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Positron-acoustic solitons with two-temperature q-non-extensive electrons in plasma

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Abstract. The propagation of solitons is studied in a tokamak plasma composed of immobile ions, hot fluid positrons and two-electron sets in hot and cold temperatures with a q-non-extensive distribution. Korteweg–de-Vries (KdV) equation is obtained via the reductive perturbation method (RPM). The dependence of positron-acoustic solitons on the ratio of hot/cold electron density to positron density, positron temperature to temperature of hot/cold electrons, the non-extensivity parameters q_{eh} and q_{ec}, as well as the angle between the magnetic field and wave propagation are investigated.

Keywords. Soliton; positron-acoustic; plasma; q-non-extensive.

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1. Introduction

The investigation of the propagation of solitons in positron–electron–ion plasmas is an interesting research topic [1–6]. High-temperature plasmas such as tokamak magnetic confinement structures are a combination of ions, positrons and electrons. Positron-acoustic solitons are waves in which the cold positron mass yields the inertia, and the restoring force is generated by the electrons as well as thermal positrons pressure [7]. Properties of ion-acoustic solitons have been investigated in plasma composed of two-temperature electrons, both experimentally [8–12] and theoretically [13–19]. If the speed of particles of plasma is more than their thermal velocity [20], the thermal distribution is no longer reliable.

In the classical particle system, the lack of correlation between the energies of individual particles allows us to consider the Maxwellian velocity distribution and the Boltzmann–Gibbs statistical mechanics. However, in plasma, there is almost no collision, and it can be considered as a correlated particle system characterised by stationary states out of thermal equilibrium which shows non-Maxwellian behaviour. Such behaviours are correctly expressed in q-non-extensive distribution [21–24], where the entropic index q determines the degree of non-extensivity of the proposed system. The parameter q, which is the basis of the generalised entropy of Tsallis, is related to the underlying dynamics of the system and measures its non-extensivity. In statistical mechanics, systems described by non-extensivity properties are systems in which the total entropy is different from the total entropy of the corresponding parts [25]. Recently, the plasmas with non-extensive components have been investigated by several researchers [26–32].

Moslem *et al* considered a three-component plasma medium (i.e., consisting of ions, non-extensive electrons and dust grains with massive negative charge) to survey the stability of the dust acoustic waves [33]. Kakoti and Saharia investigated overtaking interaction of electron-acoustic solitary waves in a plasma with hot non-extensive electrons [34]. Two-temperature electron plasmas are probable in several environments. For instance, the hot cathode discharged plasmas have a dual temperature electron distribution [15]. Jones *et al* studied the influence of two-temperature electrons on the ion-acoustic solitons in a three-component plasma composed of cold ions [35]. Baluku *et al* studied the ion-acoustic solitons in a plasma consisting of two-temperature electrons [36]. Shukla and Tagare obtained the shock solution for a plasma with two-temperature electrons by applying the KdV–Burgers equation [37]. Goswami and Buti investigated hot and cold electrons in plasma by applying KdV equation [38,39]. Buti

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by analysing Sagdeev potential in a plasma with two-temperature electrons obtained the soliton solution [40].

Here, considering the observation and study of plasmas with energy distribution of electrons with two components, hot and cold, in plasma environments, and because of the scarcity of studies in positron-acoustic solitons with particles with q-non-extensive distribution, we have investigated the characteristics of positron-acoustic soliton waves (PASWs) in a tokamak plasma having immobile ions, hot fluid positrons and two populations of electrons (hot and cold) with a non-extensive distribution in an external magnetic field. The KdV equation has been derived by the RPM. The layout of the present work is as follows: In §2, the KdV equation is obtained. In §3, the results are indicated and discussed. Lastly, conclusions are presented.

2. Calculational model

We have considered a plasma including immobile ions, hot fluid positrons and two-electron sets having hot and cold temperatures with a non-extensive distribution. The propagation of PASWs is governed by the normalised Poisson's equation, the normalised momentum equation and the normalised continuity equation in Cartesian coordinates,

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=n_{eh}+n_{ec}-n_i-n_p$$
$$=\beta_{eh}[1+(q_{eh}-1)\sigma_{eh}\phi]^{\frac{1+q_{eh}}{2(q_{eh}-1)}}$$
$$+\beta_{ec}[1+(q_{ec}-1)\sigma_{ec}\phi]^{\frac{1+q_{ec}}{2(q_{ec}-1)}}-n_i-n_p, \quad (1)$$

$$\frac{\partial v_{px}}{\partial t}+v_{px}\frac{\partial v_{px}}{\partial x}+v_{py}\frac{\partial v_{px}}{\partial y}+v_{pz}\frac{\partial v_{px}}{\partial z}$$
$$=-\frac{\partial\phi}{\partial x}+\frac{\Omega_p}{\omega_{pp}}v_{py}-\frac{5}{3}\frac{1}{n_p^{1/3}}\frac{\partial n_p}{\partial x}, \quad (2)$$

$$\frac{\partial v_{py}}{\partial t}+v_{px}\frac{\partial v_{py}}{\partial x}+v_{py}\frac{\partial v_{py}}{\partial y}+v_{pz}\frac{\partial v_{py}}{\partial z}$$
$$=-\frac{\partial\phi}{\partial y}-\frac{\Omega_p}{\omega_{pp}}v_{px}-\frac{5}{3}\frac{1}{n_p^{1/3}}\frac{\partial n_p}{\partial y}, \quad (3)$$

$$\frac{\partial v_{pz}}{\partial t}+v_{px}\frac{\partial v_{pz}}{\partial x}+v_{py}\frac{\partial v_{pz}}{\partial y}+v_{pz}\frac{\partial v_{pz}}{\partial z}$$
$$=-\frac{\partial\phi}{\partial z}-\frac{5}{3}\frac{1}{n_p^{1/3}}\frac{\partial n_p}{\partial z}, \quad (4)$$

$$\frac{\partial n_p}{\partial t}+\frac{\partial(n_pv_{px})}{\partial x}+\frac{\partial(n_pv_{py})}{\partial y}+\frac{\partial(n_pv_{pz})}{\partial z}=0, \quad (5)$$

where n_{eh} and n_{ec} represent the non-extensive distribution of the hot and cold electrons normalised by unperturbed positron number density n_{p0}. n_i and n_p are the number densities of ion and positron respectively, normalised by n_{p0}. The temperature and density ratios are considered as $\sigma_{eh}=T_p/T_{eh}$, $\sigma_{ec}=T_p/T_{ec}$, $\beta_{eh}=n_{eh0}/n_{p0}$ and $\beta_{ec}=n_{ec0}/n_{p0}$, respectively. The unperturbed hot (cold) electron number density is denoted by $n_{eh0}(n_{ec0})$. T_p and $T_{eh}(T_{ec})$ are temperatures of the positron and hot (cold) electrons. $\Omega=eB/m_pc$ is positron cyclotron frequency. e, m_p, c and $B(=3T)$ are magnitudes of the electron charge, positron mass, light speed and external static magnetic field, respectively. $v_{pj}(j=x,y,z)$ is the positron fluid velocity normalised by $c_p=(k_BT_p/m_p)^{1/2}$. Time t, electrostatic wave potential ϕ and lengths are normalised by positron plasma frequency $\omega_{pp}=(4\pi e^2n_{p0}/m_p)^{1/2}$, k_BT_p/e and positron Debye length $\lambda_{Dp}=(k_BT_p/4\pi n_{p0}e^2)^{1/2}$, respectively. The non-extensivity parameter $q_{eh}(q_{ec})$ is a real number $(q_{eh}(q_{ec})>-1)$. At equilibrium, condition of the charge neutrality is $n_{eh0}+n_{ec0}=n_{i0}+n_{p0}$, where n_{i0} is the unperturbed ion number density.

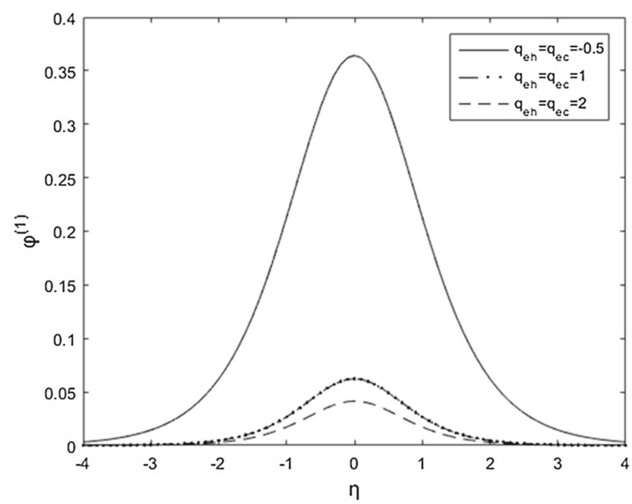

Figure 1. Potential of PASWs for different values of q. Other parameters are: $\sigma_{eh}=1, \sigma_{ec}=10$, $\beta_{eh}=\beta_{ec}=1, \Theta=60°$.

To investigate the nonlinear propagation of the PASWs and to obtain the KdV equation, we use the RPM. Independent variables can be expressed as $\xi=\varepsilon^{1/2}(l_1x+l_2y+l_3z-st)$, $\tau=\varepsilon^{3/2}t$, where $l_i(i=1,2,3)$ are the direction cosines and s is the phase velocity normalised by c_p. The expansions of the perturbed quantities in powers series in ε (small parameter) is given as

$$n_p=1+\varepsilon n_p^{(1)}+\varepsilon^2n_p^{(2)}+\varepsilon^3n_p^{(3)}+\cdots \quad (6)$$
$$n_{ei}=\beta_{ei}+\beta_{ei}A_{ei}\sigma_{ei}\varepsilon\phi^{(1)}+\beta_{ei}A_{ei}\sigma_{ei}\varepsilon^2\phi^{(2)}$$
$$+\beta_{ei}A_{ei}\sigma_{ei}\varepsilon^3\phi^{(3)}+\beta_{ei}B_{ei}\sigma_{ei}^2\varepsilon^2\phi^{(1)^2}$$
$$+2\beta_{ei}B_{ei}\sigma_{ei}^2\varepsilon^3\phi^{(1)}\phi^{(2)}$$

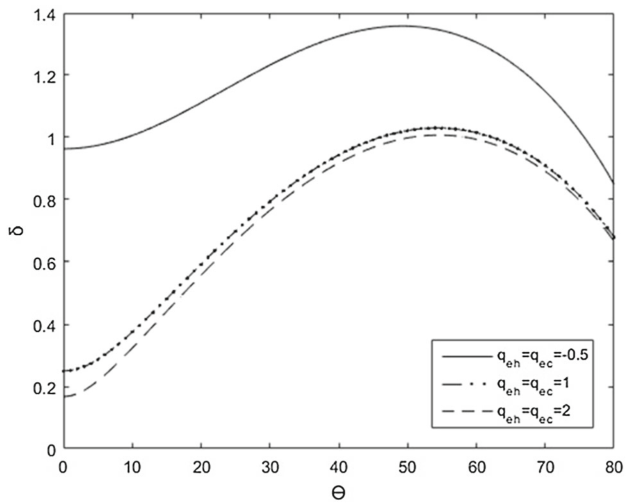

Figure 2. Plot of δ vs. Θ for different values of q. Other parameters are: $\sigma_{eh} = 1, \sigma_{ec} = 10, \beta_{eh} = \beta_{ec} = 1$.

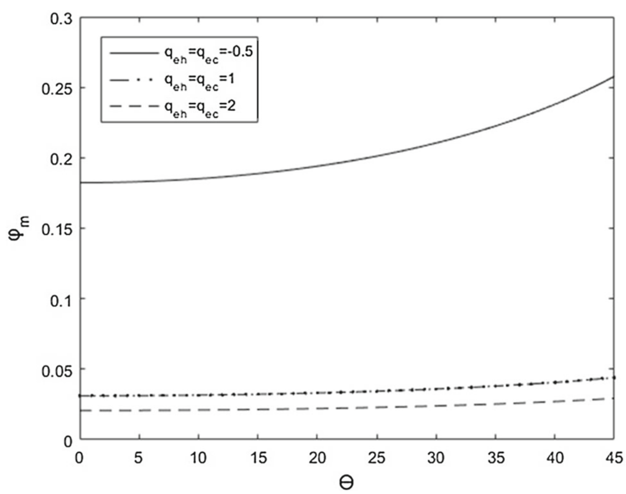

Figure 3. ϕ_{m} vs. Θ for different values of q. Other parameters are: $\sigma_{eh} = 1, \sigma_{ec} = 10, \beta_{eh} = \beta_{ec} = 1$.

$$+\beta_{ei} C_{ei} \sigma_{ei}^3 \varepsilon^3 \phi^{(1)^3} + \cdots, \quad i \equiv h, c, \tag{7}$$

$$V_{pj} = \varepsilon^{3/2} V_{pj}^{(1)} + \varepsilon^2 V_{pj}^{(2)} + \varepsilon^{5/2} V_{pj}^{(3)} + \cdots, \quad j \equiv x, y, \tag{8}$$

$$R = \varepsilon R^{(1)} + \varepsilon^2 R^{(2)} + \varepsilon^3 R^{(3)} + \cdots, \quad R \equiv v_{pz}, \phi \tag{9}$$

where

$$A_e = \frac{(1+q_e)}{2}, \; B_e = \frac{1}{8}(3 + 2q_e - q_e^2)$$

$$C_e = \frac{1}{48}(3 + 2q_e - q_e^2)(5 - 3q_e).$$

Replacing (6)–(9) into (1)–(5), we obtained the KdV equation

$$\frac{\partial}{\partial \tau}\phi^{(1)} + a\phi^{(1)} \frac{\partial}{\partial \xi}\phi^{(1)} + b\frac{\partial^3}{\partial \xi^3}\phi^{(1)} = 0. \tag{10}$$

The non-linear and dispersive coefficients are given as

$$a = \frac{\frac{l_3^4\left(3s^2 - \frac{5}{9}l_3^2\right)}{\left(s^2 - \frac{5}{3}l_3^2\right)^3} - 2\beta_{eh} B_{eh} \sigma_{eh}^2 - 2\beta_{ec} B_{ec} \sigma_{ec}^2}{\frac{2sl_3^2}{\left(s^2 - \frac{5}{3}l_3^2\right)^2}} \tag{11}$$

$$b = \frac{1 + \left(1 - l_3^2\right)\left(\frac{\omega_{pp}}{\Omega_p}\right)^2 \frac{s^4}{\left(s^2 - \frac{5}{3}l_3^2\right)^2}}{\frac{2sl_3^2}{\left(s^2 - \frac{5}{3}l_3^2\right)^2}}. \tag{12}$$

We can calculate the stationary solution of eq. (10) using the transformation ($\eta = \xi - U\tau$),

$$\phi^{(1)} = \frac{3U}{a}\operatorname{sech}^2\left[\frac{1}{2}\sqrt{\frac{U}{b}}(\xi - U\tau)\right]$$

and thus obtain the maximum amplitude ($\phi_m = 3U/a$) and width ($\delta = \sqrt{4b/U}$) of PASWs [41].

3. Results and discussion

Here, the results are investigated using figures 1–4. We have tokamak plasma parameters for numerical analysis, from various sources (see refs [42] and [43]). Figure 1 shows the profiles of PASWs for various values of non-extensivity parameters. $q_{eh}(q_{ec})$ indicates non-extensive parameter characterising the non-extensivity degree. For superthermality $-1 < q_{eh}(q_{ec}) < 1$ and for subthermality $q_{eh}(q_{ec}) > 1$ and when $q_{eh}(q_{ec}) \to 1$, the non-extensive velocity distribution function is converted into the Maxwell–Boltzmann distribution.

It is known that non-extensivity has a negative impact on the amplitude and width of the PASW, i.e. as $q_{eh}(q_{ec})$ increases, the amplitude and width of the PASWs diminish. The results of refs [40,43,44] show that the growth in $q_{eh}(q_{ec})$ results in the reduction of the amplitude and width of PASWs.

Figure 2 illustrates the variation of the width of PASWs with the angle between the magnetic field and wave propagation Θ. Figure 2 reveals that the width increases non-linearly with the angle and the width reaches its maximum when Θ is approximately 50°–55° and then decreases. It is seen that, with the increase of $q_{eh}(q_{ec})$, the width of the PASWs diminishes. The increment of the amplitude of PASWs is due to the rise

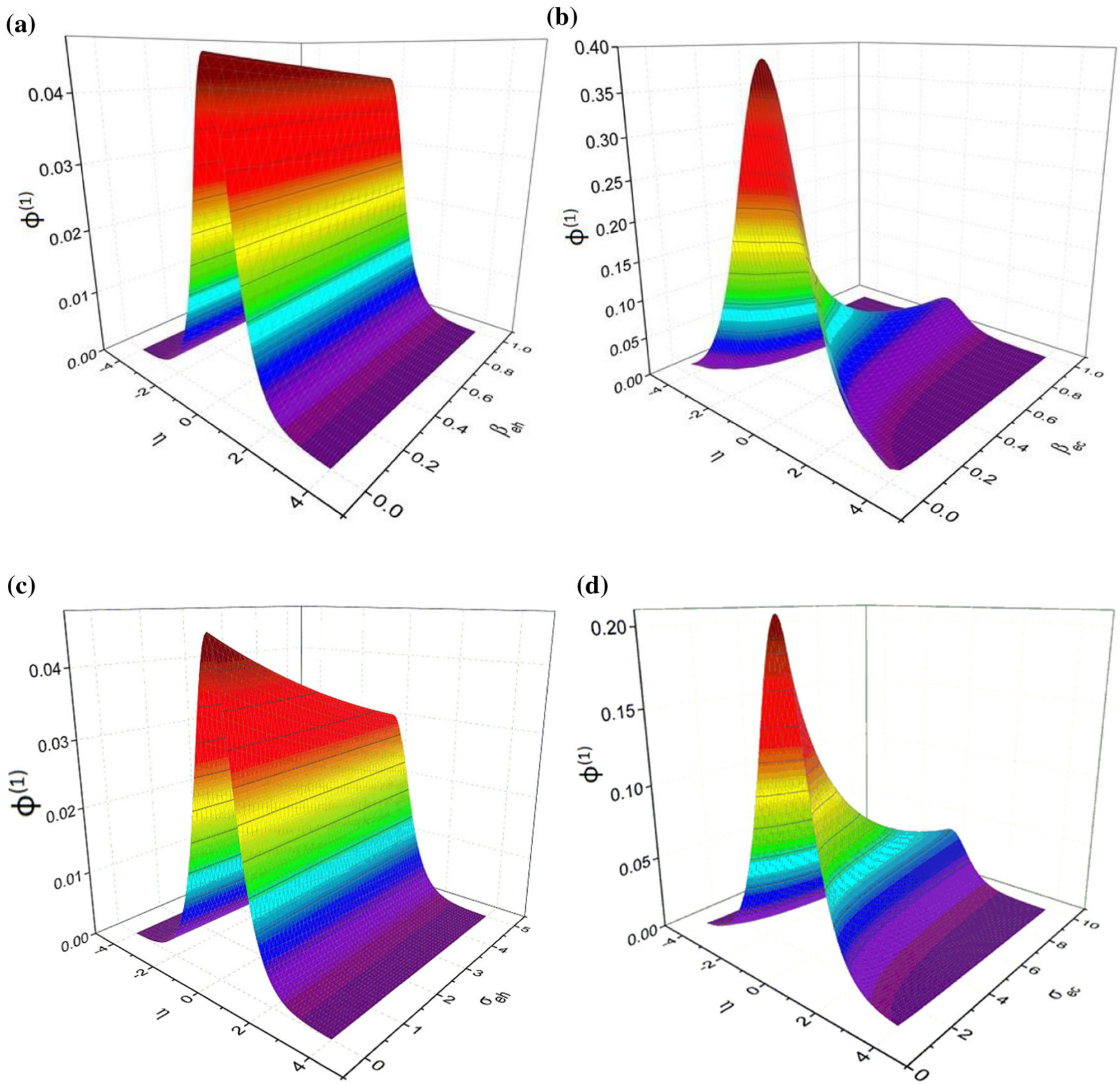

Figure 4. The evolution of $\phi^{(1)}$ with spatial coordinates η for PASWs with (**a**) β_{eh}, (**b**) β_{ec}, (**c**) σ_{eh} and (**d**) σ_{ec}. Other parameters are: $q_{eh} = q_{ec} = 2$, $\Theta = 60°$.

in Θ from 0° to 45°. Figure 3 shows that as q increases, the amplitude of the solitons decreases.

Figures 4a and 4b illustrate the profiles of PASWs for electrons-to-positron unperturbed number density ratios, which are β_{eh}, β_{ec} respectively. Figures 4c and 4d demonstrate the profiles of PASWs with proportion of the positron temperature and the electrons temperatures which are σ_{eh} and σ_{ec}, respectively. It is obvious that the width and amplitude of PASWs decrease, when β_{eh}, β_{ec}, σ_{eh} and σ_{ec} increase, which is in line with the results of ref. [40]. q values appear to have a significant effect on soliton properties and non-extensive distributed electrons affect solitary waves properties. This increase or decrease in amplitude and width in terms of q is due to the main property of soliton, i.e. the balance between coefficients of dispersion and non-linearity.

4. Conclusions

The characteristics of solitons were investigated in a plasma including immobile ions, hot fluid positrons and two-electron sets having hot and cold temperatures with a non-extensive distribution. The RPM was used to acquire the KdV equation. The main findings of the study are summarised below:

1. It was observed that as the non-extensive parameters of hot (q_{eh}) and cold (q_{ec}) electrons increase, the width and amplitude of PASWs decrease.
2. When Θ increases, the width of the PASWs reaches the peak near $\Theta = 50°$–$55°$. Subsequently, for larger angle of propagation, the width diminishes continuously.

3. As Θ increases, so did the amplitude of the PASWs.
4. Increase in β_{eh} and β_{ec} leads to reduced amplitude and width of the PASWs.
5. The growth in σ_{eh} and σ_{ec} causes a reduction in the width and amplitude of the PASWs.

We attempted to identify the impacts of the non-extensive parameter q on the characteristics of PASWs. In addition, the influence of the ratio of the density of electrons to positrons, the ratio of the temperature of positrons to electrons and the angle between the magnetic field and wave propagation were studied on width and amplitude of the PASWs.

Our theoretical research may be useful in studying the non-linear properties of propagation of PASWs in tokamak plasma.

References

[1] B B Mouhammadoul, C G L Tiofack, A Alim and A Mohamadou, *Eur. Phys. J. D* **75**, 1 (2021)
[2] J Y Wang, X P Cheng, X Y Tang, J R Yang and B Ren, *Phys. Plasmas* **21**, 032111 (2014)
[3] H Alinejad, *Astrophys. Space Sci.* **345**, 85 (2013)
[4] H Alinejad and A A Mamun, *Phys. Plasmas* **18**, 112103 (2011)
[5] R Saeed, A Shah and M Noaman-ul-Haq, *Phys. Plasmas* **17**, 102301 (2010)
[6] T S Gill, A Singh, H Kaur, N S Saini and P Bala, *Phys. Lett. A* **361**, 364 (2007)
[7] M M Rahman, M S Alam and A A Mamun, *Braz. J. Phys.* **45**, 314 (2015)
[8] R Pottelette, R E Ergun, R A Treumann, M Berthomier, C W Carlson, J P McFadden and I Roth, *Geophys. Res. Lett.* **26**, 2629 (1999)
[9] P Schippers, M Blanc, N André, I Dandouras, G R Lewis, L K Gilbert, A M Persoon, N Krupp, D A Gurnett, A J Coates and S M Krimigis, *J. Geophys. Res.* **113**, A07208 (2008)
[10] J P McFadden, C W Carlson, R E Ergun, F S Mozer, L Muschietti, I Roth and E Moebius, *J. Geophys. Res.* **108**, 8018 (2003)
[11] T Akhter, M M Hossain and A A Mamun, *Astrophys. Space Sci.* **344**, 105 (2013)
[12] Y Nakamura and H Sugai, *Chaos Solitons Fractals* **7**, 1023 (1996)
[13] H K Malik, *Phys. Plasmas* **15**, 072105 (2008)
[14] N Dashtban, S M Motevalli and T Mohsenpour, *Plasma Phys. Rep.* **44**, 854 (2018)
[15] D K Singh and H K Malik, *IEEE Trans. Plasma Sci.* **36**, 462 (2008)
[16] N S Saini, *Astrophys. Space Sci.* **346**, 155 (2013)
[17] S Sultana, I Kourakis and M A Hellberg, *Plasma Phys. Control. Fusion* **54**, 105016 (2012)
[18] T K Baluku and M A Hellberg, *Phys. Plasmas* **19**, 012106 (2012)
[19] E K El-Shewy, *Chaos Solitons Fractals* **26**, 1073 (2005)
[20] A Rényi, *Acta Math. Hung.* **6**, 285 (1995)
[21] C Tsallis, *J. Stat. Phys.* **52**, 479 (1988)
[22] I Driouch and H Chatei, *Eur. Phys. J. D* **71**, 1 (2017)
[23] S Jahan, N A Chowdhury, A Mannan and A A Mamun, *Commun. Theor. Phys.* **71**, 327 (2019)
[24] J Sarkar, J Goswami, S Chandra, C Das and B Ghosh, *Sustain. Humanosphere* **16**, 611 (2020)
[25] D Kumar Ghosh, P Chatterjee and U Narayan Ghosh, *Phys. Plasmas* **19**, 033703 (2012)
[26] P Eslami, M Mottaghizadeh and H R Pakzad, *Phys. Plasmas* **18**, 102303 (2011)
[27] M Bacha and M Tribeche, *Astrophys. Space Sci.* **337**, 253 (2012)
[28] E M Abulwafa, A M Elhanbaly, A A Mahmoud and A A Al-Araby, *Phys. Plasmas* **24**, 013704 (2017)
[29] M Shahmansouri and M Tribeche, *Astrophys. Space Sci.* **346**, 165 (2013)
[30] T S Gill, H Kaur and N S Saini, *Chaos Solitons Fractals* **30**, 1020 (2006)
[31] P Bala and T S Gill, *Pramana – J. Phys.* **95**, 1 (2021)
[32] J Tamang and A Saha, *Z. Naturforsch. A* **74**, 499 (2019)
[33] W M Moslem, R Sabry, S K El-Labany and P K Shukla, *Phys. Rev. E* **84**, 066402 (2011)
[34] R Kakoti and K Saharia, *Radiat. Eff. Defects Solids* **176**, 1129 (2021)
[35] W D Jones, A Lee, S M Gleman and H J Doucet, *Phys. Rev. Lett.* **35**, 1349 (1975)
[36] T K Baluku, M A Hellberg and F Verheest, *Europhys. Lett.* **91**, 15001 (2010)
[37] P K Shukla and S G Tagare, *Phys. Lett.* **59**, 38 (1976)
[38] B N Goswami and B Buti, *Phys. Lett.* **57**, 149 (1976)
[39] B Buti, *Phys. Lett.* **76**, 251 (1980)
[40] S M Motevalli, T Mohsenpour and N Dashtban, *Contrib. Plasma Phys.* **59**, 111 (2019)
[41] R D Smirnov, A Y Pigarov, M Rosenberg, S I Krasheninnikov and D A Mendis, *Plasma Phys. Control. Fusion* **49**, 347 (2007)
[42] LHD Experimental Group: S I Krasheninnikov *et al*, *Plasma Phys. Control. Fusion* **50**, 124054 (2008)
[43] M Ferdousi, S Sultana and A A Mamun, *Phys. Plasmas* **22**, 032117 (2015)
[44] T S Gill, P Bala and A S Bains, *Astrophys. Space Sci.* **357**, 63 (2015)