$.
2. As shown in figure 1, the metric potentials have no singularity either at the centre of the pulsar PSR B0943+10 or at the boundary.
3. Figures 2 and 3 show that the density and pressure are positive and decreasing towards the centre of the pulsar PSR B0943+10. Figure 4 shows that the gradient of the density and pressure are negative, which is a necessary condition for any real star configuration.
4. Figure 6 shows that the velocity of sound is strictly less than 1, which is required for any realistic star. Also, it is very clear from figure 5 that in our model, pressure at any fixed distance from the centre of the star is dominated by the corresponding density at that point.
5. Table 3 shows that our model satisfies all the energy conditions, i.e., NEC, WEC, DEC, SEC, supporting the fact that the matter distributed inside the star is normal (not any kind of exotic matter).
6. From figure 7, it is clear that the red-shift of our model is less than 2.
7. Figure 8 shows that the charge function is positive and increasing towards the surface of the pulsar PSR B0943+10. The charge is zero at the centre and acquires maximum value at the boundary which is $3.41784 \times 10^{18} \text{ C}$. In this model the net charge is effective to balance the mechanism of the force.

8. As can be seen in figure 9, our model is in hydrostatic equilibrium under gravitational force (F_g), hydrostatic force (F_h) and electric force (F_e).
9. Figure 10 shows that the resulting model is stable because its adiabatic index is greater than its critical value.
10. It follows from figure 11 that the mass of the pulsar PSR B0943+10 increases with increase in energy density, i.e., it satisfies the Harrison–Zeldovich–Novikov criterion for stability.

Hence, we can conclude that an analytic solution to the Einstein–Maxwell field equations which meets all the requirements of a physically and mathematically admissible solution representing a static spherically symmetric space–time described by a charged isotropic energy–momentum tensor has been obtained. We have used this solution to study the pulsar PSR B0943+10 as a Vaidya–Tikekar-type star. This model can even be useful to elucidate more compact objects apart from PSR B0943+10.

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Appendix A: Notations used in paper

To express equations in a simplified manner, notations used are as follows:

$$f_1 = \frac{2}{\sqrt{(1-K)(K+Cr^2)}} \left(\frac{a+bX}{2(1+X^2)} - \frac{a}{X^2} \right)$$

$$f_2 = \frac{a}{b^3} H(X) + \frac{B}{A}$$

$$f_3 = \frac{2}{\sqrt{(1-K)(K+Cr^2)}} \left(\frac{a+bX}{X} \right)$$

$$f_4 = \frac{a}{2b^2(a+bX)} \left[\sec^2 \left(\tan^{-1} \sqrt{\frac{bX}{a}} \right) + \cos^2 \left(\tan^{-1} \sqrt{\frac{bX}{a}} \right) - 2 \right]$$

$$f_5 = \left(\frac{a+bX}{X} \right)$$

$$g(r) = -\frac{(f_1 f_2 + f_3 f_4)(g_2 f_5 + g_5 f_2)}{f_2 f_5} + g_1 f_2 + g_2 f_1 + g_3 f_4 + g_4 f_3$$

$$g_1(r) = -\frac{f_1}{(K+Cr^2)} + \frac{2}{(1-K)(K+Cr^2)} \times \left[\frac{b}{2(1+X^2)} - \frac{X(a+bX)}{(1+X^2)^2} + \frac{2a}{X^3} \right]$$

$$g_2(r) = \frac{f_4}{\sqrt{(1-K)(K+Cr^2)}}$$

$$g_3(r) = -\frac{1}{K+Cr^2} \left[f_3 + \frac{2a}{K+Cr^2} \right]$$

$$g_4(r) = \frac{1}{\sqrt{(1-K)(K+Cr^2)}} \times \left[-\frac{b}{(a+bX)} f_4 + \frac{a}{2b(a+bX)^2} \times \left\{ \sec^2 \left(\tan^{-1} \sqrt{\frac{bX}{a}} \right) - \cos^2 \left(\tan^{-1} \sqrt{\frac{bX}{a}} \right) \right\} \right]$$

$$g_5(r) = -\frac{1}{\sqrt{(1-K)(K+Cr^2)}} \frac{a}{X^2}$$

$$g_6(r) = \frac{2(1-K)(5+Cr^2)}{K(1+Cr^2)^3}$$

$$g_7(r) = \frac{1-Cr^2}{K(1+Cr^2)^3} \left[\frac{5}{4} \frac{1-K}{1+Cr^2} + \frac{1+Cr^2}{1-K} \frac{2a}{X^2(a+bX)} + K - \frac{7}{4} \right]$$

$$g_8(r) = \frac{Cr^2}{4K(1-K)(1+Cr^2)^2} \left[5 \left(\frac{1-K}{1+Cr^2} \right)^2 - \frac{8a}{X^2(a+bX)} + \left(\frac{1+Cr^2}{1-K} \right) \frac{4a(2a+3bX)}{X^4(a+bX)^2} \right].$$

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