



Detection and optical imaging of induced convection under the action of static magnetic field gradient in a non-conducting diamagnetic fluid

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Abstract. The report elaborates, for the first time, visual observations of induced convection in a non-conducting diamagnetic fluid (water) under the action of static magnetic field gradient in the absence of thermal gradients. The convections were induced at room temperature in a suspension of deionised (DI) water and lycopodium pollen grains. The suspension was exposed to a static magnetic field gradient having a magnetic induction strength of 0.12 T. Upon application of various configurations of magnetic field gradient over the suspension, different convective flow patterns were observed in the suspension. The results show that diamagnetic interactions of water are detectable even though its value of magnetic susceptibility is small.

Keywords. Convection; diamagnetism; non-conducting fluids; gradient magnetic fields.

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1. Introduction

At the end of the previous decade, it seems that magnetic field interactions with fluids have become a topic of interest for many researchers. Generally, phenomena under these interactions are manifested based on the magnitude and type of magnetic susceptibility of the fluids. The effects are readily detectable (excluding the ferromagnetism in the current discussion) with fluids having paramagnetic susceptibilities. In most cases, these effects supersede the effects caused by the diamagnetic property of the fluids. Rather, diamagnetism is present in all the atoms of all the elements. It is the small magnitude of the susceptibility, which makes it hard to detect. Nevertheless, it has not inhibited researchers around the globe from exploring physical phenomena occurring due to the action of magnetic fields on the fluids. Not only that but ‘Lorentz Force’-based dynamics in fluids have attracted researchers as its fundamental understanding shows evidence of direct applications.

One group of researchers [1] showed that as magnetic fields are applied with various geometrical configurations on an electrolysis cell, it alters the conductivity of water. The paramagnetic and diamagnetic nature of the evolved gases, oxygen (O₂) and hydrogen (H₂), in the presence of the magnetic field, helps in increasing the effectiveness of the electrolysis. Their work shows promising results in areas like biotechnology, magneto-hydrodynamics of thrusters used in seawater, etc. The domain of metallurgical processes has also seen their fair share of implementing magnetohydrodynamics. A systematic numerical analysis was provided in [2] for one such process in which a magnetic field is used to stir the purged O₂ inside the liquid metal lead (Pb). Using controlled mixing of O₂ gas into the liquid Pb helps in forming an oxide layer on the liquid metal. This in turn gives protection to the container in which the liquid metal is kept or processed. Their study involved the application of a magnetic field over the mixture of liquid metal (Pb) and O₂ gas that was maintained under a

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thermal gradient. The results thus obtained showed that the application of the magnetic field affected the time taken for O₂ gas to mix homogeneously and to its desired concentration. This indicated an immediate application domain of advanced reactors where liquid metals are used as coolants. Similarly, in the area of chemistry, water-based reactions are ubiquitous. It would be naive to consider that presence of magnetic fields would leave chemical reactions unaffected. Bian *et al* [3] showed that using 5 T magnetic field enhances the rate of galvanic replacement (GR) reaction and simultaneously modifies the magnetic properties of the final product. Their proven method paves a path in material synthesis to use the magnetic field as one of the prominent control parameters for obtaining desired shapes and compositions.

Exploring applications is an inherent component in most of the studies. Knowing the fundamental mechanics of a system helps in evolving that exploration with novel facets. Looking into similar studies, Hirota *et al* [4] showed the effect of magnetic field on the dissolution kinetics of O₂ gas in distilled water. The onset of convection was attributed to two important factors, viz., the presence of non-uniform magnetic susceptibility gradient formed by the permeation of O₂ gas phase into the water phase and magnetic force, which is given by eq. (1). Their discussion was based on the assumption that the O₂ gas present at the surface of the sample under 2×10^4 Pa pressure, dissolves in the water sample. Thus, a magnetic susceptibility gradient is created due to the mixing of O₂ (paramagnetic) and water (diamagnetic). When the fluid was exposed to a magnetic field gradient, O₂ experiences the magnetic body force called Kelvin force [5,6]. This force is responsible for the onset of convection in their sample. Once the circulation is established, the circulating current under the surface of water enhances the dissolving rate of the molecular oxygen by carrying it towards the bottom of the sample container.

$$F = -\mu_0 \rho \chi H (\partial H / \partial Z). \quad (1)$$

Equation (1) explains the attractive and repulsive magnetic force on non-conducting fluids with positive and negative magnetic susceptibilities respectively. The magnetic force on a diamagnetic fluid is directed away from high magnetic field regions and is proportional to the square of the field gradient. The force acts in the opposite sense in the case of paramagnetic fluids. Irrespective of having such a small magnitude of force, it was shown by Huang *et al* [6] that it can be used to control thermal convection in pure water using a high static magnetic field. The team treated the same force in [7] to be the gravity analogue (termed as ‘effective gravity’) which produces magnetothermal buoyancy in differentially heated fluids.

Similarly, numerical simulations shown in [8] states that the magnetic field gradient either enhances or inhibits the thermal convection in the diamagnetic non-conducting fluid (water). They concluded with two important results: (i) thermally-driven convection are controlled by the application of the static magnetic field gradient having a critical value and (ii) above the critical value of the static magnetic field gradient, thermally-driven convection are completely replaced by the magnetically-driven convection. As described in [9], experiments were conducted using a thermal gradient of 5°C on a water sample having a volume of 0.3 ml and it was exposed to the magnetic field having a product $\{B(dB/dz)\}$ value of 1360 T²/m. One of their results indicates the presence of small downward convection inside the sample under the heater. This is the state in the system in which the magnetic force dominates the buoyancy-driven convection. Similarly, the use of the 3D computational modelling reported in [10] showed that large magnetic forces can cause the onset of axisymmetric magnetothermal convection.

In [8–10], one common concept was that the volume magnetic susceptibility of the fluid is an implicit function of temperature, i.e., $\chi_v = \chi_g(t)$ where $\chi_v = \chi_g^* \rho$ and ρ is the density of the fluid. The work described in [8–10] was based on the simultaneous interaction of the thermal and magnetic gradients in the fluid. None of them reported any observations on the onset of convection currents in water due to exposure to only static magnetic field gradients.

To date all the reported work related to the magnetothermal convection in diamagnetic fluid like water was carried out using high field superconducting magnets. This is because water has a very small diamagnetic susceptibility value of -9.1×10^{-5} . For all practical purposes, it was thought that the interaction of the magnetic field with water would be too small to produce any cognizable effect on the macroscopic level using the magnetic fields having values in thousands of Gauss. Hence, previous reports were unable to produce any experimental proofs about the direct observations of the occurrence of convection in water under the application of static magnetic field gradient in the absence of thermal gradients. The present report elaborates experimental findings of the onset of convection that is independent of the magnetic susceptibility of the dissolved gas and temperature gradients. A simple technique is employed to detect and capture induced convection in a diamagnetic fluid due to the presence of a static magnetic field gradient. The experiment was performed using DI water as the diamagnetic working fluid, rare earth magnets having 1.2 kG field strength (supplementary information, figures S1–S4), diode laser with an output power < 1 mW, travelling microscope and a web camera to

capture the convection flows. The absence of temperature gradient was noted using two calibrated resistance temperature detectors (RTDs).

2. Experimental

The experiment was performed using distilled and DI water. For both the fluid types, the results were identical. Hence, for all further work, DI water was used. A stock solution of lycopodium–water [11,12] was prepared by adding 100 μg of lycopodium powder in one litre of DI water. A suspension from the stock solution was taken in the test tube which was left undisturbed for an hour (for all the experiments) to make sure that it was stabilised.

Figures 1a–1c show the experimental and generic geometrical arrangement used for measuring the temperature gradients and for observing magnetically induced convection in water under various condition. A water bath having a volume of 900 ml was used. It was kept at ambient atmospheric pressure and temperature. An airtight sealed 5 ml test tube containing water and lycopodium suspension was kept inside the water bath. Two calibrated (supplementary information, figure S5) platinum (PT-100) RTDs were sealed inside the suspension without them touching the inner walls of the test tube. The two RTDs were connected to the Agilent's six and a half digit multimeter through a manually controlled switch. Each RTD was connected to the multimeter through the switch for 5 min and the readings were noted. The RTDs' resistance was measured with an accuracy of 0.001 Ω . For observing the convection, a simple particle image velocimetry (PIV) technique was developed based on the earlier reports [13]. The suspension was irradiated using a laser diode having 630 nm wavelength and an output power of less than 1 mW. Motions of these pollen grains were visually observed and recorded using a 10 \times microscope with an attached web camera. As shown in figure 2, a plano-convex lens having a focal length of 3 cm was added at the output of the laser diode to facilitate the detection of particles through the microscope or even with naked eyes.

In figure 2a, the test tube filled with the suspension was placed on the magnet. The convection flow started as soon as the test tube was exposed to a magnetic field. Video clips of the convective flows were recorded using the experimental set-up as described above. The obtained video clips were further processed as shown in the supplementary figures S6–S8.

The effect of different geometries of the static magnetic field gradients on the formation of convection currents was observed by applying various magnetic field convection with respect to the test tube. Figures 3

and 4 show the dual and quad configurations of the magnets. As shown in figure 3b, in the dual configuration of magnets, the observations were taken by directing the laser light and placing the microscope such that the line of sight passes just over the magnets. Two disk magnets were used to enhance the field strength and thus the field gradient for the experiment. The quad configurations of magnets were made by placing four disk magnets as shown in figure 4.

Another set of experiments was performed to observe how the placement of the single magnet affects the onset of convection in the test tube. Figure 5a shows a magnet kept at a height of 1 cm from the surface of the table in the proximity of the suspension-filled test tube. Figure 5b shows the same magnet kept on top of the test tube. An experimental arrangement was made as shown in figure 5c, which was used to observe the occurrence of convective flow based on its explicit dependence on the field gradient. Hence, a small volume (0.2 ml) of suspension was taken which in turn experienced a small magnitude of field gradient of the magnet. In figure 5d, the set-up was made to observe the contribution of the heating effect of laser on the onset of convection. It consists of a test tube filled with the suspension with the disk magnet in its proximity but 1 cm above the table surface. Initially, the suspension was exposed to laser light for an hour. After 1 h, a magnetic field was applied.

3. Results and discussions

Figures 1a and 1c show suspension-filled test tubes of 5 and 10 ml kept in a water bath and open air at room temperature. It was observed that the onset of the convection inside the test tube appears in the region that is closest to the surface (pole) of the magnet. As shown in figures 2a and 2b, nearly 100% of the laser light goes through the test tube. Some amount is scattered by the wall of the test tube. Scattering through DI water is practically negligible. As the water contains few pollen grains suspended into it, scattered laser light due to these pollen grains was observed at a scattering angle over a range of 5–10 $^\circ$ (estimation was based on visual observation) with respect to the unscattered diverged laser beam. The scattered light from pollen grains was collected through the microscope and into the web camera. Due to the use of microscope, the images obtained are inverted and are presented without any processing to their orientations. The camera was attached to a PC. Video recording free-ware recorded all the videos of the convection currents. From the captured videos, the maximum velocity of the convection currents was determined to be 2 mm/s. This magnitude was deduced from a certain section of the entire convection flow video. In actual it was found that

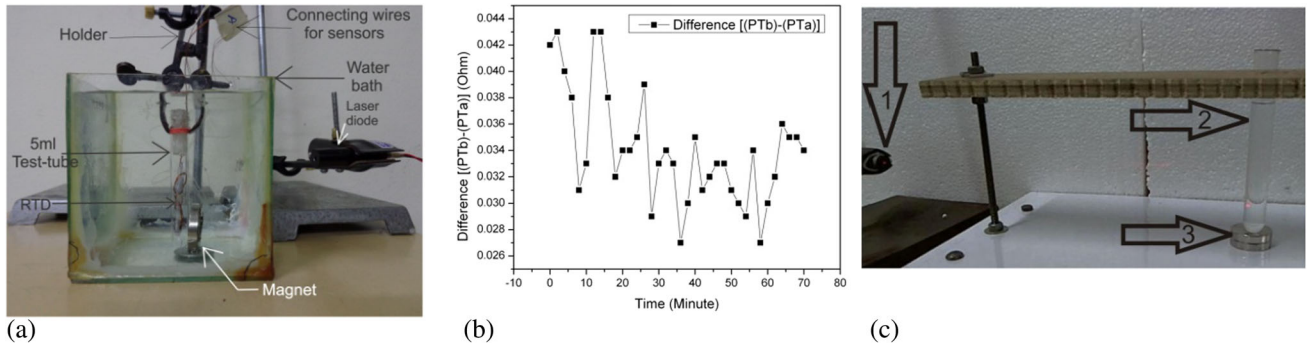


Figure 1. (a) Image of the set-up for the measurement of temperature gradient in the 5 ml test tube containing suspension, (b) graph of difference in the resistance values of two PT100 sensors vs. time and (c) image of the generic experimental set-up. Arrows 1, 2 and 3 indicate positions of laser diode, test tube filled with the suspension and the rare-earth magnets.

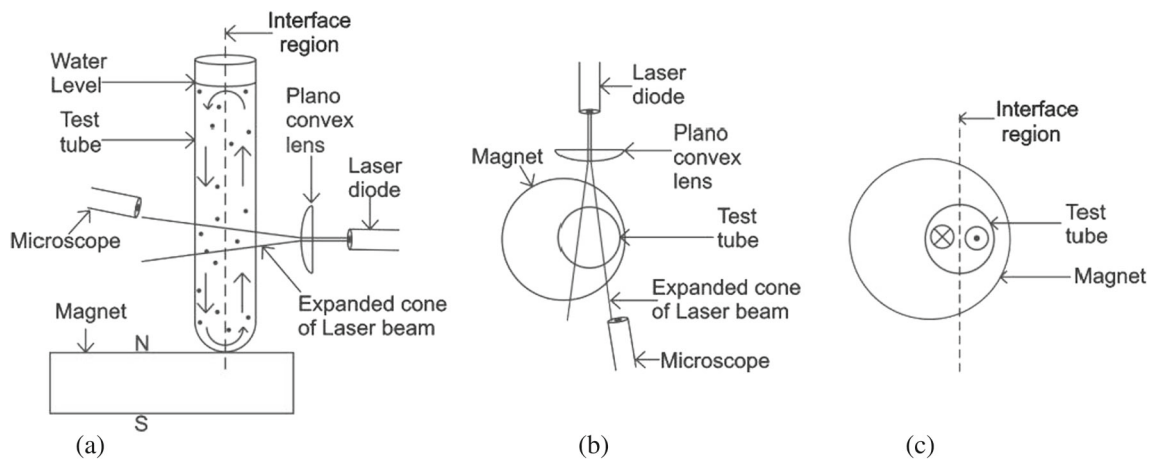


Figure 2. (a) Side view of the experimental set-up used for visual observations. The arrows inside the test tube represent the flow direction, (b) top view of the same experimental set-up and (c) upward and downward flow in the test tube as seen from the top.

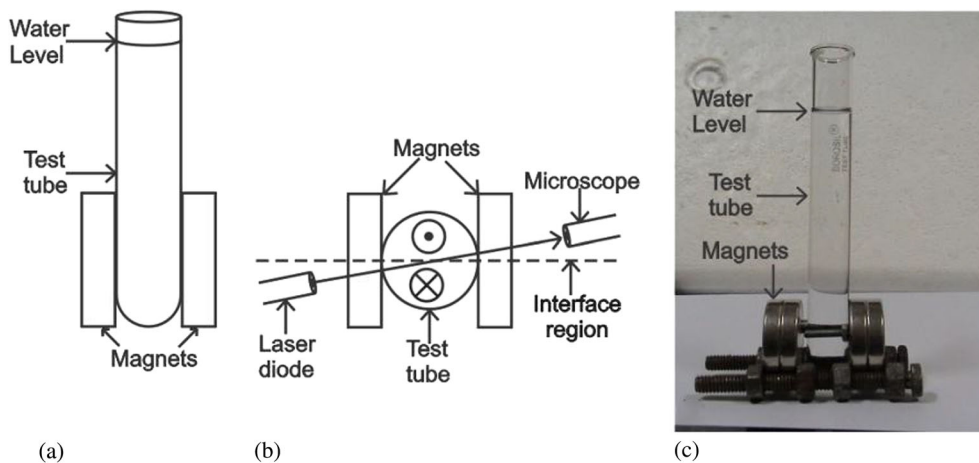


Figure 3. (a) Side view of the two (dual) magnets attached around the test tube, (b) top view, depicting the out of the page flow by the dot in the circle and downward by cross in the circle and (c) image of the set-up.

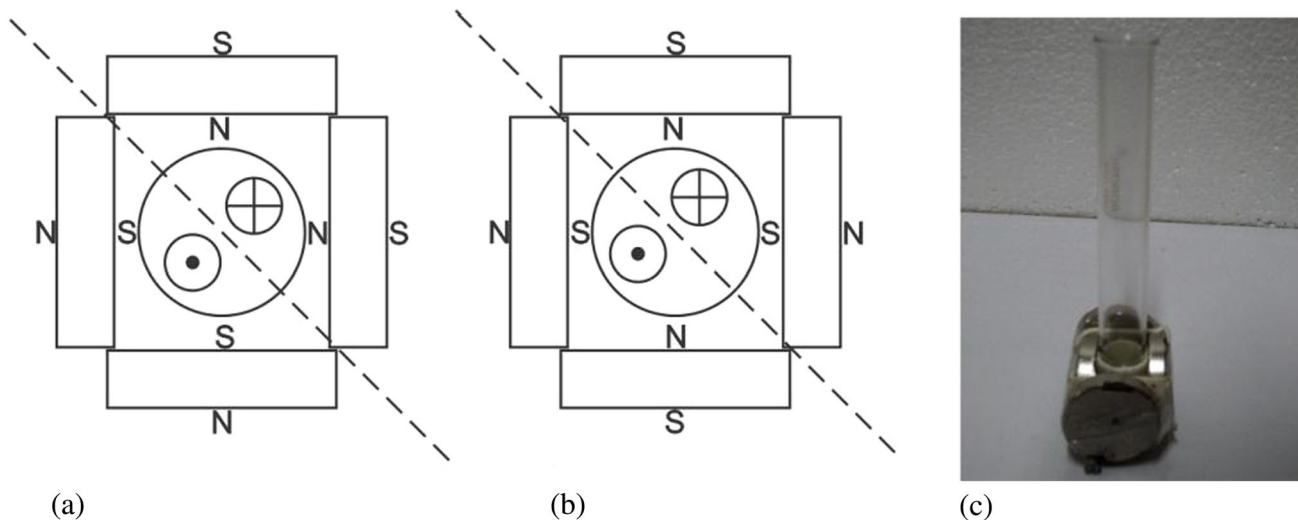


Figure 4. (a) Top view of the first configuration of four (quad) magnets attached using self-adhesive tapes around the test tube, (b) top view of the second configuration and (c) image of the set-up.

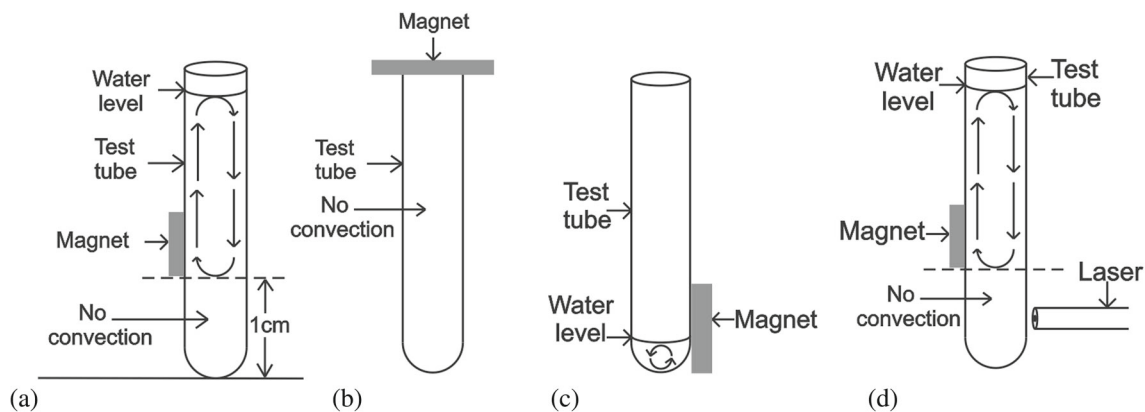


Figure 5. Side views of the set-up. (a) A magnet was placed 1 cm above the horizontal surface on which the test tube was standing, (b) the magnet was kept on the open end of the standing test tube filled with the suspension up to its brim, (c) a test tube filled with 0.2 ml of suspension was exposed to the magnet and (d) an experiment to observe whether the laser light imparts any heat energy into the test tube filled with the suspension.

the flow velocity initially was of very small magnitude, then increased to its maximum value and then again slowly decreased to zero. The total period of the convection from its onset to complete decay depends upon the volume of water taken. The magnitude of the velocities in the convection increased as the applied magnetic field was increased. This behaviour is consistent with the force equation (1). In the absence of a magnetic field, the suspension at the bottom of the test tube is in equilibrium due to the hydrostatic pressure of the suspension column over it. When it is exposed to the magnetic field, it acquires $(-\chi B^2/2\mu_0)$ amount of energy [14]. This perturbation in the suspension imparts kinetic energy to it. The motion thus obtained can be perceived as the onset of a dynamics to minimise its energy. In doing so, the suspension undergoes convective cycles. This

convection dies out after a certain time which depends upon the volume of suspension, the viscosity of the fluid, product of the field and its gradient.

The measurement of thermal gradients in the sample was done by measuring the difference in the resistance of two similar calibrated RTDs over 60 min interval. Since the resistance of any metallic conductor is an explicit function of its temperature, the resistance of both RTDs was directly considered for measuring the thermal gradients in the suspension.

Figure 1b shows the difference in the resistance values of the two RTDs. It gives an average value of 0.03Ω , which is the same as the minimum error in the calibration value of 0.03Ω . As the average values measured from the two graphs are the same, it can be concluded that there are no thermal gradients in the

5 ml volume sample within the accuracy of the measuring instrument. When the same 5 ml test tube in the water bath was exposed to the magnetic field from the disk magnet, the onset of convection was instantaneously seen as depicted in figure 2. It can be seen in figures 2b and 2c that the test tube is positioned off-centred on the disk magnet. This off-centring provides maximum field gradient to the suspension. It was easily observed that wherever the magnetic gradient was maximum, the convection flow was initiated but in the upward direction. In this case, the edge of the disk magnet has a maximum field gradient value that causes the onset of convection from the same edge and goes upward.

The imaging of the convective flows was done using readily available and simple instruments. The trajectories of pollen grains captured (supplementary information, figures S6 and S7) are no different from convection caused by the thermal gradients. The only major difference is that the convection, in this case, arises due to the force experienced by the water molecules in the space varying magnetic field. As the upward and downward flows pass each other at the centre of the test tube, it creates a region in which they mix. This region is called the 'interface region'. The mixing not only causes randomly oriented flows but also causes flows in a few specific directions. These directed flows were observed by looking at the trajectory of pollen grains (supplementary information, figure S8).

For studying the effect of different geometries of the static magnetic field gradients on convection, experiments described in figures 3 and 4 were performed. The test tube filled with the suspension was positioned in between disk magnets as shown in figure 3. The configuration of magnets that was used had two main field gradients (supplementary information, figures S9–S14). One field gradient is in the direction of the axis joining the faces of the magnets and the other one is in the direction perpendicular to this axis. The induced convection showed a unique orientation compared to the previously observed convective direction. This direction was perpendicular to the faces of the magnets as seen in figure 3b. In this case, the interface region is perpendicular to the magnet faces. Since the product of the field and its gradient (supplementary information, figure S13) is maximum in the direction perpendicular to the plane of the interface, convection was induced as depicted in figure 3b. Similarly, another arrangement containing four magnets (quad configuration), was assembled. As shown in figure 4, magnets were arranged in such a way that they covered the test tube from all sides (supplementary information, figure S15). In this case, the interface plane coincides with one of the geometrical diagonals of the square that was formed by the placement of the four magnets.

In all these observations, a particular point was noticed that the onset and behaviour of the convection were independent of the polarity of the magnet. This observation points us in the direction of diamagnetic-based gradient force driven convection in the fluid. Another set of experiments was carried out by placing the magnet at various positions near the test tube as shown in figures 5a–5d. As the formation of convection is due to the competing forces from magnetism and fluid hydrostatic pressure, the convection flows occur only up to the region where the physical diameter of the magnet extends. The entire column of the fluid was unperturbed below the dashed line, as shown in figure 5a. The convection covered the entire length of the column above the dashed line up to the maximum fluid level in the test tube. In the present phenomenon of the formation of convection in a static magnetic field gradient, if the magnetic force is kept parallel to the gravitational force, the convective flow does not occur. Figure 5b shows an arrangement where the magnet is kept on the open end of the test tube. The test tube was filled up to its brim where the fluid was in physical contact with the magnet. The system was observed continuously before and after the magnet was placed over it. The fluid did not show any convective flow irrespective of the time for which the magnet was placed over it. This in turn supports the result obtained in the experiment described in figure 5a.

To check whether the convection in the water is exclusively dependent on the magnetic field gradient, 0.2 ml of suspension was taken in a test tube and exposed to a magnetic field using a disk magnet, as shown in figure 5c. A very slow but gradual change in the position of the randomly moving single pollen grain was observed and recorded. Over the period, the pollen grain traced a circular path and eventually vanished from the view, indicating the presence of ordered convective flow. This is because the magnitude of the product of field and its gradient experienced by the 0.2 ml volume of the suspension is very small. Hence, correspondingly the force experienced by the suspension will also be small causing a low convective velocity in the system. It is shown in the time-evolved image of the pollen grain (supplementary information, figure S16) over the entire period for which the suspension was under the influence of the magnetic field.

Another experiment was done to find out whether the laser light imparts thermal energy to the suspension. Figure 5d shows the experimental arrangement. The convective flow was observed in a region confined between the dashed line and the open end of the test tube. There were no sign of any kind of ordered motion of the pollen grains in the column below the dashed line where the laser light source was irradiated for an hour. The co-existence of convective flows and stable fluid on

either side of the dashed line shows that the laser light source does not provide any cognizable thermal energy into the system.

The obtained results showed the dependence of convection on the magnetic field gradient and the geometrical configuration of magnets around the test tube containing suspension. Based on the results, a simple experiment (supplementary information, S17) was designed to check whether the suspension kept in Thiele's tube can be circulated in a closed loop.

As expected, the flow of suspension was observed over the same regions (A) and (B). It was found that suspension was flowing in from the opening at the bottom of the arm, then going through the regions (A) and (B) and then finally entering into the bigger tube through the opening at the top. As shown in figure S17b, the continuous flow in and out of the small arm and into the big tube showed the convective motion acquired by the fluid in Thiele's tube. Similar to the previous observations, the fluid flow ceases after some time even though the magnet is not removed.

In [4], the enhancement of the rate of dissolution of oxygen was attributed to the onset of magnetically induced convection in water due to susceptibility gradient formed in the fluid by the presence of pure oxygen at the surface of the fluid. They calculated the convective flow velocity using the assumption that there exists a magnetic susceptibility gradient in water. As per their experimental conditions, the concentration of oxygen at the surface of water is saturated. It decreases gradually towards the bottom. As oxygen is paramagnetic, it creates magnetic susceptibility gradient, whereas water is diamagnetic. They used an electrochemical oxygen sensor to measure the dissolved oxygen (DO) contained in the sample. Their reported values do not reflect that a thorough measurement was done at various levels in the fluid column to claim the existence of the susceptibility gradient. A simple comparison of the volume of the sample and the size of the probe would provide a roughly good estimate showing that under any normal experimental conditions, the probe will be unable to register any concentration gradients in the sample. The measured value was the average value of the concentration of the DO in the sample. Hence, it is unreasonable to assume that there is any cognizable concentration gradient of DO in the sample. This will in turn create a negligibly small magnetic susceptibility gradient that will not directly contribute to the onset of convection. To support this discussion, an experiment was carried out (Supplementary information, S18). As the magnetic field was applied as discussed earlier, the onset of convection currents was observed exactly similar to the convection observed in the fluid which was kept in the open air. The velocity of

the convection flow was measured in both the cases that came out to be of the same value as described in the initial experiments. Hence, the onset of convection cannot be attributed to the oxygen concentration gradient and thus the formed magnetic susceptibility gradient in the water sample. Another aspect that was skipped in their work was the unavailability of a result in which an inert gas atmosphere was used. In such a scenario, the results would have been as depicted in the present work.

In all the cases, the experiments were carried out at room temperature throughout the year. As the volume of the suspension was also small, based on the results, it is believed that thermal gradients, if any, are too small to set up a convection current. This is verified by the absence of ordered motion in the suspension column in the absence of magnetic fields. The exact cause for the occurrence of convection is still unclear but it can be attributed to Lenz's law for induced magnetism. The water molecule is diamagnetic, which implies that it has no net magnetic moment associated with it. Any external applied magnetic field induces a magnetic field in the molecule opposing the original field (Lenz's law). Thus under the action of gradient force, the resultant effect will move the diamagnetic molecule away from the high field region to the lower field region. As this motion of water molecules is additive, it further builds up to macroscopic instability in the fluid. To nullify this instability, the fluid undergoes rearrangement in the position of the molecules with respect to the externally applied field. An avalanche effect follows to form a convectional flow in the fluid. This way the system attains its state of equilibrium. The onset of convection, average maximum flow velocity and quenching are directly dependent on the viscosity of the diamagnetic fluid.

Based on the above discussions, the increased rate of oxygen dissolution [4] in water can now be said to be an effect which is caused by the onset of the convection currents which are induced due to the magnetic force acting on the water molecules when they are exposed to static non-uniform magnetic fields.

4. Conclusion

It was demonstrated experimentally that static magnetic field gradients can induce convection in water, a non-conducting diamagnetic fluid. Various magnetic field gradient configurations showed new dynamic orientations of convection in the test tube. Moreover, the cause of convection was established directly by the interaction of water with the magnetic field, as the experiments were carried out in ambient and controlled environments. It was also established that the laser source and the DO are not responsible for the onset of convection.

A model has been proposed for the formation and dissipation of the convection currents in the non-conducting diamagnetic fluid suspension. Further investigations are underway to quantify and model the observed effects. The presented experimental results are based on simple techniques. The results thus obtained prove the feasibility of observing such weak interactions between diamagnetic fluid and static magnetic field gradients successfully.

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