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Modified amplitude of boson star rotation by the cosmological constant and coupling term of the curvature of the Universe

BHARTI JARWAL and S SOMORENDRO SINGH*

Department of Physics and Astrophysics, University of Delhi, New Delhi, Delhi 110 007, India
*Corresponding author. E-mail: sssingh@physics.du.ac.in

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Abstract. We extend our earlier work of boson star (BS) rotation after incorporating the cosmological constant and coupling term of the curvature of the Universe with the field of boson star. Due to the cosmological constant and coupling term in the Lagrangian density of the boson star, the oscillation amplitudes of the boson star are modified for the entire study and the amplitudes are found to increase from the result obtained earlier. The introduction of cosmological constant and coupling of curvature prove the hypothetical assumption of the existence of the boson star.

Keywords. Compact stars; boson stars; quark-gluon plasma.

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1. Introduction

The concept of cosmological constant has been introduced many years ago and it has a rich history in the development of theoretical aspects of cosmology and astrophysics. In addition to this study, we need to understand the curvature of the Universe and its importance in the expansion of the Universe. The expansion of the Universe depends on its curvature. Many models had been proposed to study the cosmological constant and its effects on the expansion of the Universe. From these studies, it is concluded that the cosmological constant Λ has a very significant effect in the study of the evolution of the Universe. It is believed that the phase transition is occurred in the beginning of the evolution of the Universe [1]. There have been several attempts to provide a solution to the cosmological constant puzzle. Many astrophysical entities like compact objects (neutron stars (NS), boson star (BS) and black holes (BH)) are considered among the most exotic objects produced in the nature. They are formed in the core collapse of massive stars like supernova and, in many cases, their formations are associated with powerful astrophysical transients such as supernovae and γ-ray bursts. By studying the masses and oscillations of these objects, we can better understand the formations of these objects and their associated explosions. In the early Universe, the symmetry breaking scales were realised by larger gauge groups. As a result, the equivalence principle of quantum gravity was introduced as a fundamental symmetry in quantum gravity to reconcile the quantum mechanics and general relativity. Due to such principle, the quantum gravity is formulated in the quantum space–time–matter space with the local conformal symmetry.

From conformal theory, it is understood that the coupled scalar field is a basis for investigating the curved space quantum theory. Such conformal coupling term was introduced first by Callan *et al* [2] while investigating the divergences of the stress tensor. Since then, it has been used to investigate trace anomalies in curved space quantum theory [3] and it has been used as a matter source for the study of quantum cosmology [4]. Further, its generalisations have been used in the investigation of the curved space and its relevant properties [5].

The rotation of boson star in Newtonian approximation indirectly represents the existence and formation of these objects in our present Universe [6]. In order to give more evidence, many investigations are under process with a similar thought of gravity limit. The study tends to indicate the amplitudes of these boson star oscillations in the form of rotation in different potential backgrounds [7–10]. In this paper, we consider the Universe having Lagrangian density after coupling the curvature of the Universe with the field of boson star. Further, we investigate the effects in the solutions of ground state, first

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and second excited states due to the curvature coupling with the boson field on its rotation.

2. Newtonian treatment of boson stars with the coupling factor

It has been already reported in literature that BS is a hypothetical object made of boson particles which are represented by the complex scalar field coupled to gravity [11]. The action principle of such a system is discussed in detail under weak field approximation. The Lagrangian density of BS with the cosmological constant coupled to the field with the curvature of Universe is given by

$$£ = \frac{R-2\Lambda}{16\pi G} + g^{\mu\nu}\partial_\mu\Phi^*\partial_\nu\Phi - V(|\Phi|^2), \quad (1)$$

where $R = g^{\mu\nu}R_{\mu\nu}$ and $V(\Phi^*\Phi)$ is normally defined as $V(\Phi^*\Phi) = M^2\Phi^*\Phi$ which is represented as a mass term in the Lagrangian. This type of Lagrangian has been solved in the earlier works [2,12–15]. Again we introduce a term $\varepsilon R\Phi^*\Phi$ [3] in which the interaction of the Universe is coupled with the field of the BS. By introducing the term $\varepsilon R\Phi^*\Phi$, we like to see the changes produced in the amplitudes of BS rotation and the effects produced by the term in our earlier model of BS solutions [7,8]. So the new Lagrangian, after adding the interacting term coupled with the BS fields, is as follows:

$$£ = \frac{R-2\Lambda}{16\pi G} + g^{\mu\nu}\partial_\mu\Phi^*\partial_\nu\Phi - V(|\Phi|^2) + \varepsilon R\Phi^*\Phi. \quad (2)$$

ε is a small scaling parameter which is introduced to couple with the curvature of the Einstein term [3] and it was first introduced by Callan to soften the divergences produced by the stress tensor. For such actions, the field equation is modified and it is determined with the implication of invariance condition of Φ. The equation is

$$\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)(1-\varepsilon\Phi^*\Phi) + T_{\mu\nu} + \Lambda g_{\mu\nu} = 0. \quad (3)$$

The metric $g^{\mu\nu}$ is expanded as follows: $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$ with $|h^{\mu\nu} \ll 1|$ and $\eta^{\mu\nu} = \mathrm{diag}\left(1, -1, -r^2, -r^2\sin\theta^2\right)$. To solve the above equation for Φ from the above Lagrangian density we obtain the equation of motion (EOM) through the Euler Lagrange equation. The EOM of Φ is as follows:

$$\Box\Phi + M^2\Phi + 4\varepsilon\Lambda\Phi = 0. \quad (4)$$

Then another equation of motion through the Lagrangian density is given as

$$\Box h_{\mu\nu}(1-\varepsilon\phi^2) = -16\pi G S_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (5)$$

where $S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T$.

$$\begin{aligned} T_{\mu\nu} = {} & \Phi^*,_\nu \Phi,_\mu \\ & - \frac{1}{2}g_{\mu\nu}(\Phi^*,^\rho\Phi,_\rho + M^2\Phi^*\Phi) \\ & -\varepsilon\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\Phi^*\Phi \\ & + \varepsilon g_{\mu\nu}(\Phi^*\Box\Phi + \Phi\Box\Phi^*) \\ & - \varepsilon(\Phi^*\Phi);_{\mu\nu}. \end{aligned} \quad (6)$$

In order to satisfy the equation of motion for Φ, we assume a stationary solution, which is considered to be dependent on t and r only.

$$\Phi(\vec{r}, t) = \phi(r,\theta)e^{i\omega t}e^{im\varphi}. \quad (7)$$

Using the weak field approximation of general relativity [16], and knowing the only relevant component of $h_{\mu\nu}$, it is found that $h_{oo} = 2V(r,\theta)$, in which $V(r,\theta)$ is the Newtonian potential. In non-relativistic limit, the gravitational binding energy E per particle must be much smaller than M. However, the scalar field frequency can be written as $w = E + M$ with $|E| \ll M$. Considering all these arguments, by assuming $\phi(r,t)$ we obtain two separate equations depending upon the potential $V(r,\theta)$ and $\phi(r,t)$

$$\frac{-1}{2M}\vec{\nabla}^2\phi + E\phi = \frac{2\varepsilon\Lambda}{3}\phi \quad (8)$$

$$\vec{\nabla}^2 V = 4\pi G(\varepsilon M^2 + \varepsilon\Lambda - EM)\phi^2. \quad (9)$$

The gauge invariance of the complex scalar field implies the conservation of conserved current such as $j^\mu = i(\phi^*\partial^\mu\phi - \partial^\mu\phi^*\phi)$ with a conserved particle number: $N = 2M\int\phi^2r^2\sin(\theta)\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi$. The stationary solutions of the above equations with non-vanishing angular momentum can be obtained by usingthe associated Legendre function and orthogonality relation of $P_l^m(\theta)$. So the general solutions for the above equations are obtained as follows:

$$\phi(r,\theta) = \frac{1}{\sqrt{4\pi}}\sum_{l=m}^{\infty} R_l(r)P_l^m(\theta) \quad (10)$$

$$V(r,\theta) = \sum_{l=0}^{\infty} V_l(r)P_l(\theta). \quad (11)$$

The solution of $\phi(r,\theta)$ can be written as a coupled equation of radial and angular solutions. Similarly, $V(r,\theta)$ also can be a coupled equation of $V_l(r)$ and $P_l(\theta)$. The potential $V_l(r)$ can be taken as the normal Coulomb's potential considered earlier in our previous paper. So the most generalised form of equations for V and R using the associated Legendre polynomials are solved as follows:

$$V_{l_0}'' + \frac{2}{r} V_{l_0}' - \frac{l_0(l_0+1)}{r^2} V_{l_0} = G(2l_0+1)(\varepsilon M^2 + \varepsilon\Lambda - EM) \sum_{ll'} A_{ll'l_0} R_l R_{l'}, \quad (12)$$

$$\frac{1}{2M}\left(R_{l_0}'' + \frac{2}{r} R_{l_0}' - \frac{l_0(l_0+1)}{r^2} R_{l_0}\right) + E R_{l_0} = \frac{(2l_0+1)(l_0-m)!}{2(l_0+m)!} \sum_{l=m}^{\infty} \sum_{l'=m'}^{\infty} A_{ll'l_0} \frac{\varepsilon\Lambda}{2M} R_l, \quad (13)$$

where the prime denotes derivative with respect to r and

$$A_{ll'l_0} = \int \mathrm{d}x \, P_l^m(x) P_{l'}^m(x) P_{l_0}.$$

Now these second-order differential equations of radial and potential wave functions are found to be not trivial in solving with the existing scale. So to convert into a trivial expression, we re-scale the above parameters and make it easier to solve in the trivial way. The rescaling is done with the help of the earlier works in which the parameters are defined as follows. The typical value of ε is taken as $1/6$ and the cosmological parameter Λ is taken at around 0.02 as it is considered as a standard value in calculating the properties of Universe. Then the corresponding mass is found to be 30 GeV. Finally, the rescaled values of the parameters in terms of the old parameters are

$$\hat{r} = r\hat{N}M$$
$$\hat{V}(\hat{r},\theta) = \frac{V(r,\theta)}{\hat{N}^2},$$
$$\hat{R}(\hat{r}) = \frac{R(r)(2G)^{1/2}}{\hat{N}^2}, \quad \hat{E} = \frac{E}{M\hat{N}^2},$$
$$\hat{N} = GM^2N\frac{(2l+1)(l-m)!}{(l+m)!},$$
$$\hat{\Lambda} = \frac{\Lambda}{M^2\hat{N}},$$
$$\hat{\varepsilon} = \frac{\varepsilon}{\hat{N}}.$$

With the new scale parameters, the set of differential equations of the above equations of potential and radial wave functions for the ground and excited states are obtained with the re-scaled normalisation condition as

$$\int R_{l_0}^2(r) r^2 \mathrm{d}r = 1. \quad (14)$$

Case 1: Ground state

The ground-state solution is obtained when $l = 0$ and $m = 0$ in the Legendre polynomial. So the equations for potential and radial functions are given depending on the perturbation in the potential function.

$$\hat{V}_0'' + \frac{2}{\hat{r}}\hat{V}_0' = (\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_0^2,$$
$$\frac{1}{2}\left[\hat{R}_0'' + \frac{2}{\hat{r}}\hat{R}_0'\right] + \hat{R}_0\hat{E}_0 = \hat{\varepsilon}\hat{\Lambda}\hat{R}_0. \quad (15)$$

Case 2: First excited state

The first excited state is obtained when $l = 1$ and $m = 0, 1$. This indicates that there is a degeneracy state in the first excited state. For the degeneracy state $l = 1$ and $m = 0$, the equations for the potential and radial functions are obtained as

$$\hat{V}_0'' + \frac{2}{\hat{r}}\hat{V}_0' = \frac{1}{3}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_1^2$$
$$\hat{V}_2'' + \frac{2}{\hat{r}}\hat{V}_2' - \frac{6}{\hat{r}^2}\hat{V}_2 = \frac{2}{5}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_1^2$$
$$\frac{1}{2}\left[\hat{R}_1'' + \frac{2}{\hat{r}}\hat{R}_1' - \frac{2}{\hat{r}^2}\hat{R}_1\right] + \hat{R}_0\hat{E}_{10} = \hat{\varepsilon}\hat{\Lambda}\hat{R}_1. \quad (16)$$

Again, for the degeneracy state $l = 1$ and $m = 1$ the equations are obtained for R and potential as follows:

$$\hat{V}_0'' + \frac{2}{\hat{r}}\hat{V}_0' = \frac{2}{3}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_1^2$$
$$\hat{V}_2'' + \frac{2}{\hat{r}}\hat{V}_2' - \frac{6}{\hat{r}^2}\hat{V}_2 = \frac{-2}{3}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_1^2$$
$$\frac{1}{2}\left[\hat{R}_1'' + \frac{2}{\hat{r}}\hat{R}_1' - \frac{2}{\hat{r}^2}\hat{R}_1\right] + \hat{R}_0\hat{E}_{10} = \hat{\varepsilon}\hat{\Lambda}\hat{R}_1. \quad (17)$$

Case 3: Second excited state:

Second excited state occurs when $l = 2$ and $m = 0, 1, 2$. These different values of m show that there is degeneracy in this state. For degeneracy state $l = 2$ and $m = 0$ the equations are obtained for the potential and radial wave functions:

$$\hat{V}_0'' + \frac{2}{\hat{r}}\hat{V}_0' = \frac{1}{5}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\hat{V}_2'' + \frac{2}{\hat{r}}\hat{V}_2' - \frac{6}{\hat{r}^2}\hat{V}_2 - \frac{-6V(\hat{r})}{\hat{r}^2} = \frac{2}{7}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\hat{V}_4'' + \frac{2}{\hat{r}}\hat{V}_4' - \frac{20}{\hat{r}^2}\hat{V}_4 - \frac{20V(\hat{r})}{\hat{r}^2} = \frac{18}{35}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\frac{1}{2}\left[\hat{R}_2'' + \frac{2}{\hat{r}}\hat{R}_2' - \frac{6}{\hat{r}^2}\hat{R}_2\right] + \hat{R}_2\hat{E}_{20} = \hat{\varepsilon}\hat{\Lambda}\hat{R}_2. \quad (18)$$

Similarly, the equations can be obtained for the remaining degeneracy values m =1, 2. They can also be expressed as differential equations in terms of potential and radial functions

$$\hat{V}_0'' + \frac{2}{\hat{r}}\hat{V}_0' = \frac{1}{5}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\hat{V}_2'' + \frac{2}{\hat{r}}\hat{V}_2' - \frac{6}{\hat{r}^2}\hat{V}_2 - \frac{-6V(\hat{r})}{\hat{r}^2} = \frac{6}{7}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\hat{V}_4'' + \frac{2}{\hat{r}}\hat{V}_4' - \frac{20}{\hat{r}^2}\hat{V}_4 - \frac{20V(\hat{r})}{\hat{r}^2}$$
$$= \frac{-72}{35}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\frac{1}{2}\left[\hat{R}_2'' + \frac{2}{\hat{r}}\hat{R}_2{}' - \frac{6}{\hat{r}^2}\hat{R}_2\right] + \hat{R}_2\hat{E}_{21} = \hat{R}_2\hat{\varepsilon}\hat{\Lambda}. \tag{19}$$

For $l = 2$ and $m = 2$

$$\hat{V}_0'' + \frac{2}{\hat{r}}\hat{V}_0' = \frac{1}{5}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\hat{V}_2'' + \frac{2}{\hat{r}}\hat{V}_2' - \frac{6}{\hat{r}^2}\hat{V}_2 - \frac{-6V(\hat{r})}{\hat{r}^2} = \frac{6}{7}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\hat{V}_4'' + \frac{2}{\hat{r}}\hat{V}_4' - \frac{20}{\hat{r}^2}\hat{V}_4 - \frac{20V(\hat{r})}{\hat{r}^2}$$
$$= \frac{-72}{35}(\hat{\varepsilon} + \hat{\varepsilon}\hat{\Lambda} - \hat{E})\hat{R}_2^2$$
$$\frac{1}{2}\left[\hat{R}_2'' + \frac{2}{\hat{r}}\hat{R}_2' - \frac{6}{\hat{r}^2}\hat{R}_2\right] + \hat{R}_2\hat{E}_{21} = \hat{R}_2\hat{\varepsilon}\hat{\Lambda}. \tag{20}$$

3. Result and discussion

The analytical calculations and the numerical solutions are represented in the figures. In figure 1, we have plotted the variation of Newtonian potential with r. Figure 1a indicates the variation of potential for the ground state. Similarly, figures 1b and 1c show the amplitude change of the first and second excited states. From the figure, it is concluded that the potential variation with the size of BS shows very much similar pattern in their behaviour with the radial wave function.The radial wave of BS has significant impact on its behaviour. Their amplitude differences are affected by the size of BS. It is clearly visible from the plots of the ground, first and second excited states. So, Newtonian potential described at the ground, first and second excited states with the size show the amplitude difference among these states.

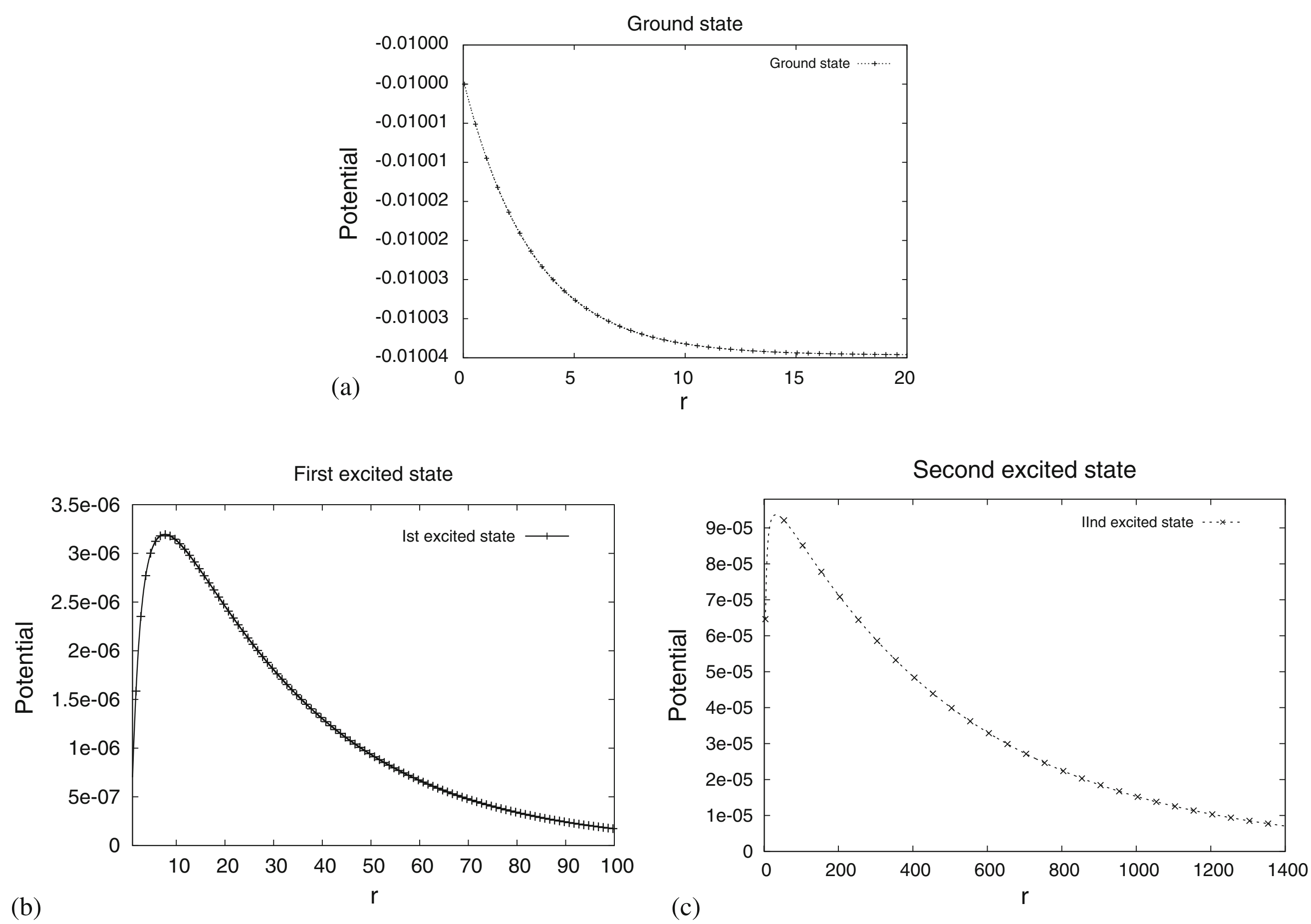

Figure 1. Variation of potential with r for the ground, first and second excited states.

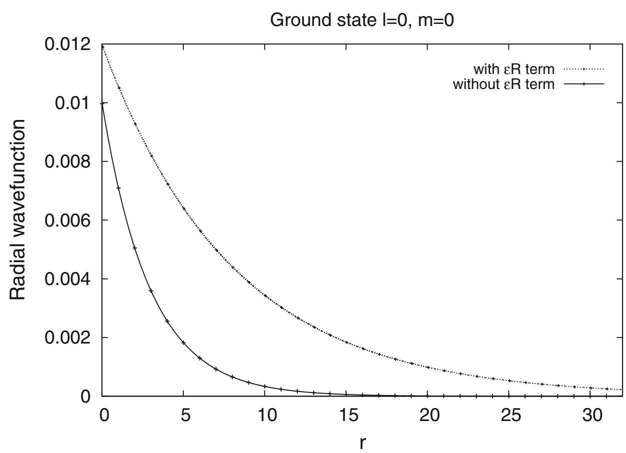

Figure 2. Effect of curvature coupling on the radial wave function *R* of the ground state.

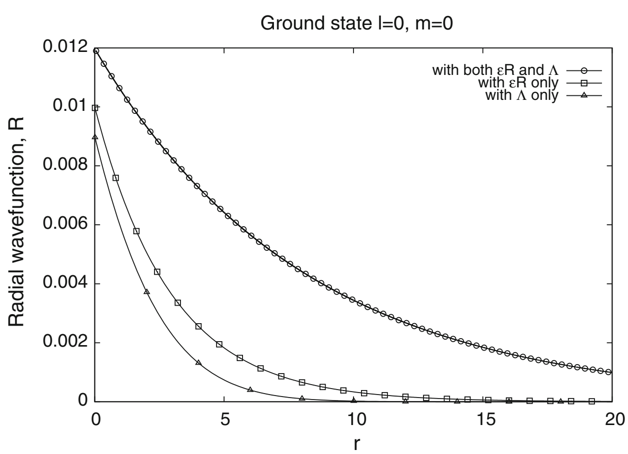

Figure 3. Effect of curvature coupling and cosmological constant on the radial wave function *R* of the ground state.

Figure 2 shows the radial wave function of the ground state. The plot with large amplitude in radial wave function with the curvature is coupled to boson field, say $R|\phi|^2$ term, in Lagrangian density whereas the other plot with lower amplitude is shown for the uncoupled system in the Lagrangian density. The inclusion of coupling shows very strong evidence of the existence of BS in comparison to the uncoupled BS with the curvature.

Now in figure 3 we have compared different amplitudes of the three processes. Maximum amplitude is produced by the introduction of coupling term and cosmological constant. Then, lesser amplitude is shown when only the coupling term is considered. So it is plotted without the cosmological constant, considering only the coupling term. Again, we have shown the lowest amplitude of BS when the system is considered with cosmological constant only. It means the system is not considered with the interaction term of the Universe.

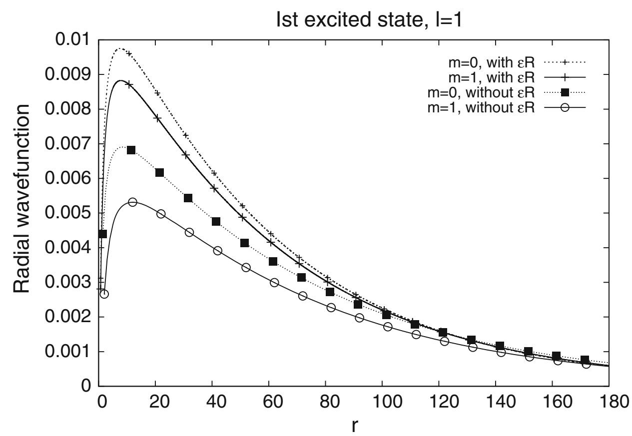

Figure 4. Effect of curvature coupling on the radial wave function *R* of the first excited state.

Figure 4 shows the first excited state of the radial wave functions of two degeneracy factors $m = 0$ and 1 after the coupled and uncoupled curvature of Universe to boson field in Lagrangian density. Even though they have the same energy, the radial wave functions have different amplitudes. The degeneracy $m = 0$ and $l = 1$ has higher amplitude than degeneracy $m = 1$ and $l = 1$. This implies that the existence of BS is clearly proved by the oscillation amplitude produced by the coupling factor. However, compared to degeneracy factors, the presence of BS is enhanced by the degeneracy of $l = 1$ and $m = 0$. The increase in amplitude with the coupling is around 20.35% more than the amplitude without coupling. The increment is large during the initial evolution of BS for both cases of coupling of curvature with boson field and without boson field. The amplitude subsequently decreases with the larger size of BS. It indicates that oscillation of radial wave function *R* increases in amplitude after coupling the field with curvature. The amplitude is found to be large during the initial state of BS creation and subsequently low with the increase of boson size. So the formation and existence of BS is clearly observed in the intermediate size of its existence. It implies that the Universe has obtained a large number of BS of intermediate size of its creation.

In figure 5a, we compare the radial oscillations of different degeneracy factors of the boson field coupling with curvature. The amplitude is large when $m = 0$ and $l = 2$ and decreases when $m = 1$ and 2. In figure 5b, we see a plot of radial wave oscillation without curvature coupling. In this figure also, the amplitude is larger with $m = 0$ than with $m = 1$ and 2. These plots are for the second excited state of BS having the orbital number $l = 2$ and $m = 0$, 1 and 2. Now, it is shown that the amplitude of oscillation with curvature coupling is more than the amplitude of radial wave function without coupling. There is so much difference in the amplitude of

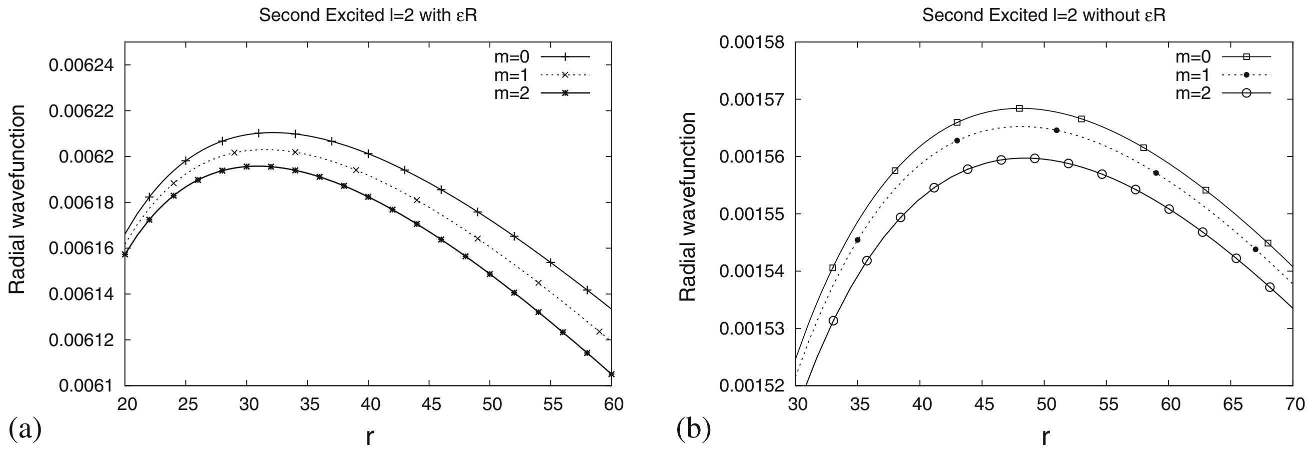

Figure 5. Radial wave function *R* of the second excited state with and without curvature coupling for $l = 2$.

BS with and without coupling. So the inclusion of the coupling has imparted more significant effects on the amplitude of BS rotation and it can give more information about the existence of these complex field particles.

However, as explained already, the degeneracy obtained provides a clear difference in the magnitude of amplitude of oscillation waves. With the increase in the value of *m*, there is a decrease in the amplitude. Similar pattern is also observed in the case of uncoupled field with curvature. However, the amplitude difference with respect to the size of BS, is clearly visible at large size in comparison with small BS. The amplitude differences among the degeneracy are very small in smaller BS whereas it is somewhat large in large BS.

From these arguments, we can conclude that there are some observable particles in the expansion of Universe and this expansion of Universe indicates the presence of a large number of bayronic and hadronic matter according to the scale of temperature. In the formation of BS and the hypothetical condensation of bosonic particles, objects like BS are formed. However, it is also observed from these issues occurred in the classical picture that the oscillations of the larger objects are less than the smaller objects subject to the application of same applied force. They have different oscillation modes with either the same or different amplitudes.

From these analytical view and numerical solutions, we can conclude that the amplitude of BS rotation after incorporating the curvature of the Universe to the field as an interaction term, enhances the formation and the hypothetical presence of BS.

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