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Study of superconductivity in rhodium

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Abstract. Superconducting behaviour of the fragile, pure, sub-milli-Kelvin temperature superconductor rhodium has been studied. Rhodium, a paramagnetic transition element, having free electron density, more than an order of magnitude larger (6.5×10^{23} cm^{-3}) than a metallic monovalent superconductor, exhibits superconductivity at an extremely low temperature of a few hundred micro-Kelvin (10^{-6} K), at ambient pressure. Rhodium exhibits traditional BCS superconductivity brought about by the residual electron–phonon interaction and shows the characteristics of Type-I superconductivity. In this work, we have studied the effect of electron–phonon, electron–electron and electron–paramagnon interactions in rhodium. Further, we have evaluated the various characteristic parameters associated with rhodium. Finally, we conclude that rhodium is explained totally by the Bardeen–Cooper–Schrieffer (BCS) theory which corresponds fundamentally to the instantaneous nature of electron–phonon interaction along with the instantaneous electron–Coulomb interaction and instantaneous electron–spin interaction.

Keywords. Sub-milli-Kelvin superconductivity; instantaneous electron–phonon interaction; electron–paramagnon interaction; transition element; rhodium.

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1. Introduction

Ever since the discovery of superconductivity, various research groups are mainly in search of high-temperature superconducting materials and are in the race to achieve room-temperature superconductivity, but very little attention is given to low-temperature superconductors discovered so far. Superconductivity in the pure crystal of the transition element, rhodium [1,2] and semimetallic element, bismuth [3–5] has been experimentally observed at ambient pressure at the extremely low temperature of a few hundred micro-Kelvins (μK) and cannot survive even in the bare Earth's magnetic field if proper precautions of magnetic shielding [3,6] are not taken.

Recently, the superconductivity in bismuth has been observed at the extremely low superconducting transition temperature, $T_C = 0.53$ milli-Kelvin ($\equiv$ mK), and exhibit $B_C(0)$, critical magnetic field at 0 K, equal to (52 $\pm$ 1) milli-Gauss ($\equiv$ mG). It exhibits all the other superconducting properties of a Type-I superconductor [4,5].

For the transition element rhodium, the number of valence electrons per atom that contribute to its Fermi sea is nine [7], because of its incomplete 4d and 5s shells [8] leading to the sharp increase in the number density of electrons to be several times that of other elemental polyatomic metallic superconductors! Hence, its superconducting transition temperature could have been higher and easily experimentally accessible. But the experimentally observed superconducting transition temperature of pure rhodium at ambient pressure turns out to be still the lowest and has the value $T_C = (0.325 \pm 0.01)$mK. The critical magnetic field that destroys superconductivity at temperature $T = 0$ K, $B_C(0) = (49 \pm 2)$ mG, is obtained by the experimental measurements of $B_C(T)$ for $T < T_C$ and extrapolated to 0 K, using the well-known expression: $B_C(T) = B_C(0)[1 - T^2/{T_C}^2]$, valid for Type-I superconductors. There is no intrusion of magnetic field in the form of vortices precluding the possibility of the presence of Type-II superconductivity [1,2].

Rhodium being the lowest temperature superconductor discovered so far, prompted us to study its

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superconducting behaviour. In the past [1] it has been studied as a weakly coupled superconductor, but later in the same paper, it was considered as a strongly coupled superconductor when electron–spin interaction (also known as electron–paramagnon interaction) is taken into account. In this communication, we have studied rhodium as a weakly coupled superconductor in the simultaneous presence of electron–phonon, electron—electron and electron–paramagnon interactions. Hence, all the parameters are calculated to show its weakly coupled behaviour which is in very good agreement with the corresponding experimental results. Further, we also compare our results with the well-known weakly (Al, Sn) and strongly (Hg, Pb) coupled superconductors. We show that rhodium exhibits traditional BCS (Bardeen–Cooper–Schrieffer) [9] superconductivity, brought about by the instantaneous residual electron–phonon interaction.

The paper is organised as follows: In §2, the methodology to evaluate the various parameters associated with this theoretical framework has been discussed. Results and discussions are explained in §3. Finally, the conclusion is highlighted in §4.

2. Methodology

We have calculated different parameters of rhodium like the effective mass of quasiparticles using Landau–Fermi liquid theory. Further, using experimental low-temperature specific heat data, we have obtained mass enhancement factor due to phonons as well as due to spin and these are compared with the corresponding experimental results of cyclotron masses. We have also obtained values of residual electron–phonon coupling constant and various BCS characteristic parameters, valid for weakly coupled superconductors. Further, we evaluate values of electron–phonon, electron–electron and electron–paramagnon interactions and compare them with those cited in the literature. All these calculations are shown in different subsections.

2.1 *Effective mass of quasiparticle of rhodium using Landau–Fermi liquid theory*

The number density of electrons in rhodium [7] is 6.53 $\times 10^{23}$ cm^{-3}, which is approximately 3.6 times that of the polyvalent superconductor aluminium and 14 times that of the monovalent superconductor, lithium, an alkali metal. The interelectron distance $2r_s$, where r_s is the radius of a sphere assigned to an electron, turns out to be 1.43 Å, which is only a fraction 0.37 of its lattice constant 3.8 Å (rhodium crystal having an fcc structure) and nearly four orders of magnitude less than that of the electron thermal de Broglie wavelength at T_C, making it a highly quantum mechanical electron gas. It is clear, therefore, that the Coulombic interaction among its electrons will be much higher than that of the other metallic superconductors and therefore leads to, among others, the enhancement of its paramagnetic susceptibility.

It is evident from the above discussion that one will have to use Landau–Fermi Liquid theory to describe such a many-body interacting electron system in terms of independent quasiparticles, i.e., quasielectrons, which result from the elementary excitation of such an interacting metallic system. It turns out, however, that a quasielectron has all the attributes of an ordinary electron except its mass m_{eff}, which is different from its normal mass m_e, and can be determined using the experimental specific heat of electrons C_V, at extremely low temperature T, in the absence of superconductivity [10]:

$$C_V = \frac{m_{eff} k_F}{3\hbar^2} k_B^2 T = \gamma(0) T, \tag{1}$$

where $\gamma(0)$ is the Sommerfeld constant and here it corresponds to its band-structure value, k_B is the Boltzmann constant, $\hbar$ is the Dirac constant and k_F is the magnitude of the Fermi wave vector of the free electron gas.

For rhodium, the value of band-structure density of states at the Fermi surface, $N(E_F) \equiv N(0)$, is 1.375 states/eV atom, which leads to the value of $\gamma(0) =$ 3.23 mJ/mole K^2 [8].

The effective mass m_{eff}, of the quasielectron, therefore turns out to be nearly $2.8m_e$. Further, the Fermi energy $E_F = 9.8$ eV is close to the value reported by Anderson [8] (9.53 eV) and Sigalas *et al* [11] (9.26 eV).

2.2 *Renormalisation of mass due to electron–phonon and electron–paramagnon interactions*

Rhodium has an incomplete d-shell having eight electrons and an s-shell having one electron and is paramagnetic. But, because of the Coulombic interaction with the spin of the electrons, there is an enhancement of its paramagnetic susceptibility by what is known as its Stoner factor [12,13] and there is a tendency for it to become weakly ferromagnetic as is evident from neutron magnetic elastic scattering experiment [14,15]. The excitation of this weakly ferromagnetic system gives rise to the generation of spin density waves, also referred to as paramagnons [16–18]. Paramagnons here, are the elementary excitations of the weakly ferromagnetic rhodium (just as ferromagnetic magnons are the elementary excitations of weakly excited ferromagnets due to its spin flips). But in contrast to quasielectrons which are fermionic elementary excitations, paramagnons like phonons, are bosonic elementary excitations. Superconductivity results because the quasielectrons (here onwards refer to as electrons) interact via exchange of virtual phonons, i.e., an electron absorbs a virtual phonon and is emitted by another electron, conserving linear momentum and spin, which gives rise to self-energy, resulting in the mass enhancement of the electron, i.e. its mass gets renormalised. Similarly, electrons can interact with paramagnons, resulting in their (electrons) mass enhancement or renormalisation of their mass [12,13]. Both these enhancements in rhodium reflect in the experimentally observed, total specific heat of electrons at extremely low temperatures and the Sommerfeld constant γ can now be written as

$$\gamma = \gamma(0)\left[1 + \frac{\delta m_{\mathrm{e-p}}}{m_{\mathrm{eff}}} + \frac{\delta m_{\mathrm{e-s}}}{m_{\mathrm{eff}}}\right] = \gamma(0)\left(1 + \lambda_{\mathrm{e-p}} + \lambda_{\mathrm{e-s}}\right), \tag{2}$$

where $\lambda_{\mathrm{e-p}}$ is referred to as electron–phonon coupling constant. Similarly, $\lambda_{\mathrm{e-s}}$ is the electron–paramagnon coupling constant.

The electron–phonon interaction is attractive and is responsible for metallic superconductivity. Electron—paramagnon interaction and electron–electron Coulombic interactions are repulsive and damaging to the occurrence of superconductivity. For rhodium, $\gamma = (4.65 \pm 0.15)$ mJ/mol K^2 [19]. Hence,

$$\frac{\gamma}{\gamma(0)} = 1.43 \pm 0.05,$$

i.e., the total enhancement of the electron mass due to both the processes is (0.43 ± 0.05). This compares well with the value given by Allen (0.4 ± 0.1) [20] and Anderson (0.44) [8]. Further, the mass enhancement determined from the ratio between the calculated and measured effective cyclotron masses is found to be 0.4 [21].

In the literature, $\lambda_{\mathrm{e-p}}$ is taken to be 0.34 [1,22] and $\lambda_{\mathrm{e-s}}$ is equal to 0.1 [1] based on the argument that in rhodium, the electron–phonon interaction is the same as that of iridium (which is the next element of the same group (9) and same block (d) as rhodium in the periodic table), because it is a superconductor with hardly any contribution from electron–paramagnon interaction ($\lambda_{\mathrm{e-s}}$ is less than 0.03 [1]).

2.3 *Residual electron–phonon interaction*

The electron–paramagnon interaction weakens the superconductivity as it may lead to the breaking of Cooper pairs. Its role is akin to the instantaneous Coulombic repulsive electron–electron interaction (represented variously by μ, μ^*, $\bar{\mu}$ depending upon the physics of the problem). Both the interactions can be viewed as the total repulsive interactions [22] that result in the diminishing of attractive electron–phonon interaction, $\lambda_{\mathrm{e-p}}$.

The well-known expression of T_{C} in the BCS theory, therefore can be modified as follows:

$$T_{\mathrm{C}} = 1.14\Theta_{\mathrm{D}} \exp(-1/\lambda_{\mathrm{BCS}}), \tag{3}$$

where Θ_{D} is the Debye temperature and λ_{BCS} is the residual attractive electron–phonon interaction which is given as

$$\lambda_{\mathrm{BCS}} = \lambda_{\mathrm{e-p}} - \mu^* - \lambda_{\mathrm{e-s}}. \tag{4}$$

One can explain the observed T_{C} (0.325 mK) of rhodium when residual electron–phonon interaction is equal to 0.069(16) with $\Theta_{\mathrm{D}} = 542$ K, but can have value 0.069(73) if $\Theta_{\mathrm{D}} = 482$ K. The experimental value of Θ_{D} near 0 K has a range: (512 ± 30) K [19].

2.4 *BCS characteristic parameters – Weakly coupled superconductor*

Various BCS characteristic parameters are calculated in this section using the established formulae mentioned in [9].

The central result of BCS theory is the appearance of a time-independent energy gap parameter, Δ, that separates the normal state of a metallic superconductor from that of its superconducting state. Even expression (3) is obtained using the fact that at $T = T_{\mathrm{C}}$, this energy gap vanishes. Using the equation.

$$\Delta_0 = 2\hbar\omega_{\mathrm{D}} \exp(-1/\lambda_{\mathrm{BCS}}) \tag{5}$$

the energy gap Δ_0 at 0 K for rhodium turns out to be 4.9×10^{-8} eV, which leads to $2\Delta_0/k_{\mathrm{B}}T_{\mathrm{C}} = 3.51$ which is very close to 3.50, the BCS result [9].

The condensation energy [9], which is the difference between the energy of the ground state of the superconductor and that of its normal electron state, for rhodium turns out to be 9.62×10^{-5} erg/cm^3.

Thus, the critical magnetic field at 0 K obtained by using the calculated condensation energy is 49.17 mG, which compares well with 49 mG, the corresponding experimental value [1], that happens to be approximately 1/5th to 1/10th of the Earth's magnetic field [23].

The ratio $\frac{\gamma(0)T_C^2}{V_{\mathrm{m}}B_c^2}$, where V_{m} is the molar volume of the superconductor and the other symbols have already been defined. For rhodium, this ratio is 0.169 which is valid for all BCS-type superconductors [9].

Yet another feature of superconductivity (in contrast to the perfect conductor) is its coherence length ξ, which is given by the expression.

$$\xi = \hbar \upsilon_F / \pi \Delta_0,$$

where $\upsilon_F = 1.859 \times 10^8$ cm/s is the band-structure value of Fermi velocity calculated using the band-structure value of Fermi energy. Δ_0 is the superconducting energy gap at 0 K.

The value of coherence length for rhodium turns out to be equal to 7.9×10^7 Å. This is also equal to the size of a Cooper pair. This value of coherence length is very close to the value calculated by Buchal *et al* [1]

Its significantly large value is due to the peculiar nature of rhodium: (i) Larger band structure value of Fermi velocity ($\upsilon_F = 1.859 \times 10^8$ cm/s) and (ii) small value of superconducting energy gap parameter ($\Delta_0 = 4.9 \times 10^{-8}$ eV) due to the presence of three interactions: attractive electron–phonon interaction which is reduced by the presence of repulsive electron–electron interaction and repulsive electron–paramagnon interaction. Further large value of υ_F is independent of the value of Δ_0.

Cooper pairs are bosons in the singlet state and are, therefore spherical shells of diameter equal to the coherence length. It is evident that these Cooper pairs are overlapping and they form an interacting charged bosonic system, even though each Cooper pair is very fragile with extremely small binding energy equal to 4.9 $\times 10^{-8}$ eV. Therefore, in the superconducting state, we have a system of interacting Cooper pairs and hence the Hamiltonian of the superconductor will not only have the kinetic energy of Cooper pairs but also the interaction among the Cooper pairs, which is also the effective Hamiltonian of BCS theory. It is quite unlike the case of Bose–Einstein condensation, where at extremely low temperature, less than the condensation temperature, the bosons are non-interacting, i.e. free and thus the Hamiltonian corresponds only to the kinetic energy of the bosons. It turns out that in rhodium even at such a low temperature of 325 μK this is not so, i.e., the Cooper pairs are not free.

One can further confirm that the superconductivity of rhodium is Type I, by evaluating the ratio κ of London's penetration depth and the coherence length, which turns out to be of the order of 10^{-7} that is much less than $1/\sqrt{2}$. For $\kappa > 1/\sqrt{2}$, the superconductivity is of Type II [24]. Thus, one may conclude that rhodium is a weakly coupled superconductor with $\lambda_{BCS} = 0.069$, much less than one, and exhibits all the experimentally observed properties of a BCS superconductor.

2.5 *Electron–phonon, electron–paramagnon and Coulomb interactions*

It is worthwhile to check that, the experimentally observed total electron–phonon interaction, λ_{e-p} too, is valid in rhodium. From eq. (4)

$$\lambda_{e-p} = \lambda_{BCS} + \mu^* + \lambda_{e-s}, \tag{6}$$

where μ^* is the Coulomb pseudopotential, the expression of which is given by eq. (7) and μ is the Coulomb interaction among electrons and can be described well, using random phase approximation [25,26], as the number density of electrons in rhodium is much higher than that of lithium, as noted earlier.

$$\mu^* = \frac{\mu}{\left[1 + \mu \ln\left(\frac{4E_F}{k_B \Theta_D}\right)\right]},$$

where

$$\mu = \frac{1}{2\pi \upsilon_F} \ln(\pi \upsilon_F).$$

υ_F is the magnitude of band structure velocity of electrons at the Fermi surface in atomic units. One may also use $\bar{\mu}$ [27] which is the harmonic mean of μ and μ^*. The calculated values of μ, μ^*, $\bar{\mu}$ for rhodium turn out to be 0.1839, 0.08215, 0.1136, respectively.

One may also calculate the electron–paramagnon coupling constant, $\lambda_{e-s} = N(0)\tilde{V}_C$ [7,28], using the ratio of the enhanced magnetic susceptibility of electron–paramagnon interaction to the enhanced specific heat due to electron–phonon interaction, which is equivalent to the ratio of susceptibility density of states at Fermi surface $N_\chi(0)$ and band structure density of state at the Fermi surface $N_\gamma(0)$, i.e.,

$$\frac{N_\chi(0)}{N_\gamma(0)} = [\left(1 + N(0)V_{ph}\right)(1 - N(0)\tilde{V}_C)]^{-1}, \tag{8}$$

where $N(0)V_{ph} = \lambda_{BCS}$ and its value is 0.069 for rhodium. $N_\chi(0) = 1.48$ states/eV atom and $N_\gamma(0) = 1.375$ states/eV atom [7,8] yielding $\lambda_{e-s} = 0.13$, which can be compared with the value 0.114 [29]. (For palladium, the theoretically quoted value for $\lambda_{e-s} = 0.73$ [29] and therefore, the Stoner factor of palladium is much higher than that of rhodium by a factor of 3.2 which may be responsible for the absence of superconductivity in palladium, so far.)

For rhodium, using these values of instantaneous Coulombic interaction and λ_{e-s}, one gets the value of λ_{e-p} as: 0.38(29), 0.28(11), 0.31(26) for μ, μ^*, $\bar{\mu}$, respectively. These values lie within the experimental quoted values (0.4 ± 0.1) [20] of λ_{e-p}, except the value 0.28(11) which is only 6% lower than the experimental value. Using these values of λ_{e-p} and the calculated values of μ, μ^*, $\bar{\mu}$ and $\lambda_{e-s} = 0.13$, in eq. (4), the values of λ_{BCS} turn out to be 0.069, 0.068 and 0.069, respectively.

3. Results and discussion

It is therefore clear that the electron–phonon coupling in the superconducting rhodium is weak and therefore BCS theory works very well. In BCS theory, the interaction between the electrons and phonons is instantaneous, as emphasised by Bogoljubov [30] and others [10,31,32] and the energy gap parameter is independent of time. However, with the discovery of superconductivity in many elemental materials with larger values of T_C, it turns out that one will have to consider the electron–phonon coupling to be strong. This implies that the electron–phonon interaction is not instantaneous but retarded, that resulted in considering the time-independent gap parameter to be dependent on time. In Eliashberg's theory [33], among other things, the energy gap parameter is frequency-dependent and the retarded nature of electron–phonon interaction is incorporated.

But Eliashberg's theory is involved and requires complicated computations to obtain the numerical results. Therefore, there have been several attempts to put these numerical results of strongly coupled superconductors like niobium etc. in the form of expression, which is similar to that given by BCS theory. McMillan's [22,34] equation happens to be a prominent and widely used equation describing strongly coupled superconductors. This and other equations served a useful role in understanding the superconductivity in strongly coupled superconductors.

It may, however, be noted that it is not easy to derive the time-independent BCS equation from Eliashberg's theory, or McMillan's-type equation, where the electron–phonon interaction is taken to be retarded [35,36]. Nonetheless, strongly coupled theory in the form of the following modified equation [1] has been used to investigate the superconductivity in rhodium:

$$T_C = \Theta_D \exp\left\{-\left(1+\lambda_{e-p}+\lambda_{e-s}\right) / \left(\lambda_{e-p}-\lambda_{e-s}-\mu*\right)\right\}.$$

When $\lambda_{e-p} = 0.34$, $\lambda_{e-s} = 0.1$, $\mu^* = 0.13$ and $\Theta_D = 542$ K, the value of T_C turns out to be 1.12 mK; a small tinkering of $\lambda_{e-s} = 0.1089$, yield $T_C = 325\ \mu$K, the observed superconducting transition temperature of rhodium.

However, we summarise our results as

- Most of the characteristic superconducting experimental results in rhodium, conform to the weakly coupled instantaneous electron–phonon interaction represented by the BCS theory, and discussed in detail earlier. All these calculated parameters are listed in table 1. (It may be pointed out that Buchel *et al* [1] have studied superconducting transition temperature of rhodium in the presence of different impurities of various elements. With the increase of impurity concentration T_C decreases, confirming that rhodium remains a weakly coupled superconductor in the presence of impurities rather than turning into a strongly coupled superconductor).

Further we support our results by the following arguments:

- If strongly coupled retarded interactions were present, several experimentally observed features should appear. For instance, the temperature-dependent variation of magnetic field in rhodium will not follow the behaviour of Type-I superconductor, just as what happens in strongly coupled superconductor, namely lead.

To show this behaviour, we have taken our calculated value of $B_C(0) = 49.17$ mG and experimental data of $B_C(T)$ vs. T from Buchal *et al* [1]. Using this, deviation of the reduced critical field from a parabolic curve as a function of the square of the reduced temperature is plotted in figure 1. The curve shows the BCS weak coupling behaviour of rhodium. For comparison, curves for weakly coupled Al, Sn and strongly coupled Hg, Pb superconductors are also shown [37].

The change in curvature in figure 1 from the linear variation near $T = 0$ K, is because of the finite entropy of superconducting electrons. Near $T = 0$ K superconducting electron entropy is negligible compared to the entropy of the normal electrons but it is not so as the temperature increases and remains finite till $T \cong T_C$.

In experimental measurements of the temperature variation of $B_C\ (T)$ in the temperature range between 0 and T_C, this entropy is present and therefore the behaviour of temperature-dependent $B_C(T)$ is different from its BCS theoretical parabolic behaviour. Thus, the curvature appears essentially because of the presence of superconducting entropy and it is present both in strongly coupled superconductors, e.g. mercury (Hg) and weakly coupled superconductors, e.g., indium (In) [38].

The difference in curvature values of different superconductors is due to their different physical characteristic parameters.

- The other characteristic superconducting parameter $2\Delta_0/k_B T_C$ will be more than what it is in the BCS theory [37,39]. There have been some attempts [40–42] to obtain an expression for the superconducting characteristic parameter $2\Delta_0/k_B T_C$, by solving

Table 1. List of characteristic parameters of rhodium.

Parameter	Notation	Value
Effective mass of quasiparticle	m_{eff}	$2.8m_e$
Total mass enhancement of the electron	$\lambda_{\text{e-p}} + \lambda_{\text{e-s}}$	0.43 ± 0.05
Residual attractive electron–phonon interaction	λ_{BCS}	0.069
Energy gap at 0 K	Δ_0	4.9×10^{-8} eV
Condensation energy	$N(0){\Delta_0}^2/4$	9.62×10^{-5} ergs/cm^3
Critical magnetic field at 0 K	$B_C(0)$ {condensation energy $= {B_C}^2/8\pi$ }	49.17 mG
Ratio valid for all BCS superconductors	$\frac{\gamma(0)T_C^2}{V_m B_C^2}$	0.169
Coherence length	$\xi = \hbar\upsilon_F/\pi\Delta_0$	7.9×10^7 Å
Binding energy of Cooper pairs	$2k_B\Theta_D$ / [exp$(1/\lambda_{\text{BCS}})$ -1]	4.9×10^{-8} eV
Ratio of London's penetration depth and coherence length	κ	10^{-7}
Instantaneous Coulombic repulsive electron–electron interaction	μ, μ^*, $\bar{\mu}$(harmonic mean of μ and μ^*)	0.1839, 0.08215, 0.1136
Electron–paramagnon coupling constant	$\lambda_{\text{e–s}} = N(0)\tilde{V}_C$	0.13
Electron–phonon coupling constant	$\lambda_{\text{e––p}}$	0.38(29), 0.28(11), 0.31(26) (Depending upon the value of Coulomb repulsive interaction μ, μ^*, $\bar{\mu}$ respectively)
Superconducting characteristic parameter	$2\Delta_0/k_BT_C$	3.50

Eliashberg equations when the frequency-dependent energy gap parameter is taken to be time-independent. One may use the following expression for $2\Delta_0/k_BT_C$ [40] to investigate the above point:

$$\frac{2\Delta_0}{k_BT_C} = \left(\frac{2\Delta_0}{k_BT_C}\right)_{\text{BCS}} \left\{1 + 12.5\left(\frac{T_C}{\omega_{\ln}}\right)^2 \ln\left(\frac{\omega_{\ln}}{2T_C}\right)\right\}, \tag{9}$$

where $\omega_{\ln}$ is the logarithmically averaged characteristic phonon frequency [29,35]. Other symbols have already been explained. On the right-hand side of eq. (9), the first term corresponds to the value of the parameter given in BCS theory and the second term corresponds to the correction for strong coupling effect. One can evaluate the second term for rhodium and it turns out to be 9.15×10^{-10} ($\omega_{\ln} = 22$ meV, $T_C = 3.25 \times 10^{-4}$ K), and for aluminium it is 4.14×10^{-3} ($\omega_{\ln} = 22.3$ meV, $T_C = 1.16$ K), which are negligible compared to the first term, confirming that strong coupling effects are indeed absent in rhodium just as they are absent in aluminium, which is the celebrated BCS superconductor. Just for the sake of comparison and completion, for lead, which is a strongly coupled superconductor, the second term is 0.908 ($\omega_{\ln} = 5.12$ meV, $T_C = 7.19$ K), not negligible compared to 3.50, indicating that the second term indeed represents the strong coupling effects. It may be pointed out that the theoretical value of $2\Delta_0/k_BT_C$ for lead is 4.408, which compares well with the corresponding experimental value (4.33 ± 0.1) [37,39].

For the sake of comparison, we have also studied bismuth, which is a recently discovered low-temperature superconductor [3]. In bismuth, the free electrons constitute a rare but highly quantum mechanically degenerate gas, with a number density of approximately five orders

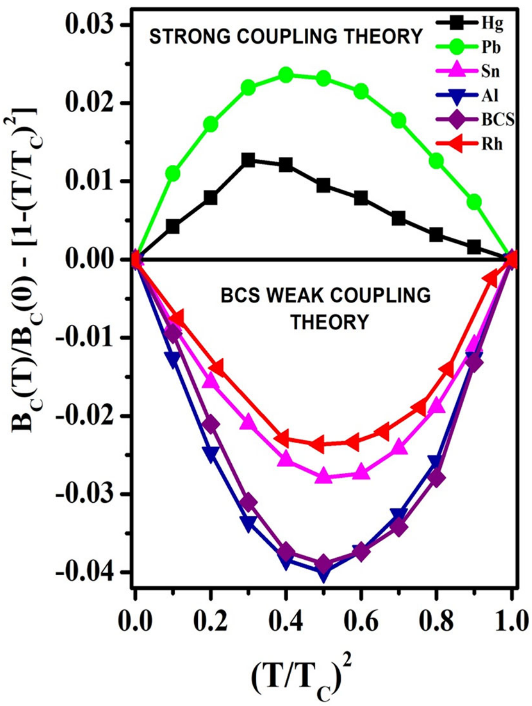

Figure 1. Deviation of the critical magnetic field from the parabolic curve for weakly coupled superconductors Al, Sn, Rh and strongly coupled superconductors Hg, Pb.

less than the other elemental superconductors because only a single electron is contributed to its Fermi sea by 10^5 of its atoms. It implies, therefore, that the volume superconductivity in pure bismuth crystal just cannot be there, based on the net attractive electron–phonon interaction. It is evident from the expression of superconducting transition temperature T_C, of the BCS theory [9] and is because of the extremely low density of states of the electrons at its Fermi surface. For bismuth, the size of the Cooper pair is 1.3×10^7 Å and binding energy is 8 $\times$ 10^{-8} eV [4] which makes it fragile. Very recently, two theoretical arguments have been advanced to explain the observed superconductivity in bismuth, based on two different non-phononic mechanisms [4,5]. In both the approaches, the well-known isotope effect, which results in superconductivity brought about basically by attractive electron–phonon interaction, is absent [9,43].

One may summarise the experimentally observed superconductivity in widely different and peculiar elements, rhodium and bismuth, as follows: First, both are very fragile as the superconductivity occurs at extremely low temperatures of 325 and 530 μK, respectively. Secondly, the superconductivity is destroyed by a tiny magnetic field of approximately 50 mG. Hence, both the elements exhibit the characteristics of a Type-I superconductor.

But it is observed recently that superconductivity in bismuth does not arise because of the electron–phonon interaction [4,5]. However, in this work, we show that the superconductivity in rhodium emerges due to the attractive residual electron–phonon interaction that survives the onslaught of repulsive electron–electron and electron–paramagnon interactions. It turns out that the experimentally observed superconducting characteristic parameters in it can be explained very well on the basis of the BCS theory of weakly coupled superconductor.

4. Conclusion

Even though in the past rhodium was investigated as weakly as well as strongly coupled superconductor, we conclude in our study that:

- Rhodium, a fragile, lowest temperature superconductor, exhibits weakly coupled superconducting behaviour, which is supported by our calculations. All our evaluated characteristic parameters are in agreement with the corresponding experimental results. Also these values are comparable with aluminium, the celebrated BCS weakly coupled superconductor.
- There are hardly any strong coupling effects due to the retarded nature of electron–phonon interaction. Therefore, superconductivity in rhodium is marked by instantaneous electron–phonon interaction, along with the instantaneous electron–Coulomb interaction and instantaneous electron–spin interaction.

References

[1] Ch Buchal, F Pobell, R M Muller, M Kubota and J R Owers-Bradley, *Phys. Rev. Lett.* **50**, 64 (1983)
[2] T A Knuuttila, J T Tuoriniemi, K Lefmann, K I Juntunen, F B Rasmussen and K K Nummila, *J. Low Temperature Phys.* **123**, 65 (2001)
[3] O Prakash, A Kumar, A Thamizhavel and S Ramakrishnan, *Science* **355**, 52 (2017)
[4] S P Tewari and C Kapoor, *Phys. Lett. A* **182**, 3131 (2018)
[5] J Ruhman and Patrick A Lee, *Phys. Rev. B* **96**, 235107 (2017)
[6] Ch Buchal, R M Mueller, F Pobell, M Kubota and H R Folle, *Solid State Commun.* **42**, 43 (1982)
[7] K Andres and M A Jensen, *Phys. Rev.* **165**, 533 (1968)
[8] O Krogh Anderson, *Phys. Rev. B* **2**, 883 (1970)
[9] J Bardeen, L N Cooper and J R Schrieffer, *Phys. Rev.* **108**, 1175 (1957)
[10] D Pines and P Nozieres, The theory of quantum liquids, in: *Neutral Fermi liquids* (Benjamin, New York, 1966) Vol. 1

[11] M Sigalas, D A Papaconstantopoulos and N C Bacalis, *Phys. Rev. B* **45**, 5777 (1992)
[12] P Fazekas, *Lecture notes on electron correlation and magnetism*, Series in Modern Condensed Matter Physics (World Scientific Publication, Singapore, New Jersey, London, Hong Kong) Vol. 5, Copyright @1999 ISBN 9810224745
[13] P Fulde, *Electron correlations in molecules and solids*, Springer Series in Solid-State Sciences (Springer-Verlag, Berlin, Heidelberg, 1991) Vol. 100
[14] J R Schrieffer, *J. Appl. Phys.* **39**, 642 (1968)
[15] G G Low, *Proceedings of International Conference on Magnetism* (Nottiagham, Sept. 1964) p. 133
[16] N F Berk and J R Schrieffer, *Phys. Rev. Lett.* **17**, 433 (1966)
[17] K Makoshi and T Moriya, *J. Phys. Soc. Jpn* **38**, 10 (1975)
[18] S Doniach and S Engelsberg, *Phys. Rev. Lett.* **17**, 750 (1966)
[19] G T Furukawa, M LReilly and J S Gallagher, *J. Phys. Chem. Ref. Data* **3**, 163 (1974)
[20] P B Allen, *Phys. Rev. B* **36**, 2920 (1987)
[21] K Carrander, M Dronjak and S P Hornfeld, *J. Phys. Chem. Solids* **38**, 289 (1977)
[22] W L McMillan, *Phys. Rev.* **167**, 331 (1968)
[23] An overview of the Earth's magnetic field, http://www.geomag.bgs.ac.uk/education/earthmag.html#_Toc2075549
[24] A A Abrikosov, *Rev. Mod. Phys.* **76**, 975 (2004)
[25] L B Dubovskii and A N Kozlov, *Sov. J. Exp. Theor. Phys.* **41**, 1113 (1975) (Russian original-ZhETF,Vol. 68, No. 6, p. 2224, August 1975)
[26] B Biswas and S P Tewari, *Phys. Rev. B* **22**, 681 (1980)
[27] L L Daemen and A W Overhauser, *Phys. Rev. B* **38**, 81 (1988)
[28] G Gladstone, M A Jensen and J R Schrieffer, Superconductivity in the transition metals: Theory and experiment, in: *Superconductivity* edited by R D Parks (Marcel Dekker, Inc., New York, Basel, 1968) Vol. 2
[29] S K Bose, *J. Phys.: Condens. Matter* **21**, 025602 (2009)
[30] N N Bogoljubov, V V Tolmachov and D V Sirko, *Fortschritte der Physik* **6**, 605 (1958)
[31] P Morel and P W Anderson, *Phys. Rev.* **125**, 1263 (1962)
[32] J Bardeen and D Pines, *Phys. Rev.* **99**, 1140 (1955)
[33] G M Eliashberg, *Sov. Phys. JETP* **11**, 696 (1960)
[34] S P Tewari and P K Gumber, *Phys. Rev. B* **41**, 2619(R) (1990)
[35] P B Allen and R C Dynes, *Phys. Rev. B* **12**, 905 (1975)
[36] C R Leavens and J P Carbotte, *Can. J. Phys.* **49**, 724 (1971)
[37] J C Swihart, D J Scalapino and Y Wada, *Phys. Rev. Lett.* **14**, 106 (1965)
[38] D K Finnemore and D E Mapother, *Phys. Rev.* **140**, A507 (1965)
[39] I Giaever and K Megerle, *Phys. Rev.* **122**, 1101 (1961)
[40] B Mitrovic, H G Zarate and J P Carbotte, *Phys. Rev. B* **29**, 184 (1984)
[41] B T Geilikman and V Z Kresin, *Phys. Lett. A* **40**, 123 (1972)
[42] G W Webb, F Marsiglio and J E Hirsch, *Physica C* **514**, 17 (2015)
[43] H Frohlich, *Phys. Rev.* **79**, 845 (1950)