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An improved efficient routing strategy on two-layer networks

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Abstract. In recent years, enhancing the network capacity of complex networks has attracted wide attention. In reality, there are two-layer networks with logical-layer and physical-layer structures. It is meaningful to do research on two-layer networks. We proposed an improved efficient routing (IER) strategy to alleviate traffic congestion in two-layer networks. The static information and tunable parameter α of the two-layer network were considered in our routing strategy. Under our routing strategy, by changing the parameter α, the two-layer networks can achieve maximum traffic capacity. The simulation results showed that compared with the shortest path (SP) routing strategy and improved static weighted routing (ISWR) strategy, the proposed routing strategy can significantly improve the network traffic capacity.

Keywords. Two-layer network; traffic capacity; routing strategy.

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1. Introduction

Complex network plays an important role in our real life, which promotes the development of society. With the discovery of small-world effects [1] and scale-free properties [2–4] of complex networks, researchers have conducted more in-depth research on complex networks [5–8]. With the rapid development of science and technology, more and more information needs to be transmitted, the network scale is also growing, and the problem of network congestion is becoming more and more complex. How to improve the network capacity of complex networks has become a hot issue for researchers [9]. At present, the research on this problem mainly has two aspects, optimising the structure of complex networks and designing more efficient routing strategies. But for many realistic networks, it is more feasible to design efficient routing strategies.

Previous studies have shown that the traditional shortest path (SP) routing strategy of scale-free networks usually leads to network congestion, because a number of packets are transmitted through the hub node with a high degree of intermediateness [10–17]. The SP routing strategy has been applied in many networks [18,19]. Scholars have proposed many new routing strategies to solve network congestion [20–23]. Naganuma and Igarashi proposed a connection weight adjustment rule and a routing strategy using neural networks [24]. Kawamoto and Igarashi proposed an effective routing strategy that relieves traffic congestion on complex networks by minimising intermediate intervals [25]. Most of the previous researches are only on one-layer networks.

The researchers, after continuous studies, discovered that the real network has many sub-layer networks, that is, the real network has a multilayer network structure [26–35]. Arenas *et al* found that under the same routing strategy, the underlying network structure affects the size of the network capacity [36]. Boccaletti *et al* found that under the same routing strategy, the more evenly distributed the network topology, the larger the network capacity [4]. Gao *et al* proposed a two-layer traffic assignment strategy. Under this routing strategy, the packet transmission speeds of different layers are different [37]. Zhuo *et al* proposed a two-layer routing strategy. The weight of the logical-layer edge is set according to the degree of the physical-layer node, and then the path with the smallest weight on the logical-layer is taken to send the packets [28]. Ma *et al* improved the routing strategy proposed by Zhuo *et al* to significantly increase network capacity on two-layer networks [38]. Zhang *et al* proposed a new multilayer network

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model, which is composed of high-speed and low-speed layers. It can significantly increase the network capacity [39].

Multilayer network structure can significantly improve network capacity. So we proposed an improved efficient routing (IER) strategy based on two-layer networks. Under our routing strategy, the probability of a packet passing through a node can be calculated by the degree of this node. The product of probability multiplication is the most optimal path, so that packet distribution can become more uniform.

The paper is organised as follows. The traffic model and two-layer network structure are introduced in §2. IER strategy is introduced in §3. In §4, the effectiveness of three routing paths (IER strategy, improved static weighted routing (ISWR) strategy and SP strategy) for some key indicators are compared with simulation results. Conclusion is given in §5.

2. The models

2.1 *Two-layer network model*

In the two-layer network, the structure is divided into logical layer and physical layer, and logical layer is the upper layer, while physical layer is the lower layer. The effective link of the logical layer is determined by the routing table of the physical layer. The entire network routing set is logically determined according to the logical path between the logical-layer nodes, and the routing table for the entire network to transmit packets is jointly determined by the two layers [40,41].

To describe the dynamic behaviour of traffic in a two-layer network, the packet flow process in a two-layer network model is studied. Packet traffic process is shown in figure 1, assuming that both the logical layer and the physical layer adopt SP routing strategy. In the logical layer, the routing path from node 1 to node 4 is $\mathrm{RP}_{1,4} = \{1, 2, 3, 4\}$. The logical-layer path maps to the physical layer corresponding to {1,6,2}, {2,5,3} and {3,4}. In the physical layer, the path from the source node to the destination node is $\mathrm{RP}'_{1,4} = \{1, 6, 2, 5, 3, 4\}$. The transmission of actual packets in the physical layer is restricted by the physical layer and the logical layer.

2.2 *Traffic model*

Nodes in the network are set up by the physical and logical layers as routers that can not only receive and send packets, but also act as packet forwarding intermediaries. Assuming that the logic layer randomly generates R packets in each time step, the packets are placed at the end of the node queue to wait for transmission. Each

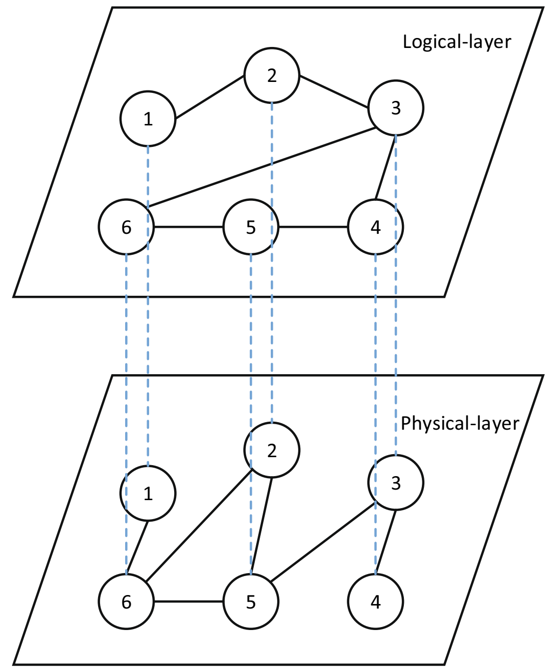

Figure 1. Two-layer network structure model. The upper layer is the logical layer, the lower layer is the physical layer. The logical layer is mapped on the physical layer.

node is processed using the first-in-first-out (FIFO) policy. Logical-layer node plays the function of routing during transmission. When the logical layer generates a packet, it will find the corresponding path between the source node and the destination node according to the set routing strategy. If there are multiple paths, one of them will be randomly selected for transmission. When the packet reaches the destination node, transmission will not be carried out.

Packets are sent from one logical end to the other end based on the physical node routing strategy at the time of the established primary function. When the packet reaches the last node of the logical link, the logical layer will select the next link until the packet reaches its destination. The information capacity of the entire network is determined by the processing capacity of nodes at the physical layer. If the processing capacity of a node is set as C, when the packets waiting in the queue of physical nodes exceed C (we set $C = 3$), the packets will be waiting for transmission in the queue of nodes. Then, the network congestion phenomenon will occur. To measure network capacity more accurately, we refer to the order parameter

$$H = \lim_{t\to\infty} \frac{C}{R} \frac{\langle \Delta W \rangle}{\Delta t}. \quad (1)$$

$\Delta W = W(t + \Delta t) - W(t)$, $W(t)$ represents, at time t, the number of packets in the network. When $R < R_c$, $H = 0$, there is no congestion in the network, and the packets can smoothly reach the destination node. When $R > R_c, H > 0$, the packets cannot reach the destination node smoothly, and there will be network congestion. R_c is the maximum packet generation rate when the network is in a free flow state.

3. Routing strategy

To avoid congestion at the central node in the network, the ISWR strategy completely avoids the hub node in the network, which does not make full use of network resources and is a phenomenon of unreasonable allocation of network resources. In the IER strategy, we assume that the logical and the physical layers have the same number of nodes. When nodes d^l and e^l are connected at the logical layer, there may be different paths between nodes d^p and e^p at the physical layer. In the physical layer, the path from node d^p to node e^p can be marked as $P(d^p \to e^p) := d^p \equiv x_0^p, x_1^p, \ldots, x_{n-1}^p, x_n^p \equiv e^p$, and assign weights to edges between two connected nodes x^p and y^p

$$q_{x^p y^p} = \log \frac{1}{P_{x^p}} + \log \frac{1}{P_{y^p}}. \tag{2}$$

'log' is a logarithmic function based on 10. P is the probability of a packet passing through a node smoothly, and

$$P = \left[\frac{1}{\ln(1+k_u)}\right]^{\ln(1+\alpha k_u)}. \tag{3}$$

'ln' is a logarithmic function based on e. In the physical layer, the path weights from nodes d^p to e^p are

$$Q_{d^p e^p} = \sum_{a=0}^{n} q_{x_a^p x_{a+1}^p} = q_{x_0^p x_1^p} + \cdots + q_{x_{n-1}^p x_n^p}. \tag{4}$$

The path length is expressed in n. The path with the smallest sum of weights is the best path to send packets. If there are multiple optimal paths, then we pick one at random. The physical-layer routing table determines the process as follows: computing degree of physical-layer network nodes, $[k_{1^p}, k_{2^p}, \ldots, k_{n-1^p}, k_{n^p}]$. Let the weight of each edge be

$$q_{x^p y^p} = \log \frac{1}{P_{x^p}} + \log \frac{1}{P_{y^p}}.$$

The logic layer is responsible for generating packets, while the physical layer is responsible for receiving and sending packets. Packets tend to pass through the hub node in the transmission process. If the packet selects the hub node, it will queue at these nodes for passage. Because the nodes with high degree bear high loads, the packets are preferentially selected to pass the nodes with high degree. Therefore, other nodes will be selected as physical-layer routing path. In the logical layer, we assign a weight to the edges between two connected nodes v^l and z^l:

$$q_{v^l z^l} = \log \frac{1}{P_{v^l}} + \log \frac{1}{P_{z^l}} + Q_{v^p z^p}. \tag{5}$$

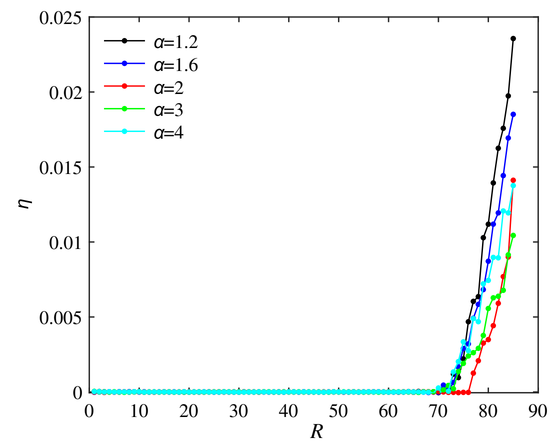

Figure 2. The relationship between η and packet generation rate R for different values of α under the IER strategy. The results show that for relatively small R and different values of α, η is approximately equal to zero, implying that the networks are in the free-flow state. When R is larger than the critical packet generating rate R_c, η suddenly increases, which means that the packets accumulate in the networks. The parameters of the BA network are $N = 400$ and $m_0 = m = 4$.

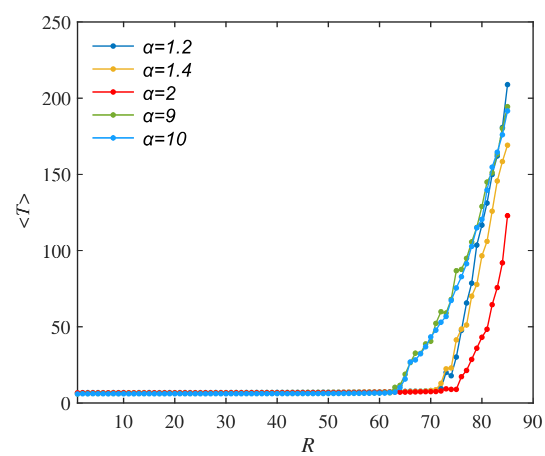

Figure 3. The relationship between the average packet transmission time T and the packet generation rate R for different values of α. When $\alpha = 2$, R reaches its maximum value and average packet travel time $\langle T \rangle$ suddenly increases. The results show that $\alpha = 2$ is the best parameter ($R = 76$). The parameters of the BA network are $N = 400$ and $m_0 = m = 4$.

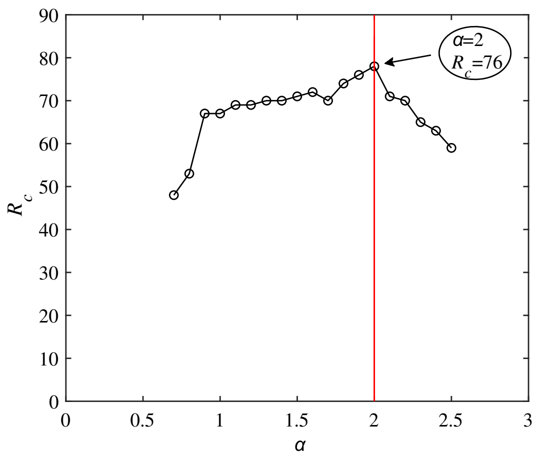

Figure 4. Different α on the BA two-layer network under IER strategy corresponds to different R_c. The results show that R_c reaches its maximum value ($R_c = 76$) when $\alpha = 2$. The BA network parameters are $N = 400$, $m_0 = m = 4$.

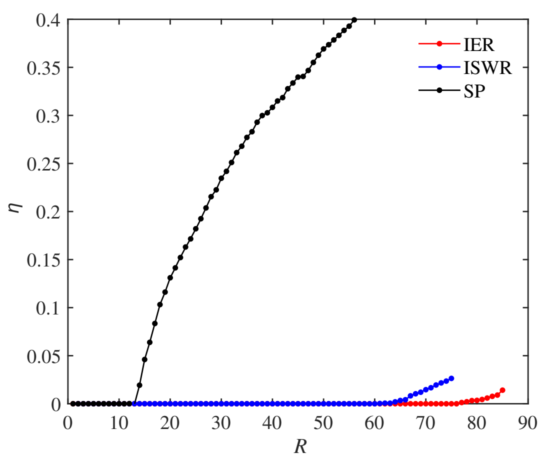

Figure 5. The relationship between η and packet generation rate R under three different routing strategies. The results show that $R_c = 13$ with SP strategy, $R_c = 63$ with ISWR strategy and $R_c = 76$ with IER strategy. Under IER strategy, R_c reaches the maximum. The BA network parameters are $N = 400$, $m_0 = m = 4$.

Then the path weight from nodes d^l to e^l is defined as

$$Q_{d^l e^l} = \sum_{a=0}^{n} q_{x_a^l x_{a+1}^l} = q_{x_0^l x_1^l} + \cdots + q_{x_{n-1}^l x_n^l}. \tag{6}$$

The best path we choose is the one that minimises the above weights. The logic-layer routing table determines the process as follows: Computing degree of logic-layer network nodes, $\left[k_{1^l}, k_{2^l}, \ldots, k_{n-1^l}, k_{n^l}\right]$. Calculate the weight of any path in the physical layer network, notated as $Q_{d^p e^p}$. Let the weight of each edge be

$$q_{d^l e^l} = \log\frac{1}{P_{d^l}} + \log\frac{1}{P_{e^l}} + Q_{d^p e^p}.$$

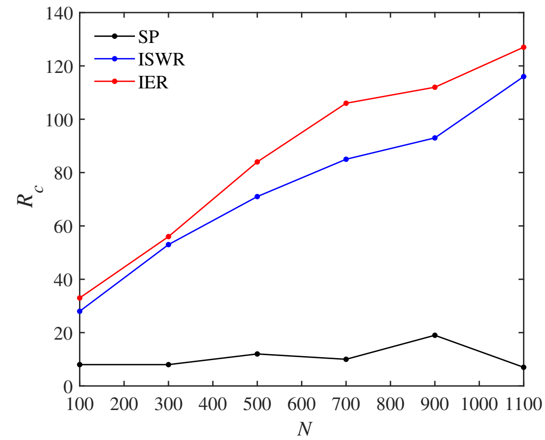

Figure 6. The relationship between network size N and R_c under three different routing strategies. The results show that under the same network size, the R_c value under IER strategy is greater than that under SP and ISWR strategies. Among them, the network average degree $\langle k \rangle = 8$.

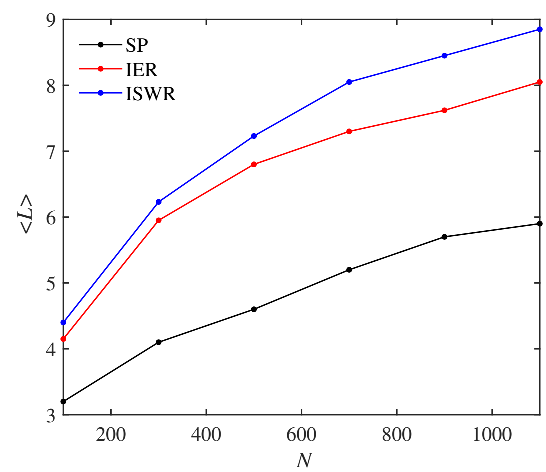

Figure 7. The relationship between average path length of different routing strategies and network size N. The results show that the average path length is the smallest under SP strategy. The average path length under IER strategy is the second and the average path length under ISWR strategy is the largest. The BA network parameters are $N = 400$, $m_0 = m = 4$.

To increase network capacity, under our strategy, the transmission load of the centre edge will be significantly reduced, and the overall transmission load of the network will be more even.

4. Simulation results and analysis

In this section, we first study the change of packet generation rate R with different α under the IER strategy in figure 2. We use Barabási-Albert (BA) scale-free network for simulation, and the network size is $N = 400$, $m_0 = m = 4$. With the increase of R, the value of η begins to increase sharply at some point, and the network congestion occurs at this time. The network capacity is maximum when $\alpha = 2$. Figure 3 shows that the average packet transmission time varies with R. We can see that the average packet transmission time is the longest at $\alpha = 2$.

Figure 4 shows that α of different parameters corresponds to different R_c on two-layer network under the IER strategy. The parameters of the BA network are $N = 400$ and $m_0 = m = 4$. From this figure, we can see that R_c is maximum at $\alpha = 2$. In other words, $\alpha = 2$ is the optimal parameter for IER strategy. We shall use this optimal parameter for the following simulation.

A series of targets were compared under IER strategy. Figure 5 shows the relationship between parameter η and packet generation rate R under three routing strategies. The other two routing strategies are SP routing strategy and ISWR strategy. In the two-layer network, the simulation parameters are $m_0 = m = 4$ and the average degree $\langle k \rangle = 8$. The critical packets R_c of SP, ISWR and IER are 13, 59 and 78 respectively. R_c under IER strategy is six times higher than R_c under SP routing strategy, R_c under ISWR strategy is 4.54 times higher than R_c under SP routing strategy. According to the simulation results, IER strategy is better than the other two routing strategies.

In figure 6, we compare R_c of the three routing strategies for different network sizes. In the two-layer network, the simulation parameters are $m_0 = m = 4$, the average degree $\langle k \rangle = 8$. The network size N goes from 100 to 1100. We can see that R_c value under SP routing strategy has little overall fluctuation under different network sizes. Under IER and ISWR strategies, R_c increases with the increase of network size N, and the overall increase is monotonic. Under the

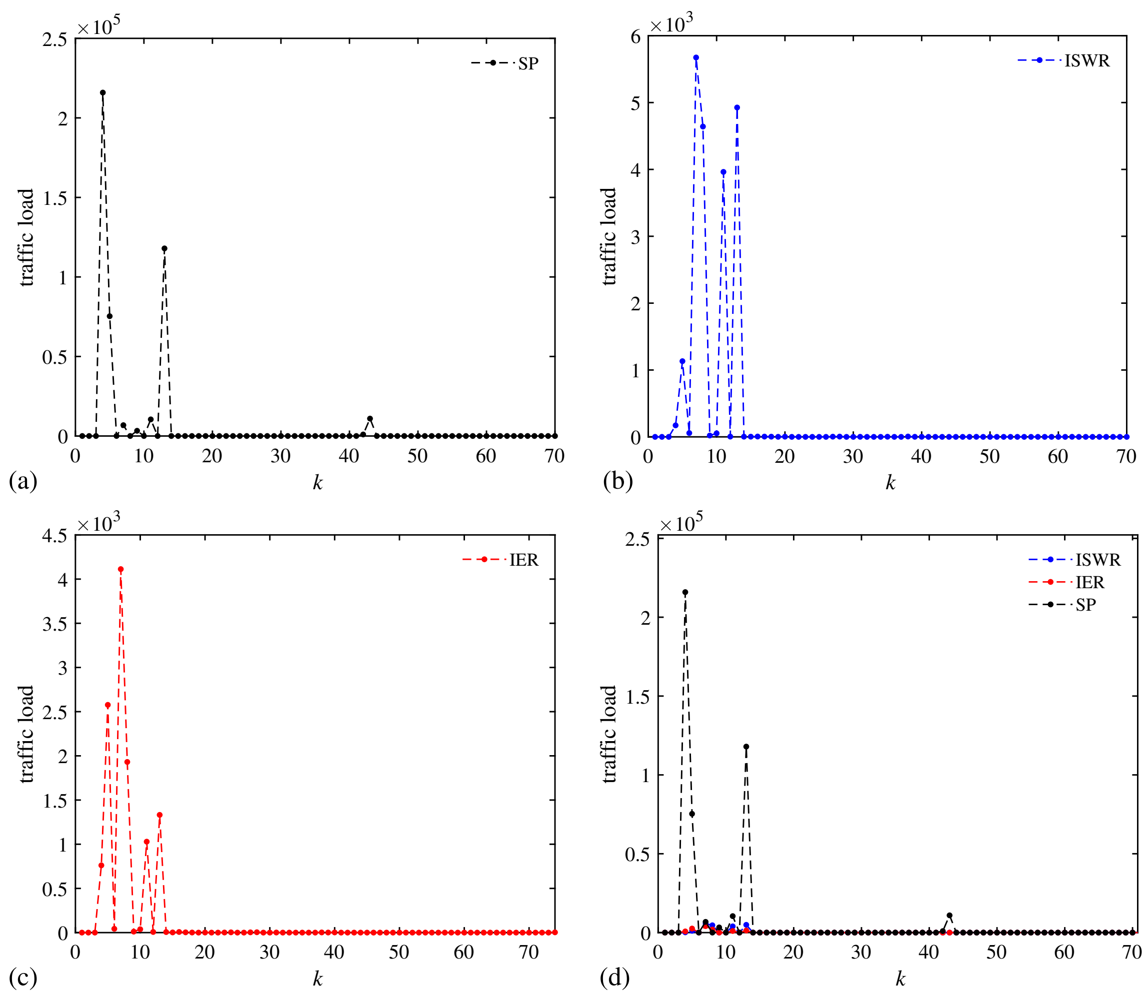

Figure 8. The average traffic load of the logical layer is distributed with node degree $\langle k \rangle$, (**a**) under SP routing strategy, (**b**) under ISWR strategy, (**c**) under IER strategy, (**d**) under the three routing strategies. The traffic load is the smallest under the IER strategy.

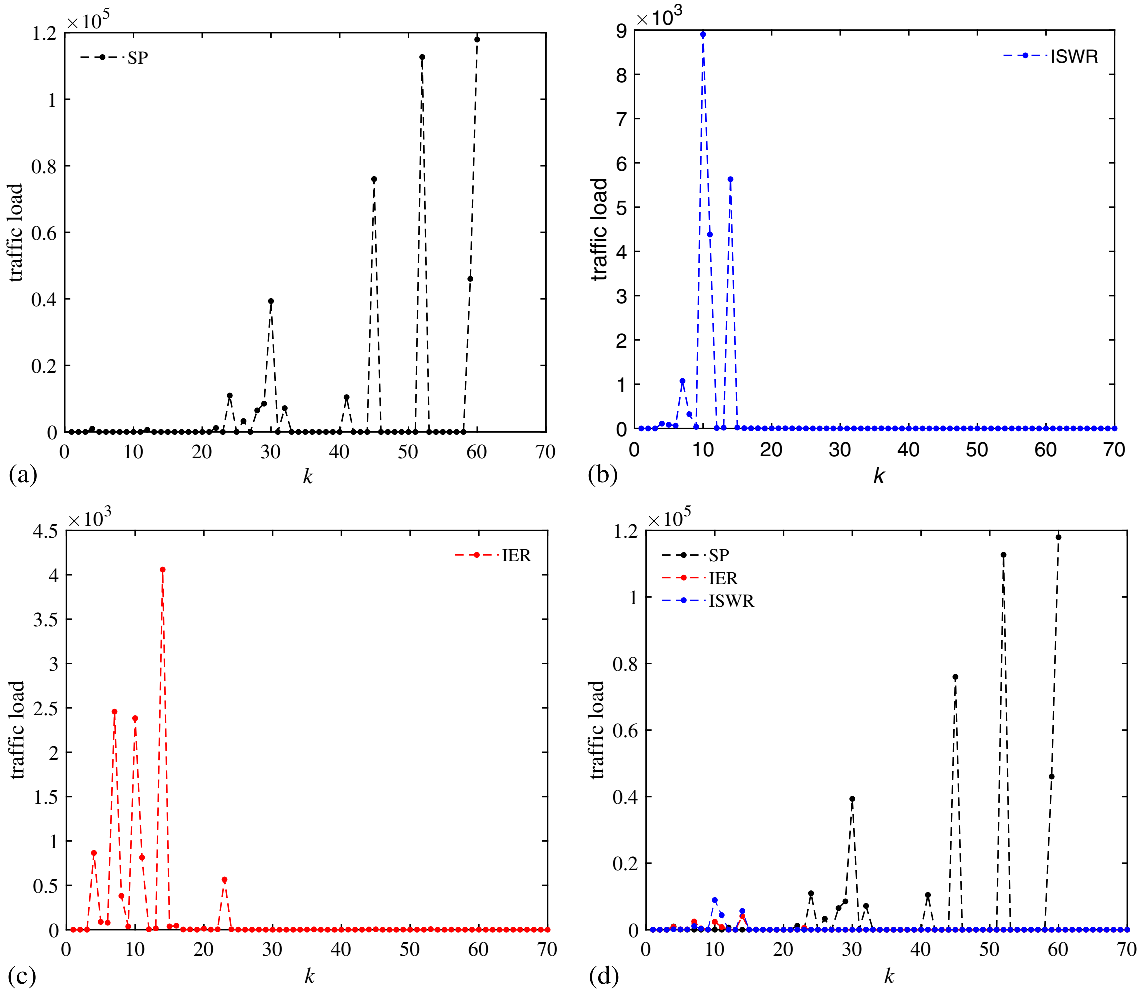

Figure 9. The average traffic load of the physical layer is distributed with node degree $\langle k \rangle$, (**a**) under SP routing strategy, (**b**) under ISWR strategy, (**c**) under IER strategy and (**d**) under the three routing strategies. The traffic load is the smallest under the IER strategy.

same network size, the R_c value under IER strategy is greater than that under SP routing strategy and ISWR strategy. Therefore, the IER strategy is better than the other two routing strategies. Under IER strategy, R_c increases with the increase of network size N. IER strategy can improve network capacity better than the other two strategies. That is to say, IER strategy can distribute transmission load better on different transmission paths. We compare the average path length under IER, ISWR and SP routing strategies. Under the parameters of network size $N = 400$ and $m_0 = m = 4$, the simulation is carried out. In figure 7 ($\langle L \rangle$ is the average path length), we can see that SP routing strategy has the shortest average path length, followed by IER strategy, and ISWR strategy has the longest average path length.

Figures 8 and 9 show the relationship between the node degree and average traffic load at the logical layer and the physical layer respectively. Network size average degree $\langle k \rangle = 8$, $N = 400$ are used to simulate the relationship between average transmission load and node degree under three routing strategies. From the simulation results, the average transmission load of IER strategy is smaller than that of the other two routing strategies. That is, the IER strategy is superior to the other two routing strategies. However, under the same parameters, IER strategy has a larger network capacity than the other two routing strategies. Therefore, the IER strategy is superior to the other two routing strategies in every way.

5. Conclusions

A local efficient routing strategy was proposed, which significantly improved the network capacity of the two-layer network. By giving different weights to the edges between nodes in the network, the probability of packets passing through different paths can be changed, which makes the network resource distribution more reasonable, reduces the probability of network congestion and greatly improves the network capacity. We set the variable parameter α to get the maximum network capacity under IER strategy by changing α. In the simulation, we analysed the critical value of R_c under three routing strategies, the number of packets in the network, the average path length and so on. Through simulation, we found that IER strategy can significantly improve network capacity compared to the other two routing strategies, and the transmission load, average path length of IER strategy are lower than the other two routing strategies. We hope that these works will be helpful to optimise and design routing strategies for two-layer networks. In future, we shall consider other aspects to further improve the performance of the two-layer network.

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