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A mentor-initiated joint remote state preparation scheme for qubits

BINAYAK S CHOUDHURY and SOUMEN SAMANTA*

Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, B. Garden, Shibpur, Howrah 711 103, India
*Corresponding author. E-mail: s.samanta.math@gmail.com

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Abstract. In this paper we design a new joint remote state preparation (JRSP) scheme for a single qubit by introducing a mentor whose role is to initiate and fix the protocol after which it is for the rest of the parties to follow some prescribed instructions. There is no initial entangled connection between the senders and the receiver. The scheme has eight branchings depending on an act of measurement by the mentor. The protocol has advantages in terms of secrecy of the information to be transmitted.

Keywords. Quantum entanglement; Bell states; measurement; mentor; unitary operators.

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1. Introduction

The science of quantum communication originated through the work of Bennett *et al* [1] in 1993 in which an arbitrary single qubit quantum state was transferred from one location to the location of a distant party using a pre-sharing maximally entangled state as quantum resource with the assistance of some classical communication. This communication process is known as quantum teleportation. Following the work of Bennett *et al* [1], a large number of teleportation schemes were designed for the transfer of various types of quantum states through appropriate entangled quantum channels between the communicating parties with the support of classical communication. Some instances of these protocols can be seen in refs [2–11]. The central feature of the teleportation schemes is that the state to be teleported is arbitrary at least from a class of prescribed states.

There is another class of protocols known as remote state preparation protocols where the central feature is that the state to be prepared is known, that is, the sender knows all the classical information about the original quantum state. These protocols were first introduced by Pati [12], following which several remote state preparation protocols have been published [13–18].

Further, for reasons of security and secrecy, it may not be desirable that a single party should be holding all possible information of the state to be sent. Also it may be the case that the complete information of the state to be sent is not available with a single party for some technical reasons, but is distributed between two parties. In this situation, for the purpose of transferring the state to a distant party, a new type of protocol known as joint remote state preparation (abbreviated as JRSP) protocol was introduced by Xia *et al* [19]. Several other such protocols are advanced subsequently to address the issue of distantly preparing known quantum states of various kinds [20–30]. Other similar works are noted in [31,32]. Some of these are illustrated now.

Yang and Xia [22] in 2012 proposed two schemes to realise the joint remote state preparation scheme of a general three-qubit state by using non-maximally GHZ states as pre-shared entangled channels. In 2017, Wang and Mo [28] created a bidirectional controlled joint remote state preparation protocol by using a seven-qubit entangled state as the quantum channel. Recently, Choudhury and Samanta [30] presented a perfect joint remote state preparation protocol which is applicable to six-qubit cluster states with the help of three quantum channels constituted with eight qubits.

In this paper we introduce a mentor in a joint remote state preparation protocol. Initially, there is no quantum entanglement sharing between the senders and the receiver. On the contrary, the mentor is connected through entangled Bell states to the senders and the receiver. The mentor initiates the protocol by an act of

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measurement at his end and classically communicates in an appropriate manner with all the other three parties. The protocol thereby is divided into eight branches, one of which has to be followed by the rest of the parties. This depends on the obtained result of the mentor in his measurement. The mentor does not have any role in the subsequent steps. We discuss the advantage of the scheme and a comparison of the role of a mentor with a controller in the last section of the paper.

2. Mentor-initiated joint remote state preparation scheme for a single qubit

Let us consider the following situation. Suppose there are two parties namely, Alice and Bob, who want to create a single qubit state at the receiver Candy's end. The three parties are situated at three spatially separated locations and also they do not enjoy any pre-sharing quantum entangled channel. In these circumstances we introduce a fourth party David as a mentor where each of the parties Alice, Bob and Candy is separately entangled with the mentor. The general single qubit state which is intended for preparation in Candy's laboratory is of the form

$$|\psi\rangle = a|0\rangle + b|1\rangle,$$

where a, b are complex numbers. But the above state is not known to a single sender, that is, the information about the complex quantities a and b are not known to a single party but is distributed amongst the two senders. To describe the situation, we write the qubit in the following form:

$$|\psi\rangle = \alpha e^{i\theta_1}|0\rangle + \beta e^{i\theta_2}|1\rangle, \tag{1}$$

where the coefficients α, β, θ_1 and θ_2 are real, $\{\theta_1, \theta_2\} \in (0, 2\pi)$ and meet the normalisation condition $|\alpha|^2 + |\beta|^2 = 1$. Both the senders know the information about the intended state partially. In particular, Alice knows the information α, β while Bob knows θ_1, θ_2 about the quantum state (1), but Candy does not have any kind of information about the state which the senders want to prepare at the receiver's laboratory.

Three quantum channels are there which are entangled Bell states shared by (David, Alice), (David, Bob) and (David, Candy) given respectively by

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{d_1a_1}$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{d_2b_1}$$

and

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{d_3c_1},$$

where the particles d_1, d_2, d_3 are in the possession of the mentor David and the particles a_1, b_1 and c_1 are in the possession of Alice, Bob and Candy respectively.

The combined state of these channels is given by

$$\begin{aligned}|\Phi\rangle &= |\phi_1\rangle_{d_1a_1} \otimes |\phi_2\rangle_{d_2b_1} \otimes |\phi_3\rangle_{d_3c_1}\\ &= \frac{1}{2\sqrt{2}}[|000000\rangle + |001100\rangle + |110000\rangle\\ &\quad + |111100\rangle + |000011\rangle\\ &\quad + |001111\rangle + |110011\rangle + |111111\rangle]_{d_1a_1d_2b_1d_3c_1}.\end{aligned} \tag{2}$$

For our purpose, we rewrite eq. (2) as

$$\begin{aligned}|\Phi\rangle &= \frac{1}{2\sqrt{2}}[|000000\rangle + |010010\rangle\\ &\quad + |100100\rangle + |110110\rangle + |001001\rangle\\ &\quad + |011011\rangle + |101101\rangle + |111111\rangle]_{d_1d_2d_3a_1b_1c_1}.\end{aligned} \tag{3}$$

The JRSP scheme is initiated and fixed by the mentor David by measuring three particles which are in his possession followed by some classical communications. Without his actions the protocol cannot be started. Also that is the end of the mentor's role.

Specifically, David executes a measurement on his three particles d_1, d_2, d_3 in the measuring basis given by

$$\begin{aligned}|\xi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle)_{d_1d_2d_3},\\ |\zeta^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|001\rangle \pm |110\rangle)_{d_1d_2d_3}\\ |\mu^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|010\rangle \pm |101\rangle)_{d_1d_2d_3},\\ |\nu^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|011\rangle \pm |100\rangle)_{d_1d_2d_3}.\end{aligned} \tag{4}$$

Equation (3) can be rewritten as

$$\begin{aligned}|\Phi\rangle &= \frac{1}{4}[|\xi^{+}\rangle \otimes (|000\rangle + |111\rangle)_{a_1b_1c_1} + |\xi^{-}\rangle\\ &\quad \otimes (|000\rangle - |111\rangle)_{a_1b_1c_1}\\ &\quad + |\zeta^{+}\rangle \otimes (|001\rangle + |110\rangle)_{a_1b_1c_1} + |\zeta^{-}\rangle\\ &\quad \otimes (|001\rangle - |110\rangle)_{a_1b_1c_1}\\ &\quad + |\mu^{+}\rangle \otimes (010\rangle + |101\rangle)_{a_1b_1c_1} + |\mu^{-}\rangle\\ &\quad \otimes (010\rangle - |101\rangle)_{a_1b_1c_1}\\ &\quad + |\nu^{+}\rangle \otimes (|011\rangle + |100\rangle)_{a_1b_1c_1} + |\nu^{-}\rangle\\ &\quad \otimes (|011\rangle - |100\rangle)_{a_1b_1c_1}].\end{aligned} \tag{5}$$

After completing the measurement in basis (4), David sends his outcomes over classical channels to Bob's laboratory as well as to Candy's laboratory. At the same time he sends a message through a 1-bit classical channel to the sender Alice indicating that he has completed his job, thereby instructs Alice to act.

After receiving David's message, Alice chooses two orthogonal vectors $|A^i\rangle (i = 1, 2)$ given by

$$|A^1\rangle_{a_1} = (\alpha|0\rangle + \beta|1\rangle)_{a_1}$$
$$|A^2\rangle_{a_1} = (\beta|0\rangle - \alpha|1\rangle)_{a_1}, \quad (6)$$

as the basis for her qubit a_1 and performs a measurement on her qubit independently in the above measurement basis. Here the choice of basis (6) is independent of the mentor's measurement result. It is possible for Alice to construct basis (6) because α and β are known to her.

After that, Alice sends her measurement result to Bob and Candy separately through two classical channels. The mentor David's outcome may belong to one of the two groups, namely,

$$P = \{|\xi^+\rangle, |\xi^-\rangle, |\zeta^+\rangle, |\zeta^-\rangle\}$$

and

$$Q = \{|\mu^+\rangle, |\mu^-\rangle, |\nu^+\rangle, |\nu^-\rangle\}.$$

In the rest of the protocol, the action of Bob depends on the belongingness of the mentor's result to the two separate groups P and Q. Precisely, Bob makes a measurement on his qubit in the basis

$$|B_1^1\rangle_{b_1} = (e^{-i\theta_1}|0\rangle + e^{-i\theta_2}|1\rangle)_{b_1}$$
$$|B_2^1\rangle_{b_1} = (e^{-i\theta_1}|0\rangle - e^{-i\theta_2}|1\rangle)_{b_1}, \quad (7)$$

if Alice's outcome is $|A^1\rangle_{a_1}$ and the mentor's result belongs to the set P, and Bob performs a measurement in the basis

$$|B_1^2\rangle_{b_1} = (e^{-i\theta_2}|0\rangle + e^{-i\theta_1}|1\rangle)_{b_1}$$
$$|B_2^2\rangle_{b_1} = (e^{-i\theta_2}|0\rangle - e^{-i\theta_1}|1\rangle)_{b_1}, \quad (8)$$

if Alice's outcome is $|A^2\rangle_{a_1}$ and the mentor's result belongs to the set P, while Bob performs a measurement in the basis $\{|B_1^2\rangle, |B_2^2\rangle\}$ given by (8) if Alice's outcome is $|A^1\rangle_{a_1}$ and the mentor's result belongs to the set Q and Bob makes measurement in the basis $|B_1^1\rangle, |B_2^1\rangle$ given by (7) if Alice's outcome is $|A^2\rangle_{a_1}$ and the mentor's result belongs to the set Q.

After that, Bob sends this measurement result to Candy over a classical channel. Then there is a further splitting of the protocol into eight branches as follows: Candy, after considering the results of Alice, Bob and David, performs an appropriate unitary operation on his qubit to prepare the desired state at his end. The appropriate unitary operations for different measurement outcomes of Alice, Bob and David are described in tables 1–8 which also give eight separate branchings of the protocol. The JRSP scheme is thereby completed.

Now we illustrate the protocol following the branching described in table 3: Suppose mentor David's measurement result is $|\zeta^+\rangle$ and he sends it to the sender Bob and receiver Candy. Then from eq. (5), the state belonging to Alice, Bob and Candy collapses into the state

$$\frac{1}{4}(|001\rangle + |110\rangle)_{a_1b_1c_1}. \quad (9)$$

With basis (6), entangled state (9) can be written as

$$|A^1\rangle_{a_1} \otimes \frac{1}{4}(\alpha|01\rangle + \beta|10\rangle)_{b_1c_1} + |A^2\rangle_{a_1} \otimes \frac{1}{4}(\beta|01\rangle - \alpha|10\rangle)_{b_1c_1}.$$

Now Alice makes a measurement on her single qubit in the basis $\{|A^1\rangle, |A^2\rangle\}$ given in (6). After that, she transmits her outcome results to the other sender Bob and the intended receiver Candy.

Suppose Alice's outcome is $|A^1\rangle_{a_1}$, then the state of particles collapses into the state

$$\frac{1}{4}(\alpha|01\rangle + \beta|10\rangle)_{b_1c_1}. \quad (10)$$

Now, according to the protocol, Bob performs measurement on his single qubit in the above basis $\{|B_1^1\rangle, |B_2^1\rangle\}$. Then, state (10) can be expressed as

$$|B_1^1\rangle_{b_1} \otimes \frac{1}{4}(\alpha e^{i\theta_1}|1\rangle + \beta e^{i\theta_2}|0\rangle)_{c_1} + |B_2^1\rangle_{b_1} \otimes \frac{1}{4}(\alpha e^{i\theta_1}|1\rangle - \beta e^{i\theta_2}|0\rangle)_{c_1}.$$

After that Bob transmits his outcome to the desired receiver Candy through a classical communication. Suppose Bob's outcome is $|B_1^1\rangle$. Then Candy applies the appropriate unitary operation $(\sigma_x)_{c_1}$ to convert his state into the state intended for preparation (see table 3). If Bob's measurement result is $|B_2^1\rangle$, then (as per table 3) Candy makes the appropriate unitary operation $(\sigma_z\sigma_x)_{c_1}$ to convert into the original state that the senders want to create at the Candy's laboratory jointly. That is the end of the protocol.

3. Discussion and conclusion

From tables 1–8, it is clear that the protocol depends on the initial measurement outcome of the mentor. For example, in row 2 of tables 5 and 6, the outcomes of Alice and Bob are the same, but the unitary operations to be executed by Candy are different, being dependent

Table 1. The mentor's measurement result is $|\xi^+\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|0\rangle + \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(I)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|0\rangle - \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(\sigma_z)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|0\rangle - \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(\sigma_x\sigma_z)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|0\rangle + \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(\sigma_x)_{c_1}$

Table 2. The mentor's measurement result is $|\xi^-\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|0\rangle - \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(\sigma_z)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|0\rangle + \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(I)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|0\rangle + \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(\sigma_x)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|0\rangle - \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(\sigma_x\sigma_z)_{c_1}$

Table 3. The mentor's measurement result is $|\zeta^+\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|1\rangle + \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(\sigma_x)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|1\rangle - \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(\sigma_z\sigma_x)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|1\rangle - \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(-\sigma_z)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|1\rangle + \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(I)_{c_1}$

Table 4. The mentor's measurement result is $|\zeta^-\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|1\rangle - \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(\sigma_z\sigma_x)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|1\rangle + \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(\sigma_x)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|1\rangle + \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(I)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|1\rangle - \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(-\sigma_z)_{c_1}$

Table 5. The mentor's measurement result is $|\mu^+\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|0\rangle + \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(I)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(-\alpha e^{i\theta_1}\|0\rangle + \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(-\sigma_z)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|0\rangle - \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(\sigma_x\sigma_z)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(-\beta e^{i\theta_2}\|0\rangle - \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(-\sigma_x)_{c_1}$

on the outcome of the mentor which are $|\mu^+\rangle$ and $|\mu^-\rangle$ respectively. Thus, Candy needs to know the measurement result of the mentor, that is, the measurement result of the mentor determines the rest of the protocol. This shows that the eight branchings of the JRSP scheme are not reducible to any less number of such branchings.

It is worthwhile to indicate the difference between a mentor and a controller of a quantum communication

Table 6. The mentor's measurement result is $|\mu^-\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|0\rangle - \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(\sigma_z)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(-\alpha e^{i\theta_1}\|0\rangle - \beta e^{i\theta_2}\|1\rangle)_{c_1}$	$(-I)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|0\rangle + \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(\sigma_x)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(-\beta e^{i\theta_2}\|0\rangle + \alpha e^{i\theta_1}\|1\rangle)_{c_1}$	$(\sigma_z\sigma_x)_{c_1}$

Table 7. The mentor's measurement result is $|\nu^+\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|1\rangle + \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(\sigma_x)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(-\alpha e^{i\theta_1}\|1\rangle + \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(\sigma_x\sigma_z)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|1\rangle - \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(-\sigma_z)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(-\beta e^{i\theta_2}\|1\rangle - \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(-I)_{c_1}$

Table 8. The mentor's measurement result is $|\nu^-\rangle$.

Alice's measurement outcomes	Bob's measurement outcomes	Collapsed state at Candy's lab	Unitary operators applied by Candy
$\|A^1\rangle_{a_1}$	$\|B_1^2\rangle_{b_1}$	$\frac{1}{4}(\alpha e^{i\theta_1}\|1\rangle - \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(\sigma_z\sigma_x)_{c_1}$
$\|A^1\rangle_{a_1}$	$\|B_2^2\rangle_{b_1}$	$\frac{1}{4}(-\alpha e^{i\theta_1}\|1\rangle - \beta e^{i\theta_2}\|0\rangle)_{c_1}$	$(-\sigma_x)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_1^1\rangle_{b_1}$	$\frac{1}{4}(\beta e^{i\theta_2}\|1\rangle + \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(I)_{c_1}$
$\|A^2\rangle_{a_1}$	$\|B_2^1\rangle_{b_1}$	$\frac{1}{4}(-\beta e^{i\theta_2}\|1\rangle + \alpha e^{i\theta_1}\|0\rangle)_{c_1}$	$(\sigma_z)_{c_1}$

protocol. In many papers, there is a controller without whose action the protocol cannot be completed. The controller acts in the middle of the protocol and allows the rest of the protocol to be executed. The controller does not communicate with the sender except in special cases of communications like bidirectional teleportation, etc. But the mentor is characteristically different. He, through his action, not only initiates but also fixes the protocol in the sense that the actions to be executed in the protocol is dependent on the act of the mentor. This finishes the role of the mentor. After that, the communicating parties are left to themselves for completing the final task of distant state preparation by following the prescribed steps. The classical communication cost involved in the protocol is 10 cbits which is the total number of classical bits required for the classical communication involved in the protocol. The cost of entanglement is 3 ebits because we use three maximally entangled Bell states.

JRSP schemes are more secure than RSP protocols because the information is divided and is not totally in the hands of a single party. By introducing the mentor in the protocol which divides the protocol into eight different branches, we make the final action at the end of the receiver dependent on the mentor in addition to the two senders Alice and Bob. It is remarkable that the mentor does not have any information of the state to be prepared. Further, the protocol cannot be executed by Alice and Bob unless the mentor acts. These factors enhance the secrecy of the protocol for which JRSP schemes are meant. We only deal with this type JRSP scheme for the case of a single qubit. It remains to be seen whether similar schemes can be designed for entangled states as well.

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