Pramana – J. Phys. (2022) 96:50
https://doi.org/10.1007/s12043-022-02298-y

Complexity factor for static cylindrical objects in $f(G, T)$ gravity

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MS received 8 October 2021; revised 27 October 2021; accepted 27 October 2021

Abstract. The purpose of this paper is to investigate the definition of complexity for static anisotropic cylindrical objects in the background of $f(G, T)$ gravity, where G and T stand for Gauss–Bonnet term and trace of the energy–momentum tensor, respectively. We develop the modified field equations, Tolman–Oppenheimer–Volkoff equation, mass distribution and structure scalars. The complexity is calculated from the splitting of the Riemann tensor in terms of a complexity factor which is associated with the physical characteristics (anisotropic pressure, inhomogeneous energy density) of the system. The zero complexity condition is derived as a constraint to estimate the behaviour of two compact objects corresponding to Gokhroo and Mehra ansatz and polytropic equation of state. We conclude that the addition of $f(G, T)$ terms contribute to the increment in the complexity of the system.

Keywords. Self-gravitating systems; $f(G, T)$ gravity; complexity factor.

PACS Nos **04.40.-b; 04.50.Kd; 04.40.Dg**

1. Introduction

Einstein's general theory of relativity (GR) has improved our perception of gravity and understanding of the basic concepts of our cosmos. The Universe contains both tiny and massive bodies which are involved in the evolution of cosmos. Einstein believed that the Universe is neither expanding nor contracting but remains in a static phase. However, Hubble unveiled the relationship between distance and recession velocity of galaxies to suggest the expansion of the Universe. It is assumed that the expanding cosmos is a result of some mysterious force called dark energy. Various astronomical phenomena like cosmic microwave background radiation and supernovae clearly revealed the rapid expansion of the Universe [1,2]. Fine-tuning and cosmic coincidence are the two fundamental problems in GR that do not sufficiently explain the current accelerated expansion of the Universe. To accomplish this, modified theories are assumed as an alternate to explain the expanding nature of the Universe.

Nojiri and Odintsov [3] proposed $f(G)$ gravity by taking an arbitrary function of the Gauss–Bonnet (GB) invariant in the Einstein–Hilbert (EH) action. This invariant was helpful in discussing the evolution of the cosmos from a decelerated to an accelerated phase. It is a combination of the Ricci tensor $(R_{\nu\mu})$, Riemann tensor $(R_{\nu\mu\beta\alpha})$ and Ricci scalar (R), i.e., $G = -4R^{\nu\mu}R_{\nu\mu} + R^{\nu\mu\beta\alpha}R_{\nu\mu\beta\alpha} + R^2$. Bamba *et al* [4] reconstructed $f(G)$ model to study the late-time acceleration and early-time bounce through the scale factor as an exponential function. Abbas *et al* [5] employed the Krori–Barua metric along with a power-law model to determine the stability and feasibility of celestial objects in $f(G)$ gravity. Sharif and Ikram [6] investigated the inflationary dynamics of an isotropic and homogeneous cosmos with the help of a viable $f(G)$ model. Shamir and Saeed [7] discussed the power-law and exponential solutions for a plane-symmetric space–time in this theory. Sharif and Saba [8] used the decoupling approach to check the stability and viability of anisotropic celestial objects. Sharif and Ramzan [9] employed the embedding class-1 approach to analyse the physical attributes of anisotropic celestial objects.

Sharif and Ikram [10] suggested a new modified theory, namely $f(G, T)$ gravity, by involving the trace of energy–momentum tensor and examined the energy conditions. They also tested the stable behaviour of the Einstein Universe by using the perturbation technique [11]. Hossienkhani *et al* [12] found that weak and null energy conditions are compatible with the anisotropic $f(G, T)$ Universe for some particular range of indicated parameters. Sharif and Naeem [13] discussed the viability and stability of cosmic structures for a spherical

Published online: 17 February 2022

system. Shamir [14] analysed the oscillating Universe with the help of two models involving linear and logarithmic trace terms.

The physical attributes of astrophysical objects such as heat, energy density, temperature, pressure, etc., play a major role in increasing the complexity of self-gravitating bodies. Therefore, the complexity of cosmic objects must be defined by a mathematical formula comprising all important components. Several efforts have been made to effectively describe the notion of complexity. However, a comprehensive definition of complexity for all disciplines has yet to be established. López-Ruiz *et al* [15] developed the idea of complexity in terms of entropy and information. Two physical objects (perfect crystal and idea gas) were primarily studied to calculate the complexity. The perfect crystal has zero entropy because its molecules are orderly arranged due to its symmetric nature while the ideal gas shows maximum entropy since its molecules are randomly distributed. Due to the symmetric nature of a perfect crystal, its probability distribution around the symmetric part contributes limited information and a small portion is sufficient to characterise all its features. Therefore, the probability distribution of a perfect crystal depends on its symmetry for all the accessible states. When we study a small portion of an ideal gas, all its accessible states provide maximum information because they possess the same probability. However, zero complexity is allotted to both the structures in physics.

Later on, a more effective way was developed to define complexity and how various probabilistic states differ from the equiprobable distribution of the system was estimated [16,17]. Both perfect crystal as well as ideal gas tend to have zero complexity using this definition. To estimate the complexity of astrophysical objects, the probability distribution was replaced with energy density in the improved approach [18–20]. However, this criterion did not include other state indicators such as heat, temperature, pressure, etc. Recently, the idea of complexity was redefined by Herrera [21] for the static matter distribution in terms of inhomogeneous energy density, pressure anisotropy and Tolman mass. The Riemann tensor was orthogonally split to form four structure scalars. The scalar that includes all the state variables was regarded as a complexity factor. Sharif and Butt [22] investigated the impact of electromagnetic field on the complexity of static anisotropic sphere. The idea of complexity was subsequently extended to non-static anisotropic dissipative sphere along with the discussion of two evolutionary modes [23]. Herrera *et al* have also specified the complexity of an axially symmetric system [24]. Sharif *et al* [25] analysed the complexity of the anisotropic spherical system in $f(G)$ gravity and identified a more complicated structure. The idea of complexity has been applied to several modified theories [26–34] using different configurations. Yousaf *et al* [35,36] computed the complexity factor for the charged and uncharged spherical systems in $f(G, T)$ gravity.

In order to figure out different physical aspects of self-gravitating configurations, it is quite possible that equations of motion are satisfied by different space–times. Under the influence of a strong gravitational field, the deviation from spherical to non-spherical symmetries becomes essential to evaluate the solution. To study the cylindrical symmetric matter configurations, Levi Civita proposed its vacuum solution that encouraged the researchers to look for the inner mysteries of different astrophysical objects. Herrera *et al* [37] proposed the field equations for cylindrically symmetric anisotropic fluid distributions and computed the structure scalars to investigate its behaviour in terms of fundamental matter properties. Houndjo *et al* [38] formulated the cylindrical field equations under modified Gauss–Bonnet gravity and developed a system of seven solutions corresponding to three respective feasible models. Sharif and Butt [39] used Herrera's technique for static matter distribution to calculate the complexity factor of a cylindrical space–time. Sharif and Butt [40] also studied the electromagnetic influence on the cylindrical structures and deduced an increment in complexity for this system.

In this paper, we examine the complexity factor for a static anisotropic cylindrical structure in the realm of $f(G, T)$ gravity. The paper is arranged as follows. In §2, the modified field equations and physical features regarding anisotropic matter are developed. Section 3 deals with the splitting of Riemann tensor to acquire structure scalars. In §4, the vanishing condition is derived to model the compact object and obtain the possible solution. In the last section, we summarise our main results.

2. $f(G, T)$ Gravity and physical features

This section deals with the physical parameters and their associated modified field equations to understand some important characteristics of self-gravitating anisotropic fluid. The action of $f(G, T)$ gravity is given as

$$I_{f(G,T)} = \frac{1}{2\kappa^2}\int [R + f(G, T)]\sqrt{-g}\mathrm{d}^4x + \int \sqrt{-g}\pounds_m \mathrm{d}^4x, \quad (1)$$

where κ^2 stands for the coupling constant, g indicates the determinant of the metric tensor $(g_{\nu\mu})$ and $\pounds_m$ is the matter Lagrangian density. The relation between the

Lagrangian density and the energy–momentum tensor is given as

$$T_{\nu\mu} = g_{\nu\mu}\pounds_m - \frac{2\partial\pounds_m}{\partial g^{\nu\mu}}. \tag{2}$$

The variation of eq. (1) with respect to the metric tensor yields the following field equations

$$\begin{aligned} G^\nu_\mu &= 8\pi T^\nu_\mu - (\Theta^\nu_\mu + T^\nu_\mu) f_T(G,T) + \frac{1}{2}\delta^\nu_\mu f(G,T) \\ &+ (4R_{\mu m}R^{\nu m} + 4R^{ml}R^\nu_{m\mu l} - 2RR^\nu_\mu \\ &- 2R^{mln\nu}R_{\mu mln}) f_G(G,T) \\ &+ (4R^\nu_\mu\nabla^2 - 4R^\nu_{m\mu l}\nabla^m\nabla^l - 2\delta^\nu_\mu R\nabla^2 \\ &+ 2R\nabla^\nu\nabla_\mu - 4R^{\nu m}\nabla_m\nabla_\mu \\ &- 4R^m_\mu\nabla^\nu\nabla_m + 4\delta^\nu_\mu R^{ml}\nabla_m\nabla_l) f_G(G,T), \end{aligned} \tag{3}$$

where $\nabla^2 = \nabla^\upsilon\nabla_\upsilon$ and $G^\nu_\mu = R^\nu_\mu - \frac{1}{2}R\delta^\nu_\mu$ represent the d'Alembert operator and Einstein tensor, respectively. Furthermore, $\Theta_{\nu\mu} = -2T_{\nu\mu} + pg_{\nu\mu}$ and partial derivatives with respect to G and T of an arbitrary function $f(G,T)$ are indicated by $f_G(G,T)$ and $f_T(G,T)$, respectively. The alternative expression for eq. (3) is given as

$$G^\nu_\mu = 8\pi T^{\nu\text{eff}}_\mu = 8\pi(T^{\nu M}_\mu + T^{\nu GT}_\mu), \tag{4}$$

where the modified terms of $f(G,T)$ are expressed as

$$\begin{aligned} T^{\nu GT}_\mu &= \frac{1}{8\pi}\big[\{(\rho+p)u^\nu u_\mu + \Pi^\nu_\mu\} f_T(G,T) \\ &+ \frac{1}{2}\delta^\nu_\mu f(G,T) + (4R_{\mu m}R^{\nu m} \\ &+ 4R^{ml}R^\nu_{m\mu l} - 2RR^\nu_\mu - 2R^{mln\nu}R_{\mu mln}) \\ &\times f_G(G,T) + (4\delta^\nu_\mu R^{ml}\nabla_m\nabla_l \\ &+ 4R^\nu_\mu\nabla^2 + 2R\nabla^\nu\nabla_\mu - 2\delta^\nu_\mu R\nabla^2 \\ &- 4R^{\nu m}\nabla_m\nabla_\mu - 4R^\nu_{m\mu l}\nabla^m\nabla^l \\ &- 4R^m_\mu\nabla^\nu\nabla_m) f_G(G,T)\big] \end{aligned} \tag{5}$$

while the energy–momentum tensor ($T^{\nu M}_\mu$) reads the anisotropic matter source as

$$T^{\nu M}_\mu = \rho u^\nu u_\mu + ph^\nu_\mu + \Pi^\nu_\mu. \tag{6}$$

Here,

$$\begin{aligned} &\Pi^\nu_\mu = \Pi\left(\chi^\nu\chi_\mu - \frac{1}{3}h^\nu_\mu\right), \quad p = \frac{p_r + 2p_\perp}{3}, \\ &\Pi = p_r - p_\perp, \quad h^\nu_\mu = \delta^\nu_\mu + u^\nu u_\mu, \\ &u^\nu u_\nu = -1, \quad \chi^\nu u_\nu = 0, \quad \chi^\nu\chi_\nu = 1, \end{aligned}$$

where the quantities $u^\nu = (\frac{1}{A}, 0, 0, 0)$, $\chi^\nu = (0, \frac{1}{B}, 0, 0)$, p_r and $p_\perp$ are the four-velocity, radial four-vector, radial and tangential pressures, respectively.

The line element describing the symmetric cylindrical object enfolded by the hypersurface (Σ) is given as follows [39]:

$$\begin{aligned} ds^2 &= -A^2(r)dt^2 + B^2(r)dr^2 \\ &+ C^2(r)d\theta^2 + C^2(r)\alpha^2 dz^2, \end{aligned} \tag{7}$$

where A, B and C are unknown functions and α being the constant term has the dimension of inverse length. This line element is the restricted form of the general cylindrically symmetric line element. Notice that the Weyl gauge allows to express the general vacuum cylindrically symmetric space–time in terms of two independent functions without loss of generality. In other words, while the Levi–Civita metric is the most general cylindrically symmetric solution to the vacuum Einstein field equations, line element (7) is not the most cylindrically symmetric line element for a fluid distribution. The modified field equations in this formalism are computed as

$$8\pi\rho^{\text{eff}} = \left[\frac{1}{rB^2}\left(\frac{2B'}{B} - \frac{1}{r}\right)\right], \tag{8}$$

$$8\pi p_r^{\text{eff}} = \left[\frac{1}{rB^2}\left(\frac{2A'}{A} + \frac{1}{r}\right)\right], \tag{9}$$

$$8\pi p_\perp^{\text{eff}} = \frac{A'}{rAB^2} + \frac{A''}{AB^2} - \frac{A'B'}{AB^3} - \frac{B'}{rB^3}, \tag{10}$$

where prime represents differentiation with respect to r and the values for ρ^{eff}, p_r^{eff} and $p_\perp^{\text{eff}}$ are evaluated as

$$\begin{aligned} \rho^{\text{eff}} &= \rho + \frac{1}{8\pi}\left[(\rho+p)f_T - \frac{f}{2} + \frac{4A''f_G}{r^2AB^4}\right. \\ &\left. + \frac{12B'f'_G}{r^2B^5} - \frac{12A'B'f_G}{r^2AB^5} - \frac{4f''_G}{r^2B^4}\right], \\ p_r^{\text{eff}} &= p_r + \frac{1}{8\pi}\left[\frac{2}{3}\Pi f_T + \frac{f}{2} + \frac{12A'B'f_G}{r^2AB^5}\right. \\ &\left. + \frac{12A'f'_G}{r^2AB^4} - \frac{4A''f_G}{r^2AB^4}\right], \\ p_\perp^{\text{eff}} &= p_\perp + \frac{1}{8\pi}\left[-\frac{\Pi}{3}f_T + \frac{f}{2} - \frac{4A''f_G}{r^2AB^4}\right. \\ &\left. - \frac{12A'B'f'_G}{rAB^5} + \frac{4A'f''_G}{rAB^4} + \frac{12A'B'f_G}{r^2AB^5} \; \frac{4A''f'_G}{rAB^4}\right]. \end{aligned}$$

The covariant divergence of eq. (3) and the hydrostatic equilibrium equation for cylindrical celestial objects can be written as

$$\begin{aligned} \nabla^\nu T_{\nu\mu} &= \frac{f_T(G,T)}{k^2 - f_T(G,T)}[(T_{\nu\mu} + \Theta_{\nu\mu})\nabla^\nu \\ &\times (\ln f_T(G,T)) + \nabla^\nu\Theta_{\nu\mu} - \frac{1}{2}g_{\nu\mu}\nabla^\nu T], \end{aligned} \tag{11}$$

$$p_r^{\prime\mathrm{eff}} = -\frac{A'(p_r^{\mathrm{eff}}+\rho^{\mathrm{eff}})}{A} + \frac{2(p_\perp^{\mathrm{eff}}-p_r^{\mathrm{eff}})}{r} + ZB^2, \quad (12)$$

where the term Z consists of the additional source terms and its value is given in Appendix A. Equation (12) can be considered as the general Tolman–Oppenheimer–Volkoff equation which is commonly used to analyse the structure of static cylindrical anisotropic matter source. The value of A'/A is obtained from eq. (9) as

$$\frac{A'}{A} = \frac{4r}{r-8m}\left(4\pi r p_r^{\mathrm{eff}} - \frac{1}{8r} + \frac{m}{r^2}\right). \quad (13)$$

Using this value in eq. (11), we get

$$p_r^{\prime\mathrm{eff}} = -\frac{4r}{r-8m}\left(4\pi r p_r^{\mathrm{eff}} - \frac{1}{8r} + \frac{m}{r^2}\right)(\rho^{\mathrm{eff}}+p_r^{\mathrm{eff}}) + \frac{2(p_\perp^{\mathrm{eff}}-p_r^{\mathrm{eff}})}{r} + ZB^2, \quad (14)$$

where m is the mass of the fluid distribution.

To compute the matter configuration of the cylindrical structure, the Misner–Sharp mass is replaced by the C-energy formula [41,42] as

$$m(r) = lE = \frac{l}{8}(1 - l^{-2}\nabla_\nu \hat{r}\nabla^\nu \hat{r}), \quad (15)$$

where $\hat{r} = \upsilon l$, $l^2 = \phi_{(2)i}\phi^{(2)i}$ and $\upsilon^2 = \phi_{(1)i}\phi^{(1)i}$. The quantities l and υ indicate the specific length and circumference radius, respectively. Also, $\phi_{(2)} = \partial/\partial z$, $\phi_{(1)} = \partial/\partial\theta$ and E is the gravitational energy per specific length. The inner mass of the cylindrical system is determined as

$$m = \frac{r}{2}\left(\frac{1}{4} - \frac{1}{B^2}\right) = \frac{r}{8} + 4\pi\int_0^r r^2\rho^{\mathrm{eff}}\mathrm{d}r. \quad (16)$$

To avoid any discontinuity, junction conditions must be satisfied at the boundary. The internal and external regions of space–time are separated by a hypersurface (Σ). In the static cylindrical symmetric case, the geometry of the external region is assessed by the metric [43]

$$\mathrm{d}s^2 = -\frac{2M}{r}\mathrm{d}\upsilon^2 - 2\mathrm{d}r\,\mathrm{d}\upsilon + r^2(\mathrm{d}\theta^2 + \alpha^2\,\mathrm{d}z^2), \quad (17)$$

where υ and M signify the retarded time and total mass, respectively. The smooth matching of internal and external space–times is achieved when the following constraints are imposed at $r = r_\Sigma =$ constant:

$$(E-M)_\Sigma = \frac{1}{8}, \quad [p_r^{\mathrm{eff}}]_\Sigma = \zeta, \quad (18)$$

where subscript Σ specifies that the values are derived at the boundary and the value of ζ is given in Appendix A.

The distortion in the celestial structures produced by the oscillating gravitational field of the surrounding bodies is computed by the Weyl tensor. The decomposition of Riemann tensor in the form of Ricci scalar, Ricci and Weyl tensor is defined as

$$R^\rho_{\nu\mu\omega} = C^\rho_{\nu\mu\omega} + \frac{1}{2}R_{\nu\omega}\delta^\rho_\mu - \frac{1}{2}R_{\nu\mu}\delta^\rho_\omega + \frac{1}{2}R^\rho_\mu g_{\nu\omega} - \frac{1}{2}R^\rho_\omega g_{\nu\mu} - \frac{1}{6}R(\delta^\rho_\mu g_{\nu\omega} - g_{\nu\mu}\delta^\rho_\omega). \quad (19)$$

The Weyl tensor is resolved into magnetic and electric parts as

$$H_{\nu\mu} = \frac{1}{2}\eta_{\nu\lambda\beta\gamma}C^{\beta\gamma}_{\mu\varrho}u^\lambda u^\varrho, \quad E_{\nu\mu} = C_{\nu\gamma\mu\delta}u^\gamma u^\delta,$$

with

$$C_{\nu\mu\lambda\kappa} = (g_{\nu\mu\beta\alpha}g_{\lambda\kappa\gamma\delta} - \eta_{\nu\mu\beta\alpha}\eta_{\lambda\kappa\gamma\delta})u^\beta u^\gamma E^{\alpha\delta}, \quad (20)$$

where $g_{\nu\mu\beta\alpha} = g_{\nu\beta}g_{\nu\alpha} - g_{\nu\alpha}g_{\mu\beta}$ and $\eta_{\nu\mu\beta\alpha}$ signifies the Levi–Civita tensor. The magnetic component of the Weyl tensor disappears for the symmetric cylindrical structure. The electric part is written as

$$E_{\nu\mu} = E\left(\chi_\nu\chi_\mu - \frac{h_{\nu\mu}}{3}\right),$$

with

$$E = \frac{1}{2AB^2}\left[A'' - \frac{A'B'}{B} + \frac{AB'}{rB} - \frac{A'}{r} + \frac{A}{r^2}\right],$$
$$E^\nu_\nu = 0 = E_{\nu\gamma}u^\gamma = 0. \quad (21)$$

We would like to mention here that due to the constrained character of the cylinder, the electric Weyl tensor is described in the form of a single scalar function, while for the general cylindrical symmetric case it is defined in terms of two scalar functions. The influence of the Weyl tensor and physical variables on the inner mass of the cylinder is obtained by employing eqs (4), (16) and (21) as

$$m(r) = \frac{4\pi r^3}{3}(\rho^{\mathrm{eff}} - p_r^{\mathrm{eff}} + p_\perp^{\mathrm{eff}}) - \frac{Er^3}{3} + \frac{r}{8} \quad (22)$$

from which we can easily extract the value of E as

$$E = -4\pi(p_r^{\mathrm{eff}} - p_\perp^{\mathrm{eff}}) + \frac{4\pi}{r^3}\int_0^r r^3\rho^{\prime\mathrm{eff}}\,\mathrm{d}r. \quad (23)$$

This describes the connection between inhomogeneous energy density, Weyl tensor and anisotropic pressure in the context of $f(G,T)$ gravity. Substituting the above value in eq. (22), we obtain

$$m(r) = \frac{4\pi r^3}{3}\rho^{\mathrm{eff}} - \frac{4\pi}{3}\int_0^r r^3\rho^{\prime\mathrm{eff}}\,\mathrm{d}r + \frac{r}{8}. \quad (24)$$

It is observed that the mass function in the above equation is influenced due to the inhomogeneity in the energy density.

Tolman [44] defined matter configuration of the static system as follows:

$$m_T = 4\pi \int_0^r ABr^2(-T_0^{0\mathrm{eff}} + T_1^{1\mathrm{eff}} + 2T_2^{2\mathrm{eff}})\mathrm{d}r. \quad (25)$$

Using the field equations in the above equation, we have

$$m_T = \frac{A'}{B}r^2. \quad (26)$$

Inserting the value of A' from eq. (13), the Tolman mass is expressed as

$$m_{\mathrm{T}} = ABm + 4\pi ABp_r^{\mathrm{eff}}r^3 - \frac{ABr}{8}.$$

This is also called the effective gravitational mass since it can be represented in terms of gravitational acceleration as

$$a = \frac{A'}{AB} = \frac{m_{\mathrm{T}}}{Ar^2}. \quad (27)$$

After some simplifications [45], we obtain

$$m_{\mathrm{T}} = (m_{\mathrm{T}})_\Sigma \left(\frac{r}{r_\Sigma}\right)^3 - r^3 \int_r^{r_\Sigma} AB\left[\frac{8\pi}{r}(p_\perp^{\mathrm{eff}} - p_r^{\mathrm{eff}}) + \frac{1}{r^4}\int_0^r 4\pi r^3 \rho'^{\mathrm{eff}}\,\mathrm{d}r\right]\mathrm{d}r. \quad (28)$$

An alternative expression for the Tolman mass is obtained using the value of E as

$$m_{\mathrm{T}} = (m_{\mathrm{T}})_\Sigma \left(\frac{r}{r_\Sigma}\right)^3 - r^3 \int_r^{r_\Sigma} \frac{AB}{r} \times[4\pi(p_\perp^{\mathrm{eff}} - p_r^{\mathrm{eff}}) + E]\,\mathrm{d}r. \quad (29)$$

These equations demonstrate the impacts of anisotropy and inhomogeneity of energy density on the Tolman mass together with the extra curvature terms.

3. Structure scalars

Structure scalars are obtained through the orthogonal decomposition of Riemann tensor to compute the complexity of the system through Herrera's technique [46]. We define the tensors $Y_{\nu\mu}$, $Z_{\nu\mu}$ and $X_{\nu\mu}$ as

$$Y_{\nu\mu} = R_{\nu\gamma\mu\delta}u^\gamma u^\delta, \quad (30)$$

$$Z_{\nu\mu} = {}_*R_{\nu\gamma\mu\delta}u^\gamma u^\delta = \frac{1}{2}\eta_{\nu\gamma\epsilon\beta}R^{\epsilon\beta}_{\mu\delta}u^\gamma u^\delta, \quad (31)$$

$$X_{\nu\mu} = {}^*R^*_{\nu\gamma\mu\delta}u^\gamma u^\delta = \frac{1}{2}\eta^{\epsilon\beta}_{\nu\gamma}, R^*_{\epsilon\beta\mu\delta}u^\gamma u^\delta, \quad (32)$$

where $\eta^{\epsilon\beta}_{\nu\gamma}$ represents the Levi–Civita symbol and $R^*_{\nu\mu\gamma\delta} = \frac{1}{2}\eta_{\epsilon\alpha\gamma\delta}R^{\epsilon\alpha}_{\nu\mu}$. The Riemann tensor can be rewritten by using eq. (19) as

$$R^{\nu\gamma}_{\mu\delta} = C^{\nu\gamma}_{\mu\delta} + 16\pi T^{\mathrm{eff}[\nu}_{[\mu}\delta^{\gamma]}_{\delta]} + 8\pi T^{\mathrm{eff}}\left(\frac{1}{3}\delta^\nu_{[\mu}\delta^\gamma_{\delta]} - \delta^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]}\right). \quad (33)$$

Inserting eqs (5) and (6) in (33), we obtain the tensorial quantities as

$$\begin{aligned} R^{\nu\gamma}_{(I)\mu\delta} &= 16\pi\rho\mu^{[\nu}\mu_{[\mu}\delta^{\gamma]}_{\delta]} + 2\rho\mu^{[\nu}\mu_{[\mu}\delta^{\gamma]}_{\delta]} \\ &\quad +16\pi ph^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]} + 2p\mu^{[\nu}\mu_{[\mu}\delta^{\gamma]}_{\delta]} \\ &\quad +8\pi(\rho+3p)\left(\frac{1}{3}\delta^\nu_{[\mu}\delta^\gamma_{\delta]} - \delta^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]}\right), \end{aligned} \quad (34)$$

$$\begin{aligned} R^{\nu\gamma}_{(II)\mu\delta} &= 16\pi\Pi^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]} + 2\Pi^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]} + \delta^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]}f \\ &\quad +8\delta^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]}R^{ml}\nabla_m\nabla_l f_G - 4R\delta^{[\nu}_{[\mu}\delta^{\gamma]}_{\delta]}\Box f_G, \end{aligned} \quad (35)$$

$$R^{\nu\gamma}_{(III)\mu\delta} = 4\rho\mu^{[\nu}_{[\mu}E^{\gamma]}_{\delta]} - \epsilon^{\nu\gamma}_{\beta}\epsilon_{\mu\delta\alpha}E^{\beta\alpha}, \quad (36)$$

$$\begin{aligned} R^{\nu\gamma}_{(IV)\mu\delta} &= 2(R_{m\mu}R^{m\nu}\delta^\gamma_\delta - R_{m\delta}R^{m\nu}\delta^\gamma_\mu + R_{m\mu}R^{m\gamma}\delta^\nu_\delta \\ &\quad +R_{m\delta}R^{m\gamma}\delta^\nu_\mu)f_G \\ &\quad +2R^{ml}(R^\nu_{m\mu l}\delta^\gamma_\delta - R^\nu_{m\delta l}\delta^\gamma_\mu - R^\gamma_{m\mu l}\delta^\nu_\delta \\ &\quad +R^\gamma_{m\delta l}\delta^\nu_\mu)f_G - R(R^\nu_\mu\delta^\gamma_\delta \\ &\quad -R^\nu_\delta\delta^\gamma_\mu - R^\gamma_\mu\delta^\nu_\delta + R^\gamma_\delta\delta^\nu_\mu)f_G \\ &\quad -2(R^\nu_{m\mu l}\delta^\gamma_\delta - R^\nu_{m\delta l}\delta^\gamma_\mu - R^\gamma_{m\mu l}\delta^\nu_\delta \\ &\quad +R^\gamma_{m\delta l}\delta^\nu_\mu)\nabla^m\nabla^l f_G + 2(R^\nu_\mu\delta^\gamma_\delta - R^\nu_\delta\delta^\gamma_\mu \\ &\quad -R^\gamma_\mu\delta^\nu_\gamma + R^\gamma_\delta\delta^\nu_\mu)\Box f_G \\ &\quad +R(\delta^\gamma_\delta\nabla^\nu_\mu - \delta^\gamma_\mu\nabla^\nu_\delta - \delta^\nu_\delta\nabla^\gamma_\mu + \delta^\nu_\mu\nabla^\gamma_\delta)f_G \\ &\quad -2(R^{m\nu}\delta^\gamma_\delta\nabla_\mu\nabla_m \\ &\quad -R^{m\nu}\delta^\gamma_\mu\nabla_\delta\nabla_m - R^{m\gamma}\delta^\nu_\delta\nabla_\mu\nabla_m \\ &\quad +R^{m\gamma}\delta^\nu_\mu\nabla_\delta\nabla_m)f_G - 2(R^m_\mu \\ &\quad \times\delta^\gamma_\delta\nabla^\nu\nabla_m - R^m_\delta\delta^\gamma_\mu\nabla^\nu\nabla_m \\ &\quad -R^m_\mu\delta^\nu_\delta\nabla^\gamma\nabla_m + R^m_\delta\delta^\nu_\mu\nabla^\gamma\nabla_m)f_G \\ &\quad -(R_{\mu lmn}R^{lmn\nu}\delta^\gamma_\delta - R_{\delta lmn}R^{lmn\nu}\delta^\gamma_\mu \\ &\quad -R_{\mu lmn}R^{lmn\gamma}\delta^\nu_\delta + R_{\delta lmn} \\ &\quad \times R^{lmn\gamma}\delta^\nu_\mu)f_G + \frac{1}{3}\big[(\rho+p)f_T + 2f \\ &\quad +4R_{l\alpha}R^{m\alpha}f_G + 4R^{lm} \\ &\quad \times R^\alpha_{l\alpha m}f_G - 2R^\beta_{lmn}R^{lmn}_\alpha f_G \\ &\quad -4R^{m\beta}\nabla_m\nabla_\beta f_G - 4R^\alpha_{l\alpha m}\nabla^m\nabla^l f_G \\ &\quad -2R\Box f_G + 16R^{lm}\nabla_m\nabla_l f_G \\ &\quad -4R^{m\alpha}\nabla_\alpha\nabla_l f_G - 2R^2 f_G\big], \end{aligned} \quad (37)$$

where we have used

$$\epsilon_{\nu\gamma\mu} = u^\beta\eta_{\beta\nu\gamma\mu}, \quad \epsilon_{\nu\gamma\mu}u^\mu = 0,$$
$$\epsilon^{\beta\gamma\alpha}\epsilon_{\alpha\nu\mu} = \delta^\beta_\nu h^\gamma_\mu - \delta^\beta_\nu h^\gamma_\mu + u_\nu(u^\beta\delta^\gamma_\mu - \delta^\beta_\mu u^\gamma).$$

The tensors in the current set-up are assessed as

$$Y_{\nu\mu} = \frac{4\pi}{3}(\rho + 3p)h_{\nu\mu} + E_{\nu\mu} + \frac{1}{6}(\rho + p)h_{\nu\mu}f_T + \frac{\Pi_{\nu\mu}}{2}f_T - 4\pi\Pi_{\nu\mu} + D^{GT}_{\nu\mu}, \quad (38)$$

$$X_{\nu\mu} = \frac{8\pi}{3}\rho h_{\nu\mu} + \frac{\Pi_{\nu\mu}}{2}f_T - E_{\nu\mu} - 4\pi\Pi_{\nu\mu} + M^{GT}_{\nu\mu}, \quad (39)$$

$$Z_{\nu\mu} = N^{GT}_{\nu\mu}. \quad (40)$$

The values of correction terms $D^{GT}_{\nu\mu}$, $M^{GT}_{\nu\mu}$ and $N^{GT}_{\nu\mu}$ are given in Appendix A.

These tensors are expressed in the form of scalar functions, known as structure scalars. Four scalar functions $(X_T, X_{TF}, Y_T, Y_{TF})$ are computed from the tensors $X_{\nu\mu}$ and $Y_{\nu\mu}$ while the fifth scalar corresponding to the tensor $Z_{\nu\mu}$ vanishes for the static cylindrical case. These scalars are given as

$$X_T = 8\pi\mu + O^{GT}, \quad (41)$$

$$X_{TF} = -4\pi\Pi - E + \frac{\Pi}{2}f_T, \quad (42)$$

$$Y_T = 4\pi(\rho + 3p_r - 2\Pi) + \frac{(\rho + p)f_T}{2} + F^{GT}, \quad (43)$$

$$Y_{TF} = E - 4\pi\Pi + \frac{\Pi}{2}f_T + I^{GT}, \quad (44)$$

where

$$I^{GT} = \frac{Q^{GT}_{\nu\mu}}{\chi_\nu\chi_\mu + \frac{1}{3}h_{\nu\mu}}.$$

The terms O^{GT}, F^{GT} and Q^{GT} are described in Appendix A. It has already been mentioned that we have imposed the restriction of the Weyl gauge. Consequently, we obtain only one structure scalar corresponding to the trace-free part of the tensor $Y_{\nu\mu}$. The scalars X_T and Y_T regulate the energy density and anisotropic stresses, respectively, in the presence of modified terms. Using eq. (23), the scalars X_{TF} and Y_{TF} are evaluated as

$$X_{TF} = \frac{4\pi}{r^3}\int_0^r \tilde{r}^3\rho'^{\text{eff}}\,\mathrm{d}r + \frac{\Pi}{2}f_T - 4\pi\Pi^{GT}, \quad (45)$$

$$Y_{TF} = \frac{\Pi}{2}f_T + 8\pi\Pi - \frac{4\pi}{r^3}\int_0^r r^3\rho'^{\text{eff}}\,\mathrm{d}r + 4\pi\Pi^{GT} + I^{GT}. \quad (46)$$

Here, X_{TF} measures the energy density inhomogeneity in the presence of correction terms while Y_{TF} is responsible for the evaluation of energy density inhomogeneity, anisotropic pressure together with extra curvature terms. Since all the fundamental physical variables with modified terms are accommodated in Y_{TF}, this is assumed to be the complexity factor for static cylindrical configuration. For the physical interpretation of Y_{TF}, we insert eq. (46) in (28) so that

$$m_T = (m_T)_\Sigma\left(\frac{r}{r_\Sigma}\right)^3 + r^3\int_r^{r_\Sigma}\frac{AB}{r} \times\left[Y_{TF} - I^{GT} - \frac{\Pi}{2}f_T + 4\pi\Pi^{GT}\right]\mathrm{d}r. \quad (47)$$

The comparison between eqs (28) and (47) shows that the impact of inhomogeneous energy density and anisotropy on the Tolman mass is characterised by Y_{TF}.

4. Fluid distributions with zero complexity factor

A large number of interrelated physical aspects influence the behaviour of stellar objects. The structures having isotropic physical parameters are regarded as the least complex such as cylinder filled with dust. On the other hand, in our work the complexity producing factors are the inhomogeneity energy density and anisotropic pressure with the contribution of dark source terms. Based on these elements, Y_{TF} is chosen as the complexity factor. Moreover, it also characterises the impact of these variables on the Tolman mass. The vanishing complexity factor condition is obtained after employing $Y_{TF} = 0$ as

$$\Pi = \frac{1}{4\pi + \frac{f_T}{2}}\left[\frac{4\pi}{r^3}\int_0^r \tilde{r}^3\rho'^{\text{eff}}\,\mathrm{d}r - 4\pi\Pi^{\text{eff}} - I^{GT}\right]. \quad (48)$$

This condition will be utilised as an additional constraint in developing the solution of the modified field equations.

Gokhroo and Mehra [47] utilised varying energy density to better understand the behaviour of celestial objects. The variable energy density in our case reads as

$$\rho^{\text{eff}} = \rho_0\left(1 - \frac{kr^2}{r_\Sigma^2}\right). \quad (49)$$

Using this value in eq. (16), we have the mass function as

$$m(r) = \frac{r}{8} - \varphi r^3\left(\frac{kr^2}{5r_\Sigma^2} - \frac{1}{3}\right), \quad (50)$$

where the constant k lies within the range (0, 1) and $\varphi = 4\pi\rho_0$. The value of metric coefficient is found by comparing eqs (16) and (50) as

$$B = \frac{1}{\sqrt{2\eta r^2\left(\frac{kr^2}{5r_\Sigma^2} - \frac{1}{3}\right)}}. \quad (51)$$

Moreover, the field equations can be written as follows:

$$8\pi(p_r^{\text{eff}} - p_\perp^{\text{eff}}) = \frac{1}{B^2}\left[\frac{A'}{rA} + \frac{1}{r^2} - \frac{A''}{A} + \frac{A'B'}{AB} + \frac{B'}{rB}\right]. \tag{52}$$

We introduce new variables

$$A^2 = \mathrm{e}^{\int(2z(r)-2/r)\mathrm{d}r}, \quad B^{-2} = y(r), \tag{53}$$

and eq. (52) turns out to be

$$y' + y\left[\frac{6}{r} - \frac{4}{zr^2} - 2z - \frac{2z'}{z}\right] = -\frac{16\pi\Pi^{\text{eff}}}{z}. \tag{54}$$

Therefore, the corresponding line element takes the form

$$\mathrm{d}s^2 = -\mathrm{e}^{\int(2z(r)-2/r)\mathrm{d}r}\mathrm{d}t^2 + \frac{r^6\mathrm{e}^{\int(-2z(r)-\frac{4}{r^2z(r)})\mathrm{d}r}}{z^2\left(16\pi\int\frac{r^6\Pi^{\text{eff}}\mathrm{e}^{\int(-2z(r)-\frac{4}{r^2z(r)})\mathrm{d}r}}{z^3}\mathrm{d}r + H\right)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \alpha^2\,\mathrm{d}z^2), \tag{55}$$

where H indicates the integration constant. The field equations in the context of $f(G,T)$ gravity can also be given as

$$4\pi p_r^{\text{eff}} = \left(\frac{z}{r} - \frac{1}{2r^2}\right)\left(\frac{1}{4} - \frac{2m}{r}\right), \tag{56}$$

$$4\pi\rho^{\text{eff}} = \frac{m'}{r^2} - \frac{1}{r^2}, \tag{57}$$

$$8\pi p_\perp^{\text{eff}} = \left(\frac{1}{8} - \frac{m}{r}\right)\left(z' + z^2 - \frac{z}{r}\left(\frac{4rm' - 4m}{r - 8m}\right) - \frac{z}{r} + \frac{1}{r^2}\right). \tag{58}$$

Physical quantities such as density, temperature, pressure etc. are helpful for understanding the internal structure of self-gravitating fluid. The dominance of some factors over others has a distinct influence on the internal structure. The relationship between energy density and pressure for anisotropic matter configuration [48,49] is described by the polytropic equation of state

$$p_r^{\text{eff}} = \rho^{\text{eff}(\omega)} = K\rho^{\text{eff}(1+1/n)}, \tag{59}$$

where K is the polytropic constant, n indicates the polytropic index and ω is known as the polytropic exponent. We proceed further by introducing the new variables

$$\sigma = \frac{p_{rc}^{\text{eff}}}{\rho_c^{\text{eff}}}, \quad r = \frac{\xi}{J}, \quad J^2 = \frac{4\pi\rho_c^{\text{eff}}}{\sigma(1+n)},$$

$$\Psi^n = \frac{\rho^{\text{eff}}}{\rho_c^{\text{eff}}}, \quad \vartheta(\xi) = m(r)J^3/4\pi\rho_c^{\text{eff}}.$$

Inserting the above values in eqs (12) and (16), we have

$$\xi^2\frac{\mathrm{d}\Psi}{\mathrm{d}\xi}\left[\frac{1-8\vartheta\sigma(n+1)/\xi}{1+\sigma\Psi}\right] + \frac{2\Pi\xi\Psi^{-n}}{p_{rc}^{\text{eff}}(n+1)} \times\left[\frac{1-8\vartheta\sigma(n+1)/\xi}{1+\sigma\Psi}\right] - \frac{\xi}{2\sigma} \times\frac{1}{n+1} - 4(-\sigma\Psi^{n+1}\xi^3 - \vartheta) = \mathrm{e}^{\lambda}Z\left[\frac{\xi^2(1-2\vartheta\sigma(n+1)/\xi)}{P_{rc}^{\text{eff}}J\Psi^n(1+\sigma\Psi)(1+n)}\right], \tag{60}$$

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}\xi} = \Psi^n\xi^2 + \frac{J^2}{32\pi\rho_c^{\text{eff}}}. \tag{61}$$

The subscript c indicates that the respective quantities are evaluated at the centre. There are three unknowns (Ψ, ϑ and Π) in the first-order equations depending upon the parameters σ and n. As the number of equations are less than the unknown functions, we need one more equation to close the system. For this purpose, the zero complexity factor condition yields

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\xi} + \frac{3\Pi}{\xi} = \frac{1}{(4\pi + \frac{f_T}{2})} \times\left[4\pi\rho_c^{\text{eff}}n\Psi^{n-1}\frac{\mathrm{d}\Psi}{\mathrm{d}\xi} - \frac{3}{\xi}(4\pi^{GT} + I^{GT}) - 4\pi\frac{\mathrm{d}\Pi^{\text{eff}}}{\mathrm{d}\xi} - I_{,\xi}^{GT} - \frac{\Pi f_{T,\xi}}{2}\right]. \tag{62}$$

Consequently, we obtain a system containing three equations (eqs (60)–(62)) that correspond to the unique solution for arbitrary values of σ and n. Moreover, the physical features of celestial objects such as density and mass can be studied by using this solution for particular values of σ and n.

5. Conclusions

The analysis of self-gravitating structures helps us to study the hidden characteristics of the giant Universe. This paper computes the complexity of anisotropic cylindrical symmetric object in the framework of $f(G,T)$ gravity. We have developed the modified field equations as well as hydrostatic equilibrium condition and found some relations between m_{T} and m using Tolman mass and C-energy formalism. The system with homogeneous energy density and isotropic pressure is regarded as less complex. It is also observed that the system is free from complexity when both the terms cancel the effect of each other. For the complexity factor, we have used the orthogonal splitting of the Riemann tensor through Herrera's approach and obtained four structure scalars. The structure scalar Y_{TF} is known as the complexity factor for the following reasons.

1. It encompasses the effects of all physical parameters including pressure anisotropy and inhomogeneous energy density with correction terms.
2. The impact of anisotropic pressure and inhomogeneous configuration on the Tolman mass is covered through this scalar.
3. This factor also addresses the electromagnetic consequences for the charged fluid [50].

This incorporates the energy density inhomogeneity, anisotropic pressure and additional curvature terms. Unlike GR, isotropic pressure and homogeneous energy density do not lead to a complexity-free structure due to the presence of modified terms. Consequently, it follows that the extra curvature terms are responsible for increasing the complexity of cylindrical object. We have also devised a condition in the form of extra curvature terms for a complex-free system ($Y_{TF} = 0$). It is worth mentioning here that isotropy, homogeneous energy density and the unavailability of modified terms in this condition correspond to complex-free structure. This condition has been utilised on the model proposed by Gokhroo and Mehra with variable energy density along with the consequences of dark source terms. The resulting equations rationally investigate the occurrence of heavenly bodies. Finally, we have found the possible solution of the system in terms of new variables by considering the polytropic equation of state and vanishing complexity condition. In both the models, it has been observed that assigning some particular values to m, z and σ, n, respectively, correspond to unique solutions. It is worthwhile to mention here that all our results reduce to GR by using the constraint $f(G, T) = 0$.

Appendix A

The terms Z and ζ are given as

$$Z = \frac{f_T}{k^2 - f_T}\left[(-p_r B^2 - pB^2)\frac{f'_T}{f_T} + (-2p_r B^2 - pB^2)' - \frac{B^2}{2}(\rho + 3p)\right],$$

$$\zeta = -\frac{1}{8\pi}\left[\frac{2}{3}\Pi f_T + \frac{f}{2} + \frac{12A'B'f_G}{r^2AB^5} + \frac{12A'f'_G}{r^2AB^4} - \frac{4A''f_G}{r^2AB^4}\right].$$

The contribution of the modified terms in the structure scalars are described by

$$N^{GT}_{\nu\mu} = \Big[2R_{m\delta}R^{m\gamma}f_G + 2R^{lm}R^{\gamma}_{l\delta m}f_G - RR^{\gamma}_{\delta}f_G - 2R_{\delta lmn}R^{lmn\gamma}f_G + 2R^{\gamma}_{\delta}\Box f_G + R\nabla^{\gamma}\nabla_{\delta}f_G - 2R^{m\gamma}\nabla_{\delta}\nabla_m f_G - 2R^{m}_{\delta}\nabla^{\gamma}\nabla_m f_G - 2R^{\gamma}_{l\delta m}\nabla^m\nabla^l f_G\Big]\epsilon_{\nu\mu\gamma}u^{\delta},$$

$$D^{GT}_{\nu\mu} = 2\Big[R_{m\mu}R^m_{\nu}f_G + R^{lm}R_{l\mu m\nu}f_G - \frac{1}{2}RR_{\nu\mu}f_G - \frac{1}{2}R_{\mu lmn}R^{lmn}_{\nu}f_G + \frac{1}{2}R\nabla_{\nu}\nabla_{\mu}f_G + R_{\nu\mu}\Box f_G - R^m_{\nu}\nabla_{\mu}\nabla_m f_G - R^m_{\mu}\nabla_{\nu}\nabla_m f_G - R_{l\mu m\nu}\nabla^m\nabla^l f_G\Big] + 4R^{lm}h_{\nu\mu}\nabla_m\nabla_l f_G - 2Rh_{\nu\mu}\Box f_G + 2\Big[-R_{m\delta}R^m_{\nu}f_G - R^{lm}R_{l\delta m\nu}f_G + \frac{1}{2}R_{\delta lmn}R^{lmn}_{\nu}f_G + \frac{1}{2}RR_{\nu\mu}f_G - R_{\delta\nu}\Box f_G + R_{l\delta m\nu}\nabla^m\nabla^m f_G - \frac{1}{2}R\nabla_{\nu}\nabla_{\delta}f_G + R^m_{\nu}\nabla_{\delta}\nabla_m f_G + R^m_{\delta}\nabla_{\nu}\nabla_m f_G\Big]u_{\mu}u^{\delta} + 2\Big[-R_{m\mu}R^{m\gamma}f_G + \frac{1}{2}R_{\mu lmn}R^{lmn\gamma}f_G - R^{lm}R^{\gamma}_{l\mu m}f_G + \frac{1}{2}RR^{\gamma}_{\mu}f_G - R^{\gamma}_{\mu}\Box f_G + R^{m\gamma}\nabla_{\mu}\nabla_m f_G - \frac{1}{2}R\nabla^{\gamma}\nabla_{\mu}f_G + R^m_{\mu}\nabla^{\gamma}\nabla_m f_G + R^{\gamma}_{m\mu l}\nabla^m\nabla^l\Big]u_{\nu}u_{\gamma} + 2\Big[R_{m\delta}R^{m\gamma}f_G - \frac{1}{2}R_{\delta lmn}R^{lmn\gamma}f_G + R^{lm}R^{\gamma}_{l\delta m}f_G - \frac{1}{2}RR^{\gamma}_{\delta}f_G + R^{\gamma}_{\delta}\Box f_G + \frac{1}{2}R\nabla^{\gamma}\nabla_{\delta}f_G - R^{\gamma}_{l\delta m}\nabla^m\nabla^l f_G - R^{m\gamma}\nabla_{\delta}\nabla_m f_G - R^m_{\delta}\nabla^{\gamma}\nabla_m f_G\Big]g_{\nu\mu}u_{\gamma}u^{\delta} - \frac{1}{3}\Big[-2R^2 f_G - 2R^{\beta}_{lmn}R^{lmn}_{\beta}f_G - 2R\Box f_G + 4R^{m\alpha}R_{m\alpha}f_G + 4R^{lm}R^{\alpha}_{m\alpha l}f_G + 16R^{lm}\nabla_m\nabla_l f_G - 4R^{m\beta}\nabla_{\beta}\nabla_m f_G - 4R^{m\alpha}\nabla_{\alpha}\nabla_m f_G - 4R^{\alpha}_{l\alpha m}\nabla^m\nabla^l f_G\Big]h_{\nu\mu} - \frac{1}{6}fh_{\nu\mu},$$

$$F^{GT} = 2\Big[R_{m\mu}R^m_{\nu}f_G + R^{lm}R_{m\mu l\nu}f_G - \frac{1}{2}RR_{\nu\mu}f_G$$

$$
\begin{aligned}
&- \frac{1}{2}R_{\mu\mu lmn}R_{\nu}^{lmn}f_G + R_{\nu\mu}\Box f_G \\
&+ \frac{1}{2}R\nabla_\nu\nabla_\mu f_G - R_\nu^m\nabla_\mu\nabla_m f_G \\
&- R_\mu^m\nabla_\nu\nabla_m f_G - R_{m\mu l\nu}\nabla^m\nabla^l f_G\bigg]g^{\nu\mu} \\
&+12R^{lm}\nabla_m\nabla_l f_G - 6R\Box f_G \\
&+2\bigg[-R_{m\delta}R_\nu^m f_G - R^{lm}R_{m\delta l\nu}f_G \\
&+\frac{1}{2}RR_{\nu\delta}f_G\frac{1}{2}R\nabla_\nu\nabla_\delta f_G - R_{\delta\nu}\Box f_G \\
&+ R_{m\delta l\nu}\nabla^m\nabla^l f_G + R_\nu^m\nabla_\delta\nabla_m f_G \\
&+R_\delta^m\nabla_\nu\nabla_m f_G + \frac{1}{2}R_{\delta lmn}R_\nu^{lmn}f_G\bigg]u_\mu u^\delta g^{\nu\mu} \\
&+2\bigg[-R_{m\mu}R^{m\gamma}f_G - R^{lm}R^\gamma_{m\mu l}f_G+\frac{1}{2}RR_\mu^\gamma f_G \\
&+ \frac{1}{2}R_{\mu lmn}R^{lmn\gamma}f_G - R_\mu^\gamma\Box f_G + R^{m\gamma}\nabla_\mu\nabla_m f_G \\
&+ R_\mu^m\nabla^\gamma\nabla_m f_G + R^\gamma_{m\mu l}\nabla^m\nabla^l f_G \\
&- \frac{1}{2}R\nabla^\gamma\nabla_\mu f_G\bigg]u_\nu u_\gamma g^{\nu\mu} + 2\bigg[R_{m\delta}R^{m\gamma}f_G \\
&+ R^{lm}R^\gamma_{m\delta l}f_G - R^{m\gamma}\nabla_\delta\nabla_m f_G \\
&- \frac{1}{2}R_{\delta lmn}R^{lmn\gamma}f_G + R_\delta^\gamma\Box f_G + \frac{1}{2}R\nabla^\gamma\nabla_\delta f_G \\
&-\frac{1}{2}RR_\delta^\gamma f_G - R_\delta^m\nabla^\gamma\nabla_m f_G \\
&- R^\gamma_{m\delta l}\nabla^m\nabla^l f_G\bigg]g_{\nu\mu}u_\gamma u^\delta g^{\nu\mu} \\
&-\bigg[4R^{l\alpha}R_{l\alpha}f_G + 4R^{lm}R^\alpha_{l\alpha m}f_G - 2R^2 f_G \\
&- 2R^\beta_{lmn}R_\beta^{lmn}f_G - 2R\Box f_G + 16R^{lm}\nabla_m\nabla_l f_G \\
&- 4R^{m\alpha}\nabla_\alpha\nabla_m f_G \\
&- 4R^{m\beta}\nabla_\beta\nabla_m f_G - 4R^\alpha_{l\alpha m}\nabla^m\nabla^l f_G\bigg] - \frac{1}{2}f,
\end{aligned}
$$

$$
\begin{aligned}
Q^{GT}_{(\nu\mu)} = &\bigg[2R_{md}R_c^m f_G + 2R^{lm}R_{ldmc}f_G \\
&- RR_{cd}f_G - R_{dlmn}R_c^{lmn}f_G + 2R_{cd}\Box f_G \\
&+ R\nabla_c\nabla_d f_G - 2R_c^m\nabla_d\nabla_m f_G \\
&- 2R_d^m\nabla_c\nabla_m f_G - 2R_{ldmc}\nabla^m\nabla^l f_G\bigg]h_\nu^c h_\mu^d \\
&+2\bigg[R_{m\delta}R^{m\gamma}f_G + R^{lm}R^\gamma_{m\delta l}f_G \\
&- \frac{1}{2}RR_\delta^\gamma f_G - \frac{1}{2}R_{\delta lmn}R^{lmn\gamma}f_G \\
&+ R_\delta^\gamma\Box f_G + \frac{1}{2}R\nabla^\gamma\nabla_\delta f_G \\
&- R^{m\gamma}\nabla_\delta\nabla_m f_G - R_\delta^m\nabla^\gamma\nabla_m f_G \\
&- R^\gamma_{m\delta l}\nabla^m\nabla^l f_G\bigg]h_{\nu\mu}u_\gamma u^\delta \\
&-2\bigg[R_{m\mu}R_\nu^m f_G + R^{lm}R_{m\mu l\nu}f_G - \frac{1}{2}RR_{\nu\mu}f_G \\
&- \frac{1}{2}R_{\mu lmn}R_\nu^{lmn}f_G + R_{\nu\mu}\Box f_G + \frac{1}{2}R\nabla_\nu\nabla_\mu f_G \\
&- R_\nu^m\nabla_\mu\nabla_m f_G \\
&- R_\mu^m\nabla_\nu\nabla_m f_G - R_{m\mu l\nu}\nabla^m\nabla^l f_G\bigg] \\
&-2\bigg[-R_{m\delta}R_\nu^m f_G - R^{lm}R_{m\delta l\nu}f_G \\
&+ \frac{1}{2}RR_{\nu\delta}f_G + \frac{1}{2}R_{\delta lmn}R_\nu^{lmn}f_G \\
&- R_{\delta\nu}\Box f_G + R_{m\delta l\nu}\nabla^m\nabla^l f_G \\
&+ R_\nu^m\nabla_\delta\nabla_m f_G + R_\delta^m\nabla_\nu\nabla_m f_G \\
&- \frac{1}{2}R\nabla_\nu\nabla_\delta f_G\bigg]u_\mu u^\delta \\
&-2\bigg[-R_{m\mu}R^{m\gamma}f_G - R^{lm}R^\gamma_{m\mu l}f_G \\
&+\frac{1}{2}RR_\mu^\gamma f_G\frac{1}{2}R_{\mu lmn}R^{lmn\gamma}f_G - R_\mu^\gamma\Box f_G \\
&+ R^{m\gamma}\nabla_\mu\nabla_m f_G + R_\mu^m\nabla^\gamma\nabla_m f_G \\
&+R^\gamma_{m\mu l}\nabla^m\nabla^l f_G - \frac{1}{2}R\nabla^\gamma\nabla_\mu f_G\bigg]u_\nu u_\gamma \\
&-2\bigg[R_{m\delta}R^{m\gamma}f_G + R^{lm}R^\gamma_{m\delta l}f_G - \frac{1}{2}RR_\delta^\gamma f_G \\
&- \frac{1}{2}R_{\delta lmn}R^{lmn\gamma}f_G + R_\delta^\gamma\Box f_G \\
&+\frac{1}{2}R\nabla^\gamma\nabla_\delta f_G - R^{m\gamma}\nabla_\delta\nabla_m f_G \\
&-R_\delta^m\nabla^\gamma\nabla_m f_G - R^\gamma_{m\delta l}\nabla^m\nabla^l f_G\bigg]u_\gamma u^\delta g_{\nu\mu},
\end{aligned}
$$

$$
\begin{aligned}
M^{GT}_{\nu\mu} = &\bigg[\frac{1}{2}R_{m\epsilon}R^{mp}f_G + \frac{1}{2}R^{lm}R^p_{m\epsilon l}f_G \\
&- \frac{1}{4}RR_\epsilon^p f_G - \frac{1}{4}R_{\epsilon lmn}R^{lmnp}f_G + \frac{1}{2}R_\epsilon^p\Box f_G \\
&+ \frac{1}{4}R\nabla^p\nabla_\epsilon f_G - \frac{1}{4}R^{lp}\nabla_\epsilon\nabla_m f_G \\
&- \frac{1}{2}R_\epsilon^m\nabla^p\nabla_m f_G - \frac{1}{2}R^p_{m\epsilon l}\nabla^m\nabla^l f_G\bigg]\epsilon_{p\delta\mu}\epsilon_\nu^{\epsilon\delta} \\
&+\bigg[-\frac{1}{2}R_{m\delta}R^{lp}f_G - \frac{1}{2}R^{lm}R^p_{m\delta l}f_G \\
&+ \frac{1}{4}RR_\delta^p f_G + \frac{1}{4}R_{\delta lmn}R^{lmnp}f_G - \frac{1}{2}R_\delta^p\Box f_G
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{4}R\nabla^{p}\nabla_{\delta}f_G+\frac{1}{4}R^{lp}\nabla_{\delta}\nabla_{m}f_G\\
&+\frac{1}{2}R^{m}_{\delta}\nabla^{p}\nabla_{m}f_G+\frac{1}{2}R^{p}_{m\delta l}\nabla^{m}\nabla^{l}f_G\Big]\epsilon_{p\epsilon\mu}\epsilon^{\epsilon\delta}_{\nu}\\
&+\Big[-\frac{1}{2}R_{m\epsilon}R^{m\gamma}f_G-\frac{1}{2}R^{lm}R^{\gamma}_{m\epsilon l}f_G\\
&+\frac{1}{4}RR^{\gamma}_{\epsilon}f_G+\frac{1}{4}R_{\epsilon lmn}R^{lmn\gamma}f_G-\frac{1}{2}R^{\gamma}_{\epsilon}\Box f_G\\
&-\frac{1}{4}R\nabla^{\gamma}\nabla_{\epsilon}f_G+\frac{1}{4}R^{m\gamma}\nabla_{\epsilon}\nabla_{m}f_G\\
&+\frac{1}{2}R^{m}_{\epsilon}\nabla^{\gamma}\nabla_{m}f_G+\frac{1}{2}R^{\gamma}_{m\epsilon l}\nabla^{m}\nabla^{l}f_G\Big]\epsilon_{\delta\gamma\mu}\epsilon^{\epsilon\delta}_{\nu}\\
&+\Big[\frac{1}{2}R_{m\delta}R^{m\gamma}f_G+\frac{1}{2}R^{lm}R^{\gamma}_{m\delta l}f_G-\frac{1}{4}RR^{\gamma}_{\delta}f_G\\
&-\frac{1}{4}R_{\delta lmn}R^{lmn\gamma}f_G+\frac{1}{2}R^{\gamma}_{\delta}\Box f_G+\frac{1}{4}R\nabla^{\gamma}\nabla_{\delta}f_G\\
&-\frac{1}{4}R^{m\gamma}\nabla_{\delta}\nabla_{m}f_G-\frac{1}{2}R^{m}_{\delta}\nabla^{\gamma}\nabla_{m}f_G\\
&-\frac{1}{2}R^{\gamma}_{m\delta l}\nabla^{m}\nabla^{l}f_G\Big]\epsilon_{\epsilon\gamma\mu}\epsilon^{\epsilon\delta}_{\nu}-4R^{lm}\nabla_{m}\nabla_{l}h_{\nu\mu}f_G\\
&+2Rh_{\nu\mu}\Box f_G+\frac{1}{3}\Big[-(\rho+p)f_T+4R^{m\alpha}R_{m\alpha}f_G\\
&+4R^{lm}R^{\alpha}_{l\alpha m}f_G-2R^{2}f_G-2R^{\beta}_{lmn}R^{lmn}_{\beta}f_G\\
&-2R\Box f_G+16R^{lm}\nabla_{m}\nabla_{l}f_G-4R^{m\alpha}\nabla_{\alpha}\nabla_{m}f_G\\
&-4R^{m\beta}\nabla_{\beta}\nabla_{m}f_G-4R^{\alpha}_{l\alpha m}\nabla^{m}\nabla^{l}f_G\Big]h_{\nu\mu}\\
&+\frac{1}{6}fh_{\nu\mu},
\end{aligned}
$$

$$
\begin{aligned}
O^{GT}=&\Big[\frac{1}{2}R_{m\epsilon}R^{mp}f_G+\frac{1}{2}R^{lm}R^{p}_{m\epsilon l}f_G\\
&-\frac{1}{4}RR^{p}_{\epsilon}f_G-\frac{1}{4}R_{\epsilon lmn}R^{lmnp}f_G+\frac{1}{2}R^{p}_{\epsilon}\Box f_G\\
&+\frac{1}{4}R\nabla^{p}\nabla_{\epsilon}f_G-\frac{1}{4}R^{lp}\nabla_{\epsilon}\nabla_{m}f_G\\
&-\frac{1}{2}R^{m}_{\epsilon}\nabla^{p}\nabla_{m}f_G\\
&-\frac{1}{2}R^{p}_{m\epsilon l}\nabla^{m}\nabla^{l}f_G\Big]g^{\nu\mu}\epsilon_{p\delta\mu}\epsilon^{\epsilon\delta}_{\nu}\\
&+\Big[-\frac{1}{2}R_{m\delta}R^{lp}f_G-\frac{1}{2}R^{lm}R^{p}_{m\delta l}f_G\\
&+\frac{1}{4}RR^{p}_{\delta}f_G+\frac{1}{4}R_{\delta lmn}R^{lmnp}f_G\\
&-\frac{1}{2}R^{p}_{\delta}\Box f_G-\frac{1}{4}R\nabla^{p}\nabla_{\delta}f_G+\frac{1}{4}R^{lp}\nabla_{\delta}\nabla_{m}f_G\\
&+\frac{1}{2}R^{m}_{\delta}\nabla^{p}\nabla_{m}f_G+\frac{1}{2}R^{p}_{m\delta l}\nabla^{m}\nabla^{l}f_G\Big]\\
&\times g^{\nu\mu}\epsilon_{p\epsilon\mu}\epsilon^{\epsilon\delta}_{\nu}\\
&+\Big[-\frac{1}{2}R_{m\epsilon}R^{m\gamma}f_G-\frac{1}{2}R^{lm}R^{\gamma}_{m\epsilon l}f_G\\
&+\frac{1}{4}RR^{\gamma}_{\epsilon}f_G+\frac{1}{4}R_{\epsilon lmn}R^{lmn\gamma}f_G\\
&-\frac{1}{2}R^{\gamma}_{\epsilon}\Box f_G-\frac{1}{4}R\nabla^{\gamma}\nabla_{\epsilon}f_G+\frac{1}{4}R^{m\gamma}\nabla_{\epsilon}\nabla_{m}f_G\\
&+\frac{1}{2}R^{m}_{\epsilon}\nabla^{\gamma}\nabla_{m}f_G+\frac{1}{2}R^{\gamma}_{m\epsilon l}\nabla^{m}\nabla^{l}f_G\Big]\\
&\times g^{\nu\mu}\epsilon_{\delta\gamma\mu}\epsilon^{\epsilon\delta}_{\nu}\\
&+\Big[\frac{1}{2}R_{m\delta}R^{m\gamma}f_G+\frac{1}{2}R^{lm}R^{\gamma}_{m\delta l}f_G\\
&-\frac{1}{4}RR^{\gamma}_{\delta}f_G-\frac{1}{4}R_{\delta lmn}R^{lmn\gamma}f_G+\frac{1}{2}R^{\gamma}_{\delta}\Box f_G\\
&+\frac{1}{4}R\nabla^{\gamma}\nabla_{\delta}f_G-\frac{1}{4}R^{m\gamma}\nabla_{\delta}\nabla_{m}f_G\\
&-\frac{1}{2}R^{m}_{\delta}\nabla^{\gamma}\nabla_{m}f_G-\frac{1}{2}R^{\gamma}_{m\delta l}\nabla^{m}\nabla^{l}f_G\Big]\\
&\times g^{\nu\mu}\epsilon_{\epsilon\gamma\mu}\epsilon^{\epsilon\delta}_{\nu}\\
&-12R^{lm}\nabla_{m}\nabla_{l}h_{\nu\mu}f_G+6R\Box f_G\\
&+\Big[-(\rho+p)f_T+4R^{l\nu}R_{l\nu}f_G\\
&+4R^{lm}R^{\nu}_{l\nu m}f_G-2R^{\mu}_{lmn}R^{lmn}_{\mu}f_G\\
&-2R^{2}f_G-2R\Box f_G\\
&+16R^{lm}\nabla_{l}\nabla_{m}f_G-4R^{l\mu}\nabla_{\mu}\nabla_{l}f_G\\
&-4R^{l\nu}\nabla_{\nu}\nabla_{l}f_G\\
&-4R^{\nu}_{l\nu m}\nabla^{l}\nabla^{m}f_G\Big]+2f.
\end{aligned}
$$

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