



Skyrme fluid in anisotropic Universe

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Abstract. Cosmological solutions are obtained in anisotropic Kantowski–Sachs (KS) and Bianchi Type-I Universes considering a cosmological constant with Skyrme fluid. Interestingly, the solutions obtained here in both the KS and Bianchi-I anisotropic Universes are found to isotropise at late time due to the presence of the Skyrme fluid, even in the absence of Λ term or any inflationary mechanism involving the inflaton field. A comparative study of both the anisotropic cosmological models are carried out here and it is found that the Bianchi-I Universe admits oscillatory solutions for a given matter configuration. We also note that the emergent Universe model can be obtained with the Skyrme fluid. The anisotropy, deceleration and jerk parameters have been studied along with the linear perturbative stability to explore the efficacy of the models. Both cosmological models are stable in the absence of cosmological constant. Besides, their predictions are compatible with the observational data. Thus, we may claim that Skyrme fluid is a possible source of isotropization of an anisotropic Universe via accelerated expansion, which is capable of reproducing some features that can be observed in the Universe.

Keywords. Kantowski–Sachs; Bianchi-I; Skyrme fluid.

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1. Introduction

It is predicted from the cosmological observations that, on a large scale the present Universe is homogeneous and isotropic. However, immediately after the Big Bang, the scenario may have been different. It is believed that the early Universe must have been highly anisotropic – the initial anisotropy being washed away in the course of evolution of the Universe with time [1], suggesting that the present state of the Universe is not affected by its initial conditions. Thus, space-times which are spatially homogeneous and anisotropic should provide a good description of the early Universe. Also, the local anisotropies observed today in galaxies and superclusters motivate us to study space-times which are anisotropic. Anisotropic cosmological models are widely studied to analyse the possible effects of anisotropy of the early Universe in the present day observations.

The Bianchi Type-I cosmological models which are homogeneous with spatially flat sections but direction dependent expansion or contraction rate, are the simplest of such anisotropic cosmological models and have

been studied in the context of present day observational signature in [2–7]. There are other types of Bianchi models as well, the whole set was named after Luigi Bianchi who classified the relevant three-dimensional spaces [8] which were first studied in the framework of the Einstein field equations (EFE) in [9–11]. Bianchi-II models with bulk viscosity in the context of string cosmology have been recently studied in [12]. Such anisotropic initial conditions have also been studied in Kantowski–Sachs (KS) cosmology [13]. Collins [14] was the first to describe the global structure of the KS space-time. There exists a class of KS solutions with a non-vanishing value of the cosmological constant in which the initial anisotropy washes out asymptotically [15]. Paul *et al* [17] studied the isotropization of an anisotropic KS Universe in the framework of chaotic inflationary scenario [16]. KS cosmologies have also been studied in details in a series of papers [18–20].

Skyrme fields in the background KS space-times have been considered in [21,22]. It may be mentioned here that in particle physics, the skyrmion is considered as a topologically stable field configuration in the framework of non-linear sigma models, which was first

used by Skyrme for pions in modelling nucleons in 1962 [23]. The model does not involve quarks and is an approximate low-energy effective theory of quantum chromodynamics (QCD). The Skyrme model is characterised by topological soliton solutions which are physically interpreted as a type of baryon termed skyrmions [24]. Thereafter, skyrmions are considered in the field theoretical framework as a topological object which is of much interest in solid-state physics and string theory. An indirect evidence of the existence of skyrmions have been reported in Bose–Einstein condensates and superconductors [25,26]. In [21,22] coupling of the Skyrme field with gravity has been considered by taking the Skyrme fluid as a source. In cosmological context, it generates bounds on both the cosmological constant and the Skyrme coupling constant [27], and hence it is worth further investigation to understand the evolution of the Universe.

In this paper, we consider the EFE for anisotropic KS and Bianchi Type-I space–times with a non-zero cosmological constant taking the Skyrme fluid as a source. The evolution of the Universe is studied for two different types of anisotropic Universes considering different permissible values of the coupling parameters accommodating different physical situations like domination of the Skyrme fluid. The paper is organised as follows. In the following section we present the field equations of the Einstein–Skyrme system. In §3 and 4, cosmological solutions are obtained for KS and Bianchi I space–times respectively taking the Skyrme fluid as a source with a cosmological constant Λ . In §5 the physical interpretations of the models are discussed. The cosmological solutions are plotted graphically for comparative study of the cosmological models. The EFE for Bianchi I background geometry is represented in the Appendix.

2. The Einstein–Skyrme action and the field equations

The Einstein–Skyrme (ES) action with non-zero Λ is given by

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + S_{\text{Skyrme}}, \quad (1)$$

where

$$S_{\text{Skyrme}} = \frac{\kappa}{2} \int d^4x \sqrt{-g} \times \text{Trace} \int \left(\frac{R_{\mu\nu} R^{\mu\nu}}{2} + \frac{\lambda F_{\mu\nu} F^{\mu\nu}}{16} \right). \quad (2)$$

Here $\kappa = f_\pi^2/4$ and $\lambda = 4/e^2 f_\pi^2$ are the skyrmion coupling constants; f_π and e denote the pion decay constant and a dimensionless parameter for stable solitons, respectively.

The equation of motion (EOM) of a Skyrme field coupled to gravity is given by

$$\nabla^\mu R_{\mu\nu} + \frac{\lambda}{4} \nabla^\mu [R_{\mu\nu}, F_{\mu\nu}] = 0. \quad (3)$$

The EFE for an ES system with non-zero Λ is given by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}^{\text{Skyrme}}. \quad (4)$$

The energy–momentum tensor for the Skyrme fluid $T_{\mu\nu}^{\text{Skyrme}}$ is expressed as

$$T_{\mu\nu}^{\text{Skyrme}} = (\rho + p_t) u_\mu u_\nu + p_t g_{\mu\nu} + (p_r - p_t) \chi_\mu \chi_\nu, \quad (5)$$

where u_μ denotes the 4-velocity and $\chi^\mu = A^{-1} \delta_r^\mu$ is a unit space-like vector in the radial direction.

This expression for the stress–energy tensor of the Skyrme field indicates the presence of an in-built anisotropy in the fluid as the pressures are different in the radial (p_r) and tangential (p_t) directions. It is found that the radial pressure is equal in magnitude but opposite in sign to the energy density. The fluid satisfies all the three energy conditions in general relativity, namely weak energy condition (WEC), dominant energy condition (DEC) and strong energy condition (SEC). However, in the presence of a Λ -term, it may or may not satisfy the strong energy condition (SEC) which is $p_t \geq \Lambda$ [22].

The gravitational constant G multiplied by the Skyrme coupling constant κ gives the effective gravitational constant $G_{\text{eff}} = G\kappa$. The parameter $8\pi G\kappa$ may lie in the range (0–1) and the other coupling constant λ is approximately of the order of $2 \times 10^{-31} \text{ m}^2$ [27]. In the anisotropic Universes considered here, the evolution of the anisotropy parameter denoted by Γ will be determined in the next sections.

3. Cosmological solution for a KS Universe with Λ and Skyrme fluid

The KS space–time is described by the line element

$$ds^2 = -dt^2 + A^2(t) dr^2 + B^2(t) [d\theta^2 + \sin^2 \theta d\phi^2], \quad (6)$$

where $A(t)$ and $B(t)$ are the scale factors and r, θ, ϕ and t are the four-dimensional coordinates. The components of $T_{\mu\nu}^{\text{Skyrme}}$ are given by [22]

$$\rho = \frac{1}{B^2} \left(1 + \frac{\lambda}{2B^2} \right), \quad (7)$$

$$p_r = -\rho, \quad (8)$$

$$p_t = \omega_t \rho = \frac{\lambda}{2B^4}, \quad (9)$$

where the EOS parameter corresponds to

$$\omega_t = \frac{\lambda}{\lambda + 2B^2}.$$

Considering the stress–energy tensor described by eqs (7)–(9), the components of the EFE given by eq. (4) can be expressed as

$$2\frac{\dot{B}\dot{A}}{BA} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} - \Lambda = 8\pi G \left[\frac{\kappa}{B^2} \left(1 + \frac{\lambda}{2B^2} \right) \right], \tag{10}$$

$$2\frac{\ddot{B}}{B} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} - \Lambda = 8\pi G \left[\frac{\kappa}{B^2} \left(1 + \frac{\lambda}{2B^2} \right) \right], \tag{11}$$

$$\frac{\dot{B}\dot{A}}{BA} + \frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} - \Lambda = -8\pi G \left[\frac{\kappa\lambda}{2B^4} \right]. \tag{12}$$

3.1 KS Model-I

Considering a connection between the scale factors of the form $\dot{B} = KA$ we get

$$\frac{\dot{A}}{A} = \frac{\ddot{B}}{\dot{B}}. \tag{13}$$

Using the first two field equations (eqs (10) and (11)) and the above relation we get

$$2\frac{\ddot{B}}{B} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} - \Lambda = \frac{\alpha_1}{B^2} + \frac{\alpha_2}{B^4}, \tag{14}$$

where $\alpha_1 = 8\pi G\kappa$ and $\alpha_2 = 4\pi G\kappa\lambda$. The EFE given by eq. (12) yields

$$2\frac{\ddot{B}}{B} + \frac{1}{B} \frac{d^3B}{dt^3} - \Lambda = -\frac{\alpha_2}{B^4}. \tag{15}$$

Multiplying both sides of the above equation by $\dot{B}B^2$ and integrating the obtained differential equation (DE), we get

$$\frac{\ddot{B}}{B} - \frac{\Lambda}{3} = \frac{\alpha_2}{B^4} + \frac{C}{B^3}, \tag{16}$$

where C denotes the constant of integration. Combining eqs (14) and (16), we get

$$\frac{\dot{B}^2}{B^2} + \frac{1 - \alpha_1}{B^2} + \frac{\alpha_2}{B^4} + \frac{2C}{B^3} - \frac{\Lambda}{3} = 0. \tag{17}$$

We introduce a time-dependent perturbation $B(t)(1 + \epsilon(t))$ to study the stability of the cosmological models. We have seen from the EFE that the components of the stress–energy tensor are expressed in terms of the scale factor B and we have also considered a relation between the two scale factors for obtaining solutions to the EFE in eq. (13) for the KS Model-I. So, involving perturbation terms up to the first power of ϵ , we obtain from eq. (17), a differential equation for perturbation $\epsilon(t)$ as

$$\frac{\dot{B}}{B}\dot{\epsilon} - \left[\frac{1 - \alpha_1}{B^2} + \frac{2\alpha_2}{B^4} \right] \epsilon = 0. \tag{18}$$

We now consider different cases.

Case A: $\alpha_2 = 0, C = 0$ and $H_0^2 = \Lambda/3$. Equation (17) reduces to

$$\frac{\dot{B}^2}{B^2} + \frac{1 - \alpha_1}{B^2} = H_0^2. \tag{19}$$

On integrating, we obtain

$$B = B_0 \cosh(H_0t), \tag{20}$$

where

$$B_0 = \frac{\sqrt{1 - \alpha_1}}{H_0}.$$

Thus, the other scale factor is given by

$$A = \frac{\dot{B}}{K} = A_0 \sinh(H_0t), \tag{21}$$

where

$$A_0 = \frac{\sqrt{1 - \alpha_1}}{\kappa}.$$

The two-directional Hubble parameters are given by

$$H_1 = \frac{\dot{A}}{A}, \quad H_2 = \frac{\dot{B}}{B}. \tag{22}$$

The average Hubble parameter is given by

$$H = \frac{H_1 + 2H_2}{3}. \tag{23}$$

We define the variation of the directional Hubble parameters as

$$\Delta H_1 = H_1 - H, \quad \Delta H_2 = H_2 - H. \tag{24}$$

The anisotropy parameter Γ is defined as

$$\Gamma = \frac{1}{3} \left[\left(\frac{\Delta H_1}{H} \right)^2 + 2 \left(\frac{\Delta H_2}{H} \right)^2 \right]. \tag{25}$$

The anisotropy parameter for the above cosmological solution yields

$$\begin{aligned} \Gamma &= \frac{1}{3} \left[\left(\frac{H_1 - H}{H} \right)^2 + 2 \left(\frac{H_2 - H}{H} \right)^2 \right] \\ &= \frac{2(\coth^2 H_0t + \tanh^2 H_0t - 2)}{\coth^2 H_0t + 4 \tanh^2 H_0t + 4}. \end{aligned} \tag{26}$$

The variation of anisotropic parameter with time is shown in figure 3. It is evident that the initial anisotropy present in the early Universe washes out to zero finally in the late Universe. The time-dependent perturbation has the form

$$\epsilon(t) = C[\tanh H_0t] \frac{1 - \alpha_1}{B_0^2 H_0^2}. \tag{27}$$

For $\alpha_1 = 1$ it is constant but for $\alpha_1 < 1$, it leads to a decreasing perturbation.

Case B: Here, we consider $\alpha_2 = 0, C = 0, H_0^2 = \Lambda/3$. However, the maximum value of the parameter α_1 may be taken as $\alpha_1 = 1$. In this case, on integrating eq. (17) the cosmological solution reduces to

$$B = B_0 e^{H_0 t}, \tag{28}$$

where B_0 is an integration constant and

$$A = \frac{B_0 H_0}{\kappa} e^{H_0 t}. \tag{29}$$

Thus, it corresponds to de-Sitter solution. As we get a vanishing anisotropy parameter ($\Gamma = 0$), the solution gives a Universe emerged from isotropic state. Also, we see in this case that the perturbation turns out to be a constant and has no time-dependence at all.

Case C: Here we consider $\alpha_1 = 1, C = 0, H_0^2 = \Lambda/3$ with a non-zero α_2 . In this case, the Skyrme fluid dominates and integrating eq. (17) we get

$$B = \frac{\alpha_2^{\frac{1}{4}}}{\sqrt{H_0}} \sqrt{\cosh 2H_0 t}. \tag{30}$$

The other scale factor is given by

$$A = \frac{\alpha_2^{\frac{1}{4}} \sqrt{H_0}}{K} \frac{\sinh 2H_0 t}{\sqrt{\cosh 2H_0 t}}. \tag{31}$$

The anisotropy parameter becomes

$$\Gamma = \frac{8 \coth^2 H_0 t + 9 \tanh^2 H_0 t - 16}{4 \coth^2 H_0 t + 4 \tanh^2 H_0 t + 8}. \tag{32}$$

The time variation of the anisotropy parameter is shown in figure 6. It is evident that an initially anisotropic Universe transforms to an isotropic Universe at late time. The perturbation on the scale factor is

$$\epsilon(t) = \frac{C}{2H_0} [\tanh H_0 t]^{2H_0} \tag{33}$$

which decreases leading to a stable Universe.

Case D: Here, Skyrme fluid is considered without cosmological constant (Λ). Thus, here we set $C = 0$ in eq. (17) which leads to

$$\frac{\dot{B}^2}{B^2} = \frac{\alpha_1 - 1}{B^2} - \frac{\alpha_2}{B^4}. \tag{34}$$

Integrating this differential equation, we get the following solutions:

$$B = \sqrt{(\alpha_1 - 1)t^2 + \frac{\alpha_2}{\alpha_1 - 1}}. \tag{35}$$

$$A = \frac{\alpha_1 - 1}{K} \frac{t}{\sqrt{(\alpha_1 - 1)t^2 + \frac{\alpha_2}{\alpha_1 - 1}}}, \tag{36}$$

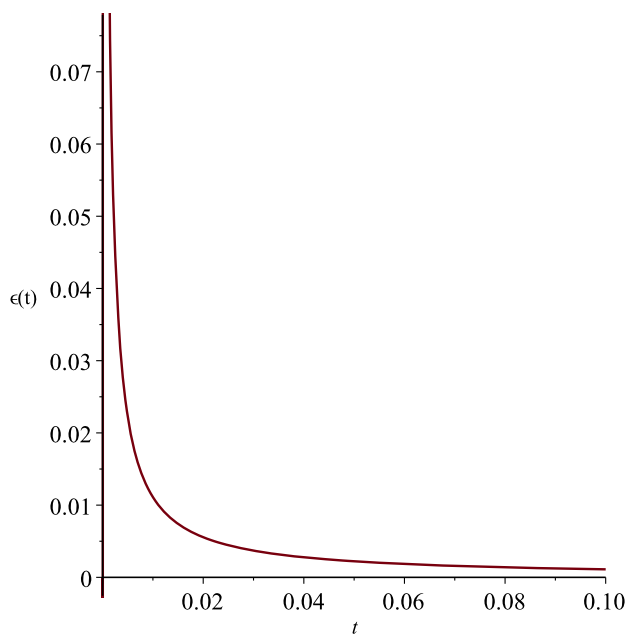


Figure 1. Variation of perturbation $\epsilon(t)$ with time t .

where $\alpha_1 > 1$, but if $0 < \alpha_1 \leq 1$ it gives unphysical result. The anisotropy parameter is given by

$$\Gamma = 2 \left[\frac{\frac{1}{t} - \frac{2t(\alpha_1 - 1)}{(\alpha_1 - 1)t^2 + \frac{\alpha_2}{(\alpha_1 - 1)}}}{\frac{1}{t} + \frac{t(\alpha_1 - 1)}{(\alpha_1 - 1)t^2 + \frac{\alpha_2}{(\alpha_1 - 1)}}} \right]^2. \tag{37}$$

The time variation of the anisotropic parameter is shown in figure 7. It is evident that the initial anisotropy washes out which in a later epoch once again will transit to an anisotropic Universe. However, this particular solution is of no physical significance. Thus, cosmological constant plays an important role in the evolution of the Universe with Skyrme fluid.

The time-dependent perturbation is found to evolve with time as

$$\epsilon(t) = \frac{C t}{(\alpha_1 - 1)t^2 + \alpha_1}. \tag{38}$$

We plot the variation of perturbation as a function of time in figure 1, and it is evident that a stable cosmological model can be obtained.

3.2 KS Model-II

Now we consider a simple case where $\dot{B} = 0$ which leads to $\frac{1}{B} = H_0$ or $B = H_0^{-1}$. Equations (10) and (11) are reduced to an identical equation which is given by

$$\frac{1}{B^2} - \Lambda = \frac{\alpha_1}{B^2} + \frac{\alpha_2}{B^4}. \tag{39}$$

The EFE given by eq. (12) reduces to

$$\frac{\ddot{A}}{A} - \Lambda = -\frac{\alpha_2}{B^4}. \tag{40}$$

In this case, the cosmological constant becomes

$$\Lambda = [(1 - \alpha_1) - \alpha_2 H_0^2] H_0^2. \tag{41}$$

Putting the value of Λ back in eq. (12), we obtain

$$\frac{\ddot{A}}{A} = [(1 - \alpha_1) - 2\alpha_2 H_0^2] H_0^2. \tag{42}$$

We note the following:

(i)

$$H_0^2 = \frac{1 - \alpha_1}{\alpha_2}, \quad \ddot{A} = 0,$$

(ii) give rise to linear evolution of A with t .

$$H_0^2 > \frac{1 - \alpha_1}{\alpha_2}, \quad \ddot{A} < 0,$$

(iii) give rise to an oscillatory Universe.

$$H_0^2 < \frac{1 - \alpha_1}{\alpha_2}, \quad \ddot{A} > 0,$$

give rise to an expanding Universe.

4. Cosmological solution for a Bianchi Type-I Universe

The line element of the Bianchi Type-I anisotropic Universe is given by

$$ds^2 = dt^2 - [R_i(t) dx_i]^2, \quad i = 1, 2, 3, \tag{43}$$

where the anisotropic Hubble parameters, $H_i = \dot{R}_i/R_i$. The average scale factor is given by $R = (R_1 R_2 R_3)^{1/3}$ and the average Hubble parameter is given by

$$H = \frac{\dot{R}}{R} = \frac{d}{dt}(\ln R) = \frac{1}{3} \frac{d}{dt}(\ln V), \tag{44}$$

where $V = R^3 = R_1 R_2 R_3$ is the volume. In this section, we consider $R_1 = A$, $R_2 = R_3 = B$, which implies $V = AB^2$ and $R = (AB^2)^{1/3}$. The EFEs are given by (refer to Appendix)

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} - \Lambda = 8\pi G \left[\frac{\kappa}{B^2} \left(1 + \frac{\lambda}{2B^2} \right) \right] \tag{45}$$

$$\frac{\ddot{B}^2}{B^2} + 2\frac{\ddot{B}}{B} - \Lambda = 8\pi G \left[\frac{\kappa}{B^2} \left(1 + \frac{\lambda}{2B^2} \right) \right] \tag{46}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \Lambda = -8\pi G \left[\frac{\kappa\lambda}{2B^4} \right]. \tag{47}$$

4.1 Bianchi-I cosmology

We consider $\dot{B} = \kappa A$. Equations (41) and (42) of the EFE reduce to an identical equation given by

$$\frac{\dot{B}^2}{B^2} + 2\frac{\ddot{B}}{B} - \Lambda = \frac{\alpha_1}{B^2} + \frac{\alpha_2}{B^4}. \tag{48}$$

Equation (43) reduces to

$$2\frac{\ddot{B}}{B} + \frac{1}{B} \frac{d^3 B}{dt^3} - \Lambda = -\frac{\alpha_2}{B^4}. \tag{49}$$

Finally, we get a differential equation for B which is given by

$$\frac{\dot{B}^2}{B^2} - \frac{\alpha_1}{B^2} + \frac{\alpha_2}{B^4} + \frac{2C}{B^3} - \frac{\Lambda}{3} = 0. \tag{50}$$

Similar to KS Model-I, a time-dependent perturbation $\epsilon(t)$ can be introduced and the above equation involving perturbation corrections up to first power in ϵ and its derivative yields

$$\frac{\dot{B}}{B} \dot{\epsilon} + \left[\frac{\alpha_1}{B^2} - \frac{2\alpha_2}{B^4} \right] \epsilon = 0. \tag{51}$$

We now study different cases of the Bianchi-I Universe (figure 2).

Case A: For $\alpha_2 = C = 0$ and $H_0^2 = \Lambda/3$, the above differential equation reduces to

$$\frac{\dot{B}^2}{B^2} - \frac{\alpha}{B^2} = H_0^2. \tag{52}$$

Integrating the above equation, we obtain the following solution:

$$B = B_0 \sinh H_0 t, \tag{53}$$

where $B_0 = \sqrt{\alpha_1}/H_0$ and

$$A = A_0 \cosh H_0 t. \tag{54}$$

Here $A_0 = \sqrt{\alpha_1}/\kappa$. The anisotropy parameter is given by

$$\Gamma = 2 \left[\frac{\coth^2 H_0 t + \tanh^2 H_0 t - 2}{4 \coth^2 H_0 t + \tanh^2 H_0 t + 4} \right]. \tag{55}$$

The time variation of the anisotropic parameter is shown in figure 3. It is evident that the initial anisotropy in the Universe decreases and finally the anisotropic Universe isotropized. The perturbation is given by

$$\epsilon(t) = C [\tanh H_0 t]^{B_0^{-2} H_0^2}. \tag{56}$$

The evolution of the perturbation with time is shown in figure 2.

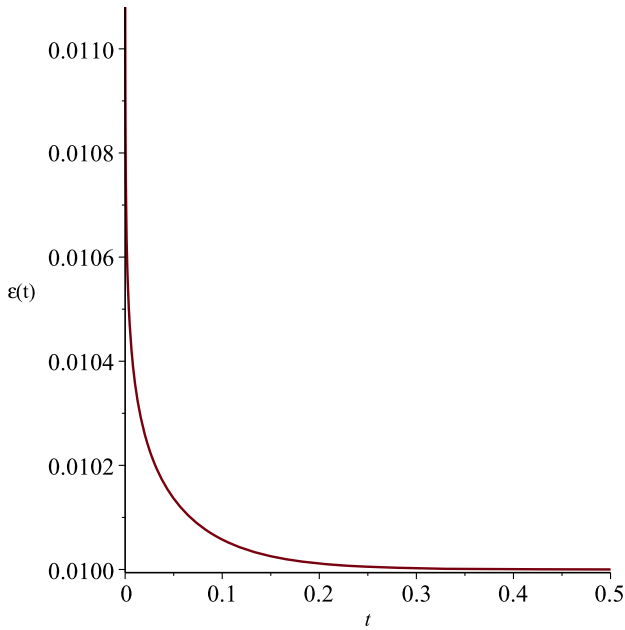


Figure 2. Variation of perturbation $\epsilon(t)$ with time t .

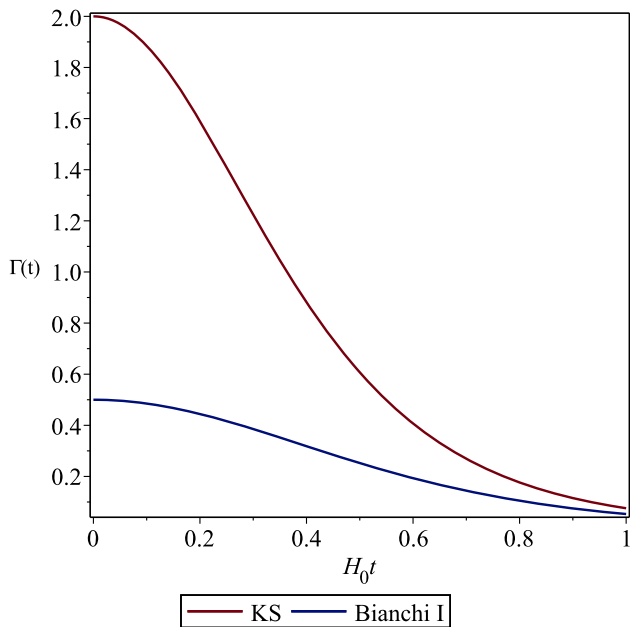


Figure 3. Variation of anisotropy parameter $\Gamma(t)$ with H_0t .

Case B: For $\alpha_1 = 1, \alpha_2 = 0 = C, H_0^2 = \Lambda/3$, eq. (43) reduces to

$$\frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = H_0^2. \tag{57}$$

The solution is given by

$$B = B_0 \sinh H_0t, \tag{58}$$

where $B_0 = 1/H_0$ and

$$A = A_0 \cosh H_0t. \tag{59}$$

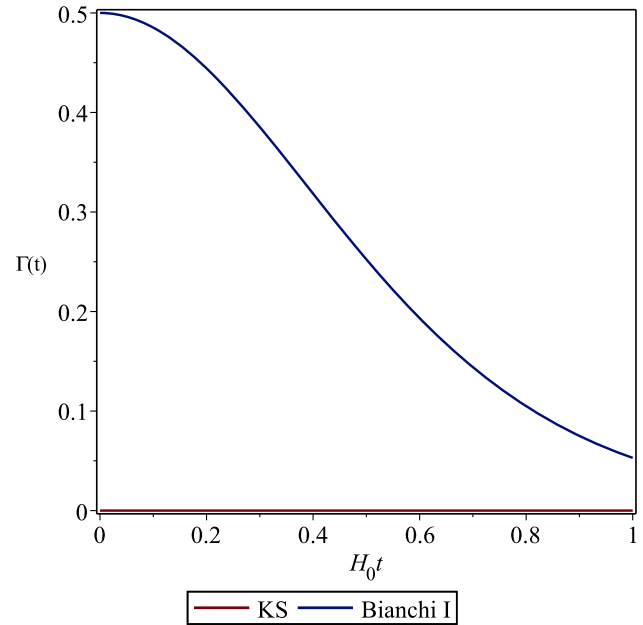


Figure 4. Variation of anisotropy parameter $\Gamma(t)$ with H_0t .

Here $A_0 = 1/\kappa$, which is similar to Case A with $\alpha_1 = 1$. In this case, the anisotropy parameter is given by

$$\Gamma = 2 \left[\frac{\coth^2 H_0t + \tanh^2 H_0t - 2}{4 \coth^2 H_0t + \tanh^2 H_0t + 4} \right]. \tag{60}$$

The time evolution of the anisotropic parameter is shown in figure 4. A Universe with initial anisotropy at a later epoch isotropizes. The perturbation takes the form

$$\epsilon(t) = C[\tanh H_0t]^{-\frac{1}{B_0^2 H_0^2}}. \tag{61}$$

The temporal evolution of the perturbation is shown in figure 5.

Case C: For $H_0^2 = \Lambda/3, C = 0$ and $\alpha_1 = 1$, the Skyrme fluid dominates as the coupling parameter κ is set to a maximum value with non-zero coupling parameter λ . The differential equation for B can be written as

$$\frac{\dot{B}^2}{B^2} - \frac{1}{B^2} + \frac{\alpha_2}{B^4} - \frac{\Lambda}{3} = 0. \tag{62}$$

On integrating once, we obtain

$$B\dot{B} = \sqrt{H_0^2 B^4 + B^2 - \alpha_2}. \tag{63}$$

In the very early Universe, the value of Λ was very large, and consequently one can approximate $(1/H_0^2) \rightarrow 0$.

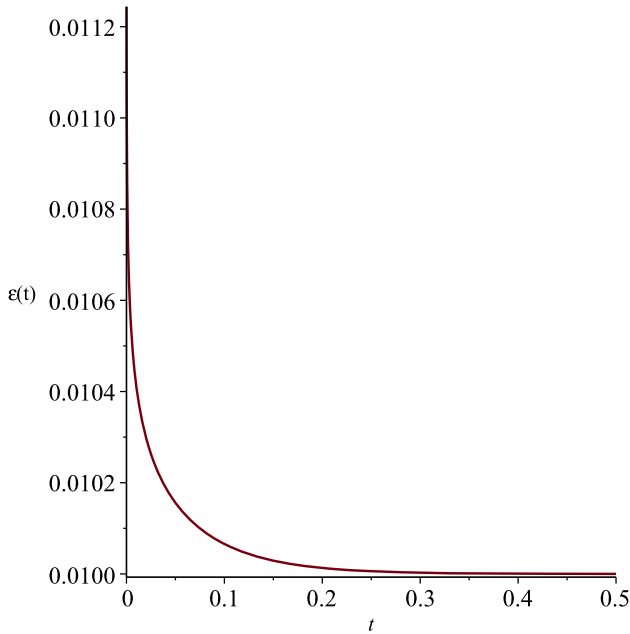


Figure 5. Variation of perturbation $\epsilon(t)$ with time t .

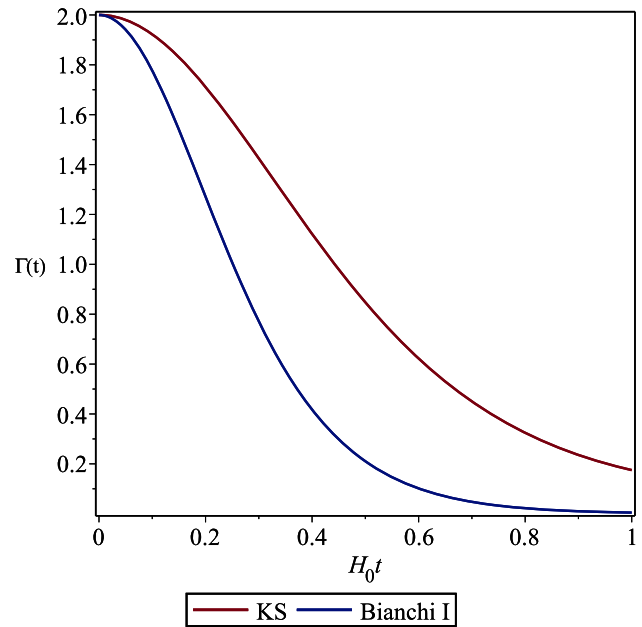


Figure 6. Variation of anisotropy parameter $\Gamma(t)$ with H_0t .

The general solution is given by

$$B = \sqrt{\frac{\sqrt{\alpha_2}}{H_0} \cosh 2H_0t - \frac{1}{2H_0^2}}. \tag{64}$$

$$A = \frac{\frac{\sqrt{\alpha_2}}{K} \sinh 2H_0t}{\sqrt{\frac{\sqrt{\alpha_2}}{H_0} \cosh 2H_0t - \frac{1}{2H_0^2}}}. \tag{65}$$

One interesting point to note is that if we consider anti-de Sitter Universe with negative Λ , for this particular Bianchi-I solution, the scale factor A will possess an additive term $1/2H_0^2$ in the denominator which can describe an emergent Universe [28,29]. Thus, an initial anisotropy with Skyrme fluid admits an emergent Universe which at a later epoch transits to an isotropic Universe which is a new solution.

$$\Gamma = \left[\frac{8H_0^2 \coth^2 2H_0t + \frac{8\alpha_2 \sinh^2 2H_0t}{\left(\frac{\sqrt{\alpha_2}}{H_0} \cosh 2H_0t - \frac{1}{2H_0^2}\right)^2} - \frac{16H_0\sqrt{\alpha_2} \cosh 2H_0t}{\left(\frac{\sqrt{\alpha_2}}{H_0} \cosh 2H_0t - \frac{1}{2H_0^2}\right)}}{4H_0^2 \coth^2 2H_0t + \frac{\alpha_2 \sinh^2 2H_0t}{\left(\frac{\sqrt{\alpha_2}}{H_0} \cosh 2H_0t - \frac{1}{2H_0^2}\right)^2} + \frac{4H_0\sqrt{\alpha_2} \cosh 2H_0t}{\left(\frac{\sqrt{\alpha_2}}{H_0} \cosh 2H_0t - \frac{1}{2H_0^2}\right)}} \right]. \tag{66}$$

The time variation of anisotropic parameter is shown in figure 6. In this case, the initial anisotropy also washes out. The perturbation on the scale factor is given by

$$\epsilon(t) = \frac{C}{2H_0} [\tanh H_0t] \left\{ \frac{\frac{\alpha_2^3}{H_0} - \frac{1}{2H_0^2} - 2\alpha_2}{\frac{\alpha_2}{H_0} - \frac{\sqrt{\alpha_2}}{2H_0^2}} \right\}$$

$$\times \left\{ \frac{\sqrt{\alpha_2}}{2H_0^2} (\tanh H_0t)^2 + \frac{1}{2H_0^2} (\tanh H_0t)^2 + \frac{\sqrt{\alpha_2}}{2H_0^2} - \frac{1}{2H_0^2} \right\} \frac{8\alpha_2 H_0^3}{4\alpha_2 H_0^2 - 1}. \tag{67}$$

Case D: For $H_0^2 = C = 0$ without a cosmological constant, eq. (47) reduces to

$$\frac{\dot{B}^2}{B^2} - \frac{\alpha_1}{B^2} + \frac{\alpha_2}{B^4} = 0. \tag{68}$$

On integrating the above equation we get

$$B = \sqrt{\alpha_1 t^2 + \frac{\alpha_2}{\alpha_1}} \tag{69}$$

and

$$A = \frac{\alpha_1 t / K}{\sqrt{\alpha_1 t^2 + \frac{\alpha_2}{\alpha_1}}}. \tag{70}$$

In this case, we obtain physically acceptable solutions even if $\alpha_2 = 0$ where B varies linearly with t and A is

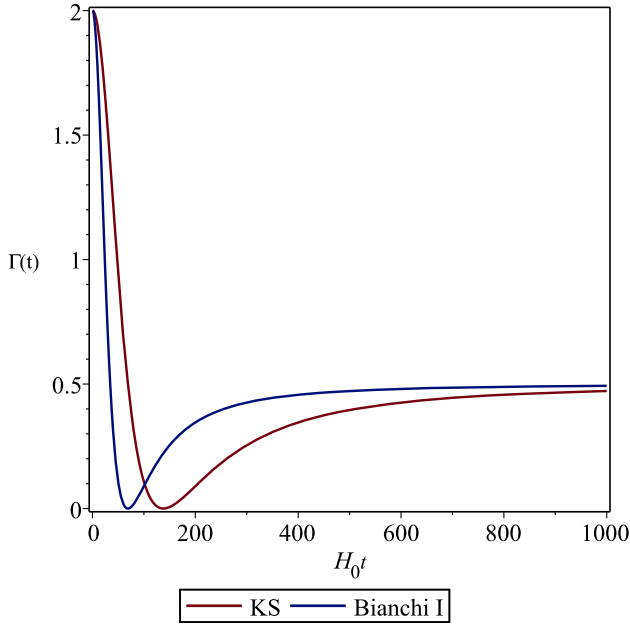


Figure 7. Variation of anisotropy parameter $\Gamma(t)$ with H_0t .

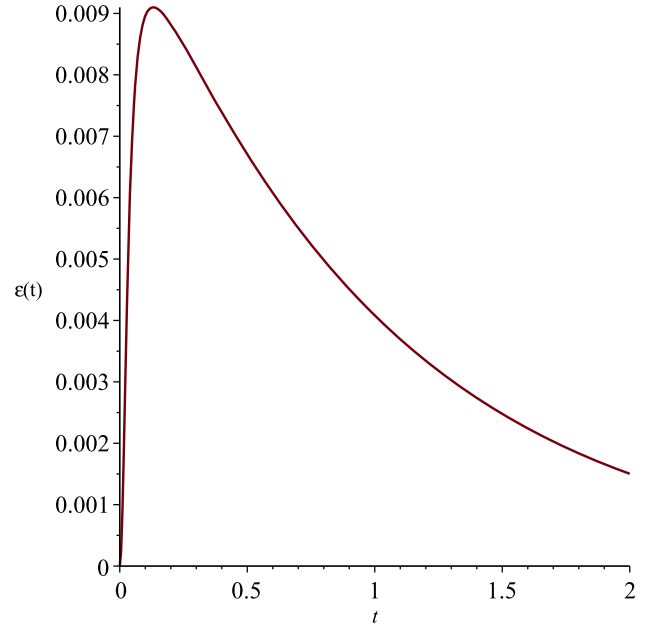


Figure 8. Variation of perturbation $\epsilon(t)$ with time t .

a constant. The anisotropy parameter is given by,

$$\Gamma = \left[\frac{\frac{2}{t^2} + \frac{8t^2\alpha_1^2}{(\alpha_1t^2 + \frac{\alpha_2}{\alpha_1})^2} - \frac{8\alpha_1}{(\alpha_1t^2 + \frac{\alpha_2}{\alpha_1})}}{\frac{1}{t^2} + \frac{t^2\alpha_1^2}{(\alpha_1t^2 + \frac{\alpha_2}{\alpha_1})^2} + \frac{2\alpha_1}{(\alpha_1t^2 + \frac{\alpha_2}{\alpha_1})}} \right]^2. \tag{71}$$

The variation of anisotropy with time is shown in figure 7. The initial anisotropy washes out but in future the Universe will transit to an anisotropic one.

The perturbation is given by

$$\epsilon(t) = \frac{Ct^2e^{-t}}{\alpha_1t^2 + \frac{\alpha_2}{\alpha_1}}. \tag{72}$$

The perturbation over time evolves as shown in figure 8.

4.2 Bianchi-I Model-II

In this model, we consider $(1/B) = H_0$ which implies $B = H_0^{-1}$. From the first two EFEs, we determine

$$\Lambda = -\alpha_1H_0^2 - \alpha_2H_0^4. \tag{73}$$

The coupling parameters κ and λ may be positive or zero, leading to both α_1 and α_2 positive quantities. In this case, the cosmological constant may be considered negative for a physically viable cosmological solution. The EFE given by eq. (43) becomes

$$\frac{\ddot{A}}{A} = -2\alpha_2H_0^2 \left[\frac{\alpha_1}{2\alpha_2} + H_0^2 \right]. \tag{74}$$

which admits an oscillatory Universe which is new and interesting.

5. Discussion

Two different anisotropic Universes described by Kantowski–Sachs and Bianchi-I metrics are considered in the presence of a cosmological constant with Skyrme fluid as the matter source. In general, for all types of solutions, the parameters associated with the Skyrme fluid play important roles in determining the evolution of the Universe. Except for Case-B of the KS Universe, we note the existence of a Universe with a large initial anisotropy, which however, is found to isotropize at a later epoch. The isotropization of the anisotropic Universe is possible in the absence of any inflationary mechanism involving the scalar inflaton field and even in the absence of the cosmological constant. This feature suggests that the Skyrme fluid as a source of matter, helps in isotropizing the anisotropic Universe leading to an accelerating expansion at the present epoch. We also note that an anisotropic Universe transits to the observed isotropic and homogeneous Universe at the present epoch with or even without Λ in the presence of Skyrme fluid.

The behaviour of the anisotropic parameter in the KS and Bianchi-I Universes are plotted in figures 3, 4, 6 and 7 for different cases. The plots of anisotropic parameter in figure 3 for both KS and Bianchi-I Universes show that the initial anisotropy may be high for the

former Universe but eventually it washes out at almost the same values of H_0t , thus indicating a significantly higher isotropization rate for a KS Universe than that of a Bianchi-I Universe. In figure 4, we draw the anisotropic parameter for both the the Universes corresponding to Case A and found that KS Universe is always anisotropic but Bianchi-I Universe isotropizes.

It is noted that when Skyrme fluid dominates in KS and Bianchi-I Universes separately, it permits a cosmology with initial large anisotropy which at a later epoch transforms to isotropic Universe. A new solution in Bianchi-I Universe is found here which permits an emergent Universe for a negative Λ which, however, cannot be realised in the KS Universe. The behaviour of the anisotropic parameters for both the Universes are plotted in figure 6. It is evident that the initial anisotropy is the same for both the Universes which eventually drops down to zero at a later epoch but the rate of isotropization is faster for the Bianchi-I Universe than for the KS Universe.

In the absence of cosmological constant (Λ), the initial large anisotropy decreases for both the Universes but it transits to a Universe which once again will transit to an anisotropic Universe at a later stage of evolution. This is possibly due to the absence of the repulsive effect of the Λ term, but physically it may be due to the structure formation. The initial anisotropy drops down faster and returns earlier for Bianchi-I Universe, while isotropization is comparatively slow for KS Universe and further growth of anisotropy is at a considerably slower rate. In this case, KS geometry gives no well-behaved acceptable solution but the Bianchi-I geometry admits an acceptable solution.

In the case of Model-II cosmological solutions for KS and Bianchi-I space–times sourced by a Skyrme fluid, we noticed an interesting feature which has a sharp contrast between the two anisotropic Universes. A KS Universe with Skyrme fluid is found to admit three types of cosmological solutions, namely linear, expanding and oscillatory, for different constraints on H_0^2 , but in the case of Bianchi-I space–time, it permits only oscillatory solution.

We consider in detail two particular solutions where we have considered the KS and Bianchi-I Universes to be filled with the Skyrme fluid, without any contribution from the cosmological constant Λ , which has been set to zero (Case D). We determine the two physical parameters, namely the deceleration and the jerk parameters for both the KS and Bianchi-I Universes in terms of the redshift parameter z and plotted the variation as a function of z to see how well they confront with observational data. If they can represent the data well, it implies that the initially anisotropic Universe described by KS and Bianchi-I space–times, sourced by a Skyrme fluid only,

not only isotropizes at late times but also can describe the observational data well enough at low red-shifts.

In the KS Universe sourced only by the Skyrme fluid, the deceleration parameter q defined as $q = -\frac{a\ddot{a}}{\dot{a}^2}$, where $a = (AB^2)^{1/3}$ gives the average scale factor, is obtained to have the form

$$q = \frac{\frac{10(z+1)^2}{9f} - \frac{(\alpha_1-1)^3(z+1)^8 f}{9K^2} + \frac{7(\alpha_1-1)^6(z+1)^{14} f^3}{9K^4}}{\frac{(z+1)^2}{9f} + \frac{2(\alpha_1-1)^3(z+1)^8 f}{9K^2} + \frac{(\alpha_1-1)^6(z+1)^{14} f^3}{9K^4}} - 2, \tag{75}$$

where we define the parameter f as

$$f = \frac{\frac{\alpha_2}{\alpha_1-1} + \sqrt{\left(\frac{\alpha_2}{\alpha_1-1}\right)^2 + \frac{4K^2}{(z+1)^6(\alpha_1-1)}}}{2(\alpha_1-1)}. \tag{76}$$

The jerk parameter j defined as

$$j = \frac{d^3B/dt^3}{aH^3}$$

is calculated for the KS Universe as

$$j(z) = \frac{d^3z/dt^3}{\dot{z}^3}(1+z)^2 - \frac{6\ddot{z}(1+z)}{\dot{z}^3} + 6 \tag{77}$$

where

$$\dot{z} = -\frac{(\alpha_1-1)^3(z+1)^7 A^{3/2}}{3K^2} - \frac{(z+1)A^{-1/2}}{3}$$

$$\ddot{z} = -\frac{7(\alpha_1-1)^3(z+1)^6 \dot{z} A^{3/2}}{3K^2} - \frac{\dot{z} A^{-1/2}}{3}$$

$$+ \frac{(z+1)A^{-1}}{3}$$

$$\frac{d^3z}{dt^3} = -\frac{14(\alpha_1-1)^3(z+1)^6 \dot{z} A}{K^2}$$

$$- \frac{2(\alpha_1-1)^3(z+1)^6 A^{1/2}}{K^2}$$

$$- \frac{14(\alpha_1-1)^3(z+1)^5 \dot{z}^2 A^{3/2}}{K^2}$$

$$- \frac{7(\alpha_1-1)^3(z+1)^6 \ddot{z} A^{\frac{3}{2}}}{K^2}$$

$$- \frac{\ddot{z} A^{-\frac{1}{2}}}{3} + \frac{2\dot{z} A^{-1}}{3} - \frac{2(z+1)A^{-\frac{3}{2}}}{3},$$

$$A = \frac{\frac{\alpha_2}{\alpha_1-1} + \sqrt{\left(\frac{\alpha_2}{\alpha_1-1}\right)^2 + \frac{4K^2}{(\alpha_1-1)(z+1)^6}}}{2(\alpha_1-1)}.$$

In the Bianchi-I Universe, the deceleration parameter takes the form

$$q = \frac{\frac{10(z+1)^2}{9f} - \frac{\alpha_1^3(z+1)^8 f}{9K^2} + \frac{7\alpha_1^6(z+1)^{14} f^3}{9K^4}}{\frac{(z+1)^2}{9f} + \frac{2\alpha_1^3(z+1)^8 f}{9K^2} + \frac{\alpha_1^6(z+1)^{14} f^3}{9K^4}} - 2, \tag{78}$$

where

$$f = \frac{\frac{\alpha_2}{\alpha_1} + \sqrt{(\frac{\alpha_2}{\alpha_1})^2 + \frac{4K^2}{(z+1)^6\alpha_1}}}{2\alpha_1} \tag{79}$$

and the jerk parameter is computed which is given by

$$j(z) = \frac{d^3z/dt^3}{\dot{z}^3} (1+z)^2 - \frac{6\ddot{z}(1+z)}{\dot{z}^3} + 6, \tag{80}$$

where

$$\begin{aligned} \dot{z} &= -\frac{\alpha_1^3(z+1)^7 A^{3/2}}{3K^2} - \frac{(z+1)A^{-1/2}}{3} \\ \ddot{z} &= -\frac{7\alpha_1^3(z+1)^6 \dot{z} A^{3/2}}{3K^2} - \frac{\dot{z} A^{-1/2}}{3} + \frac{(z+1)A^{-1}}{3} \\ \frac{d^3z}{dt^3} &= -\frac{14\alpha_1^3(z+1)^6 \dot{z} A}{K^2} - \frac{2\alpha_1^3(z+1)^6 A^{1/2}}{K^2} \\ &\quad - \frac{14\alpha_1^3(z+1)^5 \dot{z}^2 A^{3/2}}{K^2} - \frac{7\alpha_1^3(z+1)^6 \ddot{z} A^{3/2}}{K^2} \\ &\quad - \frac{\ddot{z} A^{-1/2}}{3} \\ &\quad + \frac{2\dot{z} A^{-1}}{3} - \frac{2(z+1)A^{-3/2}}{3}, \\ A &= \frac{\frac{\alpha_2}{\alpha_1-1} + \sqrt{\left(\frac{\alpha_2}{\alpha_1-1}\right)^2 + \frac{4K^2}{\alpha_1(z+1)^6}}}{2\alpha_1}. \end{aligned}$$

We plot q and j parameters as functions of the red-shift parameter z in figures 9 and 10 respectively. For the plot we have taken $K = 0.02$, $\alpha_1 = 0.5$, $\alpha_2 := 0.5 \times 10^{-31}$. All the chosen parametric values are well within the allowed ranges as mentioned in the introductory section.

Many interesting conclusions can be drawn from the plots. As we can see from the plot for q , the deceleration parameter takes an approximate value of -0.55 at the present times as the red-shift parameter approaches zero value, which is well in agreement with observational data [30]. The jerk parameter becomes unity for very small red-shift which corresponds well with the Λ CDM model. Also, a sign flip can be observed in the deceleration parameter from positive to negative which physically represents the transition from decelerated to accelerated expansion in the present times when the red-shift value $z \simeq 0.8$, which is again in good agreement with observations [30]. If we extrapolate the plot to even higher red-shifts, it is evident that q tends to 0.5, which agrees well with the predicted value [30].

We have obtained here the time-dependent perturbation introduced in the field equations. The temporal variation of the perturbation can be plotted, but for convenience we plot only those ϵ which are of physical interest. In Case D of the KS Universe and Cases A,

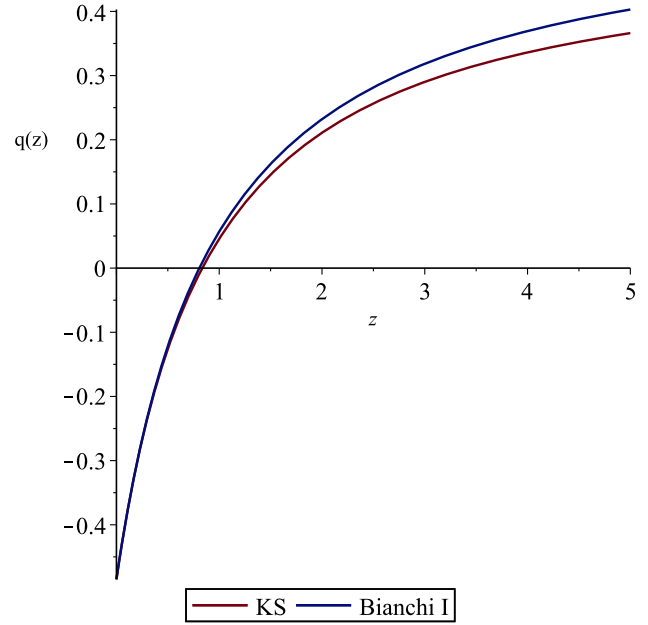


Figure 9. Variation of deceleration parameter $q(z)$ with z .

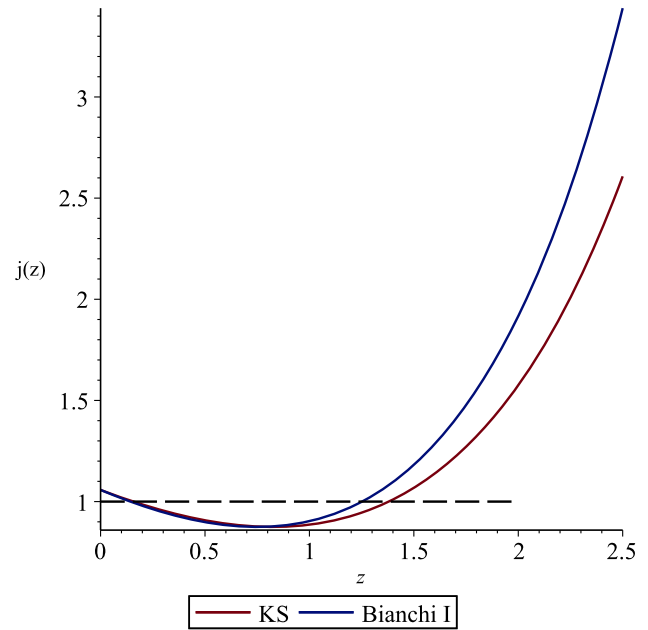


Figure 10. Variation of jerk parameter $j(z)$ with z .

B and D of the Bianchi-I Universe, the evolution of the perturbation with time relates to the physical Universe. In these cases, the small perturbation begins at a small finite maximum and then decreases monotonically before approaching a very small value which is approximately equal to zero. However, in Case D of the Bianchi Universe, the perturbation increases slightly before reaching a maximum at the early time and then falls off. All these four instances where the perturbation falls off with time indicate stable solutions. In the

other cases, the perturbation either increases with time or remains constant which does not permit stability of the cosmological models.

A very interesting observation is that, at the present epoch with red-shift parameter less than 1, both the KS and Bianchi-I Universes containing Skyrme fluid in the absence of a Λ term replicates the behaviour of the standard Λ CDM model along with the deceleration parameter which corresponds very well with observations. Also, as we can see from the perturbative stability analysis, both the cases for the KS and Bianchi-I Universes in the absence of the Λ term are characterised by stable solutions. For higher red-shift values, the behaviour of the two Universes are not exactly identical but still in close proximity such that no contrasting physical feature is obtained as in Model-II. Thus, we can conclude that both the Universes, initially anisotropic, filled with Skyrme fluid in the absence of a cosmological constant term isotropizes over time and corresponds well with the observational data at the present time, also satisfying some essential features of the standard Λ CDM model. This justifies the physical motivation for our work. Thus, we claim Skyrme fluid as a possible source which isotropizes the anisotropic Universe via the present accelerated expansion, also capable of reproducing certain features of the present observed Universe. However, the structure formation process in such a scenario has not been dealt with in the present work and we plan to do so in the near future.

Appendix

EFE for vacuum with non-zero Λ : $R_\nu^\mu = \Lambda \delta_\nu^\mu$

Computation of Ricci tensor

$$R_0^0 = \Lambda = (\ln \ddot{V}) + H_1^2 + H_2^2 + H_3^2 = \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} \tag{81}$$

$$R_1^1 = \Lambda = \frac{(V \dot{H}_1)}{V} = \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\ddot{R}_1}{R_1} \tag{82}$$

$$R_2^2 = \Lambda = \frac{(V \dot{H}_2)}{V} = \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\ddot{R}_2}{R_2} \tag{83}$$

$$R_3^3 = \Lambda = \frac{(V \dot{H}_3)}{V} = \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\ddot{R}_3}{R_3} \tag{84}$$

Computation of Ricci scalar

$$R = 2 \left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} \right] \tag{85}$$

Computation of Einstein tensor

$$G_0^0 = -\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} - \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} - \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} \tag{86}$$

$$G_1^1 = -\frac{\ddot{R}_2}{R_2} - \frac{\ddot{R}_3}{R_3} - \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} \tag{87}$$

$$G_2^2 = -\frac{\ddot{R}_1}{R_1} - \frac{\ddot{R}_3}{R_3} - \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} \tag{88}$$

$$G_3^3 = -\frac{\ddot{R}_1}{R_1} - \frac{\ddot{R}_2}{R_2} - \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} \tag{89}$$

Putting $R_1 = A$ and $R_2 = R_3 = B$, we obtain eqs (45)–(47).

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