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A mathematical model for bioconvection flow with activation energy for chemical reaction and microbial activity

MLAMULI DHLAMINI[1], HIRANMOY MONDAL[2,*], PRECIOUS SIBANDA[3], SANDILE S MOSTA[4] and SACHIN SHAW[5]

[1]Department of Applied Mathematics, National University of Science and Technology, P.O. Box AC 939, Ascot, Bulawayo, Zimbabwe
[2]Department of Applied Mathematics, Maulana Abdul Kalam Azad University of Technology, Haringhata 741 249, India
[3]School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag X01, Scottsville, Pietermaritzburg 3209, South Africa
[4]Department of Mathematics, University of Swaziland, Private Bag 4, Kwaluseni, Swaziland
[5]Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Palapye, Botswana

*Corresponding author. hiranmoymondal@yahoo.co.in

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Abstract. In most of the industrial processes, it is of paramount importance to control the heat and mass transfer rates to ensure high-quality products. Using nanofluids instead of ordinary fluids and using motile micro-organisms are some of the techniques to control heat and mass transfer rates. In some recent studies of bioconvection flow, activation energy, Brownian motion and thermophoretic effects are considered only for the solute and not for the microbes. Our current study incorporates these effects for the motile micro-organisms too. Few, if any results of this nature exist in literature. A system of partial differential equations is formulated to incorporate the effects of these parameters. The system of equations are solved numerically using the spectral quasi-linearisation method to gain an insight into the influence of key parameters on the fluid and flow properties. The thermophoretic force, the Brownian motion and activation energy are significant contributors in the microbes' dynamics. The concentration of microbes decreases with an increase in the thermophoretic force and increases with increasing microbe's Brownian motion parameter. Based on our results, we conclude that increasing activation energy leads to a decrease in microbes' velocity. The inclusion of the microbes' Brownian motion proved to be significant as this was shown to have an impact on the temperature, solute concentration and microbes' concentration in the boundary layer.

Keywords. Arrhenius activation energy; bioconvection; spectral quasi-linearisation method; micro-organisms.

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1. Introduction

The study of boundary layer flow has gained a lot of traction since its introduction in 1904 by Lewig Prandtl. Many interesting flow dynamics such as those encountered in heat, momentum and mass transfer occur in the boundary. These flow dynamics occur in many industrial and biological processes. For example, the quality of certain industrial products including in the food processing industries depend on controlling the cooling rate and the strength of any chemical reaction present [1–6]. Some scholars believe that heat transfer enhancement techniques generally reduce the thermal resistance either by increasing the effective heat transfer surface area or by generating turbulence [7]. Many innovations have been made to address the challenges of controlling heat, momentum and mass transfer rates. These include the use of nanofluids and motile micro-organisms.

The term nanofluid was first used by Choi and Eastman [8] to describe fluids with nanosized particles added to a base fluid to enhance certain fluid properties of interest. A typical nanoparticle is a stable metal (Al, Cu, Ag, Au, Fe), an oxide (Al_2O_3, CuO, TiO_2, SiO_2), carbide (SiC), nitrate (AlN, SiN), or a non-metal (graphite). The common base fluids are water, ethylene glycol, oil, polymer or biofluid [9]. Nanofluids have been used in

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a wide range of industrial applications such as cooling of electronic devices, solar energy disposal and chemical processes [9,10], optimal nanodrug delivery [11] etc. Nanofluids are believed to enhance the conductive and convection heat transfer rates [12]. This is due to the aggregation of nanoparticles, leading to percolation paths that enhance the thermal conductivity beyond what is predicted by mean-field theories for well-distributed nanoparticles [13,14]. However, there is no consensus on this theory [12]. Lately, many researchers are conducting studies on hybrid nanofluids. These are nanofluids that contain more than one nanoparticle. They believe that these fluids offer more desirable outputs than nanofluid of a single nanoparticle [15,16], renewable and sustainable colloidal suspension in photovoltaic and solar applications [17].

The quest for heat, momentum and mass transfer enhancement techniques has led to advances in scientific and engineering technologies. One technique is the use of motile micro-organisms to improve the transport rates. Bioconvection arises when instability is induced by swimming self-propelled micro-organisms interact with nanoparticles and buoyancy forces [9,10,18,19]. Some common micro-organisms include but are not limited to gravitaxis, gyrotaxis and oxytaxis micro-organisms. These micro-organisms move in response to a stimulus, oxygen concentration gradients in the case of oxytaxis, gravitational forces in the case of gravitaxis and, a combination of gravitational and viscous torques in the case of gyrotaxis [20,21]. Oxytatic micro-organisms swim up an oxygen concentration gradient to the top of the container. They form an unstable bacteria-rich layer at the top that is denser than the fluid. The bacteria-rich plumes fall thereby transporting oxygen and bacteria to the bottom of the container [10,22–25]. The use of motile micro-organisms as a transportation mechanism is encountered in processes such as aerobic fermentation and bioreactors [26,27].

This study investigates the flow dynamics of a weakly conducting fluid with chemical and microbial activation energies in a magnetic field. Arrhenius activation energy was introduced by Svante Arrhenius in 1889. It refers to the minimum energy required by the potential reactants to produce a chemical reaction. There are several studies in the literature that focus on the Arrhenius activation energy in a chemical reaction [4,28–30]. However, no study has incorporated the activation energy for micro-organisms despite reports in the literature that the microbial activity is also affected by temperature with possible activation energy for microbes [31–33]. Ross and Nichols [33] postulated that microbial growth is a consequence of complex chemical reactions occurring within the microbe's cell. They reported that the reactions are geared towards either the provision of energy and reducing power from the environment (catabolism) or the synthesis of structural and other macromolecules for growth purposes (anabolism). They used Eyring's absolute reaction rate for their model. However, these reactions are based on the Arrhenius equation and for this reason we use the Arrhenius model in the current study. Bhatti *et al* [34] and Waqas *et al* [35] presented studies with activation energy and micro-organisms. However, a closer look shows that the activation energy was only considered in the chemical reaction, and not for the microbial activity. Thus, to our knowledge, no study has investigated this phenomenon in the context of fluid flow. The other aspects that make this study unique are the thermophoretic effects from micro-organisms and the contribution of the Brownian motion of the micro-organisms. Thermophoresis, sometimes called thermomigration, is the movement of particles due to a force resulting from a temperature gradient, acting in the direction of high to low-temperature regions. The thermophoretic force for small particles is given as $F_t = -p\lambda d_p^2 \nabla T/T$, where p is the gas pressure, λ is the mean free path of the gas, d_p is the particle diameter and T is the absolute temperature of the particle. We believe that the microbes will also experience this force and we want to study the possible effect this has on the flow properties.

2. Mathematical analysis

We consider the boundary layer flow of a weakly conducting nanofluid containing motile micro-organisms with activation energy for chemical reaction and biological activity for micro-organism. The velocity is assumed to be linear, that is $u_w(x) = cx$, with $c > 0$ indicating a stretching surface. The stretching is also assumed to coincide with the plane $y = 0$.

The system of equations for the flow of a nanofluid with thermophoresis and a binary chemical reaction with Arrhenius activation energy is given by eqs (1)–(5) together with the boundary condtions (6)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_{f\infty}\left(1+N^2\right)}u \\ &+(1-C_\infty)\rho_{f_\infty} g\beta_T(T-T_\infty) \\ &-\frac{(\rho_p-\rho_{f\infty})}{\rho_{f\infty}} g\beta_C(C-C_\infty) \\ &-\frac{g\gamma(\rho_n-\rho_{f\infty})}{\rho_{f\infty}}(n-n_\infty), \end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16\sigma^* T_\infty^3}{3\rho_{f\infty}C_p}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{(\rho C_p)_{f\infty}}\left(\frac{\partial u}{\partial y}\right)^2+\tau\left\{D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+D_n\frac{\partial T}{\partial y}\frac{\partial n}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}-k_r^2\left(\frac{T}{T_\infty}\right)^n\exp\left(-\frac{E_a}{kT}\right)(C-C_\infty) \tag{4}$$

$$+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2},$$

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}+\frac{bW_c}{\Delta C}\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right)=D_n\frac{\partial^2 n}{\partial y^2}-k_m^2\left(\frac{T}{T_\infty}\right)^n\exp\left(-\frac{E_a}{kT}\right)(n-n_\infty)+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{5}$$

$$\alpha=\frac{k_m}{(\rho c)_f}\quad\text{and}\quad\tau=\frac{(\rho c)_p}{(\rho c)_f}$$

The boundary conditions for eqs (1)–(5) are given in the following form:

$$u=cx,\ v=0,\ -k_f\frac{\partial T}{\partial y}=h_f(T_W-T),$$
$$D_B\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0,$$
$$n=n_W,\ \text{at } y=0,\ t>0$$
$$u\to 0,\ \frac{\partial u}{\partial y}\to 0,\ T\to T_\infty,$$
$$C\to C_\infty\quad n\to n_\infty,\ \text{as } y\to\infty,\ t>0, \tag{6}$$

where (u, v) are the velocity components in the x and y coordinate directions, $\rho_{f\infty}$ is the fluid density, ρ_p is the density of nanoparticles, ρ_n is the density of micro-organisms, σ is the fluid conductivity, B_0 is the magnetic field, N is the Hall parameter, ν is the kinematic viscosity, β_T, β_C and γ are the thermal, solute and microbe coefficients of expansion, T and T_∞ are the temperature and ambient temperature, C and C_∞ are the nanoparticle concentration and ambient concentration, n and n_∞ are the concentrations of microbe and ambient microbe, g is the gravitational acceleration, α is the thermal diffusivity, σ^* is the Stephan–Boltzman constant, c_p is the specific heat at constant pressure, μ is the fluid viscosity, τ is the ratio of heat capacitance of nanoparticles to the base fluid, D_B is the solute Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, D_n is the Brownian diffusion coefficient of the microbe, k_r^2 is the chemical reaction rate constant, $T=(T/T_\infty)^n\exp(-E_a/\kappa T)$ is Arrhenius function, n is a constant exponent, E_a is the activation energy, b is the chemotaxis constant, W_c is the maximum cell swimming speed (bW_c is assumed to be constant), D_n is the diffusivity of the micro-organism, k_f is the conductivity of the solid, h_f is the convective heat transfer coefficient and k_m^2 is the reaction rate constant of the microbe.

3. Transforming the equations

We introduce the following dimensionless transformation to reduce the system given by eqs (1)–(5) together with boundary conditions (eq. (6)) to a system of ordinary differential eqautions:

$$\psi=\sqrt{c\nu}xf(\eta),\quad \eta=y\sqrt{\frac{c}{\nu}},$$
$$\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_W-C_\infty},$$
$$\chi(\eta)=\frac{n-n_\infty}{n_W-n_\infty}. \tag{7}$$

The system of equations (1)–(5) together with the boundary condition are transformed to the boundary value problem given by (8)–(11) and (12) respectively via the similarity transformation method.

$$f'''+ff''-f'^2-\frac{M}{1+N^2}f'+\frac{Gr}{Re^2}[\theta-Nr\phi-Rb\chi]=0, \tag{8}$$

$$\left(1+\frac{4}{3}Rd\right)\theta''+Pr\left[f\theta'+Ecf''^2+Nb\theta'\phi'+Nn\theta'\chi'+Nt\theta'^2\right]=0, \tag{9}$$

$$\phi''+Sc\left[f\phi'-\lambda^2(1+n\epsilon\theta)\exp\left(-\frac{E}{1+n\epsilon\theta}\right)\phi\right]+\frac{Nt}{Nb}\theta''=0, \tag{10}$$

$$\chi''+Sbf\chi'-Pb\left[\phi'\chi'+(\tau_0+\chi)\phi''\right]-Sb\lambda_n^2(1+n\epsilon\theta)\exp\left(-\frac{E}{1+n\epsilon\theta}\right)\chi+\frac{Nt}{Nn}\theta''=0. \tag{11}$$

$$f'(0)=1,\ f(0)=0,\ f'(\infty)\to 0,\ f''(\infty)\to 0,$$
$$\theta'(0)=-Bi\,(1-\theta(0)),\ \theta(\infty)\to 0,$$

$$Nb\phi'(0) + Nt'(0) = 0,\ \phi(\infty) \to 0,$$
$$\chi(0) = 1,\ \chi(\infty) \to 0. \quad (12)$$

The prime denotes differentiation with respect to η. The parameters in eqs (8)–(12) are given as follows: M is the magnetic field parameter (measures the applied magnetic force), N is the Hall parameter (ratio of electromagnetic forces to viscous forces), *Re* is the Reynolds number (ratio of inertial forces to viscous forces), *Gr* is the Grashof number (ratio of the buoyancy to viscous force), *Nr* is the buoyancy ratio parameter (ratio of thermal buoyancy to solute buoyancy), *Rb* is the bioconvection Rayleigh number (ratio of thermal transport due to micro-organisms to thermal transport via convection), *Rd* is the thermal radiation (relative contribution of conduction heat transfer to thermal radiation transfer), *Pr* is the Prandtl number (ratio of momentum diffusivity to thermal diffusivity), *Ec*, the Eckert number, expresses the relationship between a flow's kinetic energy and the boundary layer enthalpy difference, *Nb* is the Brownian motion parameter, that measures the random movement of particles in a fluid, *Nt* is the thermophoresis parameter that is a measure of the movement of microscopic particles due to a force of a temperature gradient, *Sc* is the Schmidt number (ratio of momentum diffusivity to mass diffusivity), λ^2 and λ_n^2 are the dimensionless chemical and microbe reaction parameters, ϵ is the temperature relative parameter, E is the dimensionless activation energy, *Sb* is the biconvection Schmidt number (ratio of momentum diffusivity to microbes diffusivity), *Pb* is the biconvection Peclet number (ratio of the movement of microbes as a result of swimming to movement of microbes due to diffusion, τ_0 is the constant micro-organism concentration difference parameter, *Nn* is the bioconvection Brownian motion parameter, which measures the random movement of microbes in a fluid and *Bi* is the Biot number (ratio of the heat transfer resistances inside and at the surface of a body). The parameters used in eqs (8)–(12) are defined as

$$M = \frac{\sigma B_0^2}{c\rho_{f\infty}}, \quad Re = \frac{cx^2}{\nu},$$
$$Gr = \frac{(1 - C_\infty)\rho_\infty g\beta_T(T_w - T_\infty)x^3}{\nu^2},$$
$$Nr = \frac{(\rho_p - \rho_{f\infty})\beta_C\Delta C}{(1 - c_\infty)\rho_{f\infty}^2\beta_T\Delta T},$$
$$Rb = \frac{(\rho_n - \rho_{f\infty})\gamma\Delta n}{(1 - c_\infty)\rho_{f\infty}^2\beta_T\Delta T},$$
$$Pr = \frac{\nu}{\alpha}, \quad Ec = \frac{(cx)^2}{c_p\Delta T},$$
$$Nb = \frac{\tau D_B\Delta C}{\nu}, \quad Nt = \frac{\tau D_T\Delta T}{\nu T_\infty},$$
$$Sc = \frac{\nu}{D_B}, \quad \lambda^2 = \frac{kr^2}{c},$$
$$E = \frac{E_a}{\kappa T_\infty}, \quad Sb = \frac{\nu}{D_n}, \quad Pb = \frac{bW_c}{D_n},$$
$$\tau_0 = \frac{n_\infty}{\Delta n}, \quad Nn = \frac{\tau D_n\Delta n}{\nu},$$
$$\lambda_m^2 = \frac{k_m^2}{c}, \quad Rd = \frac{4\sigma^* T_\infty^3}{\rho_{f\infty}\alpha c_p},$$
$$\epsilon = \frac{\Delta T}{T_\infty}, \quad Bi = \frac{h_f}{\kappa_f}\frac{x}{\sqrt{Re}}. \quad (13)$$

4. Momentum, heat and mass transfer coefficients

To obtain some insights into the impact of some key parameters on transfer enhancement we calculate some quantities of physical interest. In particular, we determine the local skin friction C_f, which measures the shear stress on the surface, the Nusselt number, Nu, which is the ratio of convective to conductive heat transfer across (normal) the boundary, the Sherwood number, Sh, which is the ratio of the convective to diffusive mass transport and the local density number of motile micro-organisms, Nm, which is the ratio of convective micro-organisms to the diffusive microorganism transport. The local skin friction coefficient is given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_w^2} = -2Re^{-1/2}f''(0), \quad (14)$$

the local Nuselt number is given as

$$Nu_x = \frac{xq_w}{\kappa\,(T_w - T_\infty)}$$
$$= -\left(1 + \frac{4}{3}Rd\right)Re^{1/2}\theta'(0), \quad (15)$$

the Sherwood number is given by

$$Sh_x = \frac{xq_c}{D_B\,(C_w - C_\infty)} - Re^{1/2}\phi'(0) \quad (16)$$

and the local density number of the motile micro-organisms is given by

$$Nm_x = \frac{xq_m}{D_n\,(n_w - n_\infty)} - Re^{1/2}\chi'(0), \quad (17)$$

where

$$\tau_w = -\mu\frac{\partial u}{\partial y}\Big|_{y=0},$$
$$q_w = -\left(\kappa + \frac{16\sigma^8 T_\infty^3}{3\rho_\infty c_p}\right)\frac{\partial T}{\partial y}\Big|_{y=0},$$
$$q_c - D_B\frac{\partial C}{\partial y}\Big|_{y=0}, \quad q_m - D_n\frac{\partial n}{\partial y}\Big|_{y=0}. \quad (18)$$

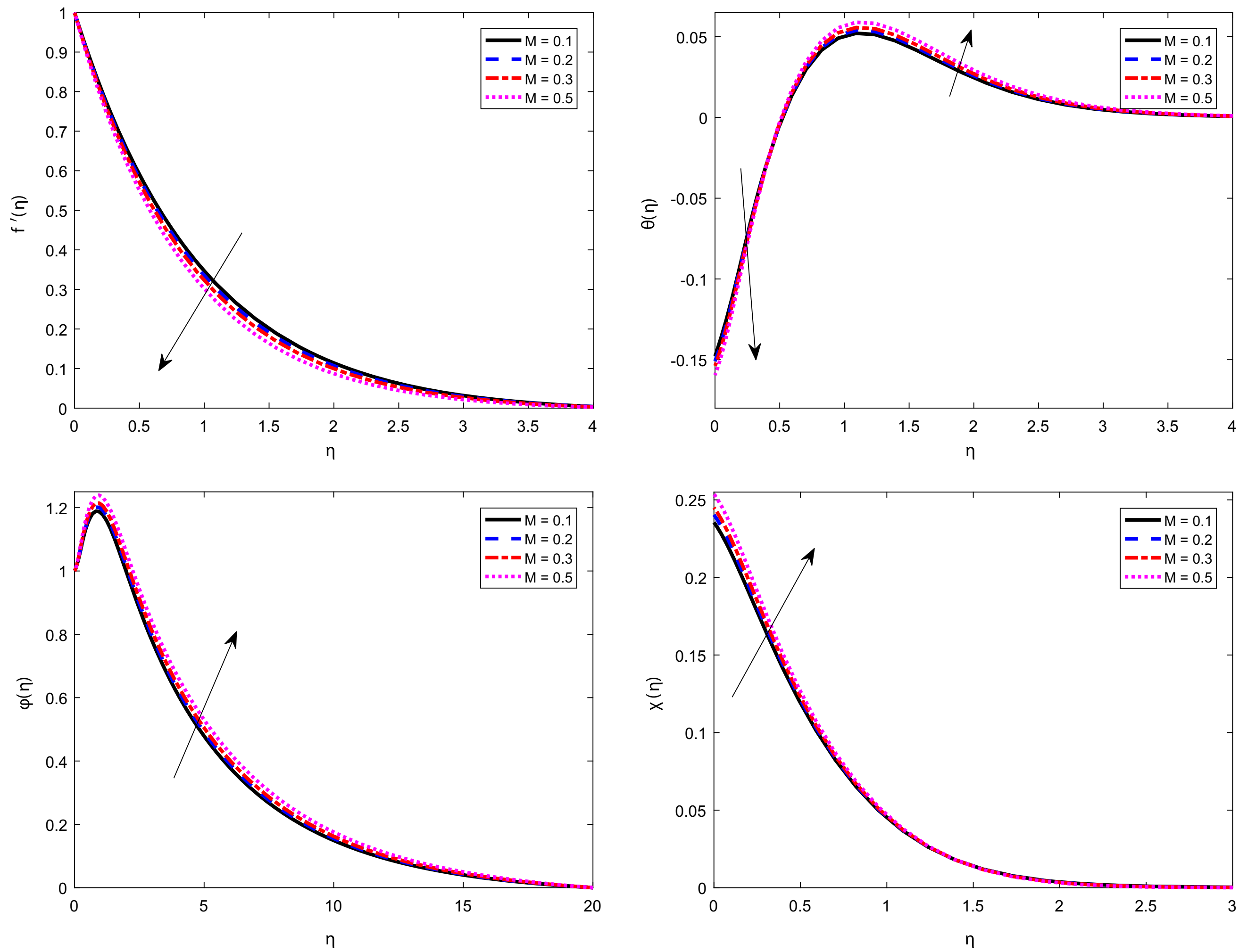

Figure 1. Effect of magnetic field parameter on the fluid and the flow properties.

5. Numerical solution using the spectral quasi-linearisation method

The nonlinear system of equations (8)–(11) together with its associated boundary condition given in eq. (12) are solved numerically using the spectral quasi-linearisation method. The spectral method offers some advantages over some standard numerical techniques, e.g., the homotopy method by Liao [36] is said to have a low convergence rate and has a limited region where solutions are valid [37,38]. The accuracy of the finite-difference method improves with increasing grid points. This has the downside that more computational resources are required, thus making the method slow [38,39]. Also, the finite methods are numerically unstable in solving nonlinear systems [40]. Stuart [40] attributes this to the bifurcation of periodic orbits. A spectral method is used because spectral-based methods, give highly accurate results despite using fewer grid points. This reduces the computational time and resources compared to other methods [37,39]. The domain given by $\eta \in [0, L_x]$ is transformed to a bi-unit interval $x \in [-1, 1]$ using the linear transformation $\eta \in L_x [x + 1]/2$. The solution is approximated in terms of a series solution using the Lagrange interpolating polynomials of the form

$$u(x) = \sum_{i=0}^{N_x} \hat{u}(x_i) L_i(x), \tag{19}$$

where $u(x)$ is interpolated at selected grid points x_i. The grid points are generated using the Gauss–Lobatto quadrature defined as

$$x_i = \left\{ \cos \frac{\pi i}{N_x} \right\}_{i=0}^{N_x}. \tag{20}$$

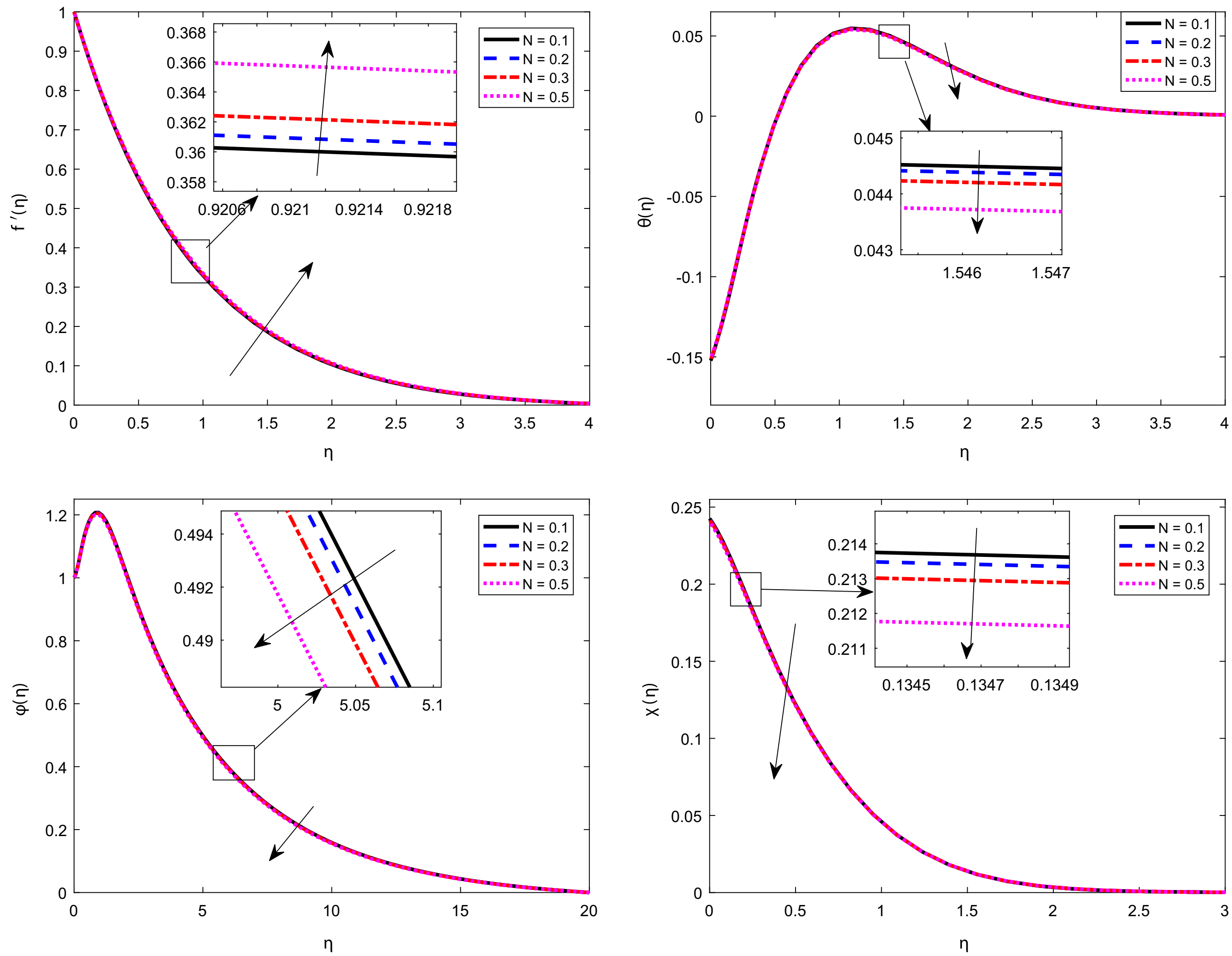

Figure 2. Effect of Hartmann number on the velocity, temperature, solute concentration and microbes' concentration.

The functions $L_i(x)$ are the Lagrange cardinal polynomials given by

$$L_i(x) = \prod_{i=0, i\neq k}^{N_x} \frac{x - x_i}{x_k - x_i}. \tag{21}$$

We define the functions F, Θ, Φ and ξ for eqs (8)–(11) as

$$F = f''' + ff'' - f'^2 - \frac{M}{1+N^2} f' + \frac{Gr}{Re^2}\left[\theta - Nr\phi - Rb\chi\right], \tag{22}$$

$$\Theta = \left(1 + \frac{4}{3}Rd\right)\theta'' + Pr\left[f\theta' + Ecf''^2 + Nb\theta'\phi' + Nn\theta'\chi' + Nt\theta'^2\right], \tag{23}$$

$$\Phi = \phi'' + Sc\left[f\phi' - \lambda^2(1+n\epsilon\theta)\exp\left(-\frac{E}{1+\epsilon\theta}\right)\phi\right] + \frac{Nt}{Nb}\theta'', \tag{24}$$

$$X = \chi'' + Sbf\chi' - Pb\left[\phi'\chi' + (\tau_0 + \chi)\phi''\right] - Sb\lambda_n^2(1+n\epsilon\theta)\exp\left(-\frac{E}{1+\epsilon\theta}\right)\chi + \frac{Nt}{Nn}\theta''. \tag{25}$$

We proceed to construct the iterative formula for the system given by eqs (8)–(11).

$$a_{0r} f'''_{r+1} + a_{1r} f''_{r+1} + a_{2r} f'_{r+1} + a_{3r} f_{r+1} + a_{4r}\theta_{r+1} + a_{5r}\phi_{r+1} + a_{6r}\chi_{r+} - F = R_f, \tag{26}$$

$$b_{0r}\theta''_{r+1} + b_{1r}\theta'_{r+} + b_{2r} f''_{r+1} + b_{3r} f_{r+1} + b_{4r}\phi'_{r+1} + b_{5r}\chi'_{r+1} - \Theta = R_\theta, \tag{27}$$

$$c_{0r}\phi''_{r+1} + c_{1r}\phi'_{r+1} + c_{2r}\phi_{r+1} + c_{3r} f_{r+1} + c_{4r}\theta''_{r+1} + c_{5r}\theta_{r+1} - \Phi = R_\phi, \tag{28}$$

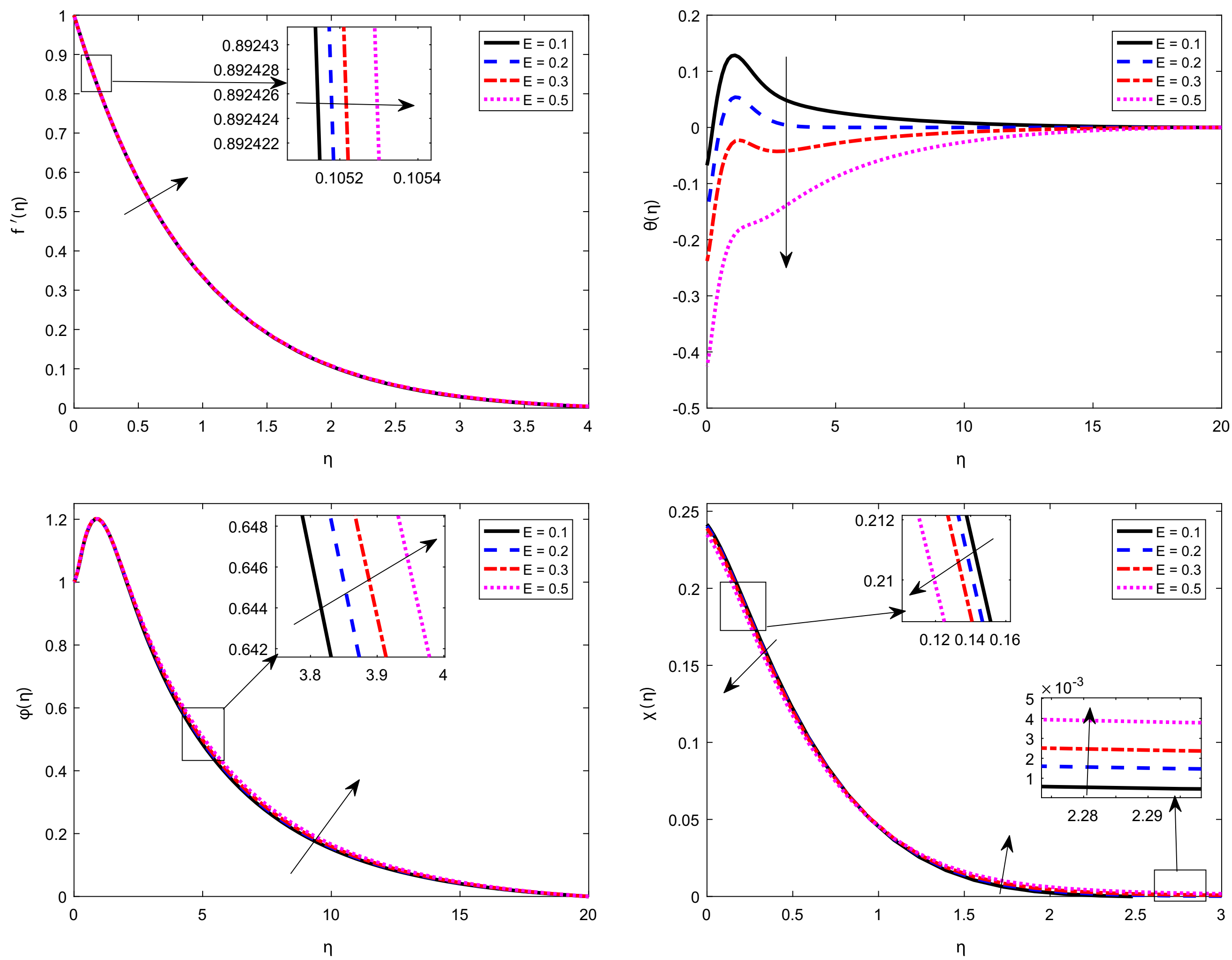

Figure 3. Effect of activation energy on the velocity, temperature, solute concentration and microbes' concentration.

$$d_{0r}\chi''_{r+1}+d_{1r}\chi'_{r+1}+d_{2r}\chi_{r+1}+b_{3r}f_{r+1}+d_{4r}\theta''_{r+1}+d_{5r}\theta_{r+1}+d_{6r}\phi''_{r+1}+d_{7r}\phi'_{r+1}-X=R_X. \tag{29}$$

The boundary conditions in eq. (12) produce the iterative process given as

$$\begin{aligned}
&f'_{r+1}(0)=1,\ \ f_{r+1}(0)=0,\\
&f'_{r+1}(\infty)\to 0,\ \ f''_{r+1}(\infty)\to 0\\
&\theta'_{r+1}(0)=-Bi\,(1-\theta_{r+1}(0)),\ \ \theta_{r+1}(\infty)\to 0\\
&Nb\phi'_{r+1}(0)+Nt\theta'_{r+1}(0)=0,\ \ \phi_{r+1}(0)\to 0\\
&\chi_{r+1}(0)=1,\ \ \chi_{r+1}(\infty)\to 0.
\end{aligned} \tag{30}$$

The coefficients in eqs (26)–(29) are given as

$$a_{0r}=1,\ \ a_{1r}=f_r,\ \ a_{2r}=f''_r,$$

$$a_{3r}=-2f'_r-\frac{M}{1+N^2},\ a_{4r}=\frac{Gr}{Re^2},$$

$$a_{5r}=-\frac{Gr\,Nr}{Re^2},\ \ a_{6r}=-\frac{Gr\,Rb}{Re^2},$$

$$b_{0r}=\left(1+\frac{4}{3}Rd\right),$$

$$b_{1r}=Pr\left[f_r+Nb\phi'_r+Nn\chi'_r+2Nt\theta'_r\right],$$

$$b_{2r}=2Pr\,Ecf''_r,\ b_{3r}=Pr\theta'_r,$$

$$b_{4r}=Pr\,Nb\theta'_r,\ b_{5r}=Pr\,Nn\theta'_r,$$

$$c_{0r}=1,\ c_{1r}=Scf_r,$$

$$c_{2r}=-Sc\lambda^2\,(1+n\epsilon\theta_r)\exp\left(-\frac{E}{1+n\epsilon\theta}\right),$$

$$c_{3r}=Sc\phi'_r,\ \ c_{4r}=\frac{Nt}{Nb},$$

$$c_{5r}=-\frac{Sc\lambda^2 n\epsilon\,(1+E+n\epsilon\theta)\exp\left(-E/(1+n\epsilon\theta)\right)}{1+n\epsilon\theta}\phi_r,$$

$$d_{0r}=1,\ d_{1r}=Sbf_r-Pb\phi'_r$$

$$d_{2r}=-Pb\phi''-Sb\lambda_n^2\,(1+n\epsilon\theta)\exp\left(-E/(1+n\epsilon\theta)\right),$$

$$d_{3r}=Sb\chi'_r,\ d_{4r}=\frac{Nt}{Nn}$$

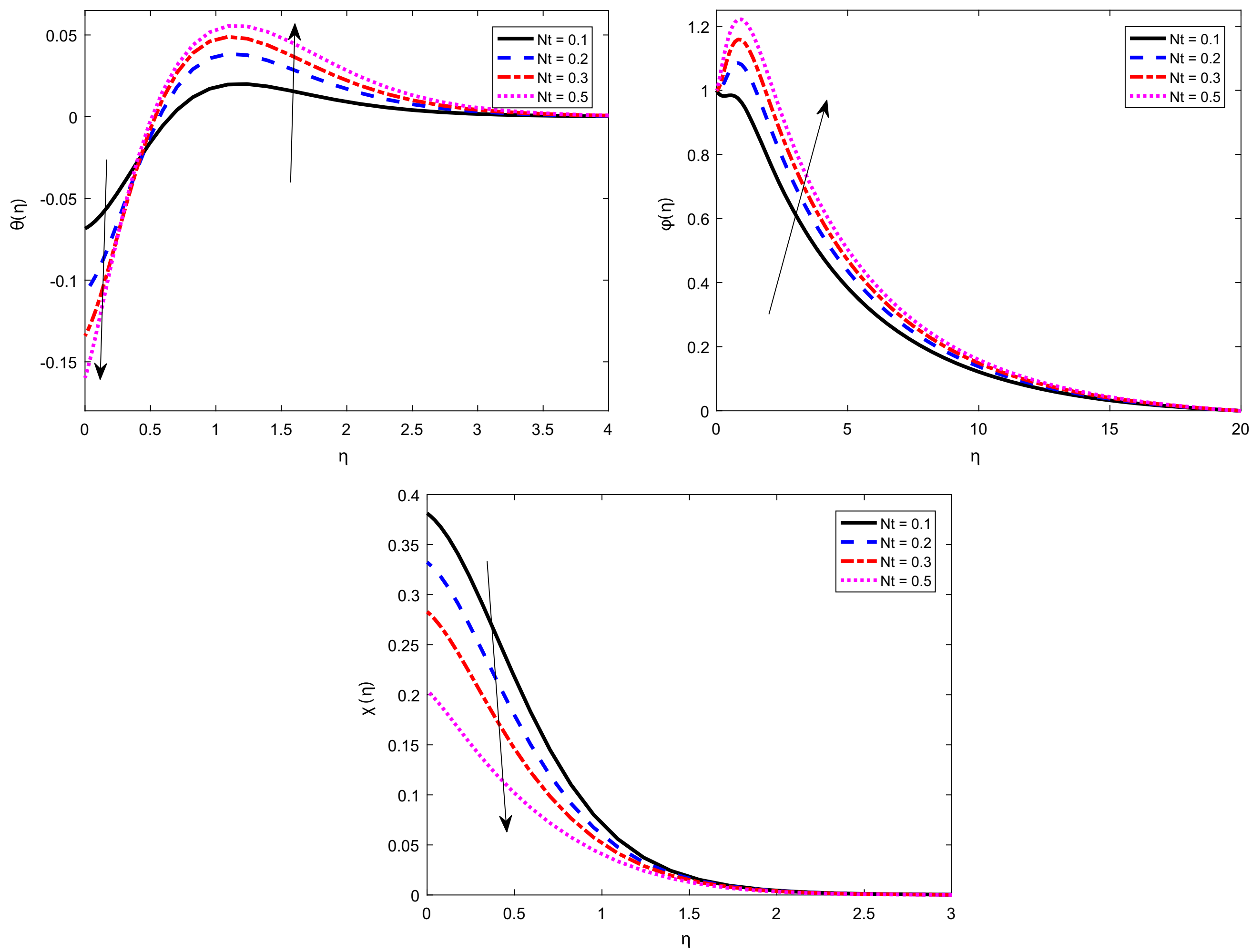

Figure 4. Effect of thermophoresis on the velocity, temperature, solute concentration and microbes' concentration.

$$d_{5r} = -\frac{Sb\lambda_n^2 n\epsilon\,(1+E+n\epsilon\theta)\exp\left(-E/(1+n\epsilon\theta)\right)}{1+n\epsilon\theta}\chi_r,$$
$$d_{6r} = -Pb\,(\tau_w + \chi_r)\,,\; d_{7r} = -Pb\chi_r'. \tag{31}$$

To run the numerical scheme, we use the following initial guess function for $f_0(\eta)$, $\theta_0(\eta)$, $\phi_0(\eta)$ and $\chi_0(\eta)$,

$$f_0(\eta) = 1 - \mathrm{e}^{-\eta},\; \theta_0(\eta) = \frac{Bi}{1+Bi}\mathrm{e}^{-\eta},$$
$$\phi_0(\eta) = -\frac{Nt}{Nb}\frac{Bi}{1+Bi}\mathrm{e}^{-\eta},$$
$$\chi_0(\eta) = \mathrm{e}^{-\eta}. \tag{32}$$

For further details and explanation on how the resulting matrices and the corresponding boundary conditions are implemented on a MATLAB code, one can refer to [37, 39,41].

6. Results and discussion

In this section, we discuss key findings of this research. We determine the fluid velocity, heat, solute concentration and and microbes' concentration against certain key parameters to find out how they are influenced by the fluid and material parameters.

In figure 1 we show the effect of magnetic parameter on the fluid and flow properties. Application of a magnetic field to an electrically conducting fluid produces a drag force called the Lorentz force. This leads to a reduction in the fluid velocity as seen in figure 1. The solute concentration, temperature and microbes' concentration increase in the boundary layer. Our results are consistent with the results in [5,42,43].

Figure 2 shows the effect of Hall parameter on the velocity, temperature, solute concentration and microbes' concentration. The velocity increases with increasing values of Hall number. This is attributed to the fact that the magnetic field on the velocity damping reduces as the Hall parameter is increased. This result

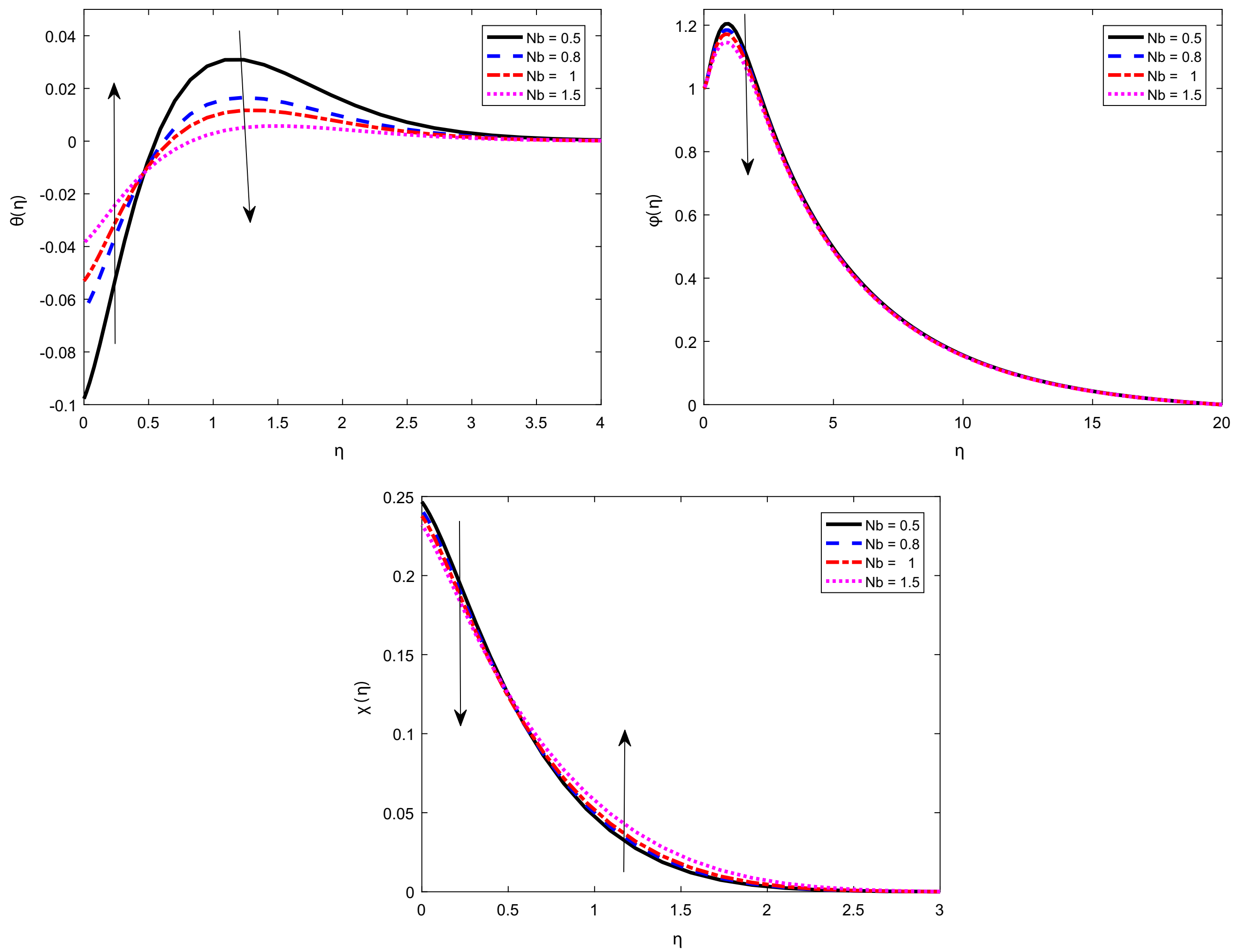

Figure 5. Effect of solute Brownian motion on the temperature, solute concentration and microbes' concentration.

is consistent with results in [10,44,45]. Other scholars, however, have reported results that show an opposite effect for the Hall parameter on the velocity profile [46, 47]. Temperature, solute concentration and microbes' concentration decrease in the boundary layer because when the fluid velocity increases these are pushed out of the boundary layer by the fast-moving fluid.

In figure 3 we investigate the effect of activation energy. Increasing the activation energy increases the solute in the boundary layer. The result on solute is in agreement with results in [4,29,48]. The temperature decreases with increasing activation energy and our results are in agreement with the results in the literature (e.g., see [4,48]). According to our results, an increase in activation energy is associated with a decrease in the microbes' concentration. Since the velocity increases with an increase in activation energy, this result is also reported by Dhlamini *et al* [4]. The microbes are pushed out of the boundary layer resulting in the observed decrease. Further into the free stream, the fluid velocity decreases coupled with the decrease in the microbes' mobility, the concentration of microbes increases. Soo and Theriot [49] showed experimentally that activation energy is correlated to microbes mobility. They showed that the speed at room temperature decreases exponentially with activation energy E by a factor of 20 across all four conditions in their experiment. Thus, they concluded that bacteria with high activation energy E move more slowly than those with low activation energy. For this phenomenon, they suggest that the differences associated with activation energy originate from biological variation among the bacteria themselves rather than from the imprecision in the measurements.

In figure 4 we explore the impact of thermophoresis. Our current results on the solute concentration are consistent with results in the literature (e.g., see [4,5,29]). However, there is a deviation when it comes to the temperature profile. As opposed to the results in literature, our current result shows a decrease in temperature for a small region close to the wall and then an

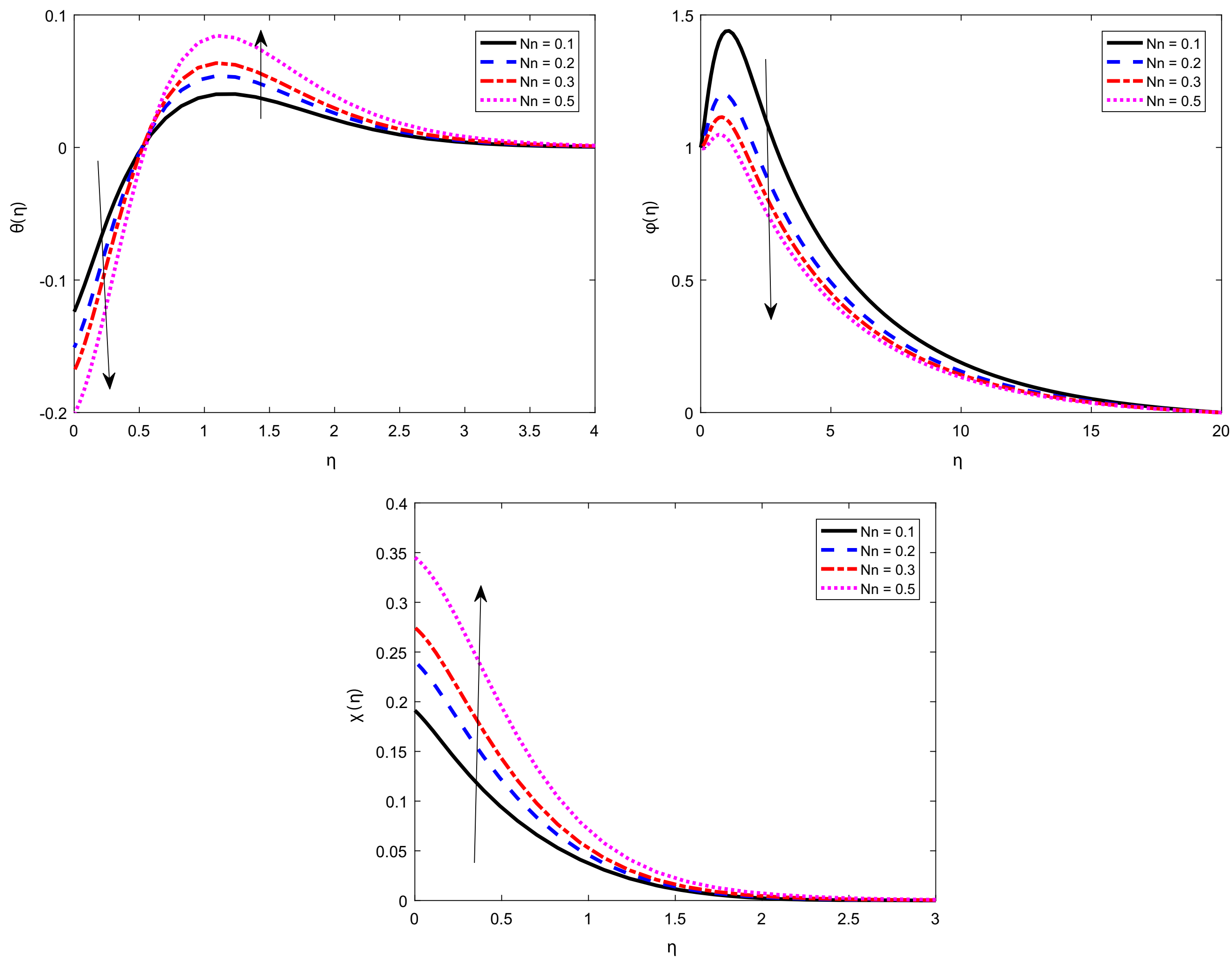

Figure 6. Effect of microbes' Brownian motion on the temperature, solute concentration and microbes' concentration.

increase in temperature further from the wall. The thermophoretic parameter is also shown to be significant for the concentration of the microbes unlike the previous studies such as [9,10,18,25,50,51] which did not report any significant effect of thermophoresis on microbes. The concentration of microbes decreases with an increase in the thermophoretic parameter. This could be due to a net force pushing the microbes out of the boundary layer, suggesting that the thermophoretic term is significant in the microbes' equation.

In figure 5 we approach the impact of solute Brownian motions terms. The solute Brownian motion causes a decrease in the solute concentration. An increase in the Brownian motion parameter increases the random movement of solute particles. This causes the warming of the boundary layer which effectively causes nanoparticles to move away from the surfaces inside the inactive fluid. This increases the deposition of the solute particles away from the surface, leading to the reduction of the concentration near the surface [52]. Similar results are reported in [4,10,53]. The temperature decreases in the boundary layer near the surface and then a reverse effect that is consistent with results in [4,53,54] is observed far from the surface. Although the mechanism is not known, the concentration of microbes is shown to decrease with increasing values of solute Brownian motion close to the surface and a reverse effect is noticed further from the surface towards the free stream.

In figure 6 we study the effect of Brownian motion parameter of the microbes. The temperature and solute profiles are similar to those observed for solute Brownian motion. The mechanism for these observations is not known. The Brownian motion of the microbes increases the concentration of the microbes in the boundary layer. This is a new result that has not been reported in any article to the best of our knowledge. We suspect that the Brownian motion of microbes would result in some microbes randomly moving back to the boundary layer as opposed to the movement due to stimulus. The net outward movement from the boundary layer is reduced compared to cases where no such movement is present.

The other parameters were found to be consistent with results in literature.

7. Conclusion

In this paper, we developed a model for the flow of a nanofluid with activation energy for solute and microbes. Microbe Brownian motion, activation energy and thermophoresis have not been studied before. The inclusion of Brownian motion, the thermophoretic force for microbes and the activation energy produced new and interesting flow dynamics. The following key observations were made in this study:

- the solute Brownian motion has an effect on the dynamics of the microbes, a finding that had not been established in previous studies,
- the microbe Brownian motion increases the microbe concentration in the boundary layer, whereas the solute Brownian motion reduces the solute in the boundary layer,
- the microbe Brownian motion reduces the solute concentration in the boundary layer,
- thermophoresis reduces the microbe concentration in the boundary layer, something that previous studies did not establish, and
- activation energy is also shown to reduce the concentration of microbes in the boundary layer close to the stretching surface.

References

[1] Y Yu, *J. Geophys. Res. Solid Earth* **116**, B9 (2011)
[2] T Orikasa *et al*, *Acta Hortic.* **1091**, 319 (2015)
[3] M A J S Van Boekel, *Compr. Rev. Food Sci. Food Saf.* **7**, 144 (2008)
[4] M Dhlamini, H Mondal, P Sibanda and S Motsa, *Int. J. Ambient Energy* **43**, 1 (2020)
[5] M Dhlamini, H Mondal, P Sibanda and S Motsa, *J. Nanofluids* **7**, 917 (2018)
[6] M Dhlamini, H Mondal, P Sibanda and S Motsa, *Int. J. Appl. Comput. Math.* **7**, 1 (2021)
[7] C Maradiya, J Vadher and R Agarwal, *Beni-Suef Univ. J. Basic Appl. Sci.* **7**, 1 (2018)
[8] S U S Choi and J A Eastman, Enhancing thermal conductivity of fluids with nanoparticles, Tech. Rep., Argonne National Lab., IL (United States, 1995)
[9] W N Mutuku and O D Makinde, *Comput. Fluids* **95**, 88 (2014)
[10] M Dhlamini, H Mondal, P Sibanda and S Motsa, *J. Cent. South Univ.* **27**, 824 (2020)
[11] R S Shawgo, A C R Grayson, Y Li and M J Cima, *Curr. Opin. Solid State Mater. Sci.* **6**, 329 (2002)
[12] R Taylor, S Coulombe, T Otanicar, P Phelan, A Gunawan, W Lv, G Rosengarten, R Prasher and H Tyagi, *J. Appl. Phys.* **113**, 1 (2013)
[13] P Keblinski, R Prasher and J Eapen, *J. Nanoparticle Res.* **10**, 1089 (2008).
[14] W Evans, R Prasher, J Fish, P Meakin, P Phelan and P Keblinski, *Int. J. Heat Mass Transf.* **51**, 1431 (2008)
[15] J Sarkar, P Ghosh and A Adil, *Renew. Sustain. Energy Rev.* **43**, 164 (2015)
[16] I Tlili, *Math. Sci.* (2021), https://doi.org/10.1007/s40096-021-00377-6
[17] T Rasheed, T Hussain, M T Anwar, J Ali, K Rizwan, M Bilal, F H Alshammari, N Alwadai and A S Almuslem, *Front. Chem.* **9** (2021), https://doi.org/10.3389/fchem.2021.737033
[18] A V Kuznetsov, *Theor. Comput. Fluid Dyn.* **26**, 291 (2012)
[19] S U Khan, K Al-Khaled, A Aldabesh, M Awais and I Tlili, *Sci. Rep.* **11**, 1 (2021)
[20] T J Pedley, *J. Fluid Mech.* **762** (2015), https://doi.org/10.1017/jfm.2014.666
[21] T J Pedley, N A Hill and J O Kessler, *J. Fluid Mech.* **195**, 223 (1988)
[22] H G Lee and J Kim, *Eur. J. Mech. B/Fluids* **52**, 120 (2015)
[23] A V Kuznetsov, *Nanoscale Res. Lett.* **6**, 1 (2011)
[24] S Ghorai and N A Hill, *J. Fluid Mech.* **400**, 1 (1999)
[25] M A Bees and N A Hill, *J. Math. Biol.* **38**, 2, 135 (1999)
[26] S M Sauid, J Krishnan, T H Ling and M V P S Veluri, *BioMed Res. Int.* **2013** (2013), https://doi.org/10.1155/2013/409675
[27] N J R Kraakman, J Rocha-Rios and M C M van Loosdrecht, *Appl. Microbiol. Biotechnol.* **91**, 873 (2011)
[28] K Maleque *et al*, *J. Thermodyn.* **2013** (2013), https://doi.org/10.1155/2013/692516
[29] M Dhlamini, P K Kameswaran, P Sibanda, S Motsa and H Mondal, *J. Comput. Des. Eng.* **6**, 2, 149 (2019)
[30] M Tencer, J S Moss and T Zapach, *IEEE Trans. Compon. Packag. Technol.* **27**, 602 (2004)
[31] H Precht, *Temperature and life* (Springer, 2013)
[32] W W Forrest, *J. Bacteriol.* **94**, 1459 (1967)
[33] T Ross and D S Nichols, *Encyclopedia of food microbiology* (Elsevier, 2014) pp. 602–609
[34] M M Bhatti, A Shahid, T Abbas, S Z Alamri and R Ellahi, *Processes* **8**, 328 (2020)
[35] H Waqas, S U Khan, S A Shehzad, M Imran and I Tlili, *Appl. Nanosci.* **10**, 4445 (2020)
[36] S Liao, *Beyond perturbation: Introduction to the homotopy analysis method* (CRC Press, 2003)
[37] S S Motsa, V M Magagula and P Sibanda, *Sci. World J.* **2014**, 581987 (2014)
[38] M Dhlamini, *A mathematical study of boundary layer and fluid flow using spectral quasilinearization methods* (2020), https://doi.org/10.13140/RG.2.2.11546.98246

[39] M S Sydney, S F Mutua and S Stanford, *Nonlinear systems: Design, analysis, estimation and control* (2016), https://doi.org/10.5772/64600
[40] A Stuart, *SIAM Rev.* **31**, 191 (1989)
[41] S S Motsa, P G Dlamini and M Khumalo, *Adv. Math. Phys.* **2014** (2014)
[42] A J Omowaye, A I Fagbade and A O Ajayi, *J. Niger. Math. Soc.* **34**, 343 (2015)
[43] S Misra and G Kamatam, *Beilstein J. Nanotechnol.* **11**, 976 (2020)
[44] A M Salem and M A El-Aziz, *Appl. Math. Model.* **32**, 1236 (2008)
[45] S Shateyi, *Math. Probl. Eng.* **2010**, 627475 (2010)
[46] H S Takhar, A J Chamkha and G Nath, *Int. J. Eng. Sci.* **40**, 13, 1511 (2002)
[47] S Noreen and M Qasim, *PLoS ONE* **10**, e0129588 (2015)
[48] Z Abbas, M Sheikh and S S Motsa, *Energy* **95**, 12 (2016)
[49] F S Soo and J A Theriot, *Proc. Natl. Acad. Sci.* **102**, 16233 (2005)
[50] S Shaw, S S Motsa and P Sibanda, *Heat Transf. Asian Res.* **47**, 718 (2018)
[51] B Mallikarjuna, A M Rashad, A Chamkha and M M M Abdou, *Front. Heat Mass Transf.* **10** (2018), https://doi.org/10.5098/hmt.10.21
[52] R Goyal and R Bhargava, arXiv:1708.05609 (2017)
[53] F Mabood, S M Ibrahim and W A Khan, *Results Phys.* **6**, 1015 (2016)
[54] A Aldabesh, M Hussain, N Khan, A Riahi, S U Khan and I Tlili, *Case Stud. Therm. Eng.* **23**, 100820 (2021)