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Ringing non-Gaussianity from inflation with a step in the second derivative of the potential

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Abstract. Inflationary model driven by a scalar field whose potential has a step in the second derivative with respect to the field is considered. For the best-fit potential parameter values, the 3-point function and the non-Gaussianity associated with the featured model is calculated. We study the shape and scale dependence of the 3-point function. The distinctive feature of this model is its characteristic ringing behaviour of $f_{\rm NL}$. We can see that the oscillations in $f_{\rm NL}$ in this model last for a much longer range of k values, than the previously studied models. In that sense, this model is potentially distinguishable from models with other features in the potential.

Keywords. Inflation; 3-point function; non-Gaussianity.

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1. Introduction

In standard slow-roll inflation, the deviation from the Gaussian distribution of the primordial perturbations [1–3] is predicted to be small. It is of the order of the slow-roll parameters [4–7]. This result does not hold if the inflaton undergoes a period of slow-roll violation during its evolution [8–11], as can happen if the inflaton potential has some localised features [12]. The resulting non-Gaussianity [13] then becomes shape and scale-dependent and modes that exit Hubble scale around the time the field crosses the feature can pick up large non-Gaussianities. An inflationary model, where the inflaton potential has a feature in its second derivative with respect to the inflaton, has been proposed in Minu Joy *et al* [14], where they showed that the power spectrum picks of small oscillations superimposed on a flat spectrum and the spectral index can experience a jump around the feature. This scenario helps to explain the local running in the spectral index observed in the WMAP and Planck data [15–18]. In the present work, we are interested in studying the non-Gaussianity predicted by this model. The analysis of the running of non-Gaussianity using Planck data is given in [19], in the context of some well-defined inflationary models. Inflationary models that predict a mildly scale-dependent bispectrum, termed as the running of the bispectrum [20–23], is discussed there. In the present work, we compute the 3-point function of the curvature perturbation and study the shape and scale dependence of the 3-point function.

The paper is organised as follows: in §2 we describe our model and give its background evolution and resulting power spectrum of curvature perturbations. In §3 we compute the 3-point function and the non-Gaussianity from the model and then conclude by summarising the results in §4.

2. The model with a step in the second derivative of the potential

A model in which the inflaton potential experiences a sudden small change in its second derivative (the effective mass of the inflaton) is considered. Let [ ] denote a jump in the relevant quantity, so that $[A] \equiv A(\varphi_c+0) - A(\varphi_c-0)$ where $\varphi=\varphi_c$ is the point at which the feature occurs. We consider the case for which $[V]=[V']=0$, $[V''] \neq 0$ and $|[V'']| \ll H^2$. The last inequality guarantees that slow-roll inflation continues during and after the phase transition, in contrast to the case of the hybrid inflation, or the case $|[V'']| \sim H^2$. A field theoretic model giving rise to such a feature of the

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Table 1. Best-fit values of potential parameters.

Parameter	Values for $N = 60$	Values for $N = 40$
M/m_{pl}	7.43×10^{-4}	8.22×10^{-4}
m/m_{pl}	4.60×10^{-7}	6.90×10^{-7}
g	2.77×10^{-4}	3.75×10^{-4}
λ	0.1	0.1

inflationary potential, based on a fast phase transition experienced by a second scalar field weakly coupled to the inflaton is described in [14].

In [14], we showed that the inflationary model with a Higgs-like potential which is used in the hybrid inflationary scenario too,

$$V(\psi, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\psi^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\psi^2 , \tag{1}$$

could successfully give rise to a step in the spectral index of primordial spectrum. At the critical value of the inflaton, $\phi_c = M/g$, the curvature of the potential $V(\psi, \phi)$ along the ψ direction vanishes and the effective mass of the ψ field $m_\psi^2 > 0$ for $\phi > \phi_c$ while $m_\psi^2 < 0$ for $\phi < \phi_c$. This implies that for large values of the inflaton ϕ, the auxiliary field ψ rolls towards $\psi = 0$. However, once the value of ϕ falls below ϕ_c, the $\psi = 0$ configuration is destabilised resulting in a rapid cascade (mini-waterfall) which takes ψ from $\psi = 0$ to its minimum value. In the original hybrid inflationary models [24,25] with waterfalls, the inflation comes to an end soon after the phase transition. In the present model, the conditions are set [14] such that the inflation field continues the slow-roll, even after the phase transition.

This potential has four parameters, namely M, m, g and λ. The model confrontation with WMAP-7 data has been done in [26] and the best-fit values of potential parameters were obtained with $N = 40$ and $N = 60$, where N represents the number of e-folds after the phase transition. Table 1 gives the potential parameters best fit with the Bicep–Keck–Planck likelihoods (combined BICEP2 and Keck Array October 2018 data in combination with the 2018 Planck data) [27–29].

2.1 Background evolution

With this potential, by initiating evolution with initial field value $\phi_i > \phi_c$, the coupled system of equations of the background inflaton and scale factor of expansion of space–time exhibits inflation with the inflaton rolling the potential, till inflation ends and ϕ finally oscillating about the potential minimum. The slow-roll conditions $\epsilon, \eta < 1$, where ϵ, η are given by

$$\epsilon \equiv 3\frac{\dot{\phi}^2/2}{\dot{\phi}^2/2 + V}, \quad \eta \equiv -3\frac{\ddot{\phi}}{3H\dot{\phi}}$$

are satisfied throughout the inflationary period. However, η has a discontinuity at $\phi = \phi_c$ as it is proportional to V''.

We explore the consequences of this discontinuity in the behaviour of the 2-point and 3-point functions of the perturbations of the dynamical variables.

2.2 Two-point function and power spectrum of scalar perturbations

The Fourier modes of the curvature perturbation satisfies the equation

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + k^2\mathcal{R}_k = 0, \tag{2}$$

where the prime denotes derivative with respect to the conformal time and the quantity z is given by

$$z \equiv \frac{a}{H}\sqrt{\rho + p} = \frac{a\dot{\phi}}{H}. \tag{3}$$

$\mathcal{R}$ is related to the Mukhanov–Sasaki variable u as $\mathcal{R} = u/z$. The scalar power spectrum is then defined as

$$\mathcal{P}_s(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \tag{4}$$

with the amplitude of the curvature perturbation $\mathcal{R}_k$ evaluated, in general, at the end of inflation.

Under slow-roll, the power spectrum is approximately given by

$$\mathcal{P}_s(k) \simeq \frac{1}{2\epsilon}\frac{H^2}{2\pi}\left(\frac{k}{aH}\right)^2 \tag{5}$$

with n_s given in terms of slow-roll parameters as

$$n_s \simeq 1 - 2\epsilon - \eta. \tag{6}$$

For the potential given in eq. (1), one can immediately notice that n_s will have discontinuity due to its dependence on η. This will result in the power spectrum having a jump in its slope at a scale set by ϕ_c. The full analytic expression for the power spectrum has small oscillations superimposed around the scale of change of slope, as shown in [14]. Figure 1 shows the quasi-flat $\mathcal{P}_k$ for this mini-waterfall hybrid model.

3. Three-point function and non-Gaussianity from the model

Our approach is based on the numerical evaluation of both the perturbation equations and the integrals, which contribute to the 3-point function described by Chen *et*

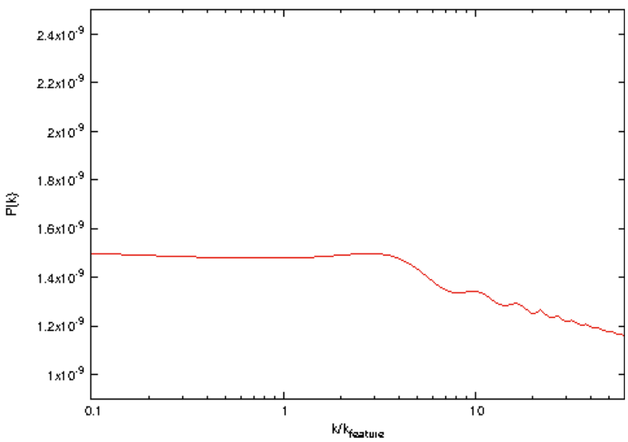

Figure 1. Primordial spectrum for a model with step in the second derivative of the potential.

al [8,9]. To compute the 3-point correlation function, one can substitute the mode solutions u_k into eq. (3.17) of [8],

$$H_{\rm int}(\tau) = -\int d^3x \left\{ a\epsilon^2\zeta\zeta'^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2\epsilon\zeta'(\partial\zeta)(\partial\chi) + \frac{a}{2}\epsilon\eta'\zeta^2\zeta' + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)(\partial^2\chi) + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 \right\} \quad (7)$$

and integrate the mode functions from τ_0 (where τ_0 is an arbitrary time when all the three modes are well inside the horizon) through to the end of inflation. This integral can be done semi-analytically for simple models, provided the slow-roll parameters are small and relatively constant. For standard single-field slow-roll inflation, the terms of order ϵ^2 in the aforementioned equation are the dominant contributors to the 3-point function and the other terms of order $\epsilon\eta'$ and ϵ^3 were neglected in refs [4–6]. In the presence of a step in the potential, the $\epsilon\eta'$ term becomes large [8,9]. The step in the second-order derivative of the potential will make the $\epsilon\eta'$ term more significant than the ϵ^2 term and hence leads to a modification of the standard slow-roll results. Thus, the term of our particular interest is the $\epsilon\eta'$ term

$$I_{\epsilon\eta'} \propto i\left(\prod_i u_i(\tau_{\rm end})\right)\int_{-\infty}^{\tau_{\rm end}} d\tau\, a^2\epsilon\eta'\,(2\pi)^3\,\delta^3 \times\left(\sum_i \mathbf{k}_i\right)\left(u_1^*(\tau)u_2^*(\tau)\frac{d}{d\tau}u_3^*(\tau) + \text{two perm}\right) + \text{c.c.}, \quad (8)$$

where 'two perm' stands for two other terms that are symmetric under permutations of the indices 1, 2 and 3, where 1, 2, 3 are short-hand for k_1, k_2 and k_3. For our featured model, $\epsilon\eta'$ can be written as

$$\epsilon\eta' = 6aH\left(2\epsilon^2 - \frac{\epsilon\eta}{2} + \frac{5}{6}\epsilon^2\eta - \frac{2}{3}\epsilon^3 - \frac{\epsilon\eta^2}{12} - \epsilon\frac{V_{\phi\phi}}{3H^2}\right). \quad (9)$$

In order to integrate eq. (8) numerically, we follow the procedure detailed in [9]. For the present model with a step in the second derivative of the potential,

$$\nu_k(\tau_0) = \frac{\sqrt{\pi\tau}}{2}H^{(2)}_{\mu_1}(k\tau_0), \quad (10)$$

where $H^{(2)}_{\mu_1}(k\tau_0)$ is the Hankel function and

$$\mu_1 = \frac{3}{2} - \frac{V''_-}{3H_0^2} + 3\epsilon_0,$$

where

$$V''_- \equiv \left(\frac{d^2V}{d\varphi^2}\right)_{\text{before phase transition}}.$$

Every 3-point correlation function has two main attributes: shape and scale. Following [8,9], we define the parameter, $\mathcal{G}$, to describe non-Gaussianities with both shape and scale dependence:

$$\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} \equiv \frac{1}{\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)}\frac{(k_1k_2k_3)^2}{P_k^2(2\pi)^7} \times\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle. \quad (11)$$

In the absence of the sharp feature, eq. (11) reduces to the local form with

$$\mathcal{G} = (3/10)f_{\rm NL}^{\rm local}\sum k_i^3. \quad (12)$$

Using eqs (11) and (12), we can calculate the non-Gaussianity parameter $f_{\rm NL}^{\rm local}$,

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left(-\frac{3}{10}f_{\rm NL}\mathcal{P}_k^2\right)\frac{\Sigma_i k_i^3}{\Pi_i k_i^3}. \quad (13)$$

For the best-fit potential parameter values given in table 1, we compute the resulting 3-point function. Figure 2 gives $f_{\rm NL}$ for our model, for the equilateral configuration ($k_1 = k_2 = k_3$). Purple line is for $N = 60$ and green line is for $N = 40$. The numerical value is not much larger than those in standard single-field slow-roll inflation, $f_{\rm NL} \sim$ the first-order slow-roll parameters $\sim \mathcal{O}(10^{-2})$. The distinctive feature of this non-Gaussianity is its characteristic ringing behaviour of $f_{\rm NL}$.

From eq. (9), we see that if $V_{\phi\phi} \ll 3H^2$ is satisfied and $V_{\phi\phi}/3H^2 \sim \epsilon$, then the $\epsilon\eta'$ term contributes three

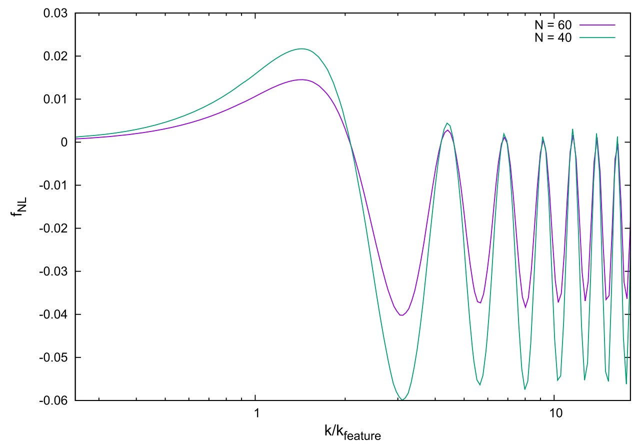

Figure 2. $f_{\rm NL}^{\rm equil}$ for the equilateral case. x-axis is $k/k_{\rm feature}$.

terms which are of second order in slow-roll parameters. These terms affect scales that exit Hubble horizon around the time the field crosses the feature in the potential. These contributions will then modify the standard slow-roll results for the non-Gaussianity. As it is clear from figure 2, the sudden small jump in $f_{\rm NL}$ occurs at a scale set by ϕ_c. The $f_{\rm NL}$ for this model is modulated by characteristic oscillations and the oscillations last for a much longer range of k values, than in the previously studied models [8,9].

4. Conclusion

The single-field, slow-roll models of inflation generically yields a negligible primordial non-Gaussianity. Thus, the bispectrum analysis of CMB data can be considered as a promising candidate for discriminating between the degenerate inflationary models. In the present work, we considered a variant of hybrid inflation where the potential has a discontinuity in its second derivative with respect to the field. This describes a fast second-order phase transition during inflation that occurs in some other scalar field weakly coupled to the inflaton. The 3-point correlation function is numerically integrated for this anomalous inflationary model where slow-roll is violated for a brief moment. The transient violation of the slow-roll leads to an oscillating and scale dependent 3-point function. The present mini-waterfall model gives an $f_{\rm NL}$ value comparable to that of the standard single-field inflationary model, $\mathcal{O}(10^{-2})$. The non-Gaussianity $f_{\rm NL}$ associated with the present model might be observable by the next generation of experiments.

For the typical potential parameter values, non-Gaussianity associated with the featured potential model is found to be oscillating. The distinctive feature of this non-Gaussianity is its characteristic ringing behaviour; $f_{\rm NL}$ oscillates between a maximum and a minimum value. The oscillations in $f_{\rm NL}$ in this model last for a much longer range of k values, than in the previously studied models. Based on the above, this model is potentially distinguishable from models with other features in the potential.

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