Pramana – J. Phys. (2022) 96:19
https://doi.org/10.1007/s12043-021-02242-6

Effect of electric field on the onset of Jeffery fluid convection in a heat-generating porous medium layer

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MS received 20 March 2021; revised 25 May 2021; accepted 10 August 2021

Abstract. In this analysis, the collective impact of external electric field and internal heat generation on the onset of thermal convection of Jeffery fluid in a porous matrix is investigated analytically. Utilising linear stability hypothesis reliant on the normal mode process, a dispersion relation is derived and this dispersion relation is investigated for stationary and oscillatory styles of convective activities. The results reveal that the stability of the system diminishes by increasing the Jeffery parameter λ, the electric field parameter R_E and the internal heating parameter S_H. It is also shown that the oscillatory style of convective movement has not been feasible for the problem.

Keywords. Convective instability; electric field; Jeffery fluid; porous matrix; internal heating; critical thermal Rayleigh–Darcy number.

PACS Nos **47.55.P; 47.20.Bp; 47.55.pb; 83.60.Np; 47.56.+r; 47.50.-d**

1. Introduction

In the past few decades, thermal convection in a layer of fluid-saturated porous medium has attracted enormous interest of the researchers due to its application in numerous fields such as geosciences, bioengineering, food processing industries, petroleum engineering, matter processing, solar power systems and chilling in processors and electronic filling [1–6]. The analysis of the creation of thermal convection in a porous layer was first made by Horton and Rogers [7] and they found that the critical estimate of the Darcy–Rayleigh number to start the convective movement is $4\pi^2$. Later, the work of Horton and Rogers [7] was extended by many researchers including Lapwood [8], Nield [9–11], Rudraiah *et al* [12,13], Rees *et al* [14–17], Shivakumara *et al* [18,19], Bhadauria *et al* [20,21], Yadav *et al* [22–26], Babu *et al* [27], Jiang *et al* [28], Celli *et al* [29], Chand *et al* [30–32], Abarzadeh and Mahian [33] and Mahajan *et al* [34,35] under diverse physical conditions. The reviews by Nield and Bejan [36], Vadasz [37], Yadav [38] and Straughan [39] may be consulted for more details.

The corresponding problem in a heat generating porous matrix has also gained significant interest in the literature. Gasser and Kazimi [40] provided a generous examination of the beginning of thermal instability in a flat porous matrix with inner heat supply effect. They studied how the critical conditions for the convective activity varies with the power of interior heat source. Khalili and Shivakumara [41] examined the combined consequences of internal heating and throughflow on the start of convection in a flat porous layer. They found the stability conditions in terms of either a Darcy-external or Darcy-internal Rayleigh number numerically applying Galerkin procedure. Impacts of Darcy number and uniform heat supply on the stability of the fluid layer in a porous medium was studied by Nouri-Borujerdi *et al* [42]. The influence of internal heating with changeable gravity field was examined by Rionero and Straughan [43], Mahabaleshwar *et al* [44] and Yadav [45,46]. Storesletten and Rees [47] investigated the beginning of convective progress in an inclined anisotropic porous matrix with interior heating and detected that the threshold Rayleigh number at the arrival of convection varies very powerfully on the anisotropy ratio as well as the inclination angle. Very recently, Hemanthkumar *et al* [48] inspected the impact of internal heating on the Darcy–Bénard convection by considering the local thermal non-equilibrium model. They observed that internal heating produces a very strong impact on the stability of the structure.

Published online: 13 January 2022

Several investigations have been done to assess the impact of electric field on the convective instability problems in a porous medium due to its numerous applications in geosciences, energy transfer concepts, plasma physics and porous substances modelling [49–53]. The electric field plays a dominant role if the working fluid is dielectric with small electric conductivity. Moreno *et al* [54] checked the effect of AC electric field on the flow of oil and water in porous matrix and observed that the application of electric field can enhance the petroleum production. The impact of temperature modulation on the electro-convection in a densely packed porous matrix layer was studied by Rudraiah and Gayathri [55]. Shivakumara *et al* [56] examined the combined influence of rotary motion and electric field on the arrival of thermal instability in a dielectric fluid in a Brinkman porous layer. Yadav *et al* [57] and Chand *et al* [58] examined the outcome of electric field on the nanofluid convective movement in a porous medium. The extension with interior heat production was also made by Yadav [59].

Recently, Jeffrey model of non-Newtonian fluid has gained increasing attention by various investigators because it is a suitable model for several applications in polymer industries and biomechanics [60–73]. The study on convective instability of Jeffery fluid convection is extremely limited. Martinez-Mardones and Perez-Garcia [74] inspected the convective instability in a Jeffrey fluid layer under different velocity boundary situations. The consequence of electric force on the convective action of Jeffery nanofluid in a porous matrix was studied by Gautam *et al* [75]. They discussed the stationary mode of convection and found that the Jeffrey factor speeds up the creation of stationary pattern of the convective motion. Very recently, Yadav [76] examined the combined impact of anisotropy and rotation on the Jeffrey fluid convection and found that the Jeffery factor and the anisotropy in permeability have dual effect in the occurrence of rotation.

The analysis of the effect of electric field with inner heat generation on the Jeffery fluid convection in porous media seems to be very important in electric machinery, chemical engineering, oil extractions and crystal growth where the convection should be controlled, and this topic has not been studied yet. Thus, the aim of this paper is to determine the salient characteristics of the joint influence of the electric field and inner heat generation on the onset of Jeffery fluid convection in porous medium. Using linear stability theory, the critical situations for the formation of convective motion are derived and discussed using figures and tables.

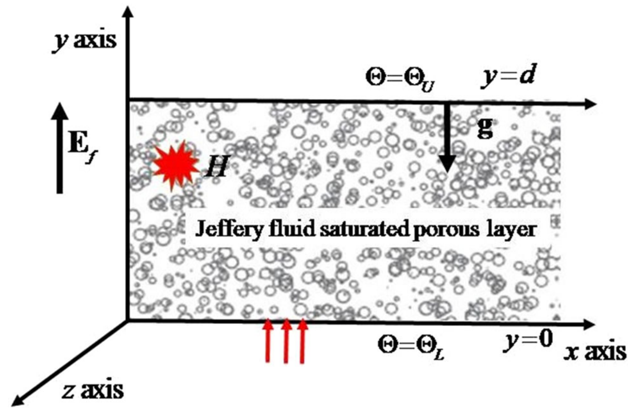

Figure 1. Physical model of the considered problem.

2. Problem formulations

dielectric Jeffery fluid convective motion in a porous medium layer of infinite extent which is heated from bottom as well as internally by a consistent distribution of heat source of power H. The layer is between two boundaries $y = 0$ and $y = d$, and the temperatures at the bottom and top boundaries are supposed to be Θ_L and Θ_U, respectively. In the gravity field g, the Jeffery fluid layer is subjected to an outer uniform upright AC electric field such that the lower edge is grounded and the upper edge is kept at a constant electrical potential ψ_1. The detailed configuration is demonstrated in figure 1. The Darcy rule is modified to include the Jeffery factor and electric force terms, while the Boussinesq estimation is employed for density digressions. The prevailing equations under this model are [56,75,77,78]:

$$\nabla \cdot \mathbf{v}_D = 0, \tag{1}$$

$$\frac{\mu}{(1+\lambda)\,K}\mathbf{v}_D = -\nabla P + \rho_0\left[1-\beta_\Theta\left(\Theta-\Theta_0\right)\right]\mathbf{g} + \mathbf{F}_E, \tag{2}$$

$$\eta\frac{\partial \Theta}{\partial \tau} + \left(\mathbf{v}_D \cdot \nabla\right)\Theta = \alpha_e \nabla^2 \Theta + H\left(\Theta-\Theta_0\right). \tag{3}$$

Here $\mathbf{v}_D\,(u_D, v_D, w_D)$ is the Darcy velocity of the Jeffery fluid, μ denotes the viscosity of the Jeffery fluid, K represents the medium permeability, λ is the Jeffery parameter and it is the ratio of relaxation to retardation times associated with the viscoelasticity of the Jeffery fluid, P indicates the pressure, $\mathbf{F}_E$ is the electric force due to electric field, ρ_0 is the reference density at $\Theta_0\,(=\Theta_U)$, β_Θ is the heat expansion coefficient, η indicates the heat capacity fraction, τ is the time, α_e is the effective thermal diffusivity and H is the strength of the internal heat source.

According to Landau *et al* [79], the elecreic force $\mathbf{F}_E$ is taken as

$$\mathbf{F}_E = \frac{1}{2}\nabla\left[\rho\frac{\partial\gamma}{\partial\rho}\left(\mathbf{E}_f\cdot\mathbf{E}_f\right)\right] - \frac{1}{2}\left(\mathbf{E}_f\cdot\mathbf{E}_f\right)\nabla\gamma. \quad (4)$$

Here, $\mathbf{E}_f$ represents the electric field, ρ shows the density of the Jeffery fluid and γ is the dielectric constant. In the right-hand side of eq. (4), the first term signifies the electrostriction force and the second term represents the dielectrophoretic force. Here, the Coulomb force is neglected in comparison to these two forces. This circumstance is valid if the frequency of the applied AC electric field is very much high compared to the reciprocal of the electric relaxation time.

For zero free charge density, the associated Maxwell equations are

$$\nabla\times\mathbf{E}_f = 0, \quad (5)$$
$$\nabla\cdot\left(\gamma\mathbf{E}_f\right) = 0, \quad (6)$$
$$\mathbf{E}_f = -\nabla\psi, \quad (7)$$

where ψ represents the electrical field potential. The dielectric constant γ is given as

$$\gamma = \gamma_0\left[1 - e\left(\Theta - \Theta_0\right)\right]. \quad (8)$$

Here, γ_0 is the dielectric constant at Θ_0 and e is the thermal expansion coefficient of γ, which is considered as very small [80]. The boundary conditions with this model are

$$\Theta = \Theta_L \text{ at } y = 0,\ \ \Theta = \Theta_U \text{ at } y = d,$$

and

$$v_D = 0 \text{ at } y = 0, d. \quad (9)$$

Now, we consider the following non-dimensional variables:

$$\bar{\mathbf{x}}=\frac{\mathbf{x}}{d},\ \bar{\tau} = \frac{\tau\alpha_e}{\eta d^2},\ \mathbf{v}=\frac{\mathbf{v}_D d}{\alpha_e},\ \bar{P} = \frac{PK}{\mu\alpha_e},\ \bar{\mathbf{E}}_f = \frac{\mathbf{E}_f}{e\Delta\Theta E_{f0}},$$
$$\bar{\Theta} = \frac{\Theta-\Theta_0}{\Delta\Theta},\ \bar{\psi} = \frac{\psi}{e\Delta\Theta E_{f0}d},\ \bar{\gamma} = \frac{\gamma}{\gamma_0}, \quad (10)$$

where E_{f0} shows the root mean square (RMS) estimate of $\mathbf{E}_f$ at $y = 0$ and $\Delta\Theta = (\Theta_L - \Theta_U)$. In that case, the dimensionless forms of eqs (1)–(9) become

$$\bar{\nabla}\cdot\bar{\mathbf{v}} = 0, \quad (11)$$
$$0 = -\bar{\nabla}\left[\bar{P} + yR_\rho\hat{\mathbf{e}}_y - \frac{1}{2}R_E\rho\frac{\partial\bar{\gamma}}{\partial\rho}\left(\bar{\mathbf{E}}_f\cdot\bar{\mathbf{E}}_f\right)\right]$$
$$-\frac{\bar{\mathbf{v}}}{(1+\lambda)} + R_\Theta\Theta\hat{\mathbf{e}}_y - \frac{1}{2}R_E\left(\bar{\mathbf{E}}_f\cdot\bar{\mathbf{E}}_f\right)\bar{\nabla}\bar{\gamma}, \quad (12)$$
$$\frac{\partial\bar{\Theta}}{\partial\bar{\tau}} + \left(\bar{\mathbf{v}}\cdot\bar{\nabla}\right)\bar{\Theta} = \left(\bar{\nabla}^2 + S_H\right)\bar{\Theta}, \quad (13)$$
$$\bar{\nabla}\times\bar{\mathbf{E}}_f = 0, \quad (14)$$
$$\bar{\nabla}\cdot\left(\bar{\gamma}\bar{\mathbf{E}}_f\right) = 0, \quad (15)$$
$$\bar{\mathbf{E}}_f = -\bar{\nabla}\bar{\psi}, \quad (16)$$
$$\bar{\gamma} = \left[1 - \Delta\Theta e\bar{\Theta}\right], \quad (17)$$
$$\bar{\Theta} = 1 \text{ at } \bar{y} = 0,\ \bar{\Theta} = 0 \text{ at } \bar{y} = 1$$

and

$$\bar{v} = 0 \text{ at } \bar{y} = 0, 1. \quad (18)$$

Here,

$$R_\Theta = \frac{\rho_0 g\beta_\Theta\Delta\Theta K d}{\mu\alpha_e}$$

is the thermal Rayleigh–Darcy number,

$$S_H = \frac{Hd^2}{\alpha_e}$$

is the internal heating parameter,

$$R_E = \frac{\gamma_0 K\left(e\Delta\Theta E_{f0}\right)^2}{\mu\alpha_e}$$

is the AC electric Rayleigh–Darcy number and

$$R_\rho = \frac{\rho_0 gKd}{\mu\alpha_e}$$

is the density Rayleigh–Darcy number.

2.1 *Basic state*

In this basic state, it is assumed that the Jeffery fluid is at rest and heat is transported only by conduction. Thus, the basic state of Jeffery fluid is expressed by

$$\bar{\mathbf{v}}_b = 0,\ \bar{\Theta}_b = \bar{\Theta}_b\left(\bar{y}\right),\ \bar{P}_b = \bar{P}_b\left(\bar{y}\right),\ \bar{\gamma}_b = \bar{\gamma}_b\left(\bar{\gamma}\right),$$
$$\bar{\mathbf{E}}_{f,b} = \bar{\mathbf{E}}_{f,b}\left(\bar{y}\right)\hat{\mathbf{e}}_y,\ \bar{\psi}_b = \bar{\psi}_b\left(\bar{y}\right). \quad (19)$$

On solving eqs (13) and (15), using basic state conditions, we have

$$\bar{\Theta}_b = \cos\left(\bar{y}\sqrt{S_H}\right) - \sin\left(\bar{y}\sqrt{S_H}\right)\cot\left(\sqrt{S_H}\right), \quad (20)$$
$$\bar{E}_{f,b} = \frac{\bar{E}_{f0}\bar{\gamma}_0}{\bar{\gamma}_b}. \quad (21)$$

Here,

$$\bar{E}_{f0} = \frac{1}{e\Delta\Theta} \text{ and } \bar{\gamma}_0 = 1.$$

With the help of eq. (17), $\bar{\gamma}_b$ can be written as

$$\bar{\gamma}_b = \left[1 - \Delta\Theta e\bar{\Theta}_b\right]. \quad (22)$$

With the help eqs (16), (21) and (22), we have

$$\bar{\psi}_b - \bar{\psi}_0 = -\bar{E}_{f0}\int_0^{\bar{y}}\left[1 - e\Delta\Theta\bar{\Theta}_b\right]^{-1}\mathrm{d}\bar{y}. \quad (23)$$

The dimensionless value of the electric filed at $\bar{y} = 0$ is given by

$$\bar{E}_{f0} = \frac{\bar{\psi}_0 - \bar{\psi}_1}{\int_0^1 \left[1 - e\Delta\Theta\bar{\Theta}_b\right]^{-1} \mathrm{d}\bar{y}}, \tag{24}$$

Here,

$$\bar{\psi}_0 = \frac{\psi_0}{e\Delta\Theta E_{f0} d} \quad \text{and} \quad \bar{\psi}_1 = \frac{\psi_1}{e\Delta\Theta E_{f0} d}.$$

In the absence of internal heating, i.e. $S_H \to 0$, eq. (20) gives

$$\bar{\Theta}_b = 1 - \bar{y}. \tag{25}$$

This result is exactly the same as found by Yadav [76].

2.2 *Perturbed equations*

We now take tiny perturbations on the basic flow as

$$\begin{aligned} &\bar{\mathbf{v}} = \bar{\mathbf{v}}', \bar{P} = \bar{P}_b + \bar{P}', \bar{\gamma} = \bar{\gamma}_b + \bar{\gamma}', \\ &\bar{\Theta} = \bar{\Theta}_b + \bar{\Theta}', \bar{\psi} = \bar{\psi}_b + \bar{\psi}', \\ &\bar{\mathbf{E}}_f = \bar{\mathbf{E}}_{b,f} + \bar{\mathbf{E}}'_f, \end{aligned} \tag{26}$$

where primes denote perturbed variables over their constancy parts. On substituting eq. (26) into eqs (11)–(17), avoiding the product of prime variables, eradicating the pressure expression from eq. (12) and considering the upright part, we achieve the subsequent linear stability equations

$$\frac{\bar{\nabla}^2 \bar{v}'}{(1+\lambda)} = R_\Theta \bar{\nabla}_H^2 \bar{\Theta}' + R_E \bar{\nabla}_H^2 \left(\bar{\Theta}' - \frac{\partial \bar{\psi}'}{\partial \bar{y}}\right), \tag{27}$$

$$\frac{\partial \bar{\Theta}'}{\partial \bar{\tau}} + \bar{v}' \bar{\Theta}_b = \left(\bar{\nabla}^2 + S_H\right) \bar{\Theta}', \tag{28}$$

$$\bar{\nabla}^2 \bar{\psi}' - \frac{\partial \bar{\Theta}'}{\partial \bar{y}} = 0, \tag{29}$$

where $\bar{\nabla}_H^2 = \left(\partial^2/\partial \bar{x}^2 + \partial^2/\partial \bar{z}^2\right)$. In perturbed form, the border situations turn into

$$\bar{v}' = \frac{\partial \bar{\psi}'}{\partial \bar{y}} = \bar{\Theta}' = 0, \text{at } \bar{y} = 0, 1. \tag{30}$$

Let the perturbed variables are as follows [23,38,77, 81–84]:

$$\left(\bar{v}', \bar{\Theta}', \bar{\psi}'\right) = [V, \ T, \ \Psi](\bar{y}) \times \mathrm{e}^{[i\delta_1 \bar{x} + i\delta_2 \bar{z} + i\sigma\bar{\tau}]}, \tag{31}$$

where δ_1 and δ_2 are the wave numbers corresponding to x and z manners, and σ is a real and non-dimensional frequency.

On applying eq (31) into eqs (27)–(29), we get

$$\frac{\left(D^2 - \delta^2\right) V}{(1+\lambda)} + R_\Theta \delta^2 T + R_E \delta^2 (T - D\Psi) = 0, \tag{32}$$

$$-V D\Theta_b + \left[D^2 - i\sigma - \delta^2 + S_H\right] T = 0, \tag{33}$$

$$\left(D^2 - \delta^2\right) \Psi - DT = 0. \tag{34}$$

Here

$$\frac{\mathrm{d}}{\mathrm{d}\bar{y}} \equiv D$$

and

$$\delta = \sqrt{\delta_1^2 + \delta_2^2}$$

is the subsequent non-dimensional wave number. In the perturbed non-dimensional structure, the boundary states convert to:

$$V = 0, T = 0, D\Psi = 0 \text{ at } \bar{y} = 0, 1, \tag{35}$$

To derive an analytical result of eqs (32)–(35), the Galerkin routine is utilised [85–88]. Thus, the support functions V, T and Ψ are taken as

$$V = \sum_{j=1}^{n} A_j V_j, \quad T = \sum_{j=1}^{n} B_j T_j, \quad \Psi = \sum_{j=1}^{n} C_j \Psi_j, \tag{36}$$

where $V_j = T_j = \sin j\pi z$, $\Psi_j = \cos j\pi z$ agreed the corresponding boundary conditions (eq (35)), A_j, B_j, and C_j are unknown coefficients. On applying the Galerkin procedure with orthogonal property, we obtained the thermal Rayleigh–Darcy number R_Θ as the eigenvalue.

3. Results and discussion

For analytical consequence, we consider $n = 1$. In that case R_Θ can be found as

$$\begin{aligned} R_\Theta = &\frac{\left(\delta^2 + \pi^2\right)\left(\delta^2 + \pi^2 - S_H\right)\left(4\pi^2 - S_H\right)}{4\delta^2\pi^2(1+\lambda)} \\ &- \frac{\delta^2 R_E}{\left(\delta^2 + \pi^2\right)} + \frac{i\sigma\left(\delta^2 + \pi^2\right)\left(4\pi^2 - S_H\right)}{4\delta^2\pi^2(1+\lambda)}. \end{aligned} \tag{37}$$

As R_Θ is a physical variable, it should be real. Accordingly, from eq. (37), we have $\sigma = 0$. This shows that the mode of convective movement will be stationary only. Therefore, eq. (37) gives the condition for the beginning of convective motion in Jeffery fluid in terms of critical thermal Rayleigh–Darcy number $R_{\Theta,c}$ at the critical value of the wave number δ_c as

$$R_{\Theta,c} = \frac{\left(\delta_c^2+\pi^2\right)\left(\delta_c^2+\pi^2-S_H\right)\left(4\pi^2-S_H\right)}{4\delta_c^2\pi^2\left(1+\lambda\right)} - \frac{\delta_c^2 R_E}{\left(\delta_c^2+\pi^2\right)}. \quad (38)$$

Here δ_c satisfies the following equation:

$$\left(S_H-4\pi^2\right)\delta_c^8+\left(2\pi^2 S_H-8\pi^4\right)\delta_c^6 + \left\{\pi^2 S_H^2+4\pi^4\left(R_E+R_E\lambda-S_H\right)\right\}\delta_c^4 + 2\pi^4\left(4\pi^4-5\pi^2 S_H+S_H^2\right)\delta_c^2 + \pi^6\left(4\pi^4-5\pi^2 S_H+S_H^2\right)=0. \quad (39)$$

From eq. (38), it is found that the internal heating parameter S_H, the Jeffery parameter λ and the electric field parameter R_E speed up the convective progress.

Now, it is essential to verify eqs (38) and (39) with the results in the literature for some particular cases. When internal heating is absent ($S_H = 0$), eqs (38) and (39) reduce to

$$R_{\Theta,c} = \frac{\left(\delta_c^2+\pi^2\right)^2}{\delta_c^2\left(1+\lambda\right)} - \frac{\delta_c^2 R_E}{\left(\delta_c^2+\pi^2\right)}, \quad (40)$$

$$\delta_c^8+2\pi^2\delta_c^6-\pi^2 R_E\left(1+\lambda\right)\delta_c^4-2\pi^6\delta_c^2-\pi^8=0. \quad (41)$$

Equation (40) is the same as that of Gautam *et al* [75] when nanoparticles are absent. In the absence of Jeffery parameter ($\lambda = 0$), eqs (38) and (39) reduce to

$$R_{\Theta,c} = \frac{\left(\delta_c^2+\pi^2\right)\left(\delta_c^2+\pi^2-S_H\right)\left(4\pi^2-S_H\right)}{4\delta_c^2\pi^2} - \frac{\delta_c^2 R_E}{\left(\delta_c^2+\pi^2\right)}. \quad (42)$$

$$\left(S_H-4\pi^2\right)\delta_c^8+\left(2\pi^2 S_H-8\pi^4\right)\delta_c^6 + \left\{\pi^2 S_H^2+4\pi^4\left(R_E-S_H\right)\right\}\delta_c^4 + 2\pi^4\left(4\pi^4-5\pi^2 S_H+S_H^2\right)\delta_c^2 + \pi^6\left(4\pi^4-5\pi^2 S_H+S_H^2\right)=0. \quad (43)$$

Equations (42) and (43) agree with Yadav [59] in the limiting case of nanoparticles. When both the Jeffery parameter and the internal heating are absent ($\lambda = S_H = 0$), eqs (38) and (39) reduce to

$$R_{\Theta,c} = \frac{\left(\delta_c^2+\pi^2\right)^2}{\delta_c^2} - \frac{\delta_c^2 R_E}{\left(\delta_c^2+\pi^2\right)}, \quad (44)$$

$$\delta_c^8+2\pi^2\delta_c^6-\pi^2 R_E\delta_c^4-2\pi^6\delta_c^2-\pi^8=0. \quad (45)$$

Equations (44) and (45) agree with the result of Shivakumara *et al* [89] for the Darcy case.

To study the influence of S_H, λ and R_E on the convective movement in the Jeffery fluid, eqs (38) and (39) are solved numerically. The results are shown in figures 2–5 and tables 1 and 2.

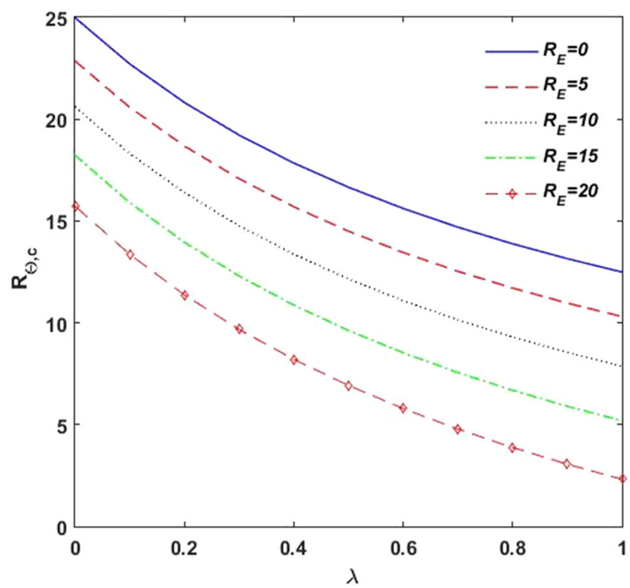

Figure 2. $R_{\Theta,c}$ vs. λ for various values of R_E when $S_H = 5$.

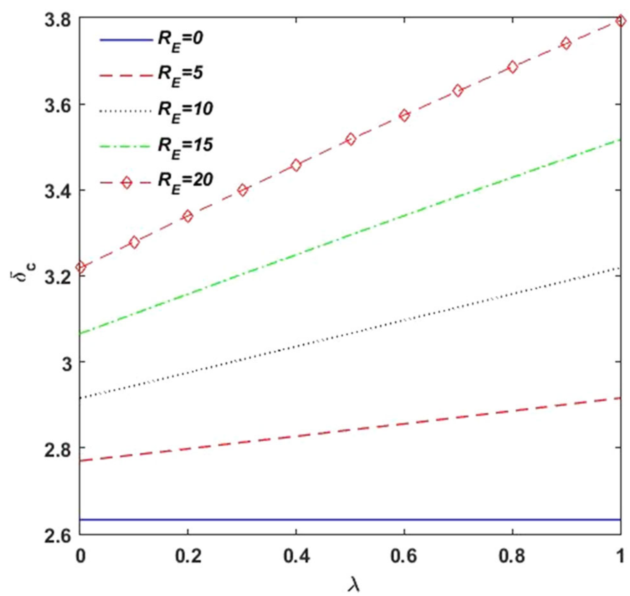

Figure 3. δ_c vs. λ for various values of R_E when $S_H = 5$.

Figures 2 and 3 present the variation of $R_{\Theta,c}$ and δ_c with λ for various values of R_E. The outcomes are also given in table 1. From figure 2, it can be seen that on increasing the values of R_E and λ, the value of $R_{\Theta,c}$ decreases. Thus, an increase in R_E and λ speed up the start of convection. This is because destabilising electrostatic power to the arrangement increases with the electric field strength, whereas the destabilising effect of λ is because an increase in the value of the Jeffery

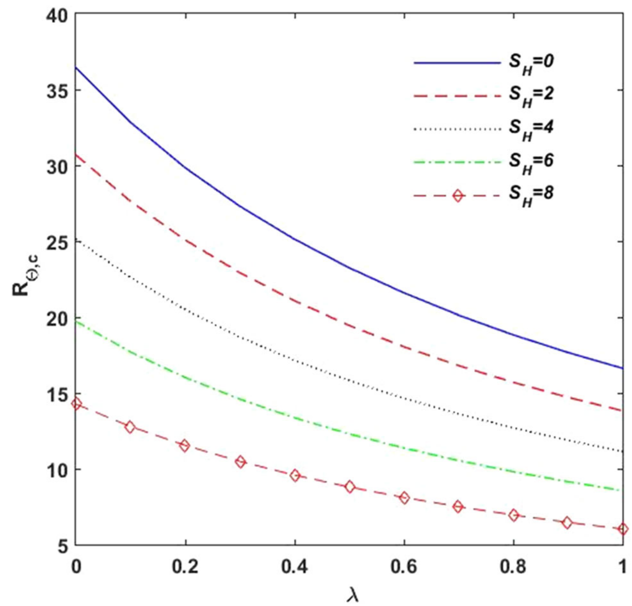

Figure 4. $R_{\Theta,c}$vs. λ for various values of S_H when $R_E = 6$.

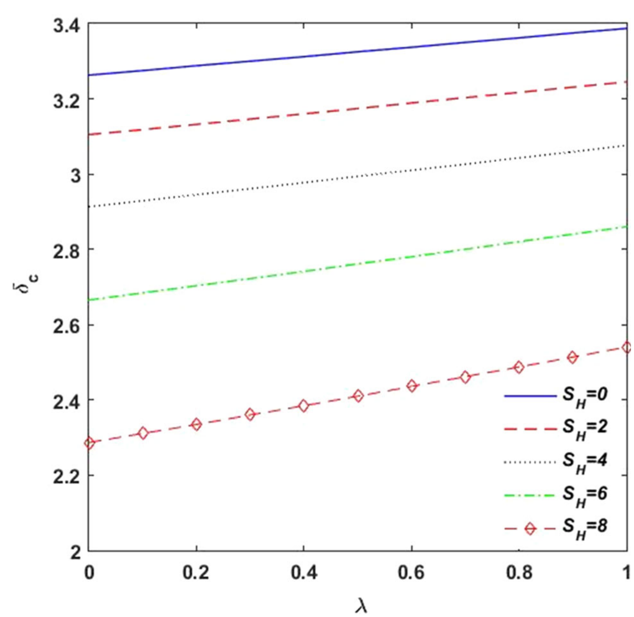

Figure 5. δ_c vs. λ for various values of S_H when $R_E = 6$.

Table 1. Contrast of $R_{\Theta,c}$ and δ_c for various values of λ and R_E when $S_H = 5$.

λ	R_E	$R_{\Theta,c}$	δ_c	λ	R_E	$R_{\Theta,c}$	δ_c
0	0	24.98	2.63	0.6	0	15.61	2.63
	5	22.86	2.77		5	13.45	2.86
	10	20.61	2.92		10	11.09	3.10
	15	18.23	3.07		15	8.53	3.34
	20	15.73	3.22		20	5.79	3.57
0.2	0	20.82	2.63	0.8	0	13.88	2.63
	5	18.68	2.80		5	11.70	2.89
	10	16.39	2.98		10	9.30	3.16
	15	13.95	3.16		15	6.68	3.43
	20	11.37	3.34		20	3.88	3.69
0.4	0	17.84	2.63	1.0	0	12.49	2.63
	5	15.69	2.83		5	10.30	2.92
	10	13.37	3.04		10	7.86	3.22
	15	10.87	3.25		15	5.19	3.52
	20	8.20	3.46		20	2.32	3.79

Table 2. Contrast of $R_{\Theta,c}$ and δ_c for various values of λ and S_H when $R_E = 6$.

λ	S_H	$R_{\Theta,c}$	δ_c	λ	S_H	$R_{\Theta,c}$	δ_c
0	0	36.42	3.26	0.6	0	21.58	3.34
	2	30.68	3.10		2	18.05	3.19
	4	25.13	2.91		4	14.65	3.01
	6	19.72	2.66		6	11.36	2.78
	8	14.27	2.29		8	8.10	2.44
0.2	0	29.83	3.29	0.8	0	18.83	3.36
	2	25.07	3.13		2	15.70	3.22
	4	20.48	2.95		4	12.70	3.04
	6	16.01	2.70		6	9.80	2.82
	8	11.54	2.34		8	6.95	2.49
0.4	0	25.12	3.31	1.0	0	16.63	3.39
	2	21.06	3.16		2	13.82	3.25
	4	17.15	2.98		4	11.14	3.08
	6	13.36	2.74		6	8.55	2.86
	8	9.58	2.38		8	6.02	2.54

parameter λ diminishes the retardation time of the Jeffery fluid. From figure 3 it is found that the value of δ_c increases with an increase in the value of both R_E and λ. Thus, R_E and λ diminish the aspect of convective cells.

The impact of internal heating parameter S_H on the beginning of convective growth in Jeffery fluid is shown in figures 4 and 5. The results are also given in table 2. From figure 4, it can be seen that $R_{\Theta,c}$ decreases on escalating the value of S_H. Thus, the influence of S_H is to speed up the start of convection. This may be recognised because enhancing S_H amounts to augment the heat energy to the arrangement which improves the turbulences in the Jeffery fluid layer and thus arrangement is less stable. Figure 5 demonstrates that increasing S_H decreases the value of δ_c and thus increases the extent of convective cells.

4. Conclusion

Applying the linear stability concept, the combined effect of the internal heating and the exterior consistent AC electric field on the onset of convective motion in a porous matrix flooded by a dielectric Jeffery fluid is examined. Using the Galerkin approach, the condition for the onset of Jeffery fluid convective development is derived analytically in terms of critical thermal Rayleigh–Darcy number $R_{\Theta,c}$, smaller which the arrangement is stable and at R_Θ somewhat above $R_{\Theta,c}$, convective motion occurs in alternating patterns of upward and downward motions. Increasing the electric field parameter R_E, internal heating parameter S_H and the Jeffery parameter λ speed up the beginning of convective activity. The size of the convective cell decreases with R_E and λ, whereas it increases with increasing S_H.

References

[1] K Vafai, *Porous media: applications in biological systems and biotechnology* (CRC Press, Boca Raton, 2010)
[2] W C Tan, L H Saw, H S Thiam, J Xuan, Z Cai and M C Yew, *Renew. Sustain. Energy Rev.* **96**, 181 (2018). https://doi.org/10.1016/j.rser.2018.07.032
[3] D B Ingham and I Pop, *Transport phenomena in porous media* (Elsevier, 1998)
[4] K Aziz, S Bories and M Combarnous, *J. Can. Pet. Technol.* **12**, 41 (1973)
[5] A Bhattacharya and R Mahajan, *J. Electron. Packag.* **128**, 259 (2006)
[6] M C Kim and D Yadav, *Transp. Porous Media* **104**, 407 (2014)
[7] C Horton and F Rogers Jr, *J. Appl. Phys.* **16**, 367 (1945)
[8] E Lapwood, *Math. Proc. Camb. Philos. Soc.* **44**, 508 (1948)
[9] D A Nield, *Water Resour. Res.* **4**, 553 (1968)
[10] D A Nield, *J. Fluid Mech.* **81**, 513 (1977)
[11] D A Nield, *Int. J. Heat Mass Transf.* **34**, 87 (1991)
[12] N Rudraiah, B Veerappa and S B Rao, *Int. J. Heat Mass Transf.* **25**, 1147 (1982)
[13] N Rudraiah, P N Kaloni and P V Radhadevi, *Rheol. Acta* **28**, 48 (1989)
[14] D A S Rees, A Selim and J Ennis-King, *Emerging topics in heat and mass transfer in porous media* edited by P Vadasz (Springer, 2008) p. 85
[15] D A S Rees and I Pop, *Transport phenomena in porous media III* (Elsevier, 2005)
[16] D A S Rees, L Storesletten and A Postelnicu, *Transp. Porous Media* **62**, 139 (2006)
[17] D A S Rees and A Postelnicu, *Int. J. Heat Mass Transf.* **44**, 4127 (2001)
[18] I S Shivakumara, M Savitha, K B Chavaraddi and N Devaraju, *Meccanica* **44**, 225 (2009)
[19] I S Shivakumara, A Mamatha and M Ravisha, *J. Eng. Math.* **67**, 317 (2010)
[20] B S Bhadauria, P G Siddheshwar, J Kumar and O P Suthar, *Transp. Porous Media* **92**, 633 (2012)
[21] B S Bhadauria and P Kiran, *Adv. Sci. Lett.* **20**, 903 (2014)
[22] D Yadav, *J. Appl. Comput. Mech.* **6**, 699 (2020), https://doi.org/10.22055/jacm.2019.31137.1833
[23] D Yadav and J Lee, *J. Nanofluids* **4**, 335 (2015), https://doi.org/10.1166/jon.2015.1159
[24] D Yadav, R Bhargava, G S Agrawal, N Yadav, J Lee and M C Kim, *Microfluid Nanofluid* **16**, 425 (2014), https://doi.org/10.1007/s10404-013-1234-5
[25] D Yadav, J Lee and H H Cho, *J. Braz. Soc. Mech. Sci. Eng.* **38**, 2299 (2016), https://doi.org/10.1007/s40430-016-0505-y
[26] D Yadav, U S Mahabaleshwar, A Wakif and R Chand, *Int. Commun. Heat Mass.* **122**, 105165 (2021), https://doi.org/10.1016/j.icheatmasstransfer.2021.105165
[27] A Benerji Babu, R Ravi and S G Tagare, *Commun. Nonlinear Sci. Numer. Simul.* **17**, 5042 (2012), https://doi.org/10.1016/j.cnsns.2012.04.014
[28] C Jiang, E Shi, Z Hu, X Zhu and N Xie, *Int. J. Heat Mass Transf.* **91**, 98 (2015)
[29] M Celli, A Barletta and D Rees, *Transp. Porous Media* **119**, 539 (2017)
[30] G C Rana, R C Thakur and S K Kango, *J. Porous Media* **17**, 657 (2014)
[31] R Chand, G C Rana and D Yadav, *J. Theor. Appl. Mech.-Pol.* **47**, 69 (2017)
[32] R Chand, D Yadav and G C Rana, *J. Porous Media* **20**, 635 (2017)
[33] P Akbarzadeh and O Mahian, *J. Mol. Liq.* **272**, 344 (2018)
[34] A Mahajan and R Nandal, *Int. J. Heat Mass Transf.* **115**, 235 (2017)
[35] A Mahajan and M K Sharma, *J. Porous Media* **17**, 439 (2014)
[36] D A Nield and A Bejan, *Convection in porous media* (Springer, 2006)
[37] P Vadasz, *Fluid flow and heat transfer in rotating porous media* (Springer, 2016)
[38] D Yadav, *Hydrodynamic and hydromagnetic instability in nanofluids* (Lap Lambert Academic Publishing, 2014)
[39] B Straughan, *Stability and wave motion in porous media* (Springer Science & Business Media, 2008)
[40] R Gasser and M Kazimi, *J. Heat Transfer* **98**, 49 (1976)
[41] A Khalili and I S Shivakumara, *Phys. Fluids* **10**, 315 (1998)
[42] A Nouri-Borujerdi, A R Noghrehabadi and D A S Rees, *Int. J. Therm. Sci.* **47**, 1020 (2008)
[43] S Rionero and B Straughan, *Int. J. Eng. Sci.* **28**, 497 (1990)
[44] U S Mahabaleshwar, D Basavaraja, S Wang, G Lorenzini and E Lorenzini, *Int. J. Heat Mass Transf.* **111**, 651 (2017)
[45] D Yadav, *Revista Cubana de Física* **37**, 24 (2020)
[46] D Yadav, *Heat Transf. Asian Res.* **49**, 1170 (2020). https://doi.org/10.1002/htj.21657
[47] L Storesletten and D A S Rees, *Fluids* **4**, 75 (2019)
[48] C Hemanthkumar, I S Shivakumara and B Rushikumar, *Darcy–Bénard convection with internal heating and a thermal nonequilibrium—A numerical study* (Advances in Fluid Dynamics, Lecture Notes in Mechanical Engineering) edited by B R Kumar, R Sivaraj, J Prakash (Springer, 2021) p. 627
[49] R F Probstein and R E Hicks, *Science* **260**, 498 (1993)

[50] F Lai and K-W Lai, *Dry. Technol.* **20**, 1393 (2002)
[51] W Gao, M Senel, G Yel, H M Baskonus and B Senel, *AIMS Math.* **5**, 1881 (2020)
[52] H Bulut, T A Sulaiman and H M Baskonus, *Opt. Quantum Electron.* **50**, 1 (2018)
[53] O A Ilhan, T A Sulaiman, H Bulut and H M Baskonus, *Eur. Phys. J. Plus* **133**, 1 (2018)
[54] R Moreno, E Bonet, O Trevisan, K Vafai and P Shivakumar, Electric alternating current effects on flow of oil and water in porous media, in: *Proceedings of the International Conference on Porous Media and Their Applications in Science, Engineering and Industry* (Hawaiir, 1996), p 147
[55] N Rudraiah and M Gayathri, *J. Heat Transfer* **131**, 101009 (2009)
[56] I S Shivakumara, C-O Ng and M Nagashree, *Int. J. Eng. Sci.* **49**, 646 (2011)
[57] D Yadav, J Lee and H H Cho, *J. Appl. Fluid Mech.* **9**, 2123 (2016), https://doi.org/10.18869/acadpub.jafm.68.236.25140
[58] R Chand, G C Rana and D Yadav, *J. Appl. Fluid Mech* **9**, 1081 (2016), https://doi.org/10.18869/acadpub.jafm.68.228.24858
[59] D Yadav, *J. Appl. Fluid Mech.* **10**, 763 (2017), https://doi.org/10.18869/acadpub.jafm.73.240.27475
[60] J A Eastman, S Choi, S Li, W Yu and L Thompson, *Appl. Phys. Lett.* **78**, 718 (2001)
[61] N Santhosh, G Radhakrishnamacharya and A J Chamkha, *J. Porous Media* **18**, 71 (2015)
[62] T Hayat, S Qayyum, M Imtiaz and A Alsaedi, *PLoS One* **11**, e0148662 (2016)
[63] O Ojjela, A Raju and N N Kumar, *J. Mech.* **35**, 657 (2019)
[64] S Nallapu and G Radhakrishnamacharya, *Int. J. Eng. Math.* **2014**, 713831 (2014)
[65] M Kahshan, D Lu and A Siddiqui, *Sci. Rep.* **9**, 1 (2019)
[66] X Guo, J Zhou, H Xie and Z Jiang, *Math. Probl. Eng.* **2018**, 6014082 (2018)
[67] K Ahmad, Z Hanouf and A Ishak, *AIP Adv.* **6**, 035024 (2016)
[68] M Bhatti and M A Abbas, *Alex. Eng. J.* **55**, 1017 (2016)
[69] K Vajravelu, S Sreenadh and P Lakshminarayana, *Commun. Nonlinear Sci. Numer. Simul.* **16**, 3107 (2011)
[70] K Mahmood, M N Sadiq, M Sajid and N Ali, *J. Braz. Soc. Mech. Sci. Eng.* **41**, 65 (2019)
[71] K Ramesh, D Tripathi, O A Bég and A Kadir, *Iran. J. Sci. Technol.-Trans. Mech. Eng.* **43**, 675 (2019)
[72] N A M Noor, S Shafie and M A Admon, *Phys. Scr.* **95**, 105213 (2020)
[73] K Naganthran, R Nazar and I Pop, *J. Braz. Soc. Mech. Sci. Eng.* **41**, 1 (2019)
[74] J Martinez-Mardones and C Perez-Garcia, *J. Phys. Condens. Matter* **2**, 1281 (1990)
[75] P K Gautam, G C Rana and H Saxena, *J. Porous Media* **23**, 1043 (2020)
[76] D Yadav, Heat *Transf. Asian Res.,* https://doi.org/10.1002/htj.22090 (2021)
[77] D Yadav, *J. Appl. Fluid Mech.* **11**, 1679 (2018), https://doi.org/10.29252/jafm.11.06.29048
[78] D Yadav, A Wakif, Z Boulahia and R Sehaqui, *J. Nanofluids* **8**, 117 (2019), https://doi.org/10.1166/jon.2019.1558
[79] L D Landau, J Bell, M Kearsley, L Pitaevskii, E Lifshitz and J Sykes, *Electrodynamics of continuous media* (Elsevier, 2013)
[80] P Roberts, *Q. J. Mech. Appl. Math.* **22**, 211 (1969)
[81] D Yadav, R Mohamed, H H Cho and J Lee, *J. Appl. Fluid Mech.* **9**, 2379 (2016), https://doi.org/10.18869/acadpub.jafm.68.236.25048
[82] D Yadav, G Agrawal and R Bhargava, *Int. J. Eng. Sci.* **49**, 1171 (2011), https://doi.org/10.1016/j.ijengsci.2011.07.002
[83] D Yadav, *Revista Cubana de Física* **35**, 108 (2018)
[84] S Chandrasekhar, *Hydrodynamic and hydromagnetic stability* (Dover Publication, 2013)
[85] D Yadav and M Maqhusi, *Asia-Pac. J. Chem. Eng.* **15**, e2514 (2020), https://doi.org/10.1002/apj.2514
[86] D Yadav, *Heat Transf. Asian Res.* **49**, 3161 (2020), https://doi.org/10.1002/htj.21767
[87] D Yadav, *Proc. Inst. Mech. Eng. C J. Mech. Eng. Sci*. **235**, 999 (2020), https://doi.org/10.1177/0954406220942551
[88] D A Nield and A V Kuznetsov, *Eur. J. Mech. B Fluids* **29**, 217 (2010)
[89] I S Shivakumara, N Rudraiah, J Lee and K Hemalatha, *Transp. Porous Media* **90**, 509 (2011)