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Theoretical study of the α-decay half-lives of $^{186-224}$Po isotopes

W A YAHYA

Department of Physics and Materials Science, Kwara State University, Malete, Nigeria
E-mail: wasiu.yahya@gmail.com

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Abstract. The α-decay half-lives for $^{186-218}$Po isotopes have been computed using Gamow-like model (GLM), Coulomb and proximity potential model (CPPM) including temperature-dependent proximity potential. The half-lives were evaluated using both experimentally and theoretically calculated Q-values. The computed α-decay half-lives are compared with experimental and empirical formulas such as Royer, universal decay law and the new Ren B formula. The standard deviations are evaluated and the results indicate that the GLM gives the best results for even–even nuclei, while the temperature-dependent CPPM gives the overall least deviation from experimental values. Among the empirical formulas, the Royer formula is the most suitable for the evaluation of α-decay half-lives for the Po isotopes considered.

Keywords. Alpha decay; half-life; Gamow-like model; proximity potential.

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1. Introduction

Alpha decay (α-decay) is a crucial decay mode that provides information about the nuclear structure and stability of heavy and superheavy nuclei [1]. It is important in the identification of new heavy and super heavy nuclei (SHN) [2], and in the study of nuclear force [3]. Investigations of the α-decay half-lives have been carried out both theoretically and experimentally using various approaches. Some of the theoretical models that have been employed to study the α-decay half-lives are the Coulomb and proximity potential model (CPPM) [4–7], the fission-like model [8], the generalised liquid drop model [9–11], the effective liquid drop model [12], the modified generalised liquid drop model [3,13,14], the preformed cluster model [15,16] and the Gamow-like model (GLM) [1,17].

An analytical formula to calculate the α-decay half-lives was developed by Royer [18] by applying a fitting procedure on a set of 373 nuclei. The universal decay law was developed by Qi *et al* [19,20]. They introduced the new universal decay law (UDL) to study α and cluster decay modes. They made use of the α-like R-matrix theory and the microscopic mechanism of the charged-particle emission. Ren *et al* [21] introduced a formula, a natural generalisation of the Viola–Seaborg formula, for cluster radioactivity half-lives. Recently, modified versions of the Ren formulas were proposed [22]. They are named new Ren A and new Ren B. The new Ren A included nuclear isospin asymmetry while new Ren B included both nuclear isospin asymmetry and angular momentum.

The half-lives of some polonium isotopes have been studied in ref. [23] through α-decay. Using nuclear potentials, the α-decays from $^{186-224}$Po have also been studied by Santhosh and Sukumaran [24]. Recently, the modified CPPM was employed to study the α-decay of $^{186-218}$Po [25]. Cheng *et al* [1] studied the α-decay half-lives of nuclei with $Z > 51$ (up to $Z = 120$) using a modified GLM. Zdeb *et al* [17] proposed a phenomenological model which is based on the Gamow theory for the calculation of α-decay half-lives. In the GLM, the square well potential is chosen as the nuclear potential, while the potential of a uniformly charged sphere is taken as the Coulomb potential.

In this study, the GLM and the CPPM (with and without temperature-dependent proximity potential) have been used to calculate the α-decay half-lives of $^{186-218}$Po isotopes using both experimentally and theoretically calculated Q_α values. The results are compared with the experimental values and values obtained from three empirical formulas, viz. the Royer formula, the universal decay law and the New Ren B formula.

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The article is organised as follows. The theory of the CPPM (with and without temperature dependence), the GLM and three empirical formulas for the calculation of α-decay half-lives are presented in §2. The results are presented and discussed in §3. The conclusion is given in §4.

2. Theory

2.1 *The Coulomb and proximity potential model (CPPM)*

The total interaction potential between the emitted and the daughter nuclei in the CPPM contains (for both the touching configuration and for separated fragments) the nuclear, the Coulomb and the centrifugal terms [26]:

$$V_{\mathrm{T}}(r) = V_{\mathrm{prox}}(z) + V_{\mathrm{C}}(r) + \frac{\hbar\ell(\ell+1)}{2\mu r^2}, \tag{1}$$

where μ is the reduced mass of the interaction system and ℓ is the angular momentum. The Coulomb potential $V_{\mathrm{C}}(r)$ is defined as

$$V_{\mathrm{C}}(r) = Z_1 Z_2 e^2 \begin{cases} \dfrac{1}{r} & \text{for } r \geq R_C, \\ \dfrac{1}{2R_C}\left[3 - \left(\dfrac{r}{R_C}\right)^2\right] & \text{for } r \leq R_C. \end{cases} \tag{2}$$

Here, Z_1 and Z_2 are the charge numbers of the daughter and emitted nuclei, respectively. The radial distance $R_C = 1.24\,(R_1 + R_2)$. The term $V_{\mathrm{prox}}(z)$ denotes the proximity potential and $z = r - C_1 - C_2$ denotes the distance between the near surfaces of the fragments, where r is the distance between the fragment centres [6]. The presence of the proximity potential causes a reduction in the height of the potential barrier. Shi and Swiatecki [27] first used the proximity potential in an empirical manner. Later, the proximity potential was used in the preformed cluster model by Malik and Gupta [28]. The proximity potential V_{prox} can be obtained by calculating the strength of the nuclear interactions between the daughter and emitted nuclei:

$$V_{\mathrm{prox}}(rz) = 4\pi b\gamma\bar{R}\phi\left(\frac{z}{b}\right)\ \mathrm{MeV}, \tag{3}$$

where the term $b\gamma\bar{R}$ depends on the geometry and shape of the two nuclei and the mean curvature radius $\bar{R}$ is given as

$$\bar{R} = \frac{C_1 C_2}{C_1 + C_2}. \tag{4}$$

The Süsmann central radii of fragments C_1 and C_2 are computed using the formula

$$C_i = R_i\left[1 - \left(\frac{b}{R_i}\right)^2 + \cdots\right], \tag{5}$$

where the diffuseness of nuclear surface $b \approx 1$ fm and R_i are given by

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}\ \mathrm{fm}\ \ (i = 1, 2). \tag{6}$$

The universal function $\phi(\epsilon = z/b)$ is given in the form [29]

$$\phi(\epsilon) = \begin{cases} -\frac{1}{2}\,(\epsilon - 2.54)^2 \\ \quad -0.0852\,(\epsilon - 2.54)^3 & \epsilon \leq 1.2511 \\ -3.437\exp\left(-\epsilon/0.75\right) & \epsilon \geq 1.2511 \end{cases}. \tag{7}$$

The nuclear surface energy coefficient γ is defined as

$$\gamma = 1.460734\left[1 - 4\left(\frac{N-Z}{N+Z}\right)^2\right]\ \mathrm{MeV/fm^2}, \tag{8}$$

where N and Z denote the neutron and proton numbers of the parent nucleus, respectively. The Prox. 2010 potential has been used in this work because it gives more accurate values of the half-lives when compared with the other proximity potentials such as Prox. 88 and the proximity potential of Zheng (Prox. Zheng) [30]. The Prox. Zheng is more appropriate for cluster decay as is evident from the results obtained in [6].

According to the WKB approximation [29,31,32], the penetration probability P of the emitted α nucleus through the potential barrier is calculated using the equation

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}}\sqrt{2\mu\left[V(r) - Q\right]}\,\mathrm{d}r\right], \tag{9}$$

where the classical turning points R_{in} and R_{out} are determined from

$$V(R_{\mathrm{in}}) = V(R_{\mathrm{out}}) = Q \tag{10}$$

and the reduced mass μ is calculated using $\mu = mA_1A_2/A$, where m is the nucleon mass, A_1 and A_2 denote the mass numbers of the emitted and daughter nuclei, respectively and A is mass number of the parent nucleus. The α-decay half-life is then computed using the formula

$$T_{1/2} = \frac{\ln 2}{\nu P}, \tag{11}$$

where the assault frequency ν has been taken to be $10^{20}\ \mathrm{s}^{-1}$.

2.1.1 *Temperature-dependent proximity potential.*

The thermal effects are studied by using the temperature-dependent forms of the parameters R, γ and b. They are given by [29]

$$R_i(T) = R_i(T=0)[1 + 0.0005T^2] \text{ fm } (i = 1, 2), \tag{12}$$

$$\gamma(T) = \gamma(T=0)\left[1 - \frac{T - T_b}{T_b}\right]^{3/2}, \tag{13}$$

$$b(T) = b(T=0)\left[1 + 0.009T^2\right], \tag{14}$$

where T_b denotes the temperature associated with near Coulomb barrier energies and $b(T = 0) = 1$. In this work, we have adopted an alternative form of the temperature-dependent surface energy coefficient in the form $\gamma(T) = \gamma(0)\,(1 - 0.07T)^2$ [33]. The temperature T (in MeV) can be obtained from

$$E^* = E_{\text{kin}} + Q_{\text{in}} = \frac{1}{9}AT^2 - T, \tag{15}$$

where E^* is the excitation energy of the parent nucleus and A is its mass number. The entrance channel Q-value of the system is denoted as Q_{in}. The kinetic energy of the emitted cluster E_{kin} is obtained from

$$E_{\text{kin}} = (A_d/A_p)Q. \tag{16}$$

2.2 *The Gamow-like model (GLM)*

A model to compute α-decay half-lives based on the Gamow theory is presented in ref. [17]. The half-life of the α-decay in the GLM is calculated using the formula

$$T_{\frac{1}{2}} = \frac{\ln 2}{\nu S_\alpha P}, \tag{17}$$

where $\nu = 10^{20}$ s^{-1} is the frequency of assaults on the barrier, the preformation probability of the α-particle at the surface $S_\alpha = 1$ and h is the α-decay hindrance factor. The penetration probability of the α-particle tunnelling through the potential barrier is obtained using the Wentzel–Kramers–Brillouin (WKB) approximation via [1,17]

$$P = \exp\left[-\frac{2}{\hbar}\int_R^b \sqrt{2\mu\,(V(r) - Q)}\,\mathrm{d}r\right], \tag{18}$$

where $\mu = mA_1A_2/(A_1+A_2)$ is the reduced mass of the emitted α-particle (with mass number A_1) and daughter nucleus (with mass number A_2) and $m = 931.5$ MeV/c^2 is the nuclear mass unit. The spherical square well radius R is obtained from

$$R = r_0(A_1^{1/3} + A_2^{1/3}) \tag{19}$$

while the classical turning point, b, is given by

$$b = \frac{Z_1Z_2e^2}{Q} \tag{20}$$

and the radius constants r_0 is taken to be 1.2 fm. Z_1 and Z_2 are the atomic numbers of the emitted cluster and daughter nucleus, respectively and $e^2 = 1.43998$ MeV fm.

In this model, the interaction potential $V(r)$ is given in the form

$$V(r) = \begin{cases} -V_0 & 0 \le r \le R, \\ \dfrac{Z_1Z_2e^2}{r} & r > R, \end{cases} \tag{21}$$

where the depth of the potential well $V_0 = 25A_1$.

2.3 *Empirical relations for α-decay*

The computed α-decay half-lives using the GLM and CPPM will be compared with empirical formulas of Royer, universal decay law and the modified Ren B formula. Brief descriptions of the empirical models are presented here.

2.3.1 *Royer empirical formula.*

In ref. [18], Royer presented an analytical formula for the calculation of α-decay half-lives given by

$$\log_{10}[T_{1/2}^{\text{Royer}}(s)] = a + bA^{1/6}Z^{1/2} + cZQ_\alpha^{-1/2}, \tag{22}$$

where Z is the atomic number of the parent nucleus, A is the mass number and Q_α is the energy released during the reaction. An improved version that contains the ℓ-dependent terms is given in refs [34,35] as

$$\begin{aligned}\log_{10}[T_{1/2}^{\text{Royer}}(s)] = {} & a + bA^{1/6}Z^{1/2} + cZQ_\alpha^{-1/2} \\ & + dAN Z\,[\ell(\ell+1)]^{1/4}\,Q^{-1} \\ & + eA[1 - (-1)^\ell].\end{aligned} \tag{23}$$

The parameters a, b, c, d and e are given in table 1.

Additionally, the parameters a, b and c for even–even heavy nuclei with $Z > 82$ and $N > 126$ were given, respectively, as -27.690, -1.0441 and 1.5702. For lighter nuclei, the parameters were given as -28.786, -1.0329 and 1.6127, respectively [34,35].

2.3.2 *The universal decay law (UDL).*

The universal decay laws (UDL) for α and cluster decays are presented in refs [19,20] starting from the microscopic mechanism of the charged-particle emission. In this model, the half-live of the α-decay or cluster decay is given by

$$\begin{aligned}\log_{10}[T_{1/2}^{\text{UDL}}(s)] &= aZ_cZ_d\sqrt{\frac{\mathcal{A}}{Q_c}} \\ &\quad + b\sqrt{\mathcal{A}Z_cZ_d\left(A_d^{1/3} + A_c^{1/3}\right)} + c \\ &= a\chi' + b\rho' + c,\end{aligned} \tag{24}$$

where

$$\mathcal{A} = \frac{A_dA_c}{A_d + A_c}. \tag{25}$$

Table 1. Values of the parameters in the Royer formula.

Set	a	b	c	d	e
Even–even	− 25.752	− 1.15055	1.5913	0.0000	0.0000
Even–odd	− 27.750	− 1.1138	1.6378	1.7383×10^{-6}	0.002457
Odd–even	− 27.915	− 1.1292	1.6531	8.9785×10^{-7}	0.002513
Odd–odd	− 26.448	− 1.1023	1.5967	1.6961×10^{-6}	0.00101

It is called the universal decay law because the relation is valid for the monopole radioactive decays of all clusters. For α-decay, the parameters a, b and c are given as $a = 0.4065$, $b = -0.4311$ and $c = -20.7889$ [19].

2.3.3 *New Ren formulae.* The modified Ren A and Ren B formulae [21] that included the nuclear isospin asymmetry term is given in ref. [22] as New Ren A (with five free parameters) and New Ren B (with six free parameters), respectively. The New Ren A (NRA) formula is given as

$$\log_{10}[T_{1/2}^{\mathrm{NRA}}(s)] = a\sqrt{\mu}Z_1Z_2\sqrt{Q} + b\sqrt{\mu Z_1 Z_2} + c + dI + eI^2, \quad (26)$$

where the nuclear isospin asymmetry $I = (N - Z)/A$, and the five parameters a, b, c, d and e are given in table 2 of ref. [22]. Z_1 and Z_2 are the atomic numbers of the cluster and the daughter nuclei, respectively, and $\mu = A_1A_2/(A_1 + A_2)$ is the reduced mass.

The New Ren B (NRB) formula included both nuclear isospin asymmetry and angular momentum. The formula is given as:

$$\log_{10}[T_{1/2}^{\mathrm{NRB}}(s)] = a\sqrt{\mu}Z_1Z_2\sqrt{Q} + b\sqrt{\mu Z_1 Z_2} + c + dI + eI^2 + f\,[\ell(\ell+1)], \quad (27)$$

where I is the nuclear isospin and the angular momentum ℓ are obtained from the selection rule given by [22,36,37]

$$\ell = \begin{cases} \delta_j & \text{for even } \delta_j \text{ and } \pi_d = \pi_p, \\ \delta_j + 1 & \text{for odd } \delta_j \text{ and } \pi_d = \pi_p, \\ \delta_j & \text{for odd } \delta_j \text{ and } \pi_d \neq \pi_p, \\ \delta_j + 1 & \text{for even } \delta_j \text{ and } \pi_d \neq \pi_p. \end{cases} \quad (28)$$

Here $\delta_j = |j_p - j_d|$, where j_d, π_d, j_p, π_p are the spin and parity values of the daughter and parent nuclei, respectively. The values of the six parameters a, b, c, d, e and f are given in table 3 of ref. [22]. In this work, we have presented results for only New Ren B because it gives less deviation from experimental values than New Ren A, especially for non even–even nuclei.

3. Results and discussions

The α-decay half-lives of the polonium isotopes ($Z = 84$) within the mass range $186 \leq A \leq 218$ have been calculated using two theoretical models, GLM and CPPM (with and without temperature dependence) and empirical formulas of Royer, UDL and NRB. In the CPPM, we have also used a temperature-dependent proximity potential (CPPMT). The proximity 2010 potential has been employed in the CPPM and CPPMT. The database has been taken from the NUBASE2016 [38–40]. The results are compared with the available experimental data [39]. The α-decay half-lives computed for 33 polonium isotopes ($^{186-218}$Po) are shown in table 2. The first four columns show, respectively, the mass number (A), the experimental Q_α values, the calculated temperature and the experimental values (Expt.) of the α-decay half-lives $\left(\log\left[T_{1/2}(s)\right]\right)$. The last six columns of the table show the computed α-decay half-lives using GLM, CPPM, CPPMT, Royer, UDL and NRB, respectively. All the models give reasonable values of the half-lives when compared to the experimental results. For all the isotopes considered here, the use of temperature-dependent proximity potential (CPPMT) improves the results when compared with the experimental values. This shows the importance of using temperature-dependent proximity potentials. Figure 1 shows the plot of the total potential $V_{\mathrm{T}}(r)$ for the α-decay of ^{188}Po using CPPM and CPPMT. The plot shows the values of the total potential $V_{\mathrm{T}}(r)$ between $r = R_{\mathrm{in}}$ and $r = R_{\mathrm{out}}$ only. The effect of temperature is only seen in the first turning point (R_{in}). There is no change in the second turning point when temperature is included. The values of the first turning point are 8.43368 and 8.44021 fm for CPPM and CPPMT, respectively, while the value of the second turning point is 29.22008 fm for both CPPM and CPPMT.

Table 3 shows the calculated α-decay half-lives using the Q value obtained from Weizsäcker–Skyrme-4+RBF (WS4+RBF) [41] mass model. The WS4+RBF mass model is known to give Q values closest to experiment when compared to other models such as the WS4 mass model [41]. The half-lives obtained using the WS4+RBF mass model is lower than that obtained using the experimental Q value. It also gives higher

Table 2. Calculated α-decay half-lives, $\log\left[T_{1/2}(s)\right]$, of polonium ($Z = 84$) using GLM, CPPM, CPPMT, Royer, UDL, NRB. The temperature is calculated in MeV.

			$\log\left[T_{1/2}(s)\right]$						
A	Q_α (MeV)	T (MeV)	Expt.	GLM	CPPM	CPPMT	Royer	UDL	NRB
186	8.5010	0.9266	− 4.398	− 4.6649	− 5.1211	− 4.9967	− 5.1005	− 4.9714	− 5.1220
187	7.9789	0.8961	− 2.854	− 3.2675	− 3.4765	− 3.3252	− 2.9050	− 3.5348	− 3.2730
188	8.0820	0.8992	− 3.561	− 3.5785	− 4.0196	− 3.8819	− 3.9721	− 3.8535	− 3.9414
189	7.6943	0.8756	− 2.420	− 2.4686	− 2.6616	− 2.5087	− 2.0180	− 2.7187	− 2.41958
190	7.6933	0.8732	− 2.609	− 2.4848	− 2.9089	− 2.764	− 2.8434	− 2.7352	− 2.7718
191	7.4933	0.8598	− 1.658	− 1.8848	− 2.2990	− 2.1521	− 1.9896	− 2.1244	− 2.2666
192	7.3196	0.8478	− 1.492	− 1.3441	− 1.7487	− 1.6010	− 1.6729	− 1.5755	− 1.5711
193	7.0938	0.8328	− 0.432	− 0.6024	− 0.9940	− 0.8465	− 0.6363	− 0.8247	− 0.8544
194	6.9871	0.8245	− 0.407	− 0.2483	− 0.6320	− 0.4854	− 0.5545	− 0.4670	− 0.4326
195	6.7499	0.8087	0.667	0.5956	0.2279	0.3714	0.6208	0.3831	0.4732
196	6.6581	0.8013	0.745	0.9240	0.5649	0.7061	0.6364	0.7133	0.7691
197	6.4116	0.7847	2.080	1.8762	1.5371	1.6714	1.9583	1.6681	1.8991
198	6.3097	0.7766	2.026	2.2762	1.9491	2.0781	2.0044	2.0684	2.1387
199	6.0743	0.7605	3.640	3.2677	2.9609	3.0818	3.4049	3.0580	3.4525
200	5.9816	0.7529	3.790	3.6637	3.3670	3.4842	3.4015	3.4527	3.5293
201	5.7993	0.7398	4.760	4.4915	4.2102	4.3219	4.6718	4.2758	4.8317
202	5.7010	0.7318	5.150	4.9470	4.6756	4.7840	4.6882	4.7281	4.8018
203	5.4960	0.7172	6.300	5.9596	5.9521	6.0516	7.1907	5.7312	6.9576
204	5.4849	0.7147	6.280	5.9983	5.7476	5.8493	5.7383	5.7700	5.8307
205	5.3247	0.7027	7.180	6.8312	6.5935	6.6912	7.0821	6.5933	7.4969
206	5.3270	0.7011	7.150	6.8000	6.5665	6.6630	6.5363	6.5630	6.6010
207	5.2159	0.6923	8.000	7.3958	7.1724	7.2660	7.6609	7.1512	8.1931
208	5.2154	0.6906	7.961	7.3798	7.1610	7.2533	7.1115	7.1359	7.1422
209	4.9792	0.6736	9.507	8.7379	8.7829	8.8680	10.2349	8.4733	10.2061
210	5.4075	0.6993	7.078	6.2923	6.0730	6.1657	6.0212	6.0635	6.0134
211	7.5946	0.8227	− 0.287	− 2.5721	− 1.7903	− 1.6830	− 0.4931	− 2.8241	− 0.0457
212	8.9542	0.8893	− 6.524	− 6.2550	− 6.6754	− 6.5341	− 6.8312	− 6.6304	− 6.8734
213	8.5361	0.8667	− 5.429	− 5.2487	− 5.6442	− 5.4998	− 5.6086	− 5.5808	− 5.3051
214	7.8335	0.8292	− 3.784	− 3.3447	− 3.6912	− 3.5560	− 3.7831	− 3.6151	− 3.8799
215	7.5263	0.8113	− 2.749	− 2.4307	− 2.7468	− 2.6234	− 2.5880	− 2.6796	− 2.0278
216	6.9063	0.7762	− 0.839	− 0.3662	− 0.6359	− 0.5279	− 0.7182	− 0.5828	− 0.8742
217	6.6621	0.7610	0.180	0.5210	0.2716	0.3736	0.5268	0.3125	1.3559
218	6.1147	0.7282	2.269	2.7386	2.5245	2.6167	2.4343	2.5366	2.2147

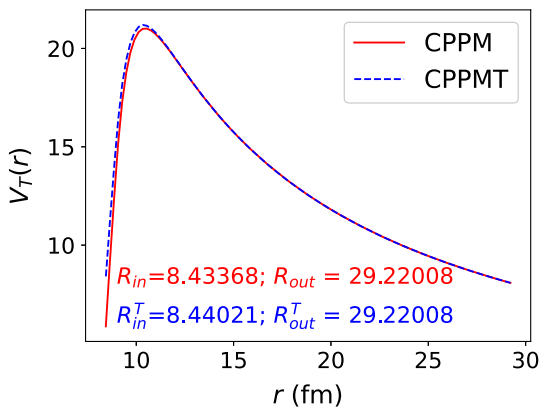

Figure 1. Plot of the total potential $V_{\mathrm{T}}(r)$ for the α-decay of ^{188}Po using CPPM and CPPMT between the range $r = R_{\mathrm{in}}$ and $r = R_{\mathrm{out}}$.

temperature values. This is the case with all the theoretical and empirical models employed in this study.

To compare the agreement between the calculated and experimental half-lives, the root mean square standard deviation σ has been evaluated using the formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(\log_{10} T^{\mathrm{Theor.}}_{1/2,i} - \log_{10} T^{\mathrm{Expt}}_{1/2,i}\right)^2\right]}, \quad (29)$$

where $T^{\mathrm{Theor.}}_{1/2,i}$ are the half-lives obtained using the six models and $T^{\mathrm{Expt}}_{1/2,i}$ are the experimental half-lives. The computed standard deviations (σ) using different models are shown in table 4. Among the theoretical models (GLM, CPPM, CPPMT), the results of the standard deviations suggest that for even–even nuclei, the GLM, with a standard deviation of 0.3558, gives the closest

Table 3. Same as in table 2 but using $Q_\alpha^{\mathrm{WS4+RBF}}$ (MeV).

			$\log\left[T_{1/2}(s)\right]$						
A	$Q_\alpha^{\mathrm{WS4+RBF}}$	T	Expt.	GLM	CPPM	CPPMT	Royer	UDL	NRB
186	8.6340	0.9337	− 4.3980	− 5.0031	− 5.4635	− 5.3435	− 5.4550	− 5.3211	− 5.4757
187	8.3570	0.9165	− 2.8540	− 4.3080	− 4.5346	− 4.3888	− 4.0445	− 4.6034	− 4.4334
188	8.0750	0.8989	− 3.5610	− 3.5589	− 3.9997	− 3.8619	− 3.9517	− 3.8334	− 3.9210
189	7.9110	0.8875	− 2.4200	− 3.1109	− 3.3151	− 3.1635	− 2.7181	− 3.3746	− 3.1318
190	7.6210	0.8692	− 2.6090	− 2.2640	− 2.6851	− 2.5394	− 2.6154	− 2.5102	− 2.5443
191	7.4930	0.8598	− 1.6580	− 1.8838	− 2.2980	− 2.1511	− 1.9885	− 2.1234	− 2.2655
192	7.3240	0.8481	− 1.4920	− 1.3585	− 1.7633	− 1.6155	− 1.6876	− 1.5900	− 1.5859
193	7.2110	0.8395	− 0.4320	− 1.0021	− 1.3996	− 1.2519	− 1.0578	− 1.2289	− 1.2934
194	7.1570	0.8342	− 0.4070	− 0.8385	− 1.2310	− 1.0837	− 1.1585	− 1.0632	− 1.0354
195	6.9030	0.8176	0.6670	0.0330	− 0.3434	− 0.1982	0.0305	− 0.1832	− 0.1417
196	6.7630	0.8074	0.7450	0.5276	0.1622	0.3050	0.2330	0.3149	0.3663
197	6.6560	0.7991	2.0800	0.9127	0.5572	0.6971	0.9514	0.7022	0.8502
198	6.4790	0.7866	2.0260	1.5855	1.2458	1.3801	1.3044	1.3773	1.4398
199	6.3150	0.7749	3.6400	2.2349	1.9119	2.0390	2.3307	2.0275	2.3333
200	6.2370	0.7683	3.7900	2.5436	2.2304	2.3536	2.2708	2.3362	2.4002
201	6.1150	0.7590	4.7600	3.0504	2.7496	2.8681	3.1776	2.8422	3.2747
202	5.9090	0.7447	5.1500	3.9580	3.6743	3.7868	3.6939	3.7461	3.8088
203	5.7470	0.7328	6.3000	4.7043	4.6818	4.7854	5.8510	4.4878	5.6073
204	5.5800	0.7206	6.2800	5.5106	5.2547	5.3581	5.2498	5.2875	5.3428
205	5.4720	0.7121	7.1800	6.0465	5.8008	5.9010	6.2743	5.8181	6.6550
206	5.5800	0.7171	7.1500	5.4730	5.2259	5.3266	5.2082	5.2511	5.2744
207	5.3380	0.7001	8.0000	6.7214	6.4918	6.5872	6.9678	6.4860	7.4707
208	5.1330	0.6853	7.9610	7.8488	7.6342	7.7252	7.5793	7.5981	7.6095
209	5.0950	0.6811	9.5070	8.0504	8.0892	8.1760	9.5032	7.7971	9.4718
210	5.5930	0.7108	7.0780	5.3328	5.1039	5.1994	5.0601	5.1140	5.0532
211	7.4470	0.8149	− 0.2870	− 2.1090	− 1.3139	− 1.2087	0.0230	− 2.3516	0.4674
212	8.8240	0.8830	− 6.5240	− 5.9449	− 6.3592	− 6.2165	− 6.5029	− 6.3059	− 6.5454
213	8.5610	0.8680	− 5.4290	− 5.3119	− 5.7087	− 5.5643	− 5.6770	− 5.6464	− 5.3764
214	8.8240	0.8788	− 3.7840	− 5.9793	− 6.3850	− 6.2413	− 6.5432	− 6.3424	− 6.6378
215	7.5410	0.8121	− 2.7490	− 2.4765	− 2.7937	− 2.6700	− 2.6368	− 2.7264	− 2.0787
216	7.1650	0.7902	− 0.8390	− 1.2702	− 1.5557	− 1.4428	− 1.6447	− 1.4983	− 1.8000
217	6.6990	0.7630	0.1800	0.3810	0.1295	0.2321	0.3799	0.1715	1.2027
218	7.1650	0.7865	2.2690	− 1.3054	− 1.5782	− 1.4691	− 1.6844	− 1.5339	− 1.9015

results to the experiment. Overall, the CPPMT, with a standard deviation of 0.4742 gives the half-lives with the least deviation from experimental values. Among the empirical formulas, the Royer model is the most suitable for the determination of the α-decay half-lives of the polonium isotopes, with a deviation of 0.4213. All the models give, overall, standard deviations less than 0.7. This suggests that the models can be used to study the α-decay half-lives of the $^{186-218}$Po isotopes. The fifth column of table 4 contains the calculated standard deviations using the WS4+RBF mass model. The values are higher than those obtained using experimental Q_α values. It is known that the half-lives are sensitive to the Q_α used in computation. However, the displayed standard deviations are lower than (not shown) those obtained using the WS4 mass model and using Q_α values derived from relativistic mean field theory binding energies.

The calculated $\log[T_{1/2}(s)]$ values using the six models have been plotted against the neutron number in figure 2. In this figure, the maximum value is obtained for $N = 125$ (^{209}Po) and the minimum is for $N = 128$ (^{212}Po). This is a reflection of the role of shell closure effects relative to the magicity of the neutron number. The difference between experimental and theoretical α-decay half-lives have been calculated using the formula

$$\Delta T_{1/2} = \log_{10}\left[T_{1/2}^{\mathrm{Theor.}}/T_{1/2}^{\mathrm{Expt}}\right]. \tag{30}$$

In figure 3, the factor $\Delta T_{1/2}$ has been plotted for all the models used in the work with the available experimental half-lives. It can be observed that most of the points are near zero and within ± 0.5.

Figure 4 shows plots of the Q_α values against neutron number using experimental and WS4+RBF mass model values. There is only a slight difference between the two.

Table 4. Calculated root means square standard deviation σ with different models.

Model	$\sigma\ (Q_\alpha^{\text{Expt}})$			$\sigma(Q_\alpha^{\text{WS4+RBF}})$
	Even–even	Even–odd	All	
GLM	0.3558	0.6698	0.5317	1.2204
CPPM	0.4724	0.6330	0.5561	1.3903
CPPMT	0.4030	0.5396	0.4742	1.2960
Royer	0.4724	0.3592	0.4213	1.2337
UDL	0.4357	0.8173	0.6493	1.3983
New Ren B	0.4437	0.4918	0.4677	1.2827

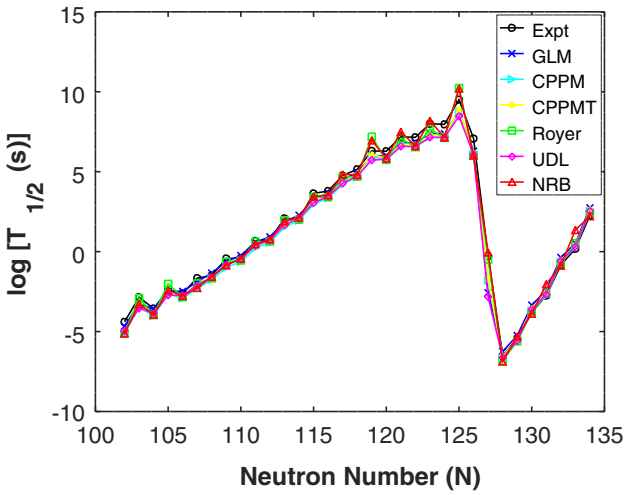

Figure 2. Comparison of the calculated α-decay half-lives of Po isotopes between the various models and experiment.

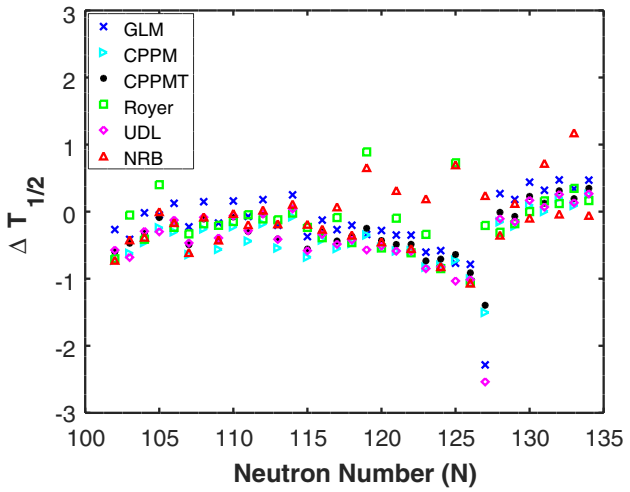

Figure 3. Plot of $\Delta T_{1/2}$ against neutron number (N) for the Po isotopes using different models.

However, the small difference has some effect on the calculated α-decay half-lives. The calculated temperature of the polonium isotopes α-decay are also plotted against the neutron number in figure 5. The inclusion of

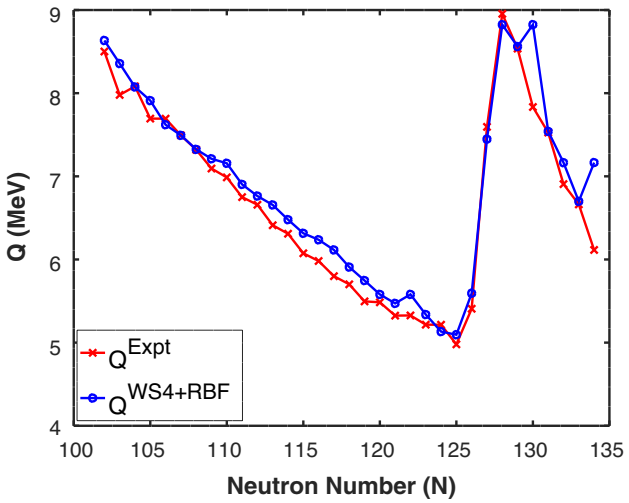

Figure 4. Plot of Q_α calculated using experimental (Expt) and theoretical (WS4+RBF) mass models against neutron number (N) for the Po isotopes.

the effect of temperature increases the values of the half-lives, as can be seen from table 2. Again, there is only a slight difference between the values of the temperature computed using experimental and theoretical Q_α values. This indicates the accuracy of the WS4+RBF mass model.

4. Conclusion

In this work, the α-decay half-lives of polonium isotopes in the mass range $186 \leq A \leq 218$ have been studied using the GLM, the CPPM including the temperature-dependent proximity potential. The Prox. 2010 proximity potential has been used. It is shown that the CPPM with temperature-dependent proximity potential (CPPMT) is the most suitable for calculating the α-decay half-lives of the polonium isotopes. Three other empirical formulas, Royer formula, the universal decay law and the new Ren B formula, were used

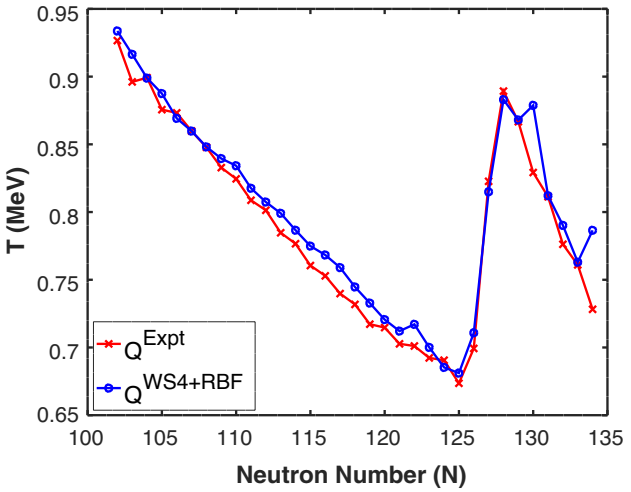

Figure 5. Plot of temperature (T in MeV) calculated using experimental (Expt) and theoretical (WS4+RBF) mass models against neutron number (N) for the Po isotopes.

in the study. The Royer formula gives the least deviation from experimental values. The α-decay half-lives were also computed using Q_α values obtained from the WS4+RBF mass model ($Q_\alpha^{\mathrm{WS4+RBF}}$). This resulted in a decrease in the calculated half-lives when compared with the results obtained using experimental Q_α values. A standard deviation less than 1.4 was obtained when $Q_\alpha^{\mathrm{WS4+RBF}}$ was used in the computation of the half-lives. In general, all the models give α-decay half-lives that are in good agreement with the experimental data.

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