Pramana – J. Phys. (2022) 96:62
https://doi.org/10.1007/s12043-022-02308-z

Regular quark star model with pressure anisotropy

AMOS V MATHIAS and JEFTA M SUNZU*

Department of Mathematics and Statistics, University of Dodoma, Dodoma, Tanzania
*Corresponding author. E-mail: jefta@aims.ac.za

MS received 14 July 2021; revised 29 October 2021; accepted 11 November 2021

Abstract. A new regular model for a stellar sphere with quark matter is found. The model satisfies a neutral quark star with pressure anisotropy. In this model, we consider an ansatz of a new form of one of the gravitational potentials, and stellar masses consistent with other findings which describe the generation of astrophysical objects. New masses, radii and surface gravitational red-shifts in acceptable ranges are also obtained using our model. The model satisfies stability and energy conditions. The state of hydrostatic equilibrium for stability is obtained by analysing the Tolman–Oppenheimer–Volkoff (TOV) equation. All other matter variables and gravitational potentials are well behaved.

Keywords. Anisotropy; neutral star; quark matter; stellar masses; stability.

PACS Nos **04.20.Jb; 04.40.Nr; 04.20.-q**

1. Introduction

The investigations on the geometries and physical characteristics of gravitating stellar spheres such as gravastars, quark stars, neutron stars, dark energy stars and black holes using Einstein–Maxwell field equations are extensively becoming an area of focus by various researchers. The work by Bowers and Liang [1] has became the benchmark for researchers to engage in searching for neutral and charged anisotropic stellar models. Various anisotropic models, such as recent models by Abdalla *et al* [2], Sunzu *et al* [3] and Bhar *et al* [4], consider space–time as static and spherically symmetric.

There are various phenomena which describe the genesis of anisotropy in the stellar fluid. Ruderman [5] contended that, when stellar interior has ultrahigh central density to the range of $10^{15}\,\mathrm{g\,cm}^{-3}$, the stellar fluid has anisotropic nature. Kippenhahn and Weigert [6] showed that the anisotropic nature of stellar sphere is contributed by its matter content being of type-3A superfluid. Other phenomena include stellar matter undergoing numerous phase transitions as indicated by Sokolov [7] and pion condensation asserted by Sawyer [8]. Several studies depict the effect of anisotropic pressure on the structure and behaviour of compact relativistic spheres. It is evident from the study by Dev and Gleiser [9] that mass and red-shift change with the variation of pressure anisotropy. The works by Murad [10], Murad and Fatema [11,12], Sunzu *et al* [13] and Ngubelanga *et al* [14] revealed the significance of including pressure anisotropy when studying the structure and properties of astrophysical stellar bodies. Karmakar *et al* [15] observed that anisotropic pressure increases the red-shifts, compactness and masses of stellar objects.

Various charged and uncharged stellar models with astrophysical significance have been developed. Uncharged stellar models are included in studies by Sunzu and Mashiku [16], Thirukkanesh and Ragel [17], Mak and Harko [18], Karmakar *et al* [15] and Kalam *et al* [19] which described the stellar models with anisotropic pressures. Charged anisotropic models include recent models by Abdalla *et al* [2], Sunzu *et al* [3], Bannerjee [20] and Mafa Takisa *et al* [21].

An equation of state plays a crucial role in the generation of exact models for stellar spheres. Stellar model by Bhar *et al* [4] applied generalised Chaplygin equation of state to obtain the solution which describes some strange stars. Other models which applied Chaplygin equation of state can be seen in [22–24]. Nasim and Azam [25] applied generalised polytropic equation of state to model compact stars with results which could be reduced to anisotropic matter with linear and quadratic equations of state. The studies in [26–28] also used the polytropic

Published online: 19 March 2022

equation of state to generate models with astrophysical significance. Thirukkanesh and Ragel [29] and Xu [30] used Van der Waals equation of state to obtain well-behaved models to describe anisotropic compact spheres. The works in [31–35] applied quadratic equation of state to model relativistic bodies with pressure anisotropy. Abdalla *et al* [2], Mak and Harko [36], Komathiraj and Maharaj [37] and Thirukkanesh and Maharaj [38] used linear equation of state to generate anisotropic quark star models. Linear equation of state has been widely used to model relativistic stellar bodies with quark matter. Sharma and Maharaj [39] reported that since the physics of stellar matter with ultrahigh density is still unclear, many studies of strange stellar spheres with quark matter are performed in the framework of MIT bag model. The bag model relates pressure with energy density in a linear form. This observation has motivated our interest to study the geometry and physical characteristics of neutral stellar objects with quark matter and anisotropy in this framework. We also employed a new form of one of the gravitational potentials to generate a quark star model. Most of the models for quark stars contain electric field. We are motivated to generate a model for a quark star with no charge. We performed detailed physical analysis that includes compactness, red-shifts, stellar masses, hydrostatic equilibrium, stability and energy conditions. Such analysis is missing in many studies that apply the same approach.

This paper is organised in six sections. In §2, the fundamental equations are given. Field equations are then transformed in §3. We develop a leading differential equation governing our model in §4 after choosing the form of one of the gravitational potentials. We then present our regular model, compactness, red-shift, hydrostatic equilibrium, stability and energy conditions in §5. Plots and discussion on profiles of graphs are given in §6. The concluding remarks are then given in §7.

2. Fundamental equations

We found a new exact model for the interior of stellar sphere. The space–time geometry is considered to be static and spherically symmetric with the interior line element

$$ds^2=-e^{2\nu(r)}dt^2+e^{2\lambda(r)}dr^2+r^2(d\theta^2+\sin^2\theta\, d\phi^2), \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are gravitational functions. We consider the Schwarzschild exterior space–time with the line element given by

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2 +r^2(d\theta^2+\sin^2\theta\, d\phi^2), \tag{2}$$

where M is the total mass of the stellar object under consideration. The energy momentum tensor for uncharged anisotropic stellar sphere is given by

$$T_{ij}=\text{diag}\,(-\rho, p_r, p_t, p_t)\,, \tag{3}$$

where ρ, p_r and p_t represent the energy density, radial pressure and tangential pressure, respectively. These quantities are determined relative to a co-moving unit time-like fluid four-velocity u^a.

The field equations which govern the stellar objects with no electric field can be written as

$$\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\lambda'}{r}e^{-2\lambda}=\rho, \tag{4a}$$

$$-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda}=p_r, \tag{4b}$$

$$e^{-2\lambda}\left(\nu''+\nu'^2-\nu'\lambda'+\frac{\nu'}{r}-\frac{\lambda'}{r}\right)=p_t, \tag{4c}$$

where primes denote the derivatives of the gravitational functions with respect to the radial distance r. We consider the coupling constant $8\pi G/c^4$ and the speed of light c to be unity. The mass function for the uncharged fluid is given by

$$M(r)=\frac{1}{2}\int_0^r \omega^2\rho\, d\omega. \tag{5}$$

The equation of state for the stellar matter is considered to admit linear equation of state given by the relation

$$p_r=\frac{1}{3}(\rho-4B), \tag{6}$$

where B is the bag constant.

3. Transformations

To simplify the integrability of the field equations in system (4), we adopt the Durgapal and Bannerji [40] transformation of the form

$$x=Cr^2,\quad Z(x)=e^{-2\lambda(r)},\quad A^2y^2(x)=e^{2\nu(r)}, \tag{7}$$

where A and C are real constants. Applying these transformations to system (4), we obtain

$$\frac{1-Z}{x}-2\dot{Z}=\frac{\rho}{C}, \tag{8a}$$

$$4Z\frac{\dot{y}}{y}+\frac{Z-1}{x}=\frac{p_r}{C} \tag{8b}$$

$$4xZ\frac{\ddot{y}}{y}+\left(4Z+2x\dot{Z}\right)\frac{\dot{y}}{y}+\dot{Z}=\frac{p_t}{C} \tag{8c}$$

and the line element

$$\mathrm{d}s^2=-A^2y^2\,\mathrm{d}t^2+\frac{1}{4xCZ}\mathrm{d}x^2+\frac{x}{C}(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2). \tag{9}$$

By subjecting mass function (5) to the transformation in eq. (7) we obtain

$$M(x)=\frac{1}{4C^{3/2}}\int_0^x\sqrt{\omega}\rho\,\mathrm{d}\omega. \tag{10}$$

Applying linear equation of state (6) to system (4), the field equations for uncharged sphere becomes

$$\rho=\left(\frac{1-Z}{x}-2\dot{Z}\right)C, \tag{11a}$$

$$p_r=\frac{1}{3}(\rho-4B), \tag{11b}$$

$$p_t=p_r+\Delta, \tag{11c}$$

$$\Delta=C\left(2x\frac{\dot{y}}{y}+\frac{5}{3}\right)\dot{Z}+C\left(4x\frac{\ddot{y}}{y}+4\frac{\dot{y}}{y}+\frac{1}{3x}\right)Z+\frac{4}{3}B-\frac{C}{3x}, \tag{11d}$$

$$\frac{\dot{y}}{y}=\frac{1}{3Z}\left(\frac{1-Z}{x}-\frac{\dot{Z}}{2}-\frac{B}{C}\right). \tag{11e}$$

We observe that (11e) can be integrated to yield

$$y(x)=K\mathrm{e}^{\int f(x)\mathrm{d}x}, \tag{12}$$

where

$$f(x)=\frac{1}{3Z}\left(\frac{1-Z}{x}-\frac{\dot{Z}}{2}-\frac{B}{C}\right)$$

and K is the constant of integration. The quantity $\Delta=p_t-p_r$ is the measure of pressure anisotropy and $2\Delta/r$ defines the anisotropic force. According to Gokhroo and Mehra [45], when $p_t>p_r$, i.e. $\Delta>0$, the anisotropic force is repulsive in nature, but has attractive nature when $p_t<p_r$, i.e. $\Delta<0$. When $\Delta=0$, we get the isotropic model. Mathematically, to make system (11) integrable, we specify one of the variables.

4. Ansatz for Z

To have a well-behaved stellar model, we need to specify any one of the variables $(\rho,\ p_r,\ p_t,\ Z,\ y,\ \Delta)$, to make the system of field equations in (11) integrable. The choice should be made on physical grounds to ensure that a well-behaved model is generated. We specify a rational form of Z with quadratic expression in numerator and a polynomial with degree 3 in the denominator expression given by

$$Z=\frac{1+\alpha x^2}{(1+\beta x)^3}, \tag{13}$$

where α and β are arbitrary constants such that $\alpha\neq\beta$ and $\beta\neq0$. This choice is regular at the centre and continuous throughout the stellar interior. The choice is an advancement of the choice made by Thirukkanesh and Ragel [41] who used a rational function with quadratic expression in the numerator and linear expression in the denominator. The choice made in this paper is different from choices made in the past. The resulting model is well behaved and satisfies the stability and energy conditions. We also generated new masses and surface red-shifts in acceptable ranges using this choice. Such analysis is missing in most of the findings using the same approach in studying uncharged stars with anisotropic pressure. This work is likely to produce new insight on the studies of quark stars with no electric field. There are studies which used choice of metric function Z to model charged static spheres. These include models generated by John and Maharaj [42], Thirukkanesh and Maharaj [43], Mafa Takisa and Maharaj [44] and Nasim and Azam [25]. Our model is generated for uncharged static sphere with quark matter.

5. The regular model

Substituting (13) into eq. (12), we obtain the metric function y in the form

$$y=K(1+\alpha x^2)^n\sqrt{(1+\beta x)}\exp[H(x)], \tag{14}$$

where K is a constant of integration and the function $H(x)$ is given by

$$H(x)=\frac{1}{6\alpha^2C}\left[-2\sqrt{\alpha}(B(\alpha-3\beta^2))\arctan(x\sqrt{\alpha})-\alpha\beta^2x(6B-2C\beta+B\beta x)\right]. \tag{15}$$

The constant n has the form

$$n=\frac{B\beta(-3\alpha+\beta^2)+\alpha C(-2\alpha+3\beta^2)}{6\alpha^2C}. \tag{16}$$

The exact solution for the field equations in system (4) can be found as follows:

$$\mathrm{e}^{2\lambda}=\frac{(1+\beta x)^3}{1+\alpha x^2}, \tag{17a}$$

$$\mathrm{e}^{2\nu}=A^2K^2(1+\alpha x^2)^{2n}(1+\beta x)\exp[2H(x)], \tag{17b}$$

$$p_r=\frac{1}{3}(\rho-4B), \tag{17c}$$

$$p_t=p_r+\Delta, \tag{17d}$$

$$\Delta = \frac{1}{3(1+\alpha x^2)(1+\beta x)}\big[4B + 11\alpha C x + 4\alpha B x^2 + 11\alpha^2 C x^3 - 12\beta C + 20B\beta x + 7\alpha\beta C x^2 + 20\alpha\beta B x^3 + 19\alpha^2\beta C x^4 - 30\beta^2 C x + 40\beta^2 B x^2 - 34\alpha\beta^2 C x^3 + 40\alpha\beta^2 B x^4 - 4\alpha^2\beta^2 C x^5 - 10\beta^3 C x^2 + 40\beta^3 B x^3 - 10\alpha\beta^3 C x^4 + 40\alpha\beta^3 B x^5 - 5\beta^4 C x^3 + 20\beta^4 B x^4 - 5\alpha\beta^4 C x^5 + 20\alpha\beta^4 B x^6 - \beta^5 C x^4 + 4\beta^5 B x^5 - \alpha\beta^5 C x^6 + 4\alpha\beta^5 B x^7 + 48\alpha n C x + 24\alpha^2 n C x^3 + 84\alpha\beta n C x^2 + 36\alpha^2\beta n C x^4 + 36\alpha\beta^2 n C x^3 + 12\alpha^2\beta^2 n C x^5 + 48\alpha^2 n^2 C x^3 + 96\alpha^2\beta n^2 C x^4 + 48\alpha^2\beta^2 n^2 C x^5 + 6C(1+\alpha x^2)(1+\beta x) \times (2 + \beta x + \alpha\beta x^3(3+8n) + 4x^2(\alpha + 2\alpha n))\dot{H}(x) + 12Cx(1+\alpha x^2)^2 \times (1+\beta x)^2 \dot{H}(x)^2 + 12Cx(1+\alpha x^2)^2 \times (1+\beta x)^2\, \ddot{H}(x)\big]. \tag{17e}$$

We find that the exact solution in system (17) of the field equations in system (11) is represented in terms of elementary functions. The mass function in eq. (10) becomes

$$M(x) = \frac{1}{6\sqrt{C}(1+\beta x)^4}\big[x\sqrt{x}(9\beta + x(\alpha(-5+\beta x) + \beta^2(6 + \beta x(4+\beta x))))\big]. \tag{18}$$

The line element becomes

$$\mathrm{d}s^2 = -A^2K^2(1+\alpha x^2)^{2n}(1+\beta x)\exp[2H(x)]\mathrm{d}t^2 + \frac{(1+\beta x)^3}{1+\alpha x^2}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2). \tag{19}$$

5.1 *Compactness and red-shift*

The compactness factor μ for the stellar sphere proposed by Buchdahl [46] is defined as

$$\mu = \frac{2M}{R}, \tag{20}$$

where M is the mass of the stellar object and R is its corresponding radius. By substituting (18) into (20) we obtain

$$\mu = \frac{x\sqrt{x}(9\beta + x(\alpha(-5+\beta x) + \beta^2(6+\beta x(4+\beta x))))}{3\sqrt{x}(1+\beta x)^4}. \tag{21}$$

The surface red-shift z_s of a stellar object is defined as

$$z_s = \frac{1}{\sqrt{1-\mu}} - 1, \tag{22}$$

where μ is the compactness factor. The expression for z_s in this model is found by substituting (21) into (22) which then becomes

$$z_s = \left(1 - \frac{x(9\beta + x(\alpha(-5+x\beta) + \beta^2(6 + x\beta(4+x\beta))))}{3(1+x\beta)^4}\right)^{-1/2} - 1. \tag{23}$$

5.2 *Hydrostatic equilibrium*

The investigation on the state of hydrostatic equilibrium in this model is done by analysing the Tolman–Oppenheimer–Volkoff (TOV) equation. The equation explains the energy conservation in the stellar matter. The forces should balance in the stellar sphere. The TOV equation is given by

$$\dot{p}_r + (\rho + p_r)\dot{\nu} - \frac{2\Delta}{r} = 0. \tag{24}$$

In this model, the three forces describing the state of hydrostatic equilibrium, including gravitational force F_g, hydrostatic force F_h and anisotropic force F_a are considered. The forces are defined by

$$F_g = -(\rho + p_r)\dot{\nu}, \tag{25a}$$

$$F_h = -\dot{p}_r, \tag{25b}$$

$$F_a = \frac{2\Delta}{r}. \tag{25c}$$

Equation (24) can therefore be written as

$$F_g + F_h + F_a = 0. \tag{26}$$

In this particular model, the expressions for the forces in (25) are given by

$$F_g = -\frac{4\sqrt{x}}{9\sqrt{C}(1+\alpha x^2)(1+\beta x)^5}[(-2B - 4C\alpha x + 9C\beta - 8B\beta x - C\alpha\beta x^2 + 12C\beta^2 x - 12C\beta^2 x - 12B\beta^2 x^2 + 8C\beta^3 x^2 - 8B\beta^3 x^3 + 2C\beta^4 x^3 - 2B\beta^4 x^4)\big(-B - 5C\alpha x + 9C\beta - 4B\beta x + C\alpha\beta x^2 + 6C\beta^2 x - 6B\beta^2 x^2 + 4C\beta^3 x^2 - 4B\beta^3 x^3 + C\beta^4 x^3 - B\beta^4 x^4\big)], \tag{27}$$

$$F_h = \frac{1}{3(1+\beta x)^5}\big[2C\sqrt{Cx}(\alpha(5 - 17\beta x + 2\beta^2 x^2) + \beta^2(30 + 10\beta x + 5\beta^2 x^2 + \beta^3 x^3))\big] \tag{28}$$

and

$$F_a = \frac{2\sqrt{C}}{3\sqrt{x}(1+\alpha x^2)(1+\beta x)}\big[4B + 11\alpha C x + 4\alpha B x^2 + 11\alpha^2 C x^3 - 12\beta C + 20B\beta x + 7\alpha\beta C x^2 + 20\alpha\beta B x^3 + 19\alpha^2\beta C x^4 - 30\beta^2 C x + 40\beta^2 B x^2 - 34\alpha\beta^2 C x^3 + 40\alpha\beta^2 B x^4 - 4\alpha^2\beta^2 C x^5$$

$$-10\beta^3Cx^2+40\beta^3Bx^3-10\alpha\beta^3Cx^4+40\alpha\beta^3Bx^5 -5\beta^4Cx^3+20\beta^4Bx^4-5\alpha\beta^4Cx^5+20\alpha\beta^4Bx^6 -\beta^5Cx^4+4\beta^5Bx^5-\alpha\beta^5Cx^6+4\alpha\beta^5Bx^7 +48\alpha nCx+24\alpha^2nCx^3+84\alpha\beta nCx^2 +36\alpha^2\beta nCx^4+36\alpha\beta^2nCx^3+12\alpha^2\beta^2nCx^5 +48\alpha^2n^2Cx^3+96\alpha^2\beta n^2Cx^4+48\alpha^2\beta^2n^2Cx^5 +6C(1+\alpha x^2)(1+\beta x)(2+\beta x+\alpha\beta x^3(3+8n) +4x^2(\alpha+2\alpha n))\dot{H}(x)+12Cx(1+\alpha x^2)^2 \times(1+\beta x)^2\dot{H}(x)^2+12Cx(1+\alpha x^2)^2 \times(1+\beta x)^2\ddot{H}(x)\big]. \tag{29}$$

5.3 *Stability conditions*

The stability of stellar models can be described using different conditions. We examine the stability of this model using Zeldovich, Chandrasekhar and Herrera conditions. The stability condition proposed by Zeldovich [47] demands the stability criterion $(p_r/\rho) < 1$. In this model

$$\frac{p_r}{\rho}=\frac{1}{3C(9\beta+\beta^4x^3+\beta x^2(\alpha+4\beta^2)+x(-5\alpha+6\beta^2))} \times\big[-4B(1+\beta x)^4+C(9\beta+\beta^4x^3+\beta x^2(\alpha+4\beta^2) +x(-5\alpha+6\beta^2))\big]. \tag{30}$$

According to Chandrasekhar [48,49], the stellar model is stable if $\Gamma > \frac{4}{3}$, where Γ is the adiabatic index defined as

$$\Gamma=\left(\frac{\rho+p_r}{p_r}\right)\frac{\mathrm{d}p_r}{\mathrm{d}\rho}. \tag{31}$$

The expression for Γ in our model is given by

$$\Gamma=\frac{4(B(1+\beta x)^4-\beta C(9+6\beta x+4x^2\beta^2+\beta^3x^3))}{3(4B(1+\beta x)^4-\beta C(9+6\beta x+4x^2\beta^2+\beta^3x^3))}. \tag{32}$$

The stability condition by Herrera [50] is referred to as cracking of the star. For stable stellar sphere, the inequality

$$|v_r^2-v_t^2|<1 \tag{33}$$

should be satisfied to avoid overturning of the star, where $v_r^2=\mathrm{d}p_r/\mathrm{d}\rho$ and $v_t^2=\mathrm{d}p_t/\mathrm{d}\rho$. In this model

$$|v_r^2-v_t^2|=\frac{1}{3}\bigg[1-\frac{1}{3C(1+\alpha x^2)^2(1+\beta)^6}(2B^2 \times(1+\beta x)^8(1-\alpha x^2+4\beta x+2\alpha\beta x^3)BC \times(1+\beta x)^4(27\beta+8\alpha\beta^4x^5+4\alpha\beta^5x^6) +x(-28\alpha+48\beta^2)+x^2(-38\alpha\beta+72\beta^3) +x^3(-52\alpha\beta^2+48\beta^4)+x^4(15\alpha^2\beta-8\alpha\beta^3 +12\beta^5))+C^2(\alpha^3x^4(5+21\beta x-69\beta^2x^2 +5\beta^3x^3)+\alpha^2x^2(30+162\beta x+99\beta^2x^2 +332\beta^3x^3+120x^4\beta^4+24\beta^5x^5-\beta^6x^6) +\beta^2(-9+90\beta x+54\beta^2x^2+132\beta^3x^3 +157\beta^4x^4+96\beta^5x^5+30\beta^6x^6+4\beta^7x^7) -\alpha(15+63\beta x+333\beta^2x^2+335\beta^3x^3 +630\beta^4x^4+468\beta^5x^5+200\beta^6x^6 +48\beta^7x^7+6\beta^8x^8))\bigg]. \tag{34}$$

5.4 *Energy conditions*

For a well-behaved anisotropic model, all energy conditions must be satisfied. The energy conditions satisfy the inequalities $\rho \geq 0$ for null energy condition (NEC), $\rho - p_r \geq 0$ and $\rho - p_t \geq 0$ for weak energy condition (WEC_r and WEC_t) and $\rho - p_r - 2p_t \geq 0$ for strong energy condition (SEC).

6. Discussion

In this section, we indicate that our exact model in §5 is well behaved throughout the stellar interior. We have performed our mathematical computations using *Mathematica* while the plots have been generated using *Python* programming language. All these graphs are plotted against the radial distance r. We generate graphs for the exact model in §5 by specifying values for the constants as $\alpha = 10.0$, $\beta = 1.9$, $B = 0.2$, $C = 1.4$ and $K = 1$.

We observe that the gravitation potentials in figures 1 and 2 are continuous, regular and finite throughout the stellar interior with positive values at the centre. In figures 3–5, the quantities ρ, p_r and p_t are decreasing functions with critical maximum values at the core of the stellar star such that $\rho' < 0$, $p_r' < 0$ and $p_t' < 0$. This behaviour is physically realistic. Some

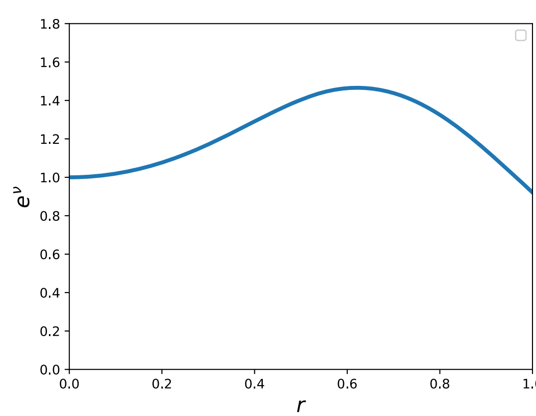

Figure 1. The potential e^{ν} against the radial distance r.

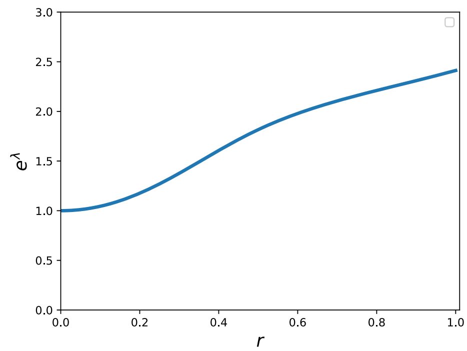

Figure 2. The potential e^{λ} against the radial distance r.

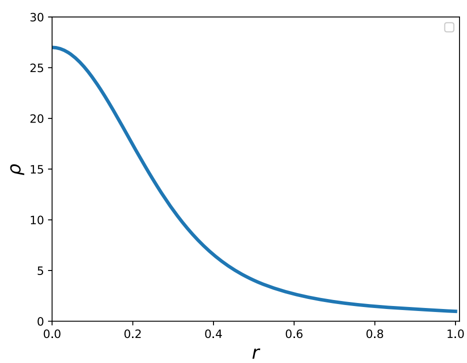

Figure 3. Energy density ρ against the radial distance r.

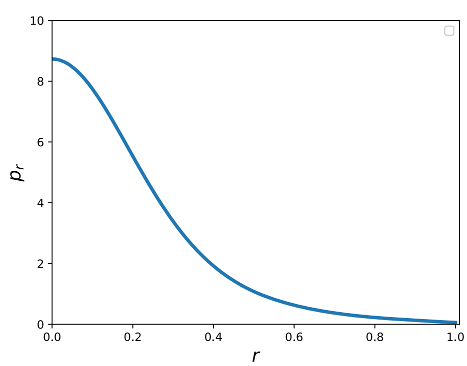

Figure 4. Radial pressure p_r against radial distance r.

recent models with similar features are found by Sunzu and Danford [51], Maharaj and Mafa Takisa [52], Mafa Takisa and Maharaj [44], Maharaj *et al* [53], Ragel and Thirukkanesh [54] and Ngubelanga *et al* [14]. Figure 6 shows that the measure of anisotropy is zero at the centre and then increases to a maximum value near the centre. This is followed by sharp decrease to a minimum value at some points within the core. As one approaches the stellar surface, the anisotropy parameter increases. We also note that $\Delta > 0$ closer to the centre of the

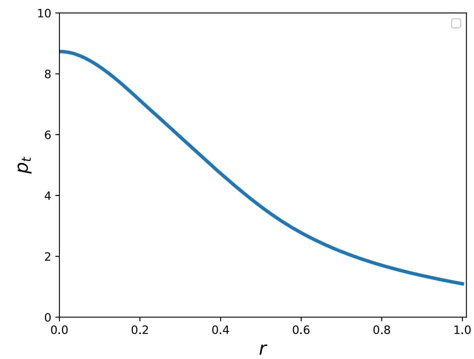

Figure 5. Tangential pressure p_t against radial distance r.

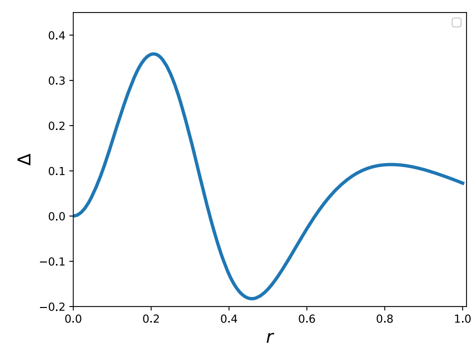

Figure 6. Measure of anisotropy Δ against radial distance r.

stellar star and then becomes negative at some points within the stellar interior and finally becomes positive as it approaches the surface. Based on Gokhroo and Mehra [45] assertion, the regions with positive anisotropic factor experience repulsive anisotropic force which acts outward and the region with negative anisotropic factor experiences attractive force which acts inward. The local maximum and local minimum critical points in the anisotropy profile successively indicate the maximum repulsive and attractive anisotropic forces within the stellar object. However, the behaviour of this force in different regions of the stellar sphere do not affect the stability of the stellar sphere. The same feature is evident in the work by Nasim and Azam [25]. It is physically significant to have $\Delta = 0$ at the centre of the stellar object for any realistic model. It is clear from figure 7 that the mass M of the stellar object increases monotonically with radial distance. Figure 8 clearly shows $v = 1/3$ which implies that the speed of sound inside the stellar sphere is less than the speed of light. This value is consistent with the speed of sound in quark star. It also agrees with eq. (6). Figure 9 shows that the compactification factor is determined to be $\mu = 0.27$ and is less than $8/9$ which is the maximum limit for uncharged compact star

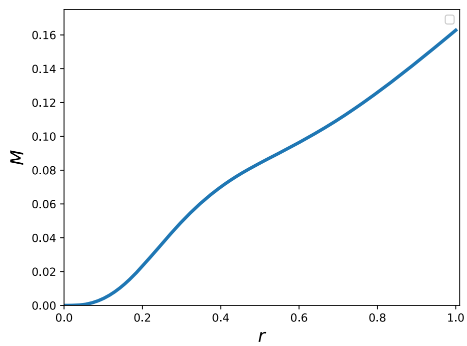

Figure 7. Mass M against radial distance r.

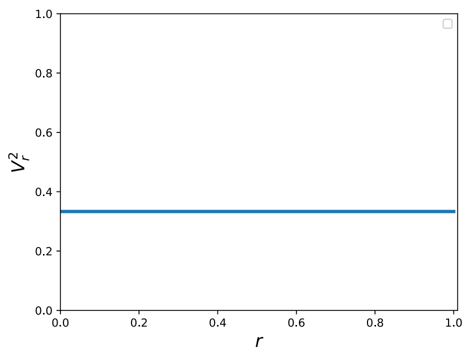

Figure 8. Speed of sound V against radial distance r.

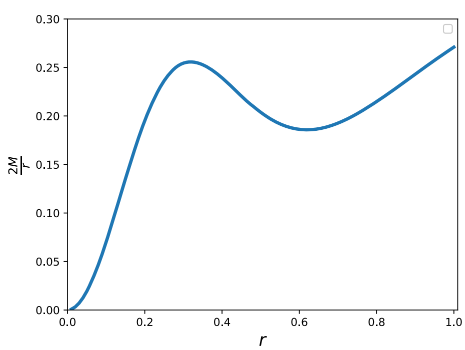

Figure 9. Compactification μ against radial distance r.

according to Buchdahl [46]. Figure 10 shows that the maximum value for the red-shift is $z_s = 0.17$. This value is less than 2 which is reported in Mafa Takisa *et al* [55] to be the maximum value for realistic compact objects. The variation of forces in TOV equation presented in figure 11 shows that the stellar sphere is in hydrostatic equilibrium. The gravitational force which is attractive in nature counterbalances the hydrostatic and anisotropic forces. Various stability conditions are also satisfied in this model. In figure 12, the equation of state is a decreasing function and its maximum value is less than 1. This implies that the Zeldovich stability condition is satisfied. Similar profile is observed in the work by Hansraj and Moodly [56]. Figure 13 shows that

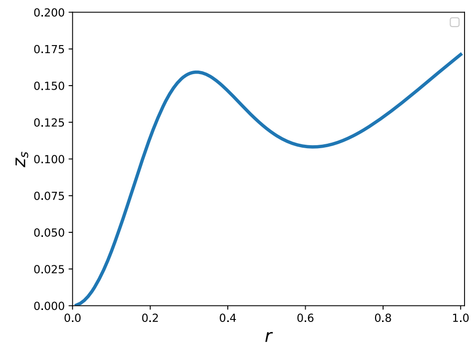

Figure 10. Surface red-shift against radial distance r.

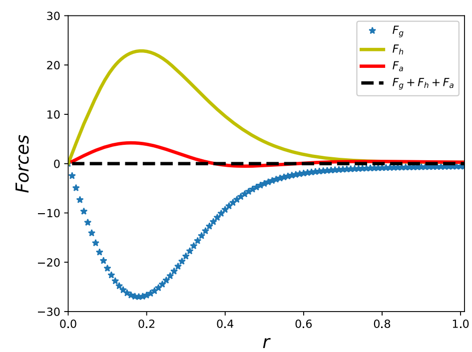

Figure 11. Variation of forces in TOV equation against radial distance r.

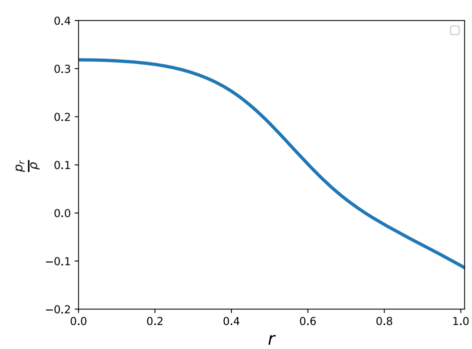

Figure 12. Equation of state p_r/ρ against radial distance r.

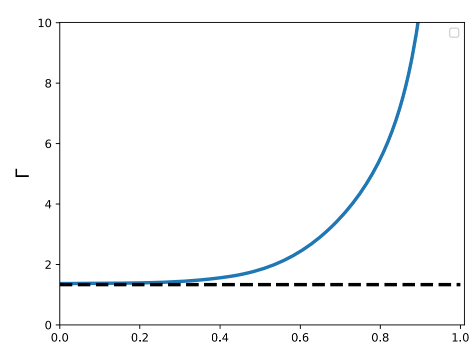

Figure 13. Adiabatic index Γ against radial distance r.

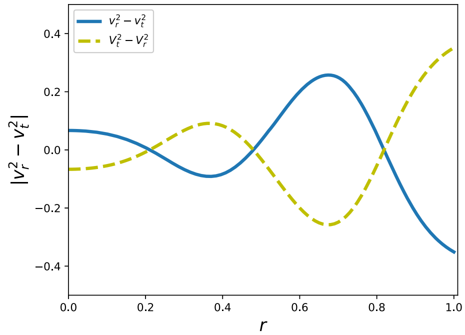

Figure 14. Herrera cracking $|v_r^2 - v_t^2|$ against radial distance r.

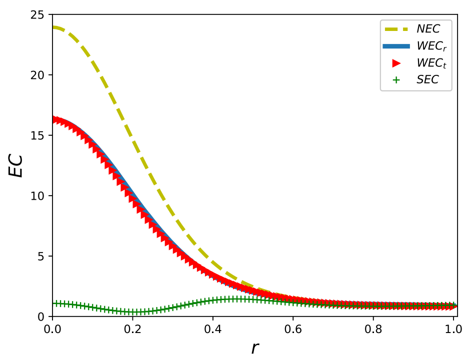

Figure 15. Energy conditions against radial distance r.

the minimum adiabatic index $\Gamma_0 \geq 1.37$ which is greater than 4/3 reported to be the lower bound according to Chandrasekhar [48,49]. Herrera [50] cracking condition ($|v_r^2 - v_t^2| \leq 1$) for stability shown in figure 14 is also satisfied. It is also evident in figure 15 that the null, weak and strong energy conditions are all satisfied with $\rho \geq 0$, $\rho - p_r \geq 0$ and $\rho - p_t \geq 0$ as well as $\rho - p_r - 2p_t \geq 0$, respectively.

We use the transformations $\tilde{C} = C\mathbb{R}^2$, $\tilde{\alpha} = \alpha\mathbb{R}^2$ and $\tilde{\beta} = \beta\mathbb{R}^2$ in mass function (18) to generate stellar masses coherent with the past observations. For computation purpose, $\mathbb{R} = 40$.

We have generated the radii and masses of stellar objects consistent with the compact stars such as 4U1538-52 with $r = 7.866$ km and $M = 0.87M_\odot$ and LMCX-4 with $r = 8.301$ km and $M = 1.04M_\odot$, SMCX-4 with $r = 8.831$ km and $M = 1.29M_\odot$, SAXJ1808.4-3658 with $r = 7.68$ km and $M = 0.903M_\odot$, HerX-1 with $r = 8.1$ km and $M = 0.85M_\odot$ as indicated in Rawls *et al* [57], Elebert *et al* [58] and Abubekerov *et al* [59], respectively. We have also generated new stellar masses in the range $(0.9417–1.3061)M_\odot$ and surface red-shifts in the range 0.1414–0.1862. These ranges are acceptable for various stellar objects. Values of the parameters for these stellar masses, radii and surface red-shifts are indicated in tables 1 and 2.

7. Conclusion

In this paper, we have generated an exact model consistent with quark stars. This model has been generated with a new choice of gravitational potential. The balance of forces within the stellar sphere for stability has been attained by analysing the TOV equation. The profiles for gravitational potentials, matter variables, speed of sound, compactness, red-shift, hydrostatic equilibrium, stability and energy conditions are well behaved. We have generated relativistic stars with stellar masses and radii compatible with those observed in the past by

Table 1. Particular relativistic stellar masses consistent with observations.

$\tilde{C}$	$\tilde{\alpha}$	$\tilde{\beta}$	r (km)	$M(M_\odot)$	Star
55	8.65	190	7.68	0.903	SAXJ1808.4-3658
920.6	5.59	100.2	7.866	0.87	4U1538-52
80	9.1	110.5	8.1	0.85	HerX-1
50	0.001	321	8.831	1.29	SMCX-4
40.5	0.05	1850	8.301	1.04	LMCX-4

Table 2. New stellar masses, radii and surface redshifts.

$\tilde{C}$	$\tilde{\alpha}$	$\tilde{\beta}$	r (km)	$M(M_\odot)$	z_s
30.5	10.5	790	7.01	1.0141	0.1862
40.5	1.5	400	8.01	1.1402	0.1823
9.8	6.5	850	8.10	0.9417	0.1414
45.8	3.5	1750	9.04	1.0956	0.1488
35.8	0.5	950	9.55	1.3061	0.1732

other researchers. These are consistent with the observations made by Rawls *et al* [57], Elebert *et al* [58] and Abubekerov *et al* [59]. Therefore, the model in this paper described astrophysical objects like 4U1538-52, LMCX-4, SMCX-4, SAXJ1808.4-3658 and HerX-1. We indicated that new stellar masses, radii and surface red-shifts generated using our model are in acceptable range for realistic stars.

Acknowledgements

The authors would like to acknowledge the University of Dodoma for providing attractive environment to conduct research and for providing facilities and resources.

References

[1] R L Bowers and E P T Liang, *Astrophys. J.* **188**, 657 (1974)
[2] A T Abdalla, J M Sunzu and J M Mkenyeleye, *Pramana – J. Phys.* **95**, 86 (2021)
[3] J M Sunzu, A K Mathias and Maharaj, *J. Astrophys. Astr.* **40**, 8 (2019)
[4] P Bhar, M Govender and R Sharma, *Pramana – J. Phys.* **90**, 5 (2018)
[5] R Ruderman, *Class. Ann. Rev. Astron. Astrophys*. **10**, 427, (1972)
[6] R Kippenhahn and A Weigert, *Stellar structure and evolution* (Springer, Berlin, 1990)
[7] A I Sokolov, *JETP* **79**, 1137 (1980)
[8] R F Sawyer, *Phys. Rev. Lett*. **29**, 823 (1972)
[9] K Dev and M Gleiser, *Gen. Relativ. Gravit.* **34**, 1793 (2002)
[10] M H Murad, *Astrophys. Space Sci.* **361**, 20 (2016)
[11] M H Murad and S Fatema, *Eur. Phys. J. C* **75**, 533 (2015)
[12] M H Murad and S Fatema, *Eur. Phys. J. Plus* **130**, 3 (2015)
[13] J M Sunzu, S D Maharaj and S Ray, *Astrophys. Space Sci.* **352**, 719 (2014)
[14] S A Ngubelanga, S D Maharaj and S Ray, *Astrophys. Space Sci.* **357**, 74 (2015)
[15] S Karmakar, S Mukherjee, R Sharma and S D Maharaj, *Pramana – J. Phys.* **68**, 881 (2007)
[16] J M Sunzu and T Mashiku, *Pramana – J. Phys.* **91**, 75 (2018)
[17] S Thirukkanesh and F C Ragel, *Pramana – J. Phys*. **78**, 687 (2012)
[18] M K Mak and T Harko, *R. Soc.* **459**, 393 (2003)
[19] M Kalam, A A Usmani, F Rahamani, S M Hossein, I Karar and R Sharma, *Int. J. Theor. Phys.* **52**, 3319 (2013)
[20] S Bannerjee, *Commun. Theor. Phys.* **70**, 585 (2018)
[21] P Mafa Takisa, S D Maharaj and L L Leeuw, *Eur. Phys. J. C* **79**, 8 (2019)
[22] F Rahaman, S Ray, A K Jafry and K Chakraborty, *Phys. Rev. D* **82**, 104055 (2010)
[23] H B Benaoum, hep-th/0205140 (2002)
[24] P Bhar, *Astrophys. Space Sci.* **359**, 41 (2015)
[25] A Nasim and M Azam, *Eur. Phys. J. C* **78**, 34 (2018)
[26] S Thirukkanesh and F C Ragel, *Pramana – J. Phys.* **78**, 687 (2012)
[27] S Thirukkanesh, F C Ragel, *Pramana – J. Phys.* **81**, 275 (2013)
[28] P Mafa Takisa, S D Maharaj, *Gen. Relat. Gravit.* **45**, 1951 (2013)
[29] S Thirukkanesh and F C Ragel, *Pramana – J. Phys*. **83**, 83 (2014)
[30] R X Xu, *Astrophys. J.* **596**, L59 (2003)
[31] K Ananda and M Bruni, *Phys. Rev. D* **74**, 023523 (2006)
[32] S Capozziello, V Cardone, E Elizalde, S Nojiri and S D Odintsov, *Phys. Rev. D* **73**, 043512 (2006)
[33] T Feroze and A A Siddiqui, *Gen. Relativ. Gravit.* **43**, 1025 (2011)
[34] P Mafa Takisa, S D Maharaj and S Ray, *Astrophys. Space Sci.* **354**, 463 (2014)
[35] P Bhar, K N Singh and N Pant, *Indian J. Phys.* **91**, 701 (2017)
[36] M K Mak and T Harko, *Int. J. Mod. Phys. D* **13**, 149 (2004)
[37] K Komathiraj and S D Maharaj, *Int. J. Mod. Phys. D* **16**, 1803 (2007)
[38] S Thirukkanesh and S D Maharaj, *Class. Quantum Grav.* **25**, 235001 (2008)
[39] R Sharma and S D Maharaj, *Mon. Not. R. Astron. Soc.* **375**, 1265 (2007)
[40] M C Durgapal and R Bannerji, *Phys. Rev. D* **27**, 328 (1983)
[41] S Thirukkanesh and F C Ragel, *Chin. Phys. C* **41**, 015102 (2017)
[42] A J John and S D Maharaj, *Pramana – J. Phys*. **77**, 461 (2011)
[43] S Thirukkanesh and S D Maharaj, *Math. Meth. Appl. Sci.* **32**, 684 (2009)
[44] P Mafa Takisa and S D Maharaj, *Gen. Relativ. Gravit.* **45**, 1951 (2013)
[45] M K Gokhroo and A L Mehra, *Gen. Relativ. Gravit.* **26**, 75 (1994)
[46] H A Buchdahl, *Phys. Rev.* **116**, 1027 (1959)
[47] Y B Zeldovich and Z Eksp, *Teor. Fiz.* **41**, 1609 [Engl. transl: *Sov. Phys. JETP* **14**, 1143 (1962)]
[48] S Chandrasekhar, *Astrophys. J.* **140**, 417 (1964)
[49] S Chandrasekhar, *Phys. Rev. Lett.* **12**, 114 (1964)
[50] L Herrera, *Phys. Lett. A* **165**, 206 (1992)
[51] J M Sunzu and P Danford, *Pramana – J. Phys.* **89**, 44 (2017)
[52] S D Maharaj and P Mafa Takisa, *Gen. Relativ. Gravit.* **44**, 1419 (2012)
[53] S D Maharaj, D K Matondo and P Mafa Takisa, *Int. J. Mod. Phys. D* **26**, 1750014 (2017)
[54] F C Ragel and S Thirukkanesh, *Eur. Phys. J. C* **79**, 306 (2019)

[55] P Mafa Takisa, S D Maharaj, A M Manjonjo and S Moopanar, *Eur. Phys. J. C* **77**, 713 (2017)
[56] S Hansraj and L Moodly, *Eur. Phys. J. C* **80**, 496 (2020)
[57] M L Rawls, J A Orosz and J E McClintock, *APJ* **730**, 25 (2011)
[58] P Elebert, M T Reynolds and P J Callanan, *MNRAS* **395**, 884 (2009)
[59] M K Abubekerov, E A Antokhina, A M Cherepashchuk and V V Shimanskii, *Astron. Rep.* **52**, 379 (2008)