



Determination of the damping coefficient of electrons in optically transparent glasses at the ultraviolet absorption region

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Abstract. The Lorentz–Maxwell model of dispersion of light has been analysed in this paper to determine the damping coefficient of the electrons in optically transparent glasses at the true resonance frequency in the ultraviolet region where glass is opaque. For this, we needed the refractive indices of glass in the optical frequency range. We argue that the true resonance condition in the absorption region prevails when the frequency at which the absorption coefficient is maximum is the same as the frequency at which the average energy per cycle of the electrons is also a maximum. We have simultaneously solved the two equations obtained from the two maxima conditions numerically to arrive at a unique solution for the true resonance frequency and the damping coefficient at this frequency. Assuming the damping coefficient to be constant over a small frequency range in the absorption region, we have determined the frequency at which the extinction coefficient is maximum using our theory. This frequency matches well with the published experimental data for silica glasses.

Keywords. Dispersion; glass; Lorentz–Maxwell model; damping coefficient; anomalous region; ultraviolet.

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1. Introduction

The Lorentz–Maxwell model of dispersion of electromagnetic (EM) waves in matter is very successful in describing the properties of matter under the action of EM waves over its whole spectrum where the wavelength is large compared to the interatomic distances. The model is generally studied in the optical frequency range where only the oscillation of electrons bound to atoms and molecules is relevant for the study of dispersion. Two important parameters of the model, namely the natural oscillation frequency and the plasma frequency of the electrons in a dielectric medium like glass, can be easily determined from the refractive indices of a glass prism measured in the optical band [1] where glass is transparent. In a condensed system like glass, one has to include the effect of the local field on the electrons apart from the field of the incident wave. This leads to another frequency which is conventionally known as the resonance frequency and is related to the plasma and the natural oscillation frequencies of the electron [2]. Although it is called the resonance frequency, there is no proof that the absorption coefficient is maximum at this frequency.

To study the absorption of EM waves in matter, a phenomenological variable called the damping coefficient is introduced in the Lorentz–Maxwell model. Glass is opaque in the ultraviolet region indicating that it has a strong absorption there. In scientific literature, there are innumerable experimental works on the interaction of silica glasses with EM waves over its whole spectrum. A summary of these works can be found in Kitamura *et al* [3]. From the experimental data on the extinction coefficient for silica glass in the ultraviolet region, we can find the frequency at which this coefficient is maximum. However, as far as we are aware, there has been no theoretical study so far which has determined this frequency by an analysis of the Lorentz–Maxwell model of dispersion. The main problem with the theoretical analysis is that it has not been possible so far to determine the value of the damping coefficient theoretically. We have determined the damping coefficient starting from the Lorentz model of dispersion. This has allowed us to probe the ultraviolet anomalous region of glass. This method may prove useful in probing other frequency bands of the dispersive media.

Our intuitive argument is that the true resonance frequency is the one at which the absorption coefficient

of the radiation inside the dielectric is maximum and at the same time this is the frequency at which the average energy per cycle of the electrons in the medium is also a maximum. In this condition, energy sharing will be maximum between the radiation and matter. We define this condition to be the true resonance condition. From the two maxima conditions, we get two equations which we solve numerically to obtain the true resonance frequency and damping coefficient at that frequency. Our only assumption is that the variation of the damping coefficient is negligibly small within the absorption band.

It is well known that the Kramers–Kronig relations [3] allow us to determine the imaginary part of the dielectric constant from an integral of the real part over the whole range of frequencies and vice versa. The theory is based on a very general causality argument and a linear response of the medium to an external perturbation. We have, on the other hand, found the derivatives of the absorption coefficient and the average energy per cycle of the electrons as a function of frequency.

In §2 we give the outline of the Lorentz–Maxwell model. In §3 we offer our physical argument for the method adopted to determine the damping coefficient and the true resonance frequency. The next four sections are just an execution of these ideas. We conclude with a summary of the work.

2. Lorentz model

In the Lorentz model [4] of dispersion of light in a dense medium like a solid or liquid, electrons execute forced simple harmonic oscillations with damping in the combined field of the incident EM wave of frequency ω and the local field. The local field arises as a result of the interaction of the electron with the fields of other atoms close by. Without any loss of generality, we can assume that the direction in which the electron is oscillating is the y direction. We can write the equation of motion as

$$\ddot{y} + \gamma \dot{y} + \omega_0^2 y = \frac{qE'_0}{m} e^{-i\omega t}, \quad (1)$$

where E'_0 is the amplitude of the effective field acting on the electrons, ω_0 is the natural oscillation frequency and γ is the damping coefficient of the electron. In the steady state, the electron will oscillate at a frequency ω of the incident wave though shifted in phase. E'_0 is related to the amplitude of the field (E_{i0}) outside from where it is incident on the medium as

$$E'_0 = \frac{1 + (\chi/3)}{1 + D\chi} E_{i0}. \quad (2)$$

The $(\chi/3)$ term in eq. (2) arises as a result of the effect of local field in the Lorentz–Lorenz theory of dielectric polarisability valid for an isotropic medium [5] where χ is the electric susceptibility. D is the depolarisation factor, a dimensionless number of the order of unity [6]. The dielectric function of the medium is given by

$$\epsilon = 1 + \chi. \quad (3)$$

Using Maxwell's phenomenological relation $\epsilon = n_c^2$ where n_c is the complex refractive index and the Lorentz–Lorenz equation [5], we arrive at the following equation for a number of resonance regions [7,8]:

$$\frac{n_c^2 - 1}{n_c^2 + 2} = \frac{Nq^2}{3\epsilon_0 m} \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 - i\gamma_j \omega}. \quad (4)$$

Here f_j is the fraction of electrons that have a natural oscillation frequency ω_{0j} and damping constant γ_j with $\sum f_j = 1$. N is the density of electrons taking part in dispersion. It is a common practice to assume a single dominant absorption frequency which is true in many practical cases, which makes the analysis simpler [9]. With this assumption $f_j = 1$ and eq. (4) can be written as

$$n_c^2 = 1 + \frac{\omega_p^2}{\omega_n^2 - \omega^2 - i\gamma\omega}, \quad (5)$$

where the plasma frequency ω_p is given by

$$\omega_p^2 = \frac{Nq^2}{\epsilon_0 m} \quad (6)$$

and we define

$$\omega_n^2 = \omega_0^2 - \frac{\omega_p^2}{3}. \quad (7)$$

In scientific literature, ω_0 is known as the natural oscillation frequency and ω_n is known conventionally as the resonance frequency [2]. So far, authors have used some chosen values of the damping coefficient γ and the plasma frequency which mimic the absorptive properties of dielectric materials, to carry out model analysis [9]. We have actually determined the damping coefficient from a prior knowledge of the natural oscillation frequency and the plasma frequency of a glass medium.

In the optical limit, where the absorption in glass is negligible, we take $\gamma = 0$. In this limit, the refractive index is real and eq. (5) reduces to

$$n^2 = 1 + \frac{\omega_p^2}{\omega_n^2 - \omega^2} \quad (8)$$

which is essentially the Sellmeier's formula [7] for dispersion in the frequency domain with one absorption band.

In the absorptive region, the dielectric function picks up an imaginary part given by

$$n_c^2 = \epsilon = \epsilon_1 + i\epsilon_2. \tag{9}$$

The optical constants, refractive index (n) and the extinction coefficient (κ), are related as

$$n_c = n + i\kappa, \tag{10}$$

where κ represents the attenuation factor of the amplitude of the EM wave in an absorptive medium. Using the last two equations and eq. (5), we obtain the real and imaginary parts of the complex dielectric function [10],

$$\epsilon_1 = n^2 - \kappa^2 = 1 + \frac{\omega_p^2(\omega_n^2 - \omega^2)}{(\omega_n^2 - \omega^2)^2 + \gamma^2\omega^2} \tag{11}$$

and

$$\epsilon_2 = 2n\kappa = \frac{\omega_p^2\gamma\omega}{(\omega_n^2 - \omega^2)^2 + \gamma^2\omega^2}. \tag{12}$$

We can express n^2 and κ^2 as functions of frequency ω using eqs (11) and (12). The details and the final expressions are shown in Appendix A.

The absorption coefficient of the incident EM wave, α , is given by [7,10]

$$\alpha = \frac{2\omega}{c}\kappa, \tag{13}$$

where c is the speed of light in vacuum and α gives the attenuation coefficient of the intensity of the incident wave. Intensity is the rate of flow of energy per unit area normal to a surface. α will be a maximum at the frequency at which the absorption of energy by the electrons from the EM field is maximum. This gives the condition of resonance. It is a general practice to consider ω_n defined in eq. (7) as the resonance frequency, although there is no proof that the energy absorption is maximum at this frequency. So we do not assume *a priori* ω_n to be the resonance frequency. In the next section we shall describe our strategy to find the true resonance frequency and in the results section we shall see that the true resonance frequency is different from both ω_0 and ω_n and lies between them. There is no real reason to call ω_n the resonance frequency. We treat the true resonance frequency as an unknown variable to be found from our analysis.

The damping coefficient γ is introduced in the Lorentz model to explain absorption. We model γ such that it is zero in the optical band and up to the frequency ω_n . In the absorption band, we assume that γ is constant from frequency ω_n to ω_0 . Above ω_0 , γ falls down and rises again to another constant value in the next resonance region if the material under study has one. With this model for γ in mind we can apply eq. (8) to find ω_n

and ω_p [1,7]. In the next section, we shall explain how to get the constant value of γ in the absorption region and the true resonance frequency.

Even if the system under study may have several absorption bands, we can study it with the assumption of a single resonance region. The optical waves oscillate the outermost electrons of atoms and molecules having the lowest natural frequencies and as a result we get the phenomenon of refraction. With an analysis of the refractive indices in the optical region under this assumption of single resonance, we are most likely to find information about the absorption band with the lowest natural oscillation frequency in the ultraviolet region closest to the optical band. This will of course depend on the strength of the resonance. The justification of the single resonance calculations with the chosen model for γ can be found from the results of our theoretical calculations which will be found to match the experimental results very well.

3. Physical arguments

From various experiments on the absorption of EM waves in matter, we know that the absorption coefficient (α) attains a maximum value at a characteristic frequency. We try to find this frequency where α is maximum.

The incident EM wave interacts with the electrons bound to the atoms and molecules. The electrons execute a forced simple harmonic oscillation with damping. The total energy of the electron is time-dependent, as the electron is being perturbed by a time-dependent harmonic force. The average energy of the electron per cycle can be worked out easily [11]. We found the frequency at which this average energy per cycle is maximum. When the frequency at which α is maximum is the same as the frequency at which the average energy per cycle of the electron is also a maximum, the EM wave will share its energy most with the electrons and will be attenuated most. This will constitute the true condition of resonance. By solving the two maxima conditions simultaneously using the numerical method, we find both the true resonance frequency ω_t and the damping coefficient at the true resonance frequency γ_t .

Heitler [12] has proposed a quantum theory of the phenomenon of damping. According to this theory, the damping coefficient is dependent on frequency though of a very slowly varying nature near resonances. This gives support to our earlier assumption that the damping coefficient is a constant within a small frequency range about the resonance frequency. However, it can be taken as zero in the optical band where glass is transparent and absorption is negligible.

4. Condition for the maximum of the absorption coefficient as a function of frequency

Our aim in this section is to find the frequency at which α is maximum. We first differentiate α with respect to ω assuming γ as constant. To find the derivative of α we first differentiate eqs (11) and (12) with respect to ω . We find two algebraic equations involving $dn/d\omega$ and $d\kappa/d\omega$. By eliminating $dn/d\omega$ from the two equations, we get the expression for $d\kappa/d\omega$ and hence by using $d\alpha/d\omega$ in eq. (13) we get

$$2 \frac{d\kappa}{d\omega} \left(n + \frac{\kappa^2}{n} \right) = \frac{A - B}{C}, \quad (14)$$

where

$$A = \omega_p^2 (\omega_n^2 - \omega^2)^2 \left[\gamma - 2 \frac{\kappa}{n} \omega \right], \quad (15)$$

$$B = \omega_p^2 \omega \gamma \left[\omega \gamma^2 - 4 (\omega_n^2 - \omega^2) \omega - 2 \frac{\kappa}{n} \omega_n^2 \gamma \right] \quad (16)$$

and

$$C = [(\omega_n^2 - \omega^2)^2 + \gamma^2 \omega^2]^2. \quad (17)$$

From this we get

$$\frac{d\alpha}{d\omega} = \frac{\kappa}{c} \left[2 + \frac{n}{\kappa} \frac{\omega}{n^2 + \kappa^2} \frac{A - B}{C} \right]. \quad (18)$$

If α is maximum then $d\alpha/d\omega$ should be zero. So we write at the maximum

$$\frac{\omega(A - B)}{C} = -2 \frac{\kappa}{n} (n^2 + \kappa^2). \quad (19)$$

It is to be noted that two sides of eq. (19) are dimensionless and later they will be compared numerically to find the solution for the true resonance frequency and the damping coefficient.

5. Condition for the maximum of the average energy per cycle of the electron as a function of frequency

In the steady state, the electron will oscillate at a frequency ω as given by the steady-state solution of eq. (1) and the total energy of the system averaged over a period is given by [11]

$$\begin{aligned} \overline{E(\omega)} &= \frac{1}{4} \frac{(qE'_0)^2}{m} \frac{(\omega^2 + \omega_0^2)}{[(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2]} \\ &= \frac{1}{4} \frac{(qE'_0)^2}{m} g(\omega), \end{aligned} \quad (20)$$

where

$$g(\omega) = \frac{(\omega^2 + \omega_0^2)}{[(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2]}. \quad (21)$$

Equation (2) shows the relationship between the incident electric field and the field acting on an electron. With the variation of frequency in the ultraviolet region, we can imagine that the amplitude of the incident field is kept constant. However, the amplitude E'_0 is dependent on χ which is frequency-dependent. Lorentz theory is based on the assumption that the response χ of the medium to the external field is small [13]. In eq. (2), χ appears both in the numerator as well as in the denominator. With the depolarisation factor D positive, any variation of χ in the numerator will be offset to some extent by the variation in χ in the denominator. So, we neglect the variation of the term E'_0 with frequency and assume it to be constant. To find the derivative of the average energy per cycle $\overline{E(\omega)}$, it is sufficient to find the derivative of the function $g(\omega)$ given by eq. (21) with respect to ω . Equating the derivative to zero, we find the condition at which the average energy per cycle is maximum. It turns out that the frequency is given by

$$\omega = \omega_0 \left[\sqrt{4 - \left(\frac{\gamma}{\omega_0} \right)^2} - 1 \right]^{1/2}. \quad (22)$$

If the incident EM wave can oscillate the bound electrons steadily at frequency ω , given by the last equation, then the wave has to deliver maximum energy per cycle and its absorption will be maximum. It is clear from eq. (22) that for real values of ω we should have the ratio

$$f = \frac{\gamma}{\omega_0} < \sqrt{3}.$$

6. Determination of the damping coefficient and the true resonance frequency

The results of an experiment performed with a prism made of flint glass have been reported by Chakrabarti [1]. In this experiment, $1/(n^2 - 1)$ has been plotted against the inverse wavelength squared at optical range following eq. (8). From this plot, we have determined the values of ω_n and ω_p . The value of ω_0 has been estimated using eq. (7). The errors in these frequencies are less than 1%. Refractive indices as a function of wavelengths are shown in table 1 and the parameters needed for this work are shown in table 2.

It has been shown in the previous section that the maximum value of $f = (\gamma/\omega_0)$ is $\sqrt{3}$. So we take trial values of f like 0.1, 0.2 up to 1.7. For all these values of f we determine γ and then ω using eq. (22). We then

determine n and κ using eqs (A.3) and (A.4) respectively and calculate A , B , C according to eqs (15)–(17) respectively. We solve eq. (19) for f numerically up to a given significant digit. From f , which satisfies eq. (19), we calculate γ_t and ω_t using eq. (22). These values are shown in table 3. The values of the damping coefficient and the true resonance frequency turn out to be

$$\gamma_t = 11.6 \times 10^{15} \text{ rad/s}$$

with an error of only 0.76% and

$$\omega_t = 16.8 \times 10^{15} \text{ rad/s}$$

with an error similar to the error in ω_0 . ω_t lies between ω_n and ω_0 . This solution is unique.

We find that ω_t differs significantly from ω_n . We have presented the parameters at true resonance frequency in table 4. The maximum value of the absorption coefficient α at the true resonance frequency has come out to be $8.18 \times 10^7 \text{ m}^{-1}$ corresponding to an attenuation length of 12.2 nm.

Jackson [14] has given the absorption coefficient α of water as a function of frequency in the ultraviolet region. He has also given the plasma frequency. In the ultraviolet region, we are concerned with electronic oscillations as the most important component responsible for dispersion. So the properties of glass will not be too different from water at these frequencies. This work [14] shows that the maximum value of α is around 10^8 m^{-1} which is approximately the same as the maximum value that we have obtained. The frequency at which the maximum occurs is close to ω_t . The full width at half maximum of the absorption curve can be read off approximately. It is determined to be approximately $15 \times 10^{15} \text{ rad/s}$ which is of the same order of magnitude that we have obtained for the value of γ_t .

The damping coefficient determined in this work is rather large. This broad absorption in the ultraviolet region is because the outer electrons in the atoms and molecules of the solid take part in the process of dispersion. The outer electrons are affected by the collisions and the electric fields of the neighbouring atoms. Consequently, an extensive region of continuous absorption is obtained in solids and liquids [8,15]. So this large value of γ_t is expected. This value of γ_t is valid only in the resonance region.

7. Some other results

We assume that the damping coefficient γ_t is constant within the small frequency range from ω_n to ω_0 in which ω_t lies. We now find frequency ω_κ at which the extinction coefficient κ is maximum, assuming ω_κ is close to ω_t . For κ to be maximum, $d\kappa/d\omega$ should be zero and

Table 1. Refractive indices as a function of wavelengths for the flint glass prism [1].

Wavelength λ (nm)	Refractive index n
706.544	1.6087
667.815	1.6108
587.574	1.6167
504.774	1.6259
501.567	1.6264
492.193	1.6277
471.314	1.6311
447.148	1.6358
438.793	1.6377

Table 2. Parameters obtained from fitting of data of refractive indices to Lorentz model [1].

ω_n (rad/s)	N (m^{-3})	ω_p (rad/s)	ω_0 (rad/s)
14.5×10^{15}	1.02×10^{29}	18.0×10^{15}	17.8×10^{15}

Table 3. Table for γ_t , ω_t and ω_κ .

$f = \gamma_t/\omega_0$	γ_t (rad/s)	ω_t (rad/s)	ω_κ (rad/s)
0.65015	11.6×10^{15}	16.8×10^{15}	15.1×10^{15}

from eq. (14) this should occur when $A = B$. Here we scan a parameter ω/ω_0 from 0.1 to 1.0. For each value of ω and known γ_t we calculate n and κ using eqs (A.3) and (A.4) respectively. We then check whether the condition $A = B$ is satisfied. Once again we find a unique solution with $(\omega_\kappa/\omega_0) = 0.84843$. This corresponds to an angular frequency $\omega_\kappa = 15.1 \times 10^{15} \text{ rad/s}$. From this we get

$$\nu_\kappa = 2.40 \times 10^{15} \text{ Hz}$$

corresponding to the wavelength

$$\lambda_\kappa = 0.125 \mu\text{m}.$$

The maximum value of κ that we have obtained at $\omega = \omega_\kappa$ is 0.769. This is of the same order of magnitude as the maximum value shown in figure 2 of Kitamura *et al* [3] which gives the maximum of κ in the ultraviolet region at a wavelength very near to $0.12 \mu\text{m}$. This is very close to the wavelength λ_κ that we have obtained. In the last column of table 3 we show the value of ω_κ . λ_κ that we have determined may be seen from figure 1 of [3] to be the nearest to the optical wavelengths on long wavelength side as we claimed in the last paragraph

Table 4. Parameters at the true resonance frequency.

Frequency ω	n	κ	α (m ⁻¹)	$1/\alpha$ (μm)	$g(\omega) = 4m\overline{E(\omega)}/(qE'_0)^2$ (s ² /rad ²)
ω_t	0.995	0.729	8.18×10^7	0.0122	0.153×10^{-31}

Table 5. Values of some parameters in the ultraviolet region.

Frequency ω	n	κ	α (m ⁻¹)	$1/\alpha$ (μm)	$g(\omega) = 4m\overline{E(\omega)}/(qE'_0)^2$ (s ² /rad ²)
ω_0	0.906	0.678	8.06×10^7	0.0124	0.149×10^{-31}
ω_κ	1.18	0.769	7.76×10^7	0.0129	0.141×10^{-31}
ω_n	1.26	0.763	7.39×10^7	0.0135	0.133×10^{-31}

of §2. The data in figure 1 of [3] may be showing other resonance/s at shorter wavelengths or larger frequencies.

In table 5 we have shown the values of n , κ , α , the absorption length $1/\alpha$ and $g(\omega)$ at frequencies close to ω_t . They have been calculated at three frequencies ω_0 , ω_κ and ω_n for the same γ_t . Comparing tables 4 and 5 we find that α and $g(\omega)$ are indeed maxima at ω_t . The anomalous nature of variation of the refractive index is evident from the values of n at frequencies around ω_t .

In §4 we determined the condition for α to be maximum. In a similar way, we can find the condition for the refractive index n to be an extremum. Interestingly, this condition gives two solutions for the frequency. The refractive index is maximum at one and minimum at the other frequency. We find that the Lorentz–Maxwell model of dispersion reproduces all the features of anomalous dispersion in the absorption region as observed in actual experiments [3].

8. Conclusion

The Lorentz–Maxwell model of dispersion of EM waves in matter has been studied here with an analysis of the phenomenon of absorption in the ultraviolet region in dielectrics like flint glass. We have shown that if we know the refractive indices of glass fairly accurately in the optical frequencies, we can explore the anomalous dispersion region in the ultraviolet region quantitatively. The key finding of this work is that the damping coefficient of the model can be determined by a simple argument. We also determine the frequency at which the absorption coefficient is maximum. We call this the true resonance frequency. In the optical region where glass is transparent, the damping coefficient can be assumed to be zero. In the absorptive part, the damping coefficient has been taken to be a constant within a short range of

frequencies. The damping coefficient matches the experimental width of the absorption coefficient curve for water available from literature.

Once the damping coefficient is determined, we can find the frequency at which the extinction coefficient is maximum. This frequency matches very well with the experimental data available in the literature for silica glasses. This indirectly shows that the value of γ determined by us is correct. Our assumption of a single resonance should give us the information of the absorption band closest to the optical frequencies. We actually observe this by comparing the peak positions of the extinction coefficient obtained by us with that found from the literature. Refractive indices estimated at different frequencies close to the true resonance frequency in the absorption region reveal the anomalous nature of dispersion. All the features of dispersion by a dielectric like glass in the ultraviolet absorption region have been reproduced from our theoretical analysis of the Lorentz–Maxwell model.

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Appendix A

Equations (11) and (12) can be easily inverted and we get [2]

$$n^2 = \frac{1}{2}[(\epsilon_1^2 + \epsilon_2^2)^{1/2} + \epsilon_1] \quad (\text{A.1})$$

and

$$\kappa^2 = \frac{1}{2}[(\epsilon_1^2 + \epsilon_2^2)^{\frac{1}{2}} - \epsilon_1]. \tag{A.2}$$

In eqs (A.1) and (A.2) we express ϵ_1 and ϵ_2 as functions of frequency using eqs (11) and (12), respectively. After a fairly straightforward algebra, we arrive at the final expressions for n^2 and κ^2 as functions of frequency ω as,

$$n^2 = \frac{[\omega_p^4 \gamma^2 \omega^2 + (\omega'^4 + \gamma^2 \omega^2 + \omega_p^2 \omega'^2)^2]^{1/2} + (\omega'^4 + \gamma^2 \omega^2 + \omega_p^2 \omega'^2)}{2(\omega'^4 + \gamma^2 \omega^2)}, \tag{A.3}$$

$$\kappa^2 = \frac{[\omega_p^4 \gamma^2 \omega^2 + (\omega'^4 + \gamma^2 \omega^2 + \omega_p^2 \omega'^2)^2]^{1/2} - (\omega'^4 + \gamma^2 \omega^2 + \omega_p^2 \omega'^2)}{2(\omega'^4 + \gamma^2 \omega^2)}, \tag{A.4}$$

where

$$\omega'^2 = \omega_n^2 - \omega^2. \tag{A.5}$$

These equations are exact and will be used for determining n and κ in the resonance region in the ultraviolet region. It can be easily checked that in the limit γ tending to zero, κ becomes zero at all frequencies and vice versa. Thus, the medium is transparent in the optical frequencies as it should. In this limit, the refractive index n satisfies a relation which has been used in the first place to get the parameters ω_n and ω_p [1].

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