



Explaining the characteristics of lateral shower age of cosmic ray extensive air showers

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Abstract. A simple analytical argument is proposed for a possible explanation of the characteristics of the lateral shower age (s) of proton (p)/nuclei-initiated showers. The argument states that the lateral density distribution (LDD) of electrons of a p -initiated shower is due to the superposition of several electromagnetic (EM) sub-showers developed at a very early stage in the atmosphere from the decay of neutral pions (π^0 s). Thanks to the superposition property of the electron LDD in a p shower, a plausible analytical parametrisation has been worked out by giving a well represented lateral density function for the electron LDDs of p - and π^0 -initiated showers. Based on cosmic ray extensive air shower simulations, we have validated how the various characteristics of s can be understood in the context of the proposed argument. The s parameter of a p shower and its correlations with the shower ages of electron- and π^0 -initiated showers support the idea that the result of superposition of several EM sub-showers initiated by π^0 s with varied energies at a very early stage might produce the LDD of electrons of a p shower. It is also noticed with the simulated data that the present conjecture works well relating to the notion of the local shower age parameter.

Keywords. Cosmic rays; extensive air showers; interactions; lateral shower age; simulations.

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1. Introduction

The study of the average longitudinal development of extensive air showers (EASs) initiated by pure electromagnetic (EM) components such as electrons (e^- or e^+) or photons (γ s) first perceived the necessity of the notion of the ‘age’ of a shower [1]. Such a concept was applied later to the lateral distribution of shower electrons around the EAS axis [2,3]. It was soon realised from the EM cascade theory that every shower has to be assigned by an age, and it remains true for showers generated by hadrons or nuclei as well.

Some current and recent works have revisited several conceptual issues in connection with the shower age, particularly for protons/nuclei-generated showers (see refs [4–8] and references therein). Basically, the trace of the shape/form of the lateral distribution of shower electrons (i.e., $e^- + e^+$) is indicated by the shower age in the EM cascade theory for showers generated by $e^-/e^+/\gamma$ s, and is fairly valid also for the hadron/nuclei-initiated showers [9].

The EAS parameters that are obtained directly from air shower experiments are densities and arrival times of shower particles. Out of all the different charged secondaries in a shower, about $\geq 90\%$ are electrons $e^- + e^+$. The important shower parameters such as the lateral shower age s , the electron/shower size (N_e) and the shower core (x_0, y_0) are obtained by fitting the electron lateral density distribution (LDD) with the Nishimura–Kamata–Greisen (NKG)-type lateral density function (LDF) [2,10]. It was found from both the experimental and simulation data that the NKG function or other type of modified LDF with a single s is not adequate to describe the LDD of electrons initiated by proton (p)/nuclei/ γ -ray primaries at all radial distances from the EAS core accurately [5,7]. In the present work, we noticed the above behaviour in the LDDs of electrons for simulated showers initiated by e^- s and π^0 s as well. This indicates that the lateral shower age varies with the radial distance, which is in principle the shower age at a point, and was labelled as the local age parameter (LAP) by Capdevielle and Gawin [11]. To assign a single age

parameter to each shower, some of the works took the minimum value of LAP from its variation against the radial distance [5,8] while others took some sort of averaging of different LAPs falling between the first minima and the second maxima [5,7,8,11].

It is noteworthy to mention that the s parameter estimated from the above two distinctive methods is adjudged as an important parameter for studying the primary cosmic rays (PCRs). These parameters have been used to correlate with other EAS observables to identify the nature of the shower initiating PCR particles [7,8]. The parameter also provides a basic relationship between the slope of the LDF and the crucial CR mass sensitive parameter ‘the depth of shower maximum’ (X_{\max}) in the ultra-high-energy (UHE) range. The parameter s characterises the development stage of an EAS albeit the parameter manifests different radial dependency. The magnitude of s is intimately connected with the degree of steepness of LDD of shower electrons. In the present work, however, we shall not re-examine any of the aforesaid characteristics of the s parameter by analysing the simulation/observed data. This work will rather discuss how the different observed properties associated with s can be understood more precisely with the proposed simple analytical argument and application of EAS simulations.

The LDD of electrons of p /nuclei-initiated shower is a result of superposition of several EM ($e-\gamma$) sub-showers generated from the decay of $\pi^0 \rightarrow \gamma + \gamma$ at the early stages of EAS development. The LDD of electrons of each EM sub-shower can be represented by the well-known NKG-type LDF,

$$\rho_i(r) = N_{e_i} C(s_i) X^{s_i-2} (1+X)^{s_i-4.5}.$$

Guided by the superposition property indicated in the present analytical argument, we express the resultant LDD of electrons of p /nuclei-initiated shower by

$$\rho(r) = \sum_i \rho_i(r)$$

with

$$\rho(r) = N_e C(s) X^{s-2} (1+X)^{s-4.5}.$$

A complete description of this analytical argument together with the superposition property is provided in the next section.

This paper is organised as follows. In §2 we present the simple analytical argument in the quest for possible physical explanation of the observed behaviour of lateral shower age. In §3 the characteristics of the air shower simulation by the CORSIKA code are described. Results concerning the concept of lateral shower age obtained from the implementation of the analytical approach

using simulation data are presented and discussed in §4. Finally, we give some conclusions in §5.

2. Elements of the simple analytical approach

The characteristics of the observed LDD of electrons produced from CR p /nuclei are imperative for understanding the PCRs, and also the EAS development in the atmosphere. It is well known that the EM content of EAS is formed by a superposition of partial EM sub-cascades initiated by γ -rays resulting from decays of (mostly) π^0 s. The dominant contribution comes from those π^0 s that are produced at very early stages of EAS development. In the cascade theory, a pure EM cascade can be expressed independently by a pair of shower age parameters [3,12]. One of them is known as the longitudinal shower age s_{\parallel} , and it is the saddle point in the inverse Melline transformation of cascade diffusion equations (see ref. [6] and references therein). The parameter manifests the development stage of an EM cascade in the atmosphere. The other one by which the LDD of e/γ s is described is termed as the lateral shower age s , which indicates the slope of the NKG type LDF of an EAS [3]. In [12], it was shown that $s_{\parallel} = s$ for pure EM showers. The work of Kamata and Nishimura [12] also validated that both the longitudinal and lateral structures of e/γ s of hadron-generated showers can be described by a single cascade with an admissible value to the slope parameter. Their study however failed to interpret the longitudinal EAS developed by the so-called slope parameter which acts as an indicator of shower age parameter.

Many EAS experiments found that s differs from s_{\parallel} for EASs generated by p /nuclei. These two age parameters were found to follow an approximate relation $s_{\parallel} - s \geq \delta$, with $\delta \approx 0.2$ [13,14]. Capdevielle and Gabinski [15] established the relation $s_{\parallel} \sim 1.3s$ using simulations. Generally, an EAS experiment equipped with fluorescence/Cherenkov detectors could estimate s_{\parallel} . On the contrary, s can be obtained by fitting the electron LDD data received from scintillation detectors placed in an EAS array with the NKG-type LDF. It should be however mentioned that the introduction of the LAP (discussed in §1) instead of s for better understanding of the radial dependency of shower age follows a different concept from what is complied by s or s_{\parallel} [5].

Now, with the help of the simple analytic route, we are aiming for deeper physical insights on the concepts of s for showers generated by p /nuclei. For the effectiveness of the adopted analytic argument, the shower data obtained from the simulations by running the CORSIKA code [16] have been exploited.

According to the present analytical argument, a hadron-initiated shower is assumed to be a result of superposition of the number of partial electron–photon subcascades started mostly from the decay of π^0 s at different developmental stages of the shower in the atmosphere. Additionally, the major contribution in the observed e /particle density comes from the $e-\gamma$ sub-cascades resulting from decays of (mostly) π^0 s generated at the early stages. The LDD of electrons of a particular $e-\gamma$ subcascade (say, the i th subcascade) is believed to be described by the NKG-type LDF with a slope/lateral age parameter s_i . Again, the overall LDD of electrons of an EAS generated by p /nuclei can be well represented by the resultant NKG-type LDF having a different slope/lateral shower age in various analyses. Hence, the superposition property applies to electron–photon subcascades in a shower requiring the following equality:

$$N_e C(s) X^{s-2} (1+X)^{s-4.5} = \sum_i |N_{e_i} C(s_i) X^{s_i-2} (1+X)^{s_i-4.5}|, \quad (1)$$

where N_e is the electron size (the sum $e^+ + e^-$) of the resultant electron LDD and N_{e_i} represents the electron size for the LDD of the i th electron–photon subcascade with shower age s_i . Let s' denotes the lateral age of an equivalent EM cascade of the hadron shower which has been generated from primary e/γ . The EM cascade also possesses the electron size N_e that is assumed to be equal to the electron size of the hadronic shower. Dividing both sides of eq. (1) by $N_e C(s') X^{s'-2} (1+X)^{s'-4.5}$, being the LDF for the LDD of electrons of an equivalent EM cascade, one may then get the following equation after taking logarithm from both sides:

$$s = s' - \frac{\ln[C(s)/C(s')] - \ln \sum_i \alpha_i C(s_i)/C(s') h^{\delta_i}}{\ln(h)}, \quad (2)$$

where $\alpha_i = N_{e_i}/N_e$, $h = X(1+X)$ with $X = r/r_m$ and $\delta_i = s_i - s'$.

It is expected that all the three lateral shower ages appearing in eq. (2) should be different from one another but the normalisation factors $C(s)$, $C(s')$ and $C(s_i)$ do differ very negligibly. The factors $C(s)$, $C(s')$ and $C(s_i)$ have the same functional form as $\frac{1}{2\pi} \frac{\Gamma(4.5-s)}{\Gamma(s)\Gamma(4.5-2s)}$ in the NKG function. s parameters in all these $C(s)$ s are obtained from the fit on the simulated densities of electrons of p -, e^- - and π^0 -initiated showers using the NKG-type LDF. We have obtained the average values of these factors $C(s)$, $C(s')$ and $C(s_i)$ along with their uncertainties from the analysis of simulated data as 0.359 ± 0.033 , 0.337 ± 0.020 and 0.330 ± 0.006 . Taking $C(s) \approx C(s') \approx C(s_i)$, we can rewrite eq. (2) as

$$s \approx s' + \frac{\ln \sum_i \alpha_i h^{\delta_i}}{\ln(h)}. \quad (3)$$

Now, we suppose that all the secondary electron–photon subcascades (say, n number of subcascades in total) have the same size n_e (or more explicitly, $N_{e_i} \approx n_e$ for $i = 1, 2, 3, \dots, n$), and also have the same age ($s_i \approx \tilde{s}$, again for $i = 1, 2, 3, \dots, n$). We assume that δ_i accounts for the difference between the lateral shower ages of two EM cascades, in which one refers to the i th electron–photon subcascade, and the rest is the effective EM cascade generated by primary e/γ . It is further assumed that δ_i will not change appreciably with the atmospheric depth, and the ratio N_{e_i}/N_e will similarly not vary from one electron–photon subcascade to the other. Then

$$s \approx s' + \frac{\ln[(nn_e/N_e)h^\delta]}{\ln(h)}, \quad (4)$$

where $\delta = \tilde{s} - s'$ and $nn_e \approx N_e$. Alternatively in the same perspective outlined above, we have obtained the following directly from eq. (2) with a positive δ ,

$$s \approx s' - \delta = 2s' - \tilde{s}. \quad (5)$$

The variation with the atmospheric depth can be

$$\frac{ds}{dt} = 2 \frac{ds'}{dt} - \frac{d\tilde{s}}{dt} > \frac{ds'}{dt}. \quad (6)$$

One can rewrite eq. (2) in an alternative way, in terms of lateral density of shower electrons:

$$\ln \left[\frac{\rho_{\text{Had}}(r)}{\rho_{\text{EM}}(r)} \right] = \ln(n\alpha_e \beta_e) + (\tilde{s} - s') \ln[h], \quad (7)$$

where

$$\alpha_e = \frac{N_{e_i}}{N_e}, \dots \approx \frac{n_e}{N_e}$$

and

$$\beta_e = C(\tilde{s}_i)/C(s'), \dots \approx C(\tilde{s})/C(s') \approx 1$$

with $i = 1, 2, \dots, n$ and n represents the total number of electron–photon subcascades produced from a hadronic primary via π^0 -decays. Then, we have

$$(s' - \tilde{s}) \approx \frac{\ln(n\alpha_e) - \ln \left[\frac{\rho_{\text{Had}}(r)}{\rho_{\text{EM}}(r)} \right]}{\ln[h]} \approx \delta. \quad (8)$$

δ can be estimated from the above relation by using simulations.

It has been reiterated that s is evaluated from the fit on the simulated densities of electrons with the NKG-type LDF for any shower ($p/e^-/\pi^0$ -initiated shower) over a certain radial distance range from the EAS core. However, it is noticed that the NKG function with a single s cannot describe the observed/simulated LDD of

EAS electrons accurately at all radial distances over the concerned radial range, which implies that the lateral shower age is a function of radial distance [5]. Later, some modifications into the NKG LDF were incorporated but the radial dependency on s could not be removed completely. From that viewpoint, the idea of LAP was emerged which is in principle the point-wise value of s [5,11]. The well-known LAP for hadron-initiated showers can be expressed as follows [11]:

$$s_{\text{local}}^{\text{Had}}(i, j) = \frac{\ln(F_{ij} X_{ij}^2 Y_{ij}^{4.5})}{\ln(X_{ij} Y_{ij})}. \quad (9)$$

For the equivalent EM cascade, the corresponding LAP is

$$s_{\text{local}}^{\text{EM}}(i, j) = \frac{\ln(F'_{ij} X'^2_{ij} Y'^{4.5}_{ij})}{\ln(X'_{ij} Y'_{ij})}. \quad (10)$$

The present argument treats the hadronic shower as a superposition of n number of electron–photon subcascades. The LAP of the hadronic shower would therefore take the following form:

$$s_{\text{local}}^{\text{Had}}(i, j) \approx \frac{\ln\left(\sum_k \tilde{F}_{ij,k} \tilde{X}_{ij,k}^2 \tilde{Y}_{ij,k}^{4.5}\right)}{\ln(\tilde{X}_{ij,k} \tilde{Y}_{ij,k})}. \quad (11)$$

It is now clear that, $\tilde{X}_{ij,1} = \tilde{X}_{ij,2}, \dots = X'_{ij} = X_{ij}$ and $\tilde{Y}_{ij,1} = \tilde{Y}_{ij,2}, \dots = Y'_{ij} = Y_{ij}$. Hence,

$$s_{\text{local}}^{\text{Had}}(i, j) \approx \frac{\ln\left(\left(\sum_k \tilde{\rho}_{ij,k}\right) \tilde{X}_{ij}^2 \tilde{Y}_{ij}^{4.5}\right)}{\ln(\tilde{X}_{ij} \tilde{Y}_{ij})}, \quad (12)$$

where $\tilde{\rho}_{ij,k}$ and ρ'_{ij} are the ratios of electron densities between two adjacent points. Here, $\tilde{\rho}_{ij,k}$ is associated with the k th subcascade, generated from π^0 -decay in a hadronic shower, while ρ'_{ij} accounts for the effective EM shower.

One may verify the relation $s - s' \approx \delta$ (eq. (5)) or $s' - \tilde{s} \approx \delta$ (eq. (8)) in the context of the LAP if we can express $s_{\text{local}}^{\text{Had}}(\text{min}) - s_{\text{local}}^{\text{EM}}(\text{min}) \approx \delta$ and also $s_{\text{local}}^{\text{EM}}(\text{min}) - \tilde{s}_{\text{local}}(\text{min}) \approx \delta$. The minimum value of the LAP that is assigned to a shower is discussed below.

Let us recall the feature of the LAP from both the simulations and observations in recent works [5,7,8]. It was found that the parameter or the LDD of shower electrons exhibits some kind of scaling behaviour if one studies the variation of the parameter with the radial distance from the EAS core [5]. Instead of taking multiple local ages belonging to a shower, a systematically chosen single-age parameter was found appropriate to deal with some important issues such as composition, energy etc. on the PCR's [5,7,8]. The minimum value of the LAP that was noticed at a particular radial distance

between the first and the second maxima in the s_{local} vs. r curves appears very effective in many of the recent works [7,8].

3. Characteristics of the air shower simulation and data analysis

The simulated showers have been generated with the combination of interaction models; EPOS(LHC)-UrQMD-EGS4 [17–19] embedded in the detailed air shower code CORSIKA ver. 7.7401 [16] for the present analysis. The hadronic interaction model EPOS-LHC looks for interaction processes above the energy 80 GeV/n, and below this energy the model UrQMD works for the hadronic processes. The EGS4 program package addresses the EM interactions in the cascade development. The CORSIKA showers have been generated at the KASCADE [10] level. The kinetic energy cut-offs for secondaries like hadrons, muons and electrons are set at 0.5, 0.025 and 0.005 GeV. The US-standard atmospheric model [20,21] is adopted for the generation of simulated showers. The shower events have been generated primarily for p and e^- primaries at a fixed primary energy 2×10^6 GeV and at the zenith angle 0° . Initially, MC showers have been generated for p and e^- primaries with about 1000 events for each primary. Required number of π^0 showers with varied energies have also been generated by maintaining some restrictions on different free parameters in the CORSIKA steering file. More details about the π^0 shower events is discussed in the following paragraph.

In the simulation, the starting altitude of the primary particle p and e^- is set corresponding to the value of the keyword 'FIXCHI' here. For these primaries, FIXCHI is set to 0 g cm^{-2} which refers the top of the atmosphere but at the immediate vicinity above the starting point of the EAS [16]. In this analytical argument, a hadronic shower is believed to be a superposition of a certain number of electron–photon subcascades. These subcascades might have produced mostly from some parent $\pi^0 \rightarrow \gamma + \gamma$ decays just below the first interaction point of the shower. We have extracted the average first interaction length from the generated p showers, and it takes a value of $\approx 65 \text{ g cm}^{-2}$ in the present analysis. Hence, the starting altitude (or FIXCHI) of the parent particles π^0 that induce electron–photon sub-cascades is set judiciously a little below the first interaction point. From the analysis, we have found that $\text{FIXCHI} \approx 75 \text{ g cm}^{-2}$ is appropriate to deliver better results.

In the primary p -air collisions, copious mesons are produced and majority of them are pions. The superposition property of the present analytical argument requires information on the total number of produced

Table 1. Energies and event numbers for π^0 showers are set from charged pions information about a depth of $\approx 75 \text{ g cm}^{-2}$ from the top in an average p -initiated shower with 2 PeV energy.

E_{π^0} -range (GeV)	$N_{\pi^0}^{\text{Event-No}}$	N_f	Simulated π^0 events
$(0.01-0.99) \times 10^5$	104.53	1	105
$(1-1.99) \times 10^5$	4.01	1	4
$(2-2.99) \times 10^5$	1.36	11	15
$(3-3.99) \times 10^5$	0.80	10	8
$(4-4.99) \times 10^5$	0.48	25	12
$(5-5.99) \times 10^5$	0.16	50	8
$(6-6.99) \times 10^5$	0.16	50	8
$(7-7.99) \times 10^5$	0.48	25	12

π^0 and their energy distribution from the analysis of p -initiated showers at around 75 g cm^{-2} from the top of the atmosphere. But a π^0 can propagate only a very short distance, about $\sim 25 \text{ nm}$ in air due to their short life time $\sim 10^{-16} \text{ s}$. So these π^0 particles cannot be counted at any given height in the simulation. They are rather tracked only by the analogy with the charged pions; all the species of pions (π^0, π^\pm) are believed to be generated with equal numbers and π^+ and π^- numbers are known from the simulation which then fixes the required π^0 number. Knowing the 3-momenta of charged pions, the energy distribution of π^0 can be found from the stated analogy. Frequency distribution of energies of π^+ or π^- will fix the π^0 numbers with different energies. Finally, the required number of π^0 showers are simulated corresponding to different energy ranges by setting $\text{FIXCHI} = 75 \text{ g cm}^{-2}$ in CORSIKA steering file. We have then combined the LDDs of electrons contributed by all the π^0 showers at the ground level for describing the LDD of an average p shower eventually.

It has been reiterated that the dominant contribution to the EM content of an EAS comes from the very early stage of its development. In p -air collisions, most of the secondary pions (π^0, π^+, π^-) are produced in the central rapidity region, and are therefore characterised by small fractions of energy, taken from their parent hadrons. The largest contributions to the EM shower content come from ‘hard’ π^0 s produced, i.e., those which acquire $\geq 10\%$ of the energy of the parent hadron. It is clear that the acquired energies (E_{π^0}) by the π^0 s are unequal. Hence, we set the event number for π^0 -initiated subshowers from the frequency distribution of average charged pions, i.e.,

$$N_{\pi^0}^{\text{event-no}} = \frac{(\pi^+ + \pi^-)}{2},$$

corresponding to different energy ranges. In table 1, the unequal energies acquired by the secondary charged pions of an average p -initiated shower (energy; $E_p = 2 \text{ PeV}$) are shown. The second column of table 1

accounts the $N_{\pi^0}^{\text{event-no}}$ derived from the generated π^\pm numbers around 75 g cm^{-2} from the top of the atmosphere from an average p -initiated shower. To simulate π^0 subshowers, we convert the fractions in column 2 of table 1 into integers by multiplying different numbers (N_f). The lateral densities so obtained from these events (column 4) at the ground level are divided finally by the respective N_f s.

4. Results concerning the properties associated with the lateral shower age

Densities of EAS electrons are measured at different radial distances from the shower core. In this work, we first discuss how the variation of the simulated electron densities with the radial distance for p -generated showers can be understood using the proposed simple arguments. Next, we briefly discuss how the correlation between s and $s_{||}$ of the p -generated showers can be understood using the present approach. We shall then discuss the main issue of the work about how the properties associated with the slope (also being an indicator of lateral shower age [6]) of the LDF describing the simulated LDD of electrons for p -generated showers can be understood with this simple analytical argument. The method gets a strong basis while this simple argument has been fruitfully applied to the radial behaviour of the LAP of EASs.

4.1 Lateral density distribution of electrons in simulated showers

It has been anticipated that the proposed argument might be effective for explaining the behaviour of the parameter s , if the method has a direct impact on the LDDs of electrons for primary p, e^- and π^0 showers. Moreover, the LDD data of EAS electrons are the basic information

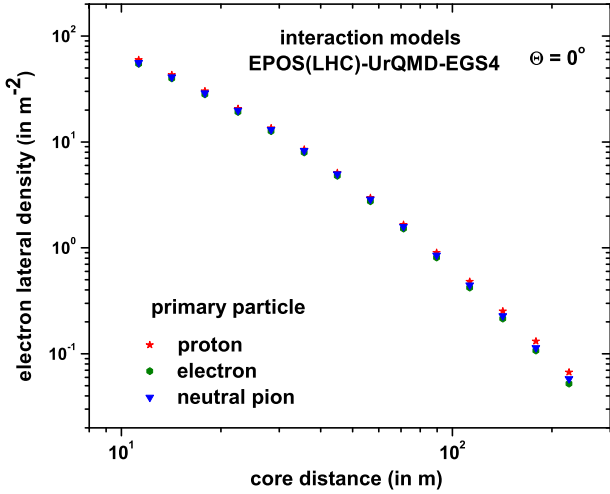


Figure 1. Mean lateral density distribution of electrons for showers generated by p , an equivalent average EM cascade initiated by e^- , and the required number of π^0 subcascades.

from which the shower data analysis starts for obtaining other important EAS parameters.

The simulated densities for secondary electrons at different radial distances from the shower core (10–225 m) are estimated in the analysis for p , an equivalent average EM cascade initiated by e^- and π^0 showers at the KASCADE level. In figure 1, we have plotted mean electron densities over the above radial range for the three types of simulated showers. For these three types of showers, a very good agreement among the LDDs of shower electrons is noted over the 10–100 m radial distance range. It ensures that the average LDD of electrons of p -initiated showers can be well represented by the superposition of a justifiable number of electron–photon subcascades from the decay of secondary π^0 s produced after the first interaction point of the same p shower. At distances beyond 100 m, a very minor contrast in the LDDs of electrons develops in the p showers with respect to the combined π^0 subshowers and the equivalent EM (e^-) shower. This might be due to the transverse momentum, the charged pions/kaons get in the first interaction. This actually gives a hint that the slope parameters of these LDDs of EAS electrons initiated by an average p/e^- , and required π^0 s, might be slightly different from each other. The generic features of showers (e.g. shower-to-shower fluctuations) generated by different types of primary particles might be the other reason for the non-vanishing difference among the slope parameters of their LDDs even though these showers were assumed to be equivalent to each other. This work reviews the effectiveness of the NKG-type function in describing the simulated LDD data of electrons generated from p , e^- and π^0 s, even though it is not shown here. It is found that

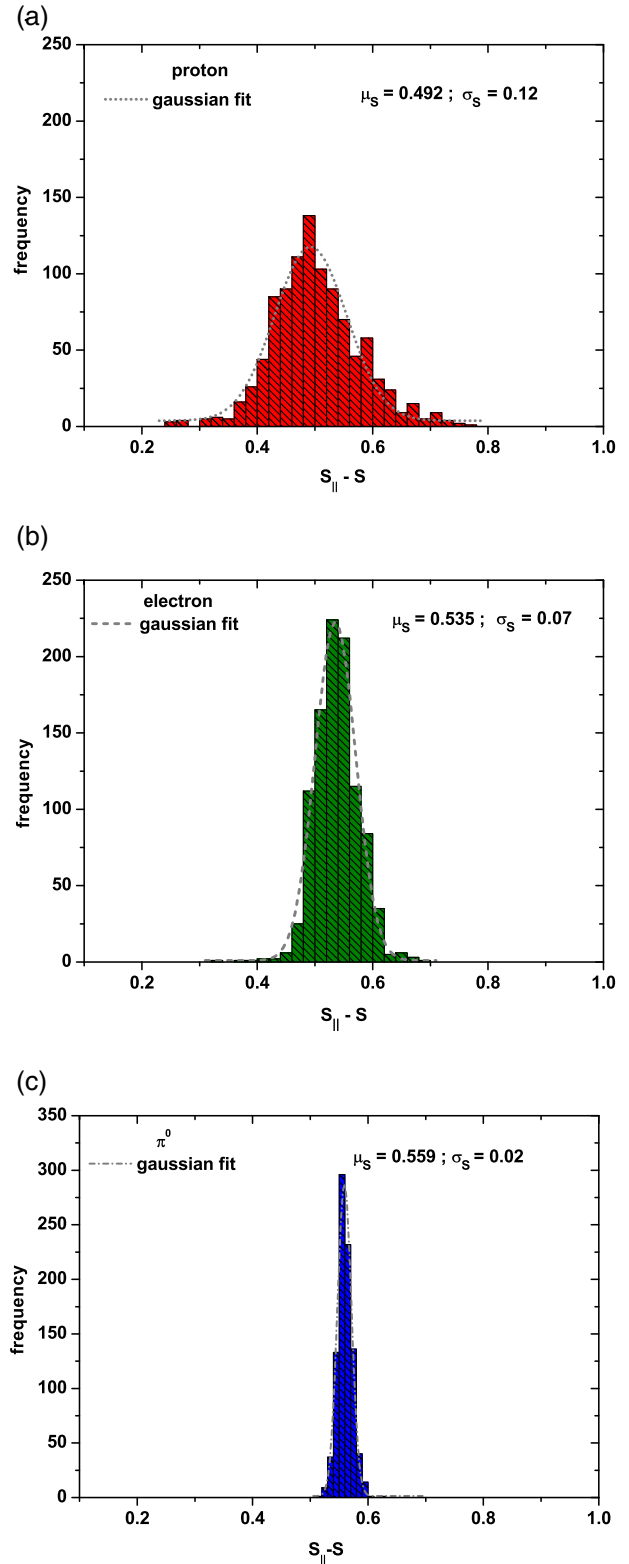


Figure 2. Distributions of differences between $s_{||}$ and s at the KASCADE site for simulated (a) p , (b) e^- and (c) π^0 showers.

the NKG function uni-vocally describes the simulated LDD data of electrons irrespective of all these primaries.

4.2 Correlation between the lateral and longitudinal shower ages

The simulated LDD data of electrons at different radial distances are fitted usually by the NKG function for obtaining s . In [9,22], the longitudinal shape/age parameter s_{\parallel} of the energy and angle distributions of shower electrons of very high-energy to UHE MC showers are well approximated to a simple form as

$$s_{\parallel} = \frac{3X}{X + 2X_{\max}}, \quad (13)$$

where X_{\max} is the depth of shower maximum and X is the atmospheric slant depth of the KASCADE level (in unit of radiation lengths). Knowing both depth parameters, s_{\parallel} of a shower can be estimated. Hence, one should use s_{\parallel} instead of the depth X referring to the shower maximum. The value of X/X_{\max} is the key for describing the EAS development stage. This work however studies the difference between s_{\parallel} and s for p -, e^- - and required number of π^0 -generated simulated showers. For these showers, the frequency distributions of differences between s_{\parallel} and s are shown in figure 2a–c. The present MC simulation gives the frequency distribution peaks within the range ≈ 0.50 – 0.55 , which is consistent with some early MC simulation predictions satisfying the general relation of the form, $s_{\parallel} - s \approx 0.5$ or equivalently $s_{\parallel} \sim 1.4s$ [15]. We also notice that the fluctuations (σ_s) of the difference between s_{\parallel} and s are sensitive to the nature of the shower initiating primary species. The correlations between s_{\parallel} and s reveal that an average p shower at least can be explained as the result of superposition of several EM subshowers generated by π^0 s.

4.3 Analytical description of the lateral shower age

Having recognised this simple analytical argument for explaining the behaviour of s through eqs (2)–(8), we shall now examine how well the LDD of electrons of simulated p -, e^- - and π^0 -initiated showers follow them. In §2, we have described how one could find eq. (5) starting from eq. (3). From the analysis of the simulated LDD data of p -, e^- - and the required number of π^0 -initiated showers, we have obtained the parameter $\delta \approx s - s' = 0.053$ and $2s' - \tilde{s} = 0.87 \approx s$. Shower ages are estimated by applying the fitting procedure (by virtue of a chi-square minimisation routine using gradient search technique) to the simulated LDD data of electrons with the NKG LDF irrespective of primary particles. We have also checked the variation of s with the atmospheric depth using simulated LDD data following eq. (6). For the purpose, we have

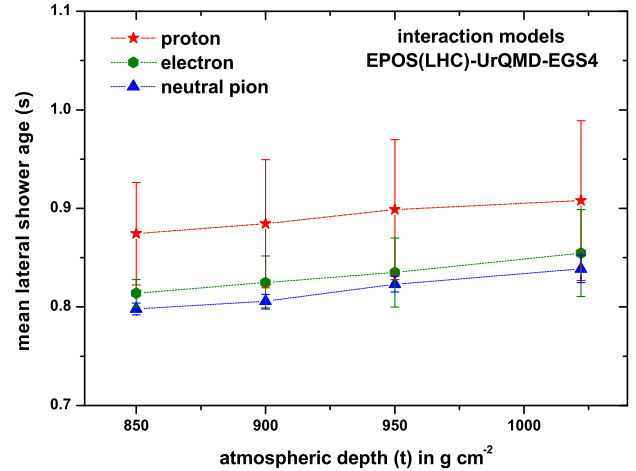


Figure 3. Variation of mean s , s' and \tilde{s} with atmospheric depth at KASCADE level. Lines are only for guiding our eyes.

generated simulated showers at three different atmospheric levels (850, 900, 950 g cm⁻²), in addition to the usual ground level 1022 g cm⁻² of the KASCADE site. Results obtained from the shower data analysis on the variation of mean lateral shower age with the atmospheric depth is shown in figure 3. The figure reveals that the shower age increases with atmospheric depth irrespective of p -, e^- - and π^0 -initiated showers, which is a generic feature of shower development.

From table 2, we obtain

$$\left(2 \cdot \frac{ds'}{dt} - \frac{d\tilde{s}}{dt} \right) = 2.76 \times 10^{-4} \text{ cm}^2 \text{ g}^{-1}$$

for the variation of atmospheric depth between 850 and 900 g cm⁻², and for the atmospheric depth interval 900–950 g cm⁻², the value is nearly 6.4×10^{-5} . Finally for the atmospheric depth interval of 950–1022 g cm⁻², we have obtained the value 3.29×10^{-4} . For the depth variations 850–900 and 950–1022 g cm⁻², the condition

$$\left(2 \cdot \frac{ds'}{dt} - \frac{d\tilde{s}}{dt} \right) > \frac{ds'}{dt}$$

predicted in the adopted analytical method, is obeyed by the simulated showers.

The work illustrated so far involves the fit procedure for estimating s for calculating $s_{\parallel} - s$, δ , ds/dt etc. An alternative attempt has been made for estimating $s' - \tilde{s} \approx \delta$ directly via eq. (8). The average densities of shower electrons for EM- and p -initiated showers at a radial distance of around 50 m from the EAS core are $\rho_{EM} \approx 3.7368$ and $\rho_p \approx 3.9342 \text{ m}^{-2}$. The corresponding electron sizes are $N_e(\text{EM}) \approx N_e(p) \approx 1.9726 \times 10^5$ and $N_e(\pi^0) \approx nn_e(\pi^0) \approx 2.0337 \times 10^5$. The value of $1/\ln[h]$ is ≈ -2.4168 with the Molière radius $r_m =$

Table 2. Variation of lateral shower age with atmospheric depth. Rows 2, 3 and 4 are respectively for the depth variation intervals, 850–900, 900–950 and 950–1022 g cm⁻².

dt in g cm ⁻²	ds	ds'	d \tilde{s}	ds/dt	ds'/dt	d \tilde{s} /dt
50	0.0100	0.0109	0.0080	2×10^{-4}	2.18×10^{-4}	1.60×10^{-4}
50	0.0144	0.0102	0.0172	2.88×10^{-4}	2.04×10^{-4}	3.44×10^{-4}
72	0.0091	0.0197	0.0156	1.26×10^{-4}	2.73×10^{-4}	2.17×10^{-4}

110 m. Using all these simulated results in eq. (8), the value of $\delta \approx s' - \tilde{s}$ takes the value of ≈ 0.05 . It deviates from the previous one, i.e. $\delta \approx s - s' \approx 0.053$ typically by $\sim 5.5\%$ with the simulated data.

4.4 Compatibility of the present analytical argument with the local age parameter

This analytical argument may receive more impetus if we compute the parameter δ in terms of the LAPs introduced in §2 for simulated p -, e^- - and π^0 -initiated showers. In the analysis, the LAP for the LDD data was computed for each shower by applying eqs (9), (10) and (11) or (12) respectively for p -, e^- - and the required number of π^0 -initiated showers. It should be mentioned that the local age obtained for the LDD of an average π^0 -initiated shower in a particular radial bin, say (r_i, r_j) would coincide with the LAP computed from the required number of π^0 -initiated showers. For computing the local age from the required number of π^0 -initiated subshowers, we shall just take the ratio of the sums of electron densities

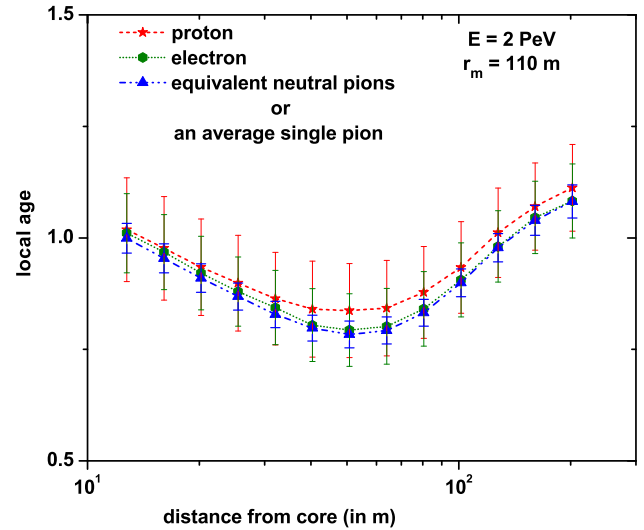
$$\sum_k \rho_{ij,k} = \sum_k \frac{\rho_{i,k}}{\rho_{j,k}}$$

of different π^0 showers between the points, r_i, r_j in eq. (12). On the other hand, the LAP computed from the electron LDD data of an average π^0 -initiated shower involves the ratio

$$\sum_k \rho_{ij,k} = \sum_k \left(\frac{\rho_{i,k}/n}{\rho_{j,k}/n} \right)$$

for n number of equivalent π^0 showers.

The variation of the LAP with the radial distance is shown in figure 4. The error of the LAP arises due to density fluctuations of simulated electron data and the uncertainties in radial distance estimation remains within 0.04 for $12 < r < 205$ m. Here, the minimum value of the LAP in the present analysis is the LAP value at the radial distance of about 50 m when $r_m = 110$ m, and is assigned to a shower as the lateral shower age parameter [5,8]. With the simulated data, we have checked the parameter δ using this alternatively defined lateral shower age parameter. We

**Figure 4.** Variation of the LAP computed from the simulated electron LDD data with radial distance for p , e^- , required number of π^0 showers or an average π^0 shower. Lines are only for guiding our eyes.

obtained $\delta \approx s_{\text{local}}(\text{min}) - s'_{\text{local}}(\text{min}) \approx 0.044$ and $\delta \approx s'_{\text{local}}(\text{min}) - \tilde{s}_{\text{local}}(\text{min}) \approx 0.01$. In this local age representation, these two δ values differ much from each other. We have also noticed that δ increases with the radial distance from the EAS core.

5. Conclusions

Investigating the characteristics of the lateral shower age with simulated shower data in detail, we conclude the following from the present analysis.

The simulated electron LDDs of p -, e^- - and π^0 -initiated showers unequivocally support the idea explained in the adopted analytical argument.

The candidature of s as a crucial EAS parameter of the longitudinal development of a shower is noticed.

The difference in the numerical values between the lateral and longitudinal shower ages can be explained by the superposition property obeyed by a number of EM subshowers initiated by π^0 s mostly from a predetermined atmospheric level at the KASCADE site. Moreover, the frequency distributions of these

differences support the fact that the longitudinal and lateral developments of a shower are closely correlated by their shape parameters. The shape of these distributions and their spreads are found to be sensitive to CR mass composition.

The numerical value of the difference $\delta \approx s - s' = 0.053 \neq 0$ indicates that the radial dependence of s is different from that of s' , because of the 2nd term in eq. (2). This otherwise supports the important underlying fact (superposition property) of this work which indeed could explain the behaviour of the shape parameter of a p /nuclei-initiated shower. The result $s' < s$ reveals the fact that for pure EM cascade, the LDD is steeper than that of the hadronic cascade. Applying simulated data to eq. (8), we have obtained $\delta \approx s' - \tilde{s} = 0.05$, which is in agreement with the fact that π^0 -initiated subshower is steeper than that of a pure EM shower. For the variation of the atmospheric depth, the condition

$$\left(2 \cdot \frac{ds'}{dt} - \frac{d\tilde{s}}{dt} \right) > \frac{ds'}{dt}$$

is found to be valid in two atmospheric depth intervals: 800–850 and 900–1022 g cm⁻² by the simulated data.

The radial variation of the LAP has reiterated the fact that a single value of s is inadequate to describe the LDD of electrons accurately by a LDF (NKG) in the entire radial distance from the EAS core. The nature of variation of the LAP for p -, e^- - and π^0 -initiated showers are portrayed as a generic feature of the LDDs of electrons in showers. A more rational single-valued minimum LAP is assigned to each shower for the purpose of using it as a good estimator in CR studies. The value of δ is almost recovered in terms of LAP (i.e. $s_{\text{local}}(\text{min}) - s'_{\text{local}}(\text{min})$) but the value $s'_{\text{local}}(\text{min}) - \tilde{s}_{\text{local}}(\text{min})$ deviates much from its earlier value in terms of lateral shower age obtained from the fitting procedure.

It is noteworthy to mention that the paper has implemented one of the main aspects of the work in a restrictive way considering a fixed number of π^0 s with varied energies produced at the same atmospheric depth (table 1). It is more justifiable from the pragmatic point of view that one has to account for at least 2–3 steps for the respective hadronic cascades. We have checked this aspect, and observed that the produced π^0 s at the lower atmospheric depths do not have noticeable contribution to reproduce the LDD for p -induced EAS because these π^0 s could pick up very little energy from high-energy hadronic interactions.

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