



Equivalence of light-front and covariant quantum electrodynamics at one-loop level and the form of the gauge boson propagator

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Abstract. We consider the three fundamental one-loop Feynman diagrams of QED, viz. vertex correction, fermion self-energy and vacuum polarisation in the light-front gauge and discuss the equivalence of their standard covariant expressions with the light-front expressions obtained using light-cone time-ordered Hamiltonian perturbation theory. Although this issue has been considered by us and others previously, our emphasis in this article is on addressing the ambiguity regarding the correct form of the gauge boson propagator to be used in the light-front gauge. We generalise our earlier results and show, using an alternative method called the asymptotic method, how integrating over the light-front energy consistently in the covariant expression of each of the three one-loop corrections leads to the propagating as well as the instantaneous diagrams of the light-front theory. In doing so, we re-establish the necessity of using the correct form of gauge boson propagator.

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1. Introduction

In the recent past, the issue of equivalence of the covariant theory and light-front time-ordered Hamiltonian perturbation theory (LFTOPT) has attracted a lot of attention [1–6]. Issue of equivalence in theories involving scalars and spin- $\frac{1}{2}$ particles has been discussed in ref. [2], whereas refs [3–5] deal with equivalence in Yukawa theory. Paston *et al* [6] have considered the issue of equivalence in QED in $(1+1)$ dimensions. The equivalence of light-front QED (LFQED) in light-front (LF) gauge and conventional QED in Coulomb gauge has been addressed in ref. [7] within the framework of Feynman–Dyson–Schwinger theory.

The recent interest in this topic is related to the issue of renormalisation of LF theories [8,9]. In light-front calculations, one uses Hamiltonian perturbation theory and starting with the LF Hamiltonian P^- and, using the Heitler method of old fashioned time-ordered perturbation theory, arrives at the expressions of LF field theory. Mustaki *et al* have obtained the LF expressions for one-loop graphs of LFQED using this method [8] and the same method has been used by Zhang and Harindranath [10] for LFQCD. An alternative method to arrive at LF expressions would be to integrate over

the light-front energy k^- in covariant expressions. Paston *et al* [11] have considered the issue of equivalence between covariant QCD and light-front QCD at the Green's function level and have shown that equivalence between the two theories can be achieved by adding non-conventional counterterms to the LF Hamiltonian. Paston *et al* [11] mentioned that the LF Hamiltonian perturbation theory can be obtained from the LFLagrangian perturbation theory by first integrating over k^- and then over other components. In the present work, we consider the issue of equivalence at the Feynman diagram level in QED and show that the light-front expressions of the one-loop diagrams of LFQED, derived using LF Hamiltonian perturbation theory [8,11], can be obtained by performing k^- -integrations in the covariant expressions by carefully taking into account the contribution of end-point singularities.

Equivalence of covariant and LFQED, at one-loop level, was discussed in detail by one of us in refs [12,13], where it was shown that the one-loop LF expressions can be obtained by performing k^- -integration in the corresponding covariant expressions of equal-time theory. Recently, the issue was revisited in ref. [14], where Mantovani *et al* raised certain issues in our first work [12] and also pointed out correctly that our calculation for

one-loop vertex correction was only for the ‘+’ component of Λ^μ .

An important issue in these proofs of equivalence is that of the form of the gauge boson propagator in light-front gauge, which has been a topic of keen interest in [14–20]. We consider this issue too in our present work. One-loop renormalisation of LFQED in LF gauge was discussed extensively in ref. [8] using gauge boson propagator of the form [21]

$$d_{\alpha\beta}(k) = -g_{\alpha\beta} + \frac{\delta_{\alpha+k\beta} + \delta_{\beta+k\alpha}}{k^+} \quad (1)$$

which we shall refer to as the ‘two-term propagator’ in this work. A method of arriving at this gauge propagator, without using the Cauchy principal value prescription to deal with the pole in the propagator, was developed using the gauge choice $A_+ = 0$ in ref. [22] for QED and in ref. [23] for Yang–Mills theory. Srivastava and Brodsky, while discussing the LF quantisation of Hamiltonian QCD in detail, constructed S-matrix expansion in LF time-ordered products [24]. They showed that the free field gauge boson propagator is transverse with respect to both its 4-momentum and the gauge condition, and should have the form

$$d'_{\alpha\beta}(k) = -g_{\alpha\beta} + \frac{\delta_{\alpha+k\beta} + \delta_{\beta+k\alpha}}{k^+} - \frac{\delta_{\alpha+}\delta_{\beta+}k^2}{(k^+)^2}. \quad (2)$$

This form has been used in refs [7,15–17,25,26] and we shall refer to this doubly transverse gauge boson propagator as the ‘three-term propagator’ in this work. Using a causal approach, it was shown in ref. [27] that the three-term propagator can be arrived at without making use of any specific prescription to handle the poles. Vacuum polarisation calculation was performed using this approach in ref. [28].

Suzuki and Sales obtained, at the classical level, the gauge conditions that can lead to the three-term gauge boson propagator [17,18]. The third term in this propagator is traditionally dropped on the grounds that it is exactly cancelled by the ‘instantaneous’ term in the LF interaction Hamiltonian [18] and it is argued that this term is unphysical and does not propagate any information. However, the physical significance of this term has subsequently been stressed [17]. It was shown in ref. [17] using the method of Lagrange’s multiplier consistently that the correct form of the gauge boson propagator necessarily has the third, contact term. The importance of this term in renormalisation was also stressed by these authors. The equivalence of the manifestly covariant photon propagator to the sum of contributions from the transverse and longitudinal polarisation of the virtual photon has been explicitly shown in ref. [19].

The issue of which form of photon propagator should be used in the proof of equivalence has been addressed by us [12,13] as well as by Mantovani *et al* [14]. It was shown in ref. [14] that the equivalence with the expressions of Mustaki *et al* can be achieved using the method of performing k^- -integration only if one uses the two-term photon propagator. An important ingredient in this calculation consists of splitting the photon propagator into on-shell and off-shell parts. In ref. [13], we had used an alternative method, called the asymptotic method proposed by Bakker *et al* [1], to prove equivalence and had shown that, in the case of vacuum polarisation, the instantaneous photon exchange diagrams of ref. [8] can be generated using the two-term propagator by carefully evaluating the contribution of the arc at infinity in contour integrations.

The present work is motivated by the need to clarify the issue of form of photon propagator in LF gauge used in these proofs of equivalence. We present an alternative proof of equivalence for fermion self-energy and vertex correction using the asymptotic method [13] and also extend our earlier proof of equivalence to a general component of Λ^μ . We stress the fact that the form of the propagator needed to achieve equivalence actually depends on whether one has used both the Lorenz condition and light-front gauge condition $A^+ = 0$ to arrive at the LF Hamiltonian or not.

The starting point for our discussion is the equal-time QED Lagrangian in LF gauge given by

$$\mathcal{L} = \frac{i}{2} \bar{\psi} \overleftrightarrow{\not{\partial}} \psi - m \bar{\psi} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \bar{\psi} \gamma^\mu \psi A_\mu - \frac{1}{2\alpha} (2n_\mu A^\mu)(\partial_\nu A^\nu). \quad (3)$$

As shown by Suzuki *et al*, this Lagrangian leads to the three-term propagator. The Lagrange’s multiplier in the above Lagrangian takes care of the Lorenz condition $\partial_\mu A^\mu = 0$ as well as the LF gauge condition $n_\mu A^\mu = 0$. However, the Hamiltonian in ref. [8] has been obtained by using only the LF gauge condition. Hence, in order to establish the equivalence of covariant expressions with the results of Mustaki *et al* [8], it is appropriate only to use the two-term propagator in the covariant expression also, as was done by Mantovani *et al* [14], as the third term of the photon propagator arises from the Lorenz condition in \mathcal{L} .

On the other hand, if we derive the LF Hamiltonian on the lines of Mustaki *et al* but making use of the Lorenz condition as well then, as we shall show in §2, the 4-point vertex involving instantaneous photon exchange is not present in the Hamiltonian and hence the diagrams involving the instantaneous photon exchange will be absent in the one-loop calculations. We establish

equivalence of this theory with the covariant theory in subsequent sections.

In §3, we consider the fermion self-energy correction. We start with its covariant expression with the three-term propagator and show, by performing the k^- -integration, that indeed only the regular diagram and the instantaneous fermion exchange diagram of LFQED are generated using the methods of Mantovani *et al.* For the sake of completeness, we also show that in the case of vacuum polarisation too, the procedure of k^- -integration leads to the regular and instantaneous fermion exchange diagrams.

In §4, we revisit, for the case of fermion self-energy, our earlier proof of equivalence using the asymptotic method [1,13]. We establish, using this method also, that the covariant expression with the two term-propagator, on performing the k^- -integration, leads to all the diagrams in ref. [8], while only regular and instantaneous fermion exchange diagrams are generated if the three-term propagator is used.

In §5, we calculate the vertex correction contributions of the instantaneous fermion exchange diagrams in LF QED that were not considered in our previous work [12] because of their matrix structure. This was briefly discussed by us recently in ref. [20].

In §6, we establish the equivalence between the covariant and LF expressions for a general component of the one-loop vertex correction Λ^μ , by performing k^- -integration in the covariant expression.

Finally, in §7, we summarise our results and comment on the issue of form of the photon propagator. Appendix A contains the conventions and some basic formulae and Appendix B contains details of the calculations presented in §5.

2. Form of the photon propagator and the light-front Hamiltonian

There has been a great deal of discussion on the form of the photon propagator to be used in LF gauge as mentioned in the Introduction. Suzuki *et al* have shown that classically the propagator derived from LF gauge Lagrangian in eq. (3) has the third term also. Brodsky and Srivastava obtained this form in LF field theory and also showed that one necessarily has to introduce an instantaneous interaction term in the Hamiltonian if one eliminates the unphysical degrees of freedom. We give below the form of the interaction Hamiltonian in this case for the sake of completeness [14]:

$$\mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{LFint}} = e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{e^2}{2} \left(\frac{1}{i\partial_-} \bar{\psi}\gamma^+\psi \right) \left(\frac{1}{i\partial_-} \bar{\psi}\gamma^+\psi \right) \quad (4)$$

which is the QED analog of the LF QCD Hamiltonian derived by Brodsky *et al* [24]. In the above equation, ∂_- is defined as the partial derivative with respect to the coordinate x^- where

$$x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}.$$

Similarly, the Dirac matrices are defined as

$$\gamma^\pm = \frac{\gamma^0 \pm \gamma^3}{\sqrt{2}}.$$

Brodsky *et al* have also shown that when the free gauge field satisfies both the Lorenz condition as well as the light-cone gauge condition, then its propagator is doubly transverse, i.e. transverse to both its four-momentum and the gauge direction. The LF quantised QED Lagrangian in LF gauge in eq. (4) differs from the covariant form due to the presence of the second term representing an additional instantaneous interaction [24]. As pointed out by Mantovani *et al*, if one starts with the $\mathcal{L}_{\text{LFint}}$ in eq. (4), then the contribution of the third term in the propagator cancels the contribution of the instantaneous vertex and therefore, it is sufficient to work with the two-term propagator. Thus, the proof of equivalence as presented by Mantovani *et al* deals with proving the equivalence between Lagrangian formulation of LF quantised QED based on the Lagrangian in eq. (4) and the corresponding Hamiltonian version as given by Mustaki *et al* [8].

In this work, we investigate the issue of equivalence of equal-time quantised QED in standard covariant formulation based on the Lagrangian in eq. (3) and LF quantised Hamiltonian QED as given in eq. (10). The main point that we stress in this work is that the Lagrangian in eq. (3) leads to a doubly transverse three-term propagator obtained using the fact that the gauge field satisfies the Lorenz condition as well as the light-cone gauge condition, whereas the derivation of LF Hamiltonian in ref. [8] uses only the LF gauge condition and the Lorenz condition is not taken into account. In the following, we re-visit their derivation, but taking into account the Lorenz condition as well, and show that the resulting Hamiltonian does not have the instantaneous photon interaction.

We start with the QED Lagrangian

$$\mathcal{L} = \frac{i}{2} \bar{\psi} \overleftrightarrow{\not{\partial}} \psi - m \bar{\psi} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \bar{\psi} \gamma^\mu \psi A_\mu \quad (5)$$

which, after applying the light-cone gauge condition $A^+ = 0$, leads to the LF Hamiltonian [8]

$$P^- = P_G^- + P_F^-, \quad (6)$$

where P_G^- and P_F^- are the bosonic and fermionic parts given by

$$P_G^- = \int d^2\mathbf{x}_\perp dx^- \left[(\partial_- A_k)(\partial_k A_+) - \frac{1}{2}(\partial_- A_+)^2 + \frac{1}{2}(F_{12})^2 \right] \quad (7)$$

and

$$P_F^- = \int d^2\mathbf{x}_\perp dx^- \left[\bar{\psi} \left[-\frac{i}{2}\gamma^- \overleftrightarrow{\partial}_- - \frac{i}{2}\gamma^k \overleftrightarrow{\partial}_k + m \right] \psi + J^\mu A_\mu \right] \quad (8)$$

($k = 1, 2$).

Mustaki *et al* obtained the Euler–Lagrange equation for A_+ which turns out to be a constraint equation

$$\partial_-^2 A_+ = \partial_- \partial_k A_k - J^+ \quad (9)$$

using which A_+ is eliminated. The Hamiltonian can then be expressed in terms of only physical transverse degrees of freedom of the photon and a non-local effective four-point vertex corresponding to the instantaneous photon exchange is generated.

However, if one applies the light-cone gauge condition $A^+ = 0$ as well as the Lorenz condition $\partial \cdot A = 0$, A_+ does not appear in P_G^- and eq. (9) leads to $J^+ = 0$. As a result, when the LF Hamiltonian is expressed in terms of only independent degrees of freedom, one obtains

$$P^- = H_0 + V_1 + V_2. \quad (10)$$

Here,

$$H_0 = \int d^2\mathbf{x}_\perp dx^- \left[\frac{i}{2}\bar{\xi}\gamma^- \overleftrightarrow{\partial}_- \xi + \frac{1}{2}(F_{12})^2 - \frac{1}{2}a_+ \partial_- \partial_k a_k \right] \quad (11)$$

is the free Hamiltonian,

$$V_1 = e \int d^2\mathbf{x}_\perp dx^- \bar{\xi} \gamma^\mu \xi a_\mu \quad (12)$$

is the standard, order- e , three-point interaction and

$$V_2 = -\frac{i}{4}e^2 \int d^2\mathbf{x}_\perp dx^- dy^- \epsilon(x^- - y^-) \times (\bar{\xi} a_k \gamma^k)(x) \gamma^+ (a_j \gamma^j \xi)(y) \quad (13)$$

is order- e^2 non-local four-point vertex corresponding to an instantaneous fermion exchange where $y = (x^+, y^-, \mathbf{x}^\perp)$. In eqs (11)–(13), the fields ψ and A_μ

have been replaced with ξ and a_μ which are respectively the fermionic and photonic fields in which the dependent degrees of freedom have been eliminated using the equations of constraint retaining only the independent degrees, as has been worked out in detail in §II of ref. [8].

Note that this Hamiltonian differs from the Hamiltonian obtained by Mustaki *et al* by the absence of the non-local instantaneous photon exchange interaction. The non-local interaction involving instantaneous fermion exchange is still present though.

To summarise, we give below the different Lagrangians in equal-time theory which lead to different forms of (off-shell) propagators in covariant theory and different H_{LF} :

1. \mathcal{L}_{LF} in ref. [8] has been obtained from the following Lagrangian of the covariant theory:

$$\mathcal{L}_{LF}^{(1)} = i\bar{\psi} \not{\partial} \psi - m\bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\bar{\psi} \gamma^\mu \psi A_\mu - \frac{1}{2\alpha} (n_\mu A^\mu)^2$$

which leads to the 2-term propagator and \mathcal{H}_{LF} of ref. [8].

2. Following Suzuki and Sales [17], if we define the LF gauge by

$$\partial_\mu A^\mu = n_\mu A^\mu = 0$$

then this, in covariant formulation, leads to the Lagrangian

$$\mathcal{L}_{LF}^{(2)} = i\bar{\psi} \not{\partial} \psi - m\bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\bar{\psi} \gamma^\mu \psi A_\mu - \frac{1}{2\alpha} (n_\mu A^\mu)^2 - \frac{1}{\beta} (\partial_\mu A^\mu)^2$$

which leads to the 3-term propagator and \mathcal{H}_{LF} given by eq. (10).

The point that we make in this paper is that if we start from $\mathcal{L}_{LF}^{(1)}$, we obtain H_{LF} of ref. [8] obtained by Mustaki *et al* and therefore, in order to establish equivalence with their results, we must use the two-term propagator as done in ref. [14]. On the other hand, if we use $\mathcal{L}_{LF}^{(2)}$, we do not get the instantaneous photon exchange term in H_{LF} as shown in eq. (10) and therefore, while using the three-term propagator corresponding to $\mathcal{L}_{LF}^{(2)}$, we need to establish equivalence with only the regular and instantaneous fermion exchange diagrams. The additional instantaneous term that is mentioned by Mustaki *et al* in ref. [14] in the context of covariant theory (see ref. [22]) gives rise to a term in the derivation of the LF Hamiltonian which actually cancels the instantaneous photon exchange part of the LF Hamiltonian. One

can either keep this instantaneous interaction in equal-time theory and use the two-term photon propagator or drop it and use the three-term photon propagator. Both amount to using both the Lorenz condition and light-cone gauge condition and both lead to H_{LF} of eq. (10), i.e. the LF Hamiltonian without the instantaneous photon exchange potential.

In the next section, we shall draw all the basic one-loop graphs in LFQED resulting from the Hamiltonian in eq. (10). The expressions for these diagrams were obtained in ref. [8]. We shall then show that all these can be generated by performing k^- -integration in the covariant expressions with the three-term photon propagator.

3. Equivalence of covariant and LF one-loop expressions

In this section, we first give expressions for one-loop corrections in our formulation of LFQED which have been obtained using the techniques of old fashioned time-ordered perturbation theory in the LF framework. The one-loop diagrams considered here are a subset of diagrams given in ref. [8] due to the absence of instantaneous photon exchange interaction in our formulation. We shall then show that these expressions can be obtained from the corresponding covariant expressions by performing k^- -integration consistently while using the three-term photon propagator. We use the method of performing the k^- -integrations [12] to establish equivalence. In this section, we use the procedure followed in ref. [14] for dealing with the divergences coming from the photon propagator. The results will be reproduced in §4 using the asymptotic method. In §3.1, we show the equivalence of one-loop fermion self-energy graph in covariant QED described by the Lagrangian in eq. (3) with the LFQED fermion self-energy graphs resulting from the Hamiltonian in eq. (10). As explained in §2, we use the three-term propagator in place of the two-term propagator and compare the results with those in ref. [14]. Section 3.2 is a review of the work done in ref. [12]. This is included for the sake of completeness. We defer the proof of equivalence for vertex correction to §6 till after calculating the one-loop vertex correction in §5 for a general component of Λ^μ .

3.1 Fermion self-energy

Starting with the LF Hamiltonian in eq. (10), one obtains the one-loop corrections to the fermion self-energy in the LF time-ordered perturbation theory which consist of the ‘regular’ diagram and an instantaneous fermion exchange diagram shown in figure 1.

Following the procedure in ref. [8], one obtains the following expressions for these diagrams which are eqs (3.9) and (3.10) in ref. [8]:

$$\bar{u}_{p',s'} \Sigma_1 u_{p,s} = \frac{e^2}{m} \int \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \times \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \times \frac{\bar{u}_{p',s'} \gamma^\mu (\not{k}' + m) \gamma^\nu u_{p,s} d_{\mu\nu}(k)}{p^- - k^- - k'^-} \quad (14)$$

for the regular diagram and

$$\bar{u}_{p',s'} \Sigma_2 u_{p,s} = \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \int_0^\infty \frac{dk^+}{k^+(p^+ - k^+)} \quad (15)$$

for the instantaneous fermion exchange diagram where (p, s) , (p', s') , (k, λ) and (k', σ') are the momentum and spin/polarisation of the incoming fermion, outgoing fermion, intermediate photon and intermediate fermion respectively, as shown in figure 1.

In the standard covariant formulation of equal-time quantised QED, only figure 1a is present and the expression in LF gauge is

$$\Sigma(p) = \frac{ie^2}{2m} \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^\mu (\not{p} - \not{k} + m) \gamma^\nu d'_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]}, \quad (16)$$

where

$$d'_{\alpha\beta}(k) = d_{\alpha\beta}(k) - \frac{\delta_\alpha + \delta_\beta + k^2}{(k^+)^2} = -g_{\alpha\beta} + \frac{\delta_\alpha + k_\beta + \delta_\beta + k_\alpha}{k^+} - \frac{\delta_\alpha + \delta_\beta + k^2}{(k^+)^2}.$$

We define the on-shell and off-shell parts of momentum p of a massive particle as

$$p_{\text{on}} = \left(p^+, \frac{p_\perp^2 + m^2}{2p^+}, \mathbf{p}_\perp \right)$$

and

$$p_{\text{off}} = \left(0, \frac{p^2 - m^2}{2p^+}, \mathbf{0} \right)$$

respectively. Thus, to show equivalence of covariant and LF expressions, one rewrites $\not{p} - \not{k}$ in eq. (16) as a sum of an on-shell part and an off-shell part [12]:

$$\begin{aligned} \not{p} - \not{k} &= \gamma^+(p^- - k^-) + \gamma^-(p^+ - k^+) \\ &\quad + \gamma^\perp(\mathbf{p}_\perp - \mathbf{k}_\perp) \\ &= \gamma^+ \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2}{2(p^+ - k^+)} \right] \\ &\quad + \gamma^-(p^+ - k^+) + \gamma^\perp(\mathbf{p}_\perp - \mathbf{k}_\perp) \end{aligned}$$

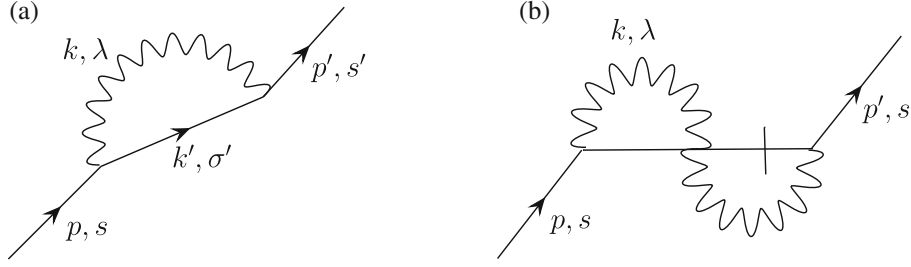


Figure 1. ‘Regular’ and instantaneous fermion exchange self-energy diagrams.

$$\begin{aligned}
 & +\gamma^+ \left[(p^- - k^-) - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2}{2(p^+ - k^+)} \right] \right] \\
 & = \mathcal{H}'_{\text{on}} + \frac{\gamma^+ [(p - k)^2 - m^2]}{2(p^+ - k^+)} \quad (17)
 \end{aligned}$$

which leads to

$$\Sigma(p) = \Sigma_1(p) + \Sigma_2(p), \quad (18)$$

where

$$\begin{aligned}
 & \Sigma_1(p) \\
 & = \frac{ie^2}{2m} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu (\mathcal{H}'_{\text{on}} + m) \gamma^\nu d_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \\
 & - \frac{ie^2}{2m} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu (\mathcal{H}'_{\text{on}} + m) \gamma^\nu \delta_{\mu+\delta\nu} k^2}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon](k^+)^2} \quad (19)
 \end{aligned}$$

and

$$\Sigma_2(p) = \frac{ie^2}{2m} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu \gamma^+ \gamma^\nu d'_{\mu\nu}(k)}{2(p^+ - k^+) [k^2 - \mu^2 + i\epsilon]}. \quad (20)$$

Hence, $\Sigma_1(p)$ corresponds to the on-shell part of the fermion propagator whereas $\Sigma_2(p)$ corresponds to its off-shell part. Using the identities

$$\gamma^\alpha \gamma^\mu \gamma^\beta d_{\alpha\beta}(k) = \frac{2}{k^+} [\gamma^+ k^\mu + g^{+\mu} \mathcal{H}], \quad (\gamma^+)^2 = 0$$

and

$$\bar{u}_{p,s'} \gamma^\mu u_{p,s} = 2p^\mu \delta_{ss'},$$

the last expression leads to

$$\begin{aligned}
 \bar{u}_{p,s'} \Sigma_2(p) u_{p,s} & = \frac{ie^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp dk^+}{(2\pi)^4 k^+ (p^+ - k^+)} \\
 & \times \int \frac{dk^-}{(k^- - \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})}. \quad (21)
 \end{aligned}$$

k^- -integral in this equation has a pole at

$$k_1^- = \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+}$$

which approaches infinity as $k^+ \rightarrow 0$. In order to deal with the pole at infinity, we use the method of u -integration [1,12]. We make the change of variable $u = 1/k^-$, thus modifying the integral to

$$\int_{-\infty}^{+\infty} \frac{du}{u [1 - u (\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})]}. \quad (22)$$

The u -integral needs to be regulated and hence we write

$$\frac{1}{u} = \frac{1}{2} \left[\frac{1}{u + i\delta} + \frac{1}{u - i\delta} \right]$$

which leads to

$$\begin{aligned}
 & \int \frac{dk^-}{(k^- - \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})} \\
 & = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{du}{(u + i\delta) [1 - u (\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})]} \\
 & + \frac{1}{2} \int_{-\infty}^{+\infty} \frac{du}{(u - i\delta) [1 - u (\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})]}. \quad (23)
 \end{aligned}$$

In eq. (23), the first u -integral has poles at

$$u_1 = -i\delta$$

and

$$u_2 = \frac{2k^+}{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}.$$

For $k^+ < 0$, the integral is zero since both poles lie below the real axis. For $k^+ > 0$, u_1 lies below and u_2 above the real axis. Closing the contour below gives the value of the integral as $-2\pi i \theta(k^+)$ as $\delta \rightarrow 0$. The u -integral in the second term has poles at

$$u_1 = i\delta$$

and

$$u_2 = \frac{2k^+}{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}.$$

For $k^+ > 0$, both poles lie above the real axis and the integral vanishes on closing the contour in the lower

half-plane. For $k^+ < 0$, we close the contour in the upper half-plane as u_1 lies above and u_2 below the real line. The value of the integral is $2\pi i\theta(-k^+)$ as $\delta \rightarrow 0$. Thus,

$$\int \frac{dk^-}{(k^- - \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})} = -\pi i\theta(k^+) + \pi i\theta(-k^+) \quad (24)$$

and

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_2(p) u_{p,s} &= \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \\ &\times \left[\frac{1}{2} \int_0^\infty \frac{dk^+}{k^+(p^+ - k^+)} - \frac{1}{2} \int_{-\infty}^0 \frac{dk^+}{k^+(p^+ - k^+)} \right] \end{aligned} \quad (25)$$

which can be shown to be the same as

$$\bar{u}_{p,s'} \Sigma_2(p) u_{p,s} = \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \int_0^\infty \frac{dk^+}{k^+(p^+ - k^+)}. \quad (26)$$

This is nothing but the expression for instantaneous fermion exchange diagram as given in eq. (15).

$\Sigma_1(p)$ can be written as

$$\Sigma_1(p) = \Sigma_1^{(a)}(p) + \Sigma_1^{(b)}(p), \quad (27)$$

where

$$\begin{aligned} \Sigma_1^{(a)}(p) &= \frac{ie^2}{2m} \\ &\times \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^\mu (\not{H}'_{\text{on}} + m) \gamma^\nu d_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \end{aligned} \quad (28)$$

and

$$\begin{aligned} \Sigma_1^{(b)}(p) &= -\frac{ie^2}{2m} \\ &\times \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^\mu (\not{H}'_{\text{on}} + m) \gamma^\nu \delta_{\mu+\delta_\nu+k^2}}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon](k^+)^2}. \end{aligned} \quad (29)$$

Thus, these two terms of $\Sigma_1(p)$, viz. $\Sigma_1^{(a)}(p)$ and $\Sigma_1^{(b)}(p)$, correspond to the first two terms and the third term of the photon propagator respectively. Using the identities $\gamma^+ \gamma^- \gamma^+ = 2\gamma^+$, $(\gamma^+)^2 = 0$ and $\bar{u}_{p,s'} \gamma^\mu u_{p,s} = 2p^\mu \delta_{ss'}$, we get

$$\begin{aligned} \bar{u}_{ps'} \Sigma_1^{(b)}(p) u_{ps} &= -\frac{2ie^2 p^+ \delta_{ss'}}{2m} \\ &\times \int \frac{d^2 \mathbf{k}_\perp dk^+}{(2\pi)^4 (k^+)^2} \int \frac{dk^-}{[p^- - k^- - \frac{(p_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}]}. \end{aligned} \quad (30)$$

The k^- -integral in this equation has a pole at

$$k_1^- = p^- - \frac{(p_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}$$

and at infinity as $k^+ \rightarrow p^+$. Evaluating the k^- -integral along the same lines as in the case of $\Sigma_2(p)$, we get

$$\begin{aligned} &\int \frac{dk^-}{(p^- - k^- - \frac{(p_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)})} \\ &= -\pi i\theta(p^+ - k^+) + \pi i\theta(k^+ - p^+). \end{aligned} \quad (31)$$

Thus,

$$\begin{aligned} \bar{u}_{ps'} \Sigma_1^{(b)}(p) u_{ps} &= \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \\ &\times \left[\int_{p^+}^\infty \frac{dk^+}{(k^+)^2} - \int_{-\infty}^{p^+} \frac{dk^+}{(k^+)^2} \right]. \end{aligned} \quad (32)$$

Changing the variable $k^+ \rightarrow (p^+ + k^+)$ in the first integral and $k^+ \rightarrow (p^+ - k^+)$ in the second gives

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_1^{(b)}(p) u_{p,s} &= \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \\ &\times \left[\int_0^\infty \frac{dk^+}{(p^+ + k^+)^2} - \int_0^\infty \frac{dk^+}{(p^+ - k^+)^2} \right]. \end{aligned} \quad (33)$$

$\Sigma_1^{(a)}(p)$ can be evaluated using the method of splitting $d_{\mu\nu}$ into on-shell and off-shell parts as done in ref. [14]. This leads, after performing the k^- -integration, to the following two expressions:

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_1^{(a1)}(p) u_{p,s} &= \frac{e^2}{m} \int \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \\ &\times \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \frac{\bar{u}_{p,s'} \gamma^\mu (\not{H}'_{\text{on}} + m) \gamma^\nu u_{p,s} d_{\mu\nu}(k_{\text{on}})}{p^- - k_{\text{on}}^- - k_{\text{on}}'^-} \end{aligned} \quad (34)$$

and

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_1^{(a2)}(p) u_{p,s} &= -\frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \\ &\times \left[\int_0^\infty \frac{dk^+}{(p^+ + k^+)^2} - \int_0^\infty \frac{dk^+}{(p^+ - k^+)^2} \right]. \end{aligned} \quad (35)$$

Equation (34), which is the same as eq. (57) in ref. [14], is the expression for the ‘regular’ diagram and eq. (35) cancels the contribution of eq. (33).

In conclusion, using the three-term photon propagator and employing the method of splitting the propagator into on-shell and off-shell parts as given in ref. [14], the ‘regular’ and instantaneous fermion exchange diagrams are generated by performing k^- -integration in the covariant expression. Had we started with the two-term propagator instead, as was done by Mantovani *et al*, eq.

(29) and hence eq. (33) would have been absent leaving eq. (35) intact, which, in fact, is the expression for the instantaneous photon diagram given in figure 2. This diagram is not present in our formulation based on the Hamiltonian in eq. (10). These findings are consistent with the discussion in §2.

In the next section, we shall employ an alternative method called the asymptotic method, in place of split-

$$p - k - m = k'_{\text{on}} - m + \frac{\gamma^+[(p-k)^2 - m^2]}{2(p^+ - k^+)}.$$

Doing so, eq. (38) splits up as follows:

$$\Pi^{\mu\nu}(p) = \Pi_1^{\mu\nu}(p) + \Pi_2^{\mu\nu}(p) + \Pi_3^{\mu\nu}(p) + \Pi_4^{\mu\nu}(p), \quad (39)$$

where

$$\Pi_1^{\mu\nu}(p) = ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\gamma^\mu(k_{\text{on}} + m)\gamma^\nu(k'_{\text{on}} - m)]}{2k^+ 2(p^+ - k^+)(k^- - \frac{\mathbf{k}_\perp^2 + m^2 - i\epsilon}{2k^+})(p^- - k^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)})}, \quad (40)$$

ting the propagator into on-shell and off-shell parts, to achieve the same results.

3.2 Vacuum polarisation

The one-loop diagrams that contribute to vacuum polarisation in LFTOPT are given in figure 3. The expressions for these diagrams, as given in eqs (4.6) and (4.5) of ref. [8], are as follows:

$$\epsilon_\mu^\lambda(p)\Pi_1^{\mu\nu}\epsilon_\nu^{\lambda'}(p) = 2e^2 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \times \frac{\text{Tr}[\not{d}^\lambda(p)(k+m)\not{d}^{\lambda'}(p)(k' - m)]}{p^- - k^- - k'^-} \quad (36)$$

for the ‘regular’ diagram, and

$$\epsilon_\mu^\lambda(p)\Pi_2^{\mu\nu}\epsilon_\nu^{\lambda'}(p) = e^2 \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^3} \times \int_0^\infty dk^+ \left[\frac{1}{p^+ - k^+} - \frac{1}{p^+ + k^+} \right] \quad (37)$$

for the two instantaneous diagrams where (p, λ) , (p', λ') , (k, σ) and (k', σ') are the momentum and spin/polarisation of the incoming photon, outgoing photon and the intermediate fermions, as shown in figure 3.

The standard covariant expression for vacuum polarisation is given by

$$\Pi^{\mu\nu}(p) = ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\gamma^\mu(k+m)\gamma^\nu(p-k-m)]}{(k^2 - m^2 + i\epsilon)[(p-k)^2 - m^2 + i\epsilon]}. \quad (38)$$

Employing a similar scheme as in the case of self-energy, we split $(k+m)$ and $(p-k-m)$ in eq. (38) into the on-shell and off-shell parts, i.e.

$$k+m = k_{\text{on}} + m + \frac{\gamma^+(k^2 - m^2)}{2k^+},$$

$$\Pi_2^{\mu\nu}(p) = ie^2 \int \frac{d^4k}{(2\pi)^4} \times \frac{\text{Tr}[\gamma^\mu(k_{\text{on}} + m)\gamma^\nu\gamma^+]}{2k^+ 2(p^+ - k^+)(k^- - \frac{\mathbf{k}_\perp^2 + m^2 - i\epsilon}{2k^+})}, \quad (41)$$

$$\Pi_3^{\mu\nu}(p) = ie^2 \times \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\gamma^\mu\gamma^+\gamma^\nu(k'_{\text{on}} - m)]}{2k^+ 2(p^+ - k^+)(p^- - k^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)})}, \quad (42)$$

$$\Pi_4^{\mu\nu}(p) = ie^2 \times \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\gamma^\mu\gamma^+\gamma^\nu\gamma^+]}{2k^+ 2(p^+ - k^+)}. \quad (43)$$

On using the fact that $\epsilon_- = 0$, the identity $(\gamma^+)^2 = 0$, and the anticommutation relations of γ -matrices, we can see that the contribution of $\Pi_4^{\mu\nu}(p)$ to the transition amplitude, viz. $\epsilon_\mu^\lambda\Pi_4^{\mu\nu}(p)\epsilon_\nu^{\lambda'}$ is null.

The k^- -integral of $\Pi_1^{\mu\nu}(p)$, i.e.

$$\int \frac{dk^-}{(k^- - \frac{\mathbf{k}_\perp^2 + m^2 - i\epsilon}{2k^+})(p^- - k^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)})}$$

has poles at

$$k_1^- = \frac{\mathbf{k}_\perp^2 + m^2 - i\epsilon}{2k^+}$$

and at

$$k_2^- = p^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}.$$

For $k^+ < 0$, both poles lie above the real axis and for $k^+ > p^+$, both lie below it. Hence, on closing the contour on the opposite side of the position of the poles, both the ranges $k^+ < 0$ and $k^+ > p^+$ provide no contribution to the integral. For $0 < k^+ < p^+$, k_1^- lies below the

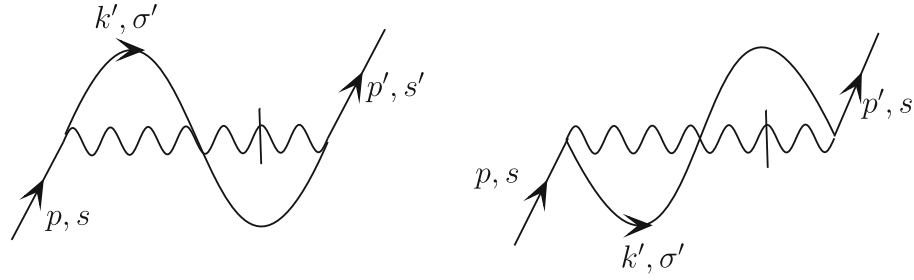


Figure 2. Instantaneous photon exchange self-energy diagram.

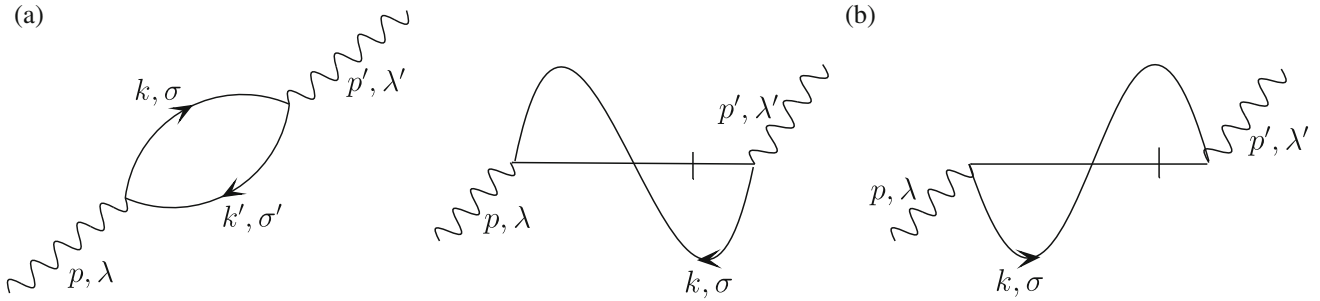


Figure 3. ‘Regular’ and instantaneous vacuum polarisation diagrams.

real axis and k_2^- lies above. Closing the contour below, the value of this k^- -integral is

$$\frac{-2\pi i\theta(k^+)\theta(p^+ - k^+)}{\left(p^- - \frac{\mathbf{k}_\perp^2 + m^2 - i\epsilon}{2k^+} - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}\right)}$$

and hence

$$\begin{aligned} \epsilon_\mu^\lambda(p)\Pi_1^{\mu\nu}\epsilon_\nu^{\lambda'}(p) &= 2e^2 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \\ &\times \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \frac{Tr[d^\lambda(p)(\not{H}_{on} + m)d^{\lambda'}(p)(\not{H}'_{on} - m)]}{p^- - k_{on}^- - k'_{on}^-} \end{aligned} \quad (44)$$

which is the same as the expression for the diagram in figure 3a given by eq. (36).

Using the trace properties, $\Pi_2^{\mu\nu}(p)$ further reduces to

$$\begin{aligned} \Pi_2^{\mu\nu}(p) &= ie^2 \int \frac{d^4k}{(2\pi)^4} \\ &\times \frac{k_{on}^- Tr[\gamma^\mu\gamma^+\gamma^\nu\gamma^+] + k^+ Tr[\gamma^\mu\gamma^-\gamma^\nu\gamma^+]}{2k^+ 2(p^+ - k^+)(k^- - \frac{\mathbf{k}_\perp^2 + m^2 - i\epsilon}{2k^+})}. \end{aligned} \quad (45)$$

The first term of the numerator in the above integral provides no contribution to $\epsilon_\mu^\lambda(p)\Pi_2^{\mu\nu}\epsilon_\nu^{\lambda'}(p)$ as $(\gamma^+)^2 = 0$, $\epsilon_- = 0$ and $\{\gamma^i, \gamma^+\} = 0$. Thus,

$$\epsilon_\mu^\lambda(p)\Pi_2^{\mu\nu}\epsilon_\nu^{\lambda'}(p) = ie^2 \int \frac{d^2\mathbf{k}_\perp dk^+}{(2\pi)^4}$$

$$\times \frac{\epsilon_\mu^\lambda Tr[\gamma^\mu\gamma^-\gamma^\nu\gamma^+]\epsilon_\nu^{\lambda'}}{4(p^+ - k^+)} \int \frac{dk^-}{(k^- - \frac{\mathbf{k}_\perp^2 + m^2 - i\epsilon}{2k^+})}. \quad (46)$$

The numerator in the above integral,

$$\epsilon_\mu^\lambda Tr[\gamma^\mu\gamma^-\gamma^\nu\gamma^+]\epsilon_\nu^{\lambda'} = 4,$$

which can be shown using $\epsilon_\mu^\lambda p^\mu = 0$, $\epsilon_\mu^{\lambda'} \epsilon^{\lambda\mu} = -\delta_{\lambda'\lambda}$, $\epsilon_-^\lambda = 0$. The k^- -integral is the same as evaluated in the previous subsection. Thus, we have

$$\begin{aligned} \epsilon_\mu^\lambda(p)\Pi_2^{\mu\nu}\epsilon_\nu^{\lambda'}(p) &= \frac{e^2}{2} \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^3} \\ &\times \int_0^\infty dk^+ \left[\frac{1}{p^+ - k^+} - \frac{1}{p^+ + k^+} \right]. \end{aligned} \quad (47)$$

The calculation of $\Pi_3^{\mu\nu}$ follows exactly on the lines of $\Pi_2^{\mu\nu}$ and we have

$$\begin{aligned} \epsilon_\mu^\lambda(p)\Pi_3^{\mu\nu}\epsilon_\nu^{\lambda'}(p) &= \frac{e^2}{2} \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^3} \\ &\times \int_0^\infty dk^+ \left[\frac{1}{p^+ - k^+} - \frac{1}{p^+ + k^+} \right]. \end{aligned} \quad (48)$$

Equations (47) and (48) add up to give the LF expression for instantaneous fermion diagrams, i.e. eq. (37). It can be inferred from the above calculations that

- (i) the regular diagram in LFTOPT corresponds to the situation where both the fermions in the loop are on-shell, and

- (ii) the additional (instantaneous) diagrams that contribute to the photon self-energy can be looked at as being the result of one of the fermions going off-shell.

4. The asymptotic method

Asymptotic method was introduced in ref. [1] by Bakker *et al* in the context of (1 + 1)-theories as an alternative to explicit evaluation of arc contribution. In this method, one deals directly with the linear divergences as $k^+ \rightarrow 0$ and as $k^+ \rightarrow p^+$ and isolates the divergent part by evaluating the integrand at the asymptotic values of k^- . The method was used by us in ref. [13] in the context of QED. In this section, we shall employ the asymptotic method first using the two-term gauge boson propagator

$$d_{\alpha\beta}(k) = -g_{\alpha\beta} + \frac{\delta_{\alpha^+k\beta} + \delta_{\beta^+k\alpha}}{k^+},$$

and shall show that the regular, instantaneous fermion exchange as well as the instantaneous photon exchange diagrams of LFTOPT are generated by this method also. The two major points to be noted here are: (i) this method is being used to carry out the k^- -integration because of the non-vanishing arc contributions to the contour integral and (ii) it is the two-term propagator that generates the instantaneous photon diagram (along with the rest of the diagrams). This is due to the presence of the interaction vertex

$$V_3 = -\frac{e^2}{4} \int d^2\mathbf{x}_\perp dx^- dy^- \times (\bar{\xi}\gamma^+\xi)(x)|x^- - y^-|(\bar{\xi}\gamma^+\xi)(y) \quad (49)$$

in the LF Hamiltonian [8], which is otherwise absent if we apply Lorenz condition also in the derivation of LF Hamiltonian as argued by us in §2. In this section, we shall use the asymptotic method to prove equivalence for fermion self-energy.

The covariant expression for fermion self-energy with the two-term photon propagator is given as follows:

$$\begin{aligned} \Sigma(p) &= \frac{ie^2}{2m} \\ &\times \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{k} + m)\gamma^\nu d_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \\ &= \Sigma_1^{(a)}(p) + \Sigma_2(p), \end{aligned} \quad (50)$$

where

$$\begin{aligned} \Sigma_1^{(a)}(p) &= \frac{ie^2}{2m} \\ &\times \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{k}_{\text{on}} + m)\gamma^\nu d_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \end{aligned} \quad (51)$$

and

$$\Sigma_2(p) = \frac{ie^2}{2m} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu\gamma^+\gamma^\nu d_{\mu\nu}(k)}{2(p^+ - k^+)[k^2 - \mu^2 + i\epsilon]} \quad (52)$$

as per the notations used in §3.1. $\bar{u}_{p,s}\Sigma_2(p)u_{p,s}$ has already been evaluated in that section and it was seen that it gives the expression for the instantaneous fermion exchange diagram. However, in the contour integration over k^- in the expression for $\Sigma_1^{(a)}(p)$, the key observation is that, due to the presence of a factor of k^- in $d_{\mu\nu}(k)$, there are possible arc contributions for the cases (i) $k^- \rightarrow \infty$ as $k^+ \rightarrow 0$ and (ii) $k^- \rightarrow \infty$ as $k^+ \rightarrow p^+$ because in these cases, the integrand does not go to zero as $k^- \rightarrow \infty$.

The asymptotic method consists of taking the asymptotic limits of the integrand and subtracting it from the integrand which reduces the degree of divergence. The asymptotic parts are then evaluated separately and added to the integral which can now be evaluated using the method of residues. Thus, we rewrite $\Sigma_1^{(a)}(p)$ as

$$\begin{aligned} \Sigma_1^{(a)}(p) &= \left[\Sigma_1^{(a)}(p) - \Sigma_{1(1)}^{(a)\text{asy}}(p) - \Sigma_{1(2)}^{(a)\text{asy}}(p) \right] \\ &+ \Sigma_{1(1)}^{(a)\text{asy}}(p) + \Sigma_{1(2)}^{(a)\text{asy}}(p), \end{aligned} \quad (53)$$

where

$$\Sigma_{1(1)}^{(a)\text{asy}}(p) = \lim_{\substack{k^+ \rightarrow 0 \\ k^- \rightarrow \infty}} \Sigma_1^{(a)}(p) \quad (54)$$

and

$$\Sigma_{1(2)}^{(a)\text{asy}}(p) = \lim_{\substack{k^+ \rightarrow p^+ \\ k^- \rightarrow \infty}} \Sigma_1^{(a)}(p). \quad (55)$$

Using the identities $\gamma^\alpha\gamma^\mu\gamma^\beta d_{\alpha\beta}(k) = \frac{2}{k^+}(\gamma^+k^\mu + g^{+\mu}k)$ and $\bar{u}_{p,s}\gamma^\mu u_{p,s'} = 2p^\mu\delta_{ss'}$, we see that the numerator of the integrand in $\bar{u}_{p,s}\Sigma_1^{(a)}u_{p,s}$ is

$$\begin{aligned} \text{Num} &= \frac{4p^+[(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2]}{(p^+ - k^+)} \\ &+ \frac{8p^+(p^+ - k^+)k^-}{k^+} - \frac{4p^+(\mathbf{p}_\perp - \mathbf{k}_\perp)\cdot\mathbf{k}_\perp}{k^+} \\ &+ 4p^-(p^+ - k^+) - \frac{4\mathbf{p}_\perp\cdot\mathbf{k}_\perp(p^+ - k^+)}{k^+} - 4m^2 \end{aligned} \quad (56)$$

which in the asymptotic limit (i) $k^- \rightarrow \infty$ as $k^+ \rightarrow 0$, reduces to

$$\text{Num}_{(1)}^{\text{asy}} = \frac{8p^+(p^+ - k^+)k^-}{k^+} \quad (57)$$

and the denominator of $\bar{u}_{p,s}\Sigma_1^{(a)}u_{p,s}$ reduces to

$$\text{Den}_{(1)}^{\text{asy}} = -2k^-(p^+ - k^+)D_1, \quad (58)$$

where

$$D_1 = k^2 - \mu^2 + i\epsilon.$$

Thus,

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_{1(1)}^{(a)\text{asy}}(p) u_{p,s} &= \frac{-2ie^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^4} \\ &\times \int \frac{dk^+}{(k^+)^2} \int \frac{dk^-}{(k^- - \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})}. \end{aligned} \quad (60)$$

The k^- -integral is evaluated in §3.1 and is

$$\int \frac{dk^-}{(k^- - \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+})} = -\pi i \theta(k^+) + \pi i \theta(-k^+).$$

Hence,

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_{1(1)}^{(a)\text{asy}}(p) u_{p,s} &= \frac{e^2 p^+ \delta_{ss'}}{2m} \\ &\times \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \left[\int_{-\infty}^0 \frac{dk^+}{(k^+)^2} - \int_0^\infty \frac{dk^+}{(k^+)^2} \right]. \end{aligned} \quad (61)$$

Changing the variable k^+ to $-k^+$ in the first k^+ -integral gives

$$\bar{u}_{p,s'} \Sigma_{1(1)}^{(a)\text{asy}}(p) u_{p,s} = 0. \quad (62)$$

In the asymptotic limit (ii) $k^- \rightarrow \infty$ as $k^+ \rightarrow p^+$, eq. (56) reduces to

$$\text{Num}_{(2)}^{\text{asy}} = \frac{8p^+(p^+ - k^+)k^-}{k^+} \quad (63)$$

and the denominator of $\bar{u}_{p,s} \Sigma_1^{(a)} u_{p,s}$ reduces to

$$\text{Den}_{(2)}^{\text{asy}} = (2k^+ k^-) D_2, \quad (64)$$

where

$$D_2 = (p - k)^2 - m^2 + i\epsilon. \quad (65)$$

So,

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s} &= \frac{2ie^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^4} \\ &\times \int \frac{dk^+}{(k^+)^2} \int \frac{dk^-}{(p^- - k^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)})}. \end{aligned} \quad (66)$$

This k^- -integral too is evaluated in §3.1 and is

$$\begin{aligned} \int \frac{dk^-}{(p^- - k^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)})} \\ = -\pi i \theta(p^+ - k^+) + \pi i \theta(k^+ - p^+). \end{aligned}$$

Thus,

$$\bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s} = \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3}$$

$$\times \left[\int_{-\infty}^{p^+} \frac{dk^+}{(k^+)^2} - \int_{p^+}^{\infty} \frac{dk^+}{(k^+)^2} \right]. \quad (67)$$

Changing the variable k^+ to $(p^+ - k^+)$ in the first k^+ -integral and to $(p^+ + k^+)$ in the second gives

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s} &= \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^3} \\ &\times \left[\int_0^\infty \frac{dk^+}{(p^+ - k^+)^2} - \int_0^\infty \frac{dk^+}{(p^+ + k^+)^2} \right] \end{aligned} \quad (68)$$

which is the same as the expression for instantaneous photon exchange diagrams of figure 2.

Thus, we see that the asymptotic method correctly generates the instantaneous photon exchange diagrams too when the two-term gauge boson propagator is used. Now we go on to show how the regular diagram is generated. Separating the asymptotic part, we get

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_1^{(a)}(p) u_{p,s} &= [\bar{u}_{p,s'} \Sigma_1^{(a)}(p) u_{p,s} \\ &- \bar{u}_{p,s'} \Sigma_{1(1)}^{(a)\text{asy}}(p) u_{p,s} - \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s}] \\ &+ \bar{u}_{p,s'} \Sigma_{1(1)}^{(a)\text{asy}}(p) u_{p,s} + \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s} \\ &= [\bar{u}_{p,s'} \Sigma_1^{(a)}(p) u_{p,s} - \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s}] \\ &+ \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s} \end{aligned} \quad (69)$$

because $\bar{u}_{p,s'} \Sigma_{1(1)}^{(a)\text{asy}}(p) u_{p,s} = 0$ (see eq. (62)).

It can be easily shown that

$$\begin{aligned} \bar{u}_{p,s'} \Sigma_1^{(a)}(p) u_{p,s} - \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s} \\ = \frac{ie^2}{2m} \int \frac{d^2 k_\perp dk^+ N}{(2\pi)^4} \int \frac{dk^-}{D_1 D_2}, \end{aligned} \quad (70)$$

where D_1, D_2 are defined in eqs (59 and 65) and

$$\begin{aligned} N &= 4p^+ k_{\text{on}}^- - \frac{4p^+(\mathbf{p}_\perp - \mathbf{k}_\perp) \cdot \mathbf{k}_\perp}{k^+} + 4p^-(p^+ - k^+) \\ &- \frac{4\mathbf{p}_\perp \cdot \mathbf{k}_\perp (p^+ - k^+)}{k^+} - 4m^2 \\ &+ \frac{4p^+(p^+ - k^+)(\mathbf{k}_\perp^\perp)^2}{(k^+)^2}. \end{aligned} \quad (71)$$

The k^- -integral in eq. (70) has poles at

$$k_1^- = \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+}$$

and

$$k_2^- = p^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}.$$

The integral goes to zero for $k^+ < 0$ and $k^+ > p^+$ because in each of these cases, both poles lie on one side of the real axis. For $0 < k^+ < p^+$, k_1^- lies below

whereas k_2^- lies above the real axis. Closing the contour below, we find

$$\begin{aligned} & \bar{u}_{p,s'} \Sigma_1^{(a)}(p) u_{p,s} - \bar{u}_{p,s'} \Sigma_{1(2)}^{(a)\text{asy}}(p) u_{p,s} \\ &= \frac{e^2}{m} \int \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+ N}{k^+(p^+ - k^+) [p^- - k_{\text{on}}^- - k'_{\text{on}}^-]}. \end{aligned} \quad (72)$$

Using the identities

$$\gamma^\alpha \gamma^\mu \gamma^\beta d_{\alpha\beta}(k) = \frac{2}{k^+} (\gamma^+ k^\mu + g^{+\mu} k)$$

and

$$\bar{u}_{p,s} \gamma^\mu u_{p,s'} = 2p^\mu \delta_{ss'},$$

it can be seen that

$$\bar{u}_{p,s'} \gamma^\mu (\not{k}'_{\text{on}} + m) \gamma^\nu d_{\mu\nu}(k_{\text{on}}) u_{p,s} = N. \quad (73)$$

Thus,

$$\begin{aligned} \bar{u}_{p',s'} \Sigma_1 u_{p,s} &= \frac{e^2}{m} \int \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \\ &\times \frac{\bar{u}_{p,s'} \gamma^\mu (\not{k}' + m) \gamma^\nu u_{p,s} d_{\mu\nu}(k_{\text{on}})}{p^- - k_{\text{on}}^- - k'_{\text{on}}^-} \end{aligned} \quad (74)$$

which is the expression for the regular diagram (eq. (14)). Thus, we see that using the two-term propagator and employing the asymptotic method to consistently take into account the arc contributions, all the one-loop self-energy diagrams in ref. [8], viz. the regular diagram, the instantaneous fermion exchange diagram and the instantaneous photon exchange diagrams are generated. As shown in §3.1, if one uses the three-term gauge boson propagator, there is an extra contribution due to the third term of the propagator, which will cancel eq. (68) and thus in this method also, the three-term propagator generates only the regular and instantaneous fermion exchange diagrams. This is consistent with the arguments presented in §2.

5. One-loop vertex correction in LFTOPT

One-loop renormalisation of LFQED has been discussed in detail in ref. [8], where Mustaki *et al* have enlisted all the one-loop diagrams contributing to Λ^μ . We have presented, in figure 4, all the connected diagrams that contribute to the process [8,12]. The rest of the diagrams for vertex correction given in ref. [8] are corrections to external legs and hence can be absorbed in renormalisation constants. Thus, the only diagrams relevant here are those given in figure 4. Figures 4a and 4b, which we call the regular diagrams, contain only the

standard QED vertex. These two have been evaluated for the ‘+’ component in ref. [8] using LFTOPT. Figures 4c and 4d contain the instantaneous fermion vertex and were not evaluated by Mustaki *et al* and by us [12] as these two diagrams do not contribute to Λ^+ because of the structure of γ -matrices. Both the works discussed the evaluation and equivalence of the ‘+’ component only. Here, we extend this study to a general Λ^μ . In the following equations in this section, (p, s) , (p', s') , $(q, \tilde{\lambda})$, (k, λ) , (k', σ') and (k'', σ'') are the momentum and spin/polarisation of the incoming fermion, outgoing fermion, outgoing photon, intermediate photon and intermediate fermions respectively, as shown in figure 4.

Contributions of the regular diagrams in figures 4a and 4b are given by

$$\begin{aligned} \Lambda_{(a)}^\mu &= \lambda \int_{-\infty}^{+\infty} \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_0^{p^+ - q^+} \frac{dk^+}{k^+ k'^+ k''^+} \\ &\times \frac{\gamma^\alpha (\not{k}'' + m) \gamma^\mu (\not{k}' + m) \gamma^\beta d_{\alpha\beta}(k)}{(p^- - k^- - k'^-)(p^- - q^- - k^- - k''^-)} \end{aligned} \quad (75)$$

and

$$\begin{aligned} \Lambda_{(b)}^\mu &= -\lambda \int_{-\infty}^{+\infty} \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_{p^+ - q^+}^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \\ &\times \frac{\gamma^\alpha (\not{k}'' + m) \gamma^\mu (\not{k}' + m) \gamma^\beta d_{\alpha\beta}(k)}{(p^- - k^- - k'^-)(p^- - p'^- - k'^- + k''^-)} \end{aligned} \quad (76)$$

respectively where $\lambda^{-1} = (2\pi)^{3/2} \sqrt{2p^+} \sqrt{2p'^+} \sqrt{2q^+}$. As mentioned earlier, figures 4c and 4d have not been evaluated earlier and hence we present the calculation of these in detail now.

In perturbation theory, the transition amplitude has the expansion

$$T = V + V \frac{1}{p^- - H_0} V + \dots \quad (77)$$

For figure 4c, the transition amplitude upto order e^3 is written as

$$\begin{aligned} T_{p,p',q}^{(c)} &= e^3 \bar{u}_{p',s'} \Lambda_{(c)}^\mu u_{p,s} \epsilon_{\tilde{\lambda}}^{\tilde{\lambda}}(q) \delta^3[\mathbf{p} \\ &\quad - (\mathbf{p}' + \mathbf{q})] \theta(p^+) \theta(p^+ - q^+) \end{aligned} \quad (78)$$

whereas for figure 4d, we write

$$\begin{aligned} T_{p,p',q}^{(d)} &= e^3 \bar{u}_{p',s'} \Lambda_{(d)}^\mu u_{p,s} \epsilon_{\tilde{\lambda}}^{\tilde{\lambda}}(q) \delta^3[\mathbf{p} \\ &\quad - (\mathbf{p}' + \mathbf{q})] \theta(p^+) \theta(p^+ - q^+). \end{aligned} \quad (79)$$

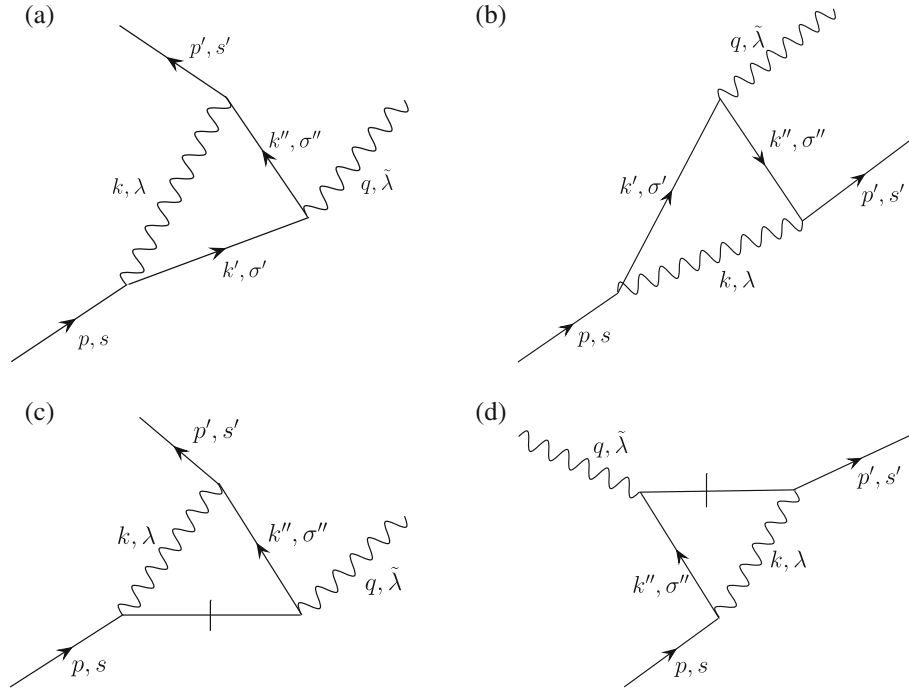


Figure 4. ‘Regular’ and instantaneous fermion exchange vertex correction diagrams.

The transition amplitudes due to the instantaneous fermion exchange diagrams are obtained, following the standard procedure, by inserting complete sets of states which lead to the following expressions (details are presented in Appendix B):

$$\begin{aligned}
 T_{p,p',q}^{(c)} &= \langle p', s'; q, \tilde{\lambda} | V_1 \frac{1}{p^- - H_0} V_2 | p, s \rangle \\
 &= \int d^3\mathbf{k}'' d^3\mathbf{k} d^3\mathbf{k}'_1 d^3\mathbf{k}_1 \theta(k''^+) \theta(k^+) \theta(k_1'^+) \theta(k_1^+) \\
 &\quad \times \sum_{\sigma'', \lambda, \sigma'_1, \lambda_1} \langle p', s'; q, \tilde{\lambda} | V_1 | k'', \sigma''; k, \lambda; q, \tilde{\lambda} \rangle \\
 &\quad \times \langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} | \frac{1}{p^- - H_0} | k'_1, \sigma'_1; k_1, \lambda_1; q, \tilde{\lambda} \rangle \\
 &\quad \times \langle k'_1, \sigma'_1; k_1, \lambda_1; q, \tilde{\lambda} | V_2 | p, s \rangle \\
 &= \int \frac{d^3\mathbf{k}'' d^3\mathbf{k} \theta(k''^+) \theta(k^+)}{p^- - k''^- - k^- - q^-} \sum_{\sigma'', \lambda} \langle p', s'; q, \tilde{\lambda} | V_1 | k'', \\
 &\quad \sigma''; k, \lambda; q, \tilde{\lambda} \rangle \langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} | V_2 | p, s \rangle \quad (80)
 \end{aligned}$$

for figure 4c and

$$\begin{aligned}
 T_{p,p',q}^{(d)} &= \langle p', s'; q, \tilde{\lambda} | V_2 \frac{1}{p^- - H_0} V_1 | p, s \rangle \\
 &= \int d^3\mathbf{k}'' d^3\mathbf{k} d^3\mathbf{k}'_1 d^3\mathbf{k}_1 \theta(k''^+) \theta(k^+) \theta(k_1'^+) \theta(k_1^+) \\
 &\quad \times \sum_{\sigma'', \lambda, \sigma'_1, \lambda_1} \langle p', s'; q, \tilde{\lambda} | V_2 | k'', \sigma''; k, \lambda \rangle \\
 &\quad \times \langle k'', \sigma''; k, \lambda | \frac{1}{p^- - H_0} | k'_1, \sigma'_1; k_1, \lambda_1 \rangle \\
 &\quad \times \langle k'_1, \sigma'_1; k_1, \lambda_1 | V_1 | p, s \rangle \\
 &= \int \frac{d^3\mathbf{k}'' d^3\mathbf{k} \theta(k''^+) \theta(k^+)}{p^- - k''^- - k^-} \sum_{\sigma'', \lambda} \langle p', s'; q, \tilde{\lambda} | V_2 | k'', \\
 &\quad \sigma''; k, \lambda \rangle \langle k'', \sigma''; k, \lambda | V_1 | p, s \rangle \quad (81)
 \end{aligned}$$

for figure 4d, where $d^3\mathbf{k} = dk^+ d^2\mathbf{k}_\perp$. Calculating each of these matrix elements by substituting the appropriate term of the interaction Hamiltonian and expanding the fields in their Fourier components, one obtains the following expressions for the transition amplitudes:

$$T_{p,p',q}^{(c)} = e^3 \bar{u}_{p',s'} \left[\lambda \int \frac{d^3 \mathbf{k} \theta(k^+) \theta(p'^+ - k^+)}{(4\pi)^3 k^+ k'^+ (p^+ - k^+)} \frac{\gamma^\alpha (\not{H}'' + m) \gamma^k \gamma^+ \gamma^j d_{\alpha j}(k)}{(p^- - k'^- - k^- - q^-)} \right] \\ \times u_{p,s} \tilde{\epsilon}_k^\lambda(q) \delta^3[\mathbf{p} - (\mathbf{p}' + \mathbf{q})] \theta(p^+) \theta(p^+ - q^+) \quad (82)$$

and

$$T_{p,p',q}^{(d)} = e^3 \bar{u}_{p',s'} \left[\lambda \int \frac{d^3 \mathbf{k} \theta(k^+) \theta(p^+ - k^+)}{(4\pi)^3 k^+ k'^+ (p^+ - k^+ - q^+)} \frac{\gamma^k \gamma^+ \gamma^j (\not{H}' + m) \gamma^\beta d_{k\beta}(k)}{(p^- - k'^- - k^-)} \right] \\ \times u_{p,s} \tilde{\epsilon}_j^\lambda(q) \delta^3[\mathbf{p} - (\mathbf{p}' + \mathbf{q})] \theta(p^+) \theta(p^+ - q^+) \quad (83)$$

respectively for the two diagrams. Using eq. (B.4), we obtain

$$T_{p,p',q}^{(c)} = e^3 \bar{u}_{p',s'} \left[\lambda \int_{-\infty}^{+\infty} \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \right. \\ \left. \times \frac{\gamma^\alpha (\not{H}'' + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k)}{(p'^- - k^- - k''^-)} \right] \\ \times u_{p,s} \tilde{\epsilon}_\mu^\lambda(q) \delta^3[\mathbf{p} - (\mathbf{p}' + \mathbf{q})] \theta(p^+) \theta(p^+ - q^+) \quad (84)$$

and hence,

$$\Lambda_{(c)}^\mu = \lambda \int_{-\infty}^{+\infty} \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \\ \times \frac{\gamma^\alpha (\not{H}'' + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k)}{(p'^- - k^- - k''^-)}. \quad (85)$$

In a similar fashion, using eq. (B.5), we obtain

$$T_{p,p',q}^{(d)} = e^3 \bar{u}_{p',s'} \left[\lambda \int_{-\infty}^{+\infty} \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \right. \\ \left. \times \frac{\gamma^\alpha \gamma^+ \gamma^\mu (\not{H}' + m) \gamma^\beta d_{\alpha\beta}(k)}{(p^- - k^- - k'^-)} \right] \\ \times u_{p,s} \tilde{\epsilon}_\mu^\lambda(q) \delta^3[\mathbf{p} - (\mathbf{p}' + \mathbf{q})] \theta(p^+) \theta(p^+ - q^+) \quad (86)$$

and hence,

$$\Lambda_{(d)}^\mu = \lambda \int_{-\infty}^{+\infty} \frac{d^2 \mathbf{k}_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \\ \times \frac{\gamma^\alpha \gamma^+ \gamma^\mu (\not{H}' + m) \gamma^\beta d_{\alpha\beta}(k)}{(p^- - k^- - k'^-)}. \quad (87)$$

6. Equivalence of covariant and LF expressions of one-loop vertex correction

In this section, we present the proof of equivalence for the one-loop vertex correction. In ref. [12], we established the equivalence of covariant and LF expressions for Λ^+ , i.e. the ‘+’ component of the one-loop vertex correction Λ^μ . Here, we present a more general proof which is valid for all components of Λ^μ .

The standard covariant expression for vertex correction in the LF gauge comes from figure 4a which is the only diagram that contributes to Λ^μ in covariant theory. It is given by

$$\Lambda^\mu(p, p', q) = i e^3 \int \frac{d^4 k}{(2\pi)^4} \\ \times \frac{\gamma^\alpha (\not{p}' - \not{k} + m) \gamma^\mu (\not{p} - \not{k} + m) \gamma^\beta d'_{\alpha\beta}(k)}{[(p-k)^2 - m^2 + i\epsilon][(p'-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]}, \quad (88)$$

where we have used the three-term photon propagator

$$d'_{\alpha\beta}(k) = d_{\alpha\beta}(k) - \frac{\delta_{\alpha+} \delta_{\beta+} k^2}{(k^+)^2} \\ = -g_{\alpha\beta} + \frac{\delta_{\alpha+} k_\beta + \delta_{\beta+} k_\alpha}{k^+} - \frac{\delta_{\alpha+} \delta_{\beta+} k^2}{(k^+)^2}.$$

In order to show that this standard covariant expression for vertex correction is equivalent to the expressions calculated in the LF time-ordered perturbation theory diagrams given in figure 4, we split the fermion momenta into on-shell and off-shell parts as was done in the case of fermion self-energy and vacuum polarisation. Similar to eq. (17), $\not{p}' - \not{k}$ can be written as

$$p' - k = k''_{\text{on}} + \frac{\gamma^+ [(p' - k)^2 - m^2]}{2(p'^+ - k^+)}. \quad (89)$$

Using eqs (17) and (89), eq. (88) becomes

There are sufficient powers of k^- in the denominators of $\Lambda_{1,p,p',q}^\mu$ and $\Lambda_{2,p,p',q}^\mu$ to make the k^- -integral vanish on the arc at infinity and hence, there are no arc contri-

$$\begin{aligned} \Lambda^\mu(p, p', q) = & ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha (k''_{\text{on}} + m) \gamma^\mu (k'_{\text{on}} + m) \gamma^\beta d'_{\alpha\beta}(k)}{[(p - k)^2 - m^2 + i\epsilon][(p' - k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \\ & + ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha (k''_{\text{on}} + m) \gamma^\mu \gamma^+ \gamma^\beta d'_{\alpha\beta}(k)}{2(p^+ - k^+) [(p' - k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \\ & + ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha \gamma^+ \gamma^\mu (k'_{\text{on}} + m) \gamma^\beta d'_{\alpha\beta}(k)}{2(p'^+ - k^+) [(p - k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \\ & + ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha \gamma^+ \gamma^\mu \gamma^+ \gamma^\beta d'_{\alpha\beta}(k)}{2(p^+ - k^+) 2(p'^+ - k^+) [k^2 - \mu^2 + i\epsilon]}. \end{aligned} \quad (90)$$

The last integral in eq. (90) does not contribute to the transition amplitude $T_{p,p',q} = \bar{u}_{p,s'} \Lambda_{p,p',q}^\mu u_{p,s} \epsilon_{\tilde{\lambda}}(q)$ as can be seen using $\epsilon_{\tilde{\lambda}} = 0$, $(\gamma^+)^2 = 0$ and the anticommutation relations of γ -matrices. The identity $(\gamma^+)^2 = 0$ also leads to the fact that the third term of the photon propagator, viz. $-\frac{\delta_{\alpha+\beta+k^2}}{(k^+)^2}$ provides null contributions to the second and third integrals of eq. (90). Hence, eq. (90) reduces to

$$\Lambda_{p,p',q}^\mu = \Lambda_{1,p,p',q}^\mu + \Lambda_{2,p,p',q}^\mu + \Lambda_{3,p,p',q}^\mu + \Lambda_{4,p,p',q}^\mu, \quad (91)$$

butions in the case of $\Lambda_{1,p,p',q}^\mu$ and $\Lambda_{2,p,p',q}^\mu$. Thus, a naive contour integration using the method of residues gives the required result. The integrals are explicitly evaluated below.

$\Lambda_{1,p,p',q}^\mu$ can be written as

$$\Lambda_{1,p,p',q}^\mu = ie^3 \int \frac{d^2\mathbf{k}_\perp dk^+}{(2\pi)^4} \frac{\gamma^\alpha (k''_{\text{on}} + m) \gamma^\mu (k'_{\text{on}} + m) \gamma^\beta}{2k^+ 2(p^+ - k^+) 2(p'^+ - k^+)} I_1, \quad (96)$$

where

$$I_1 = \int \frac{dk^- d_{\alpha\beta}(k)}{\left[k^- - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] \right] \left[p^- - k^- - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] \right] \left[p'^- - k^- - \left[\frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)} \right] \right]} \quad (97)$$

where

$$\Lambda_{1,p,p',q}^\mu = ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha (k''_{\text{on}} + m) \gamma^\mu (k'_{\text{on}} + m) \gamma^\beta d_{\alpha\beta}(k)}{[(p - k)^2 - m^2 + i\epsilon][(p' - k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]}, \quad (92)$$

$$\Lambda_{2,p,p',q}^\mu = -ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^+ (k''_{\text{on}} + m) \gamma^\mu (k'_{\text{on}} + m) \gamma^+}{(k^+)^2 [(p - k)^2 - m^2 + i\epsilon][(p' - k)^2 - m^2 + i\epsilon]}, \quad (93)$$

$$\Lambda_{3,p,p',q}^\mu = ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha (k''_{\text{on}} + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k)}{2(p^+ - k^+) [(p' - k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]}, \quad (94)$$

$$\Lambda_{4,p,p',q}^\mu = ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha \gamma^+ \gamma^\mu (k'_{\text{on}} + m) \gamma^\beta d_{\alpha\beta}(k)}{2(p'^+ - k^+) [(p - k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]}. \quad (95)$$

which has poles at

$$k_1^- = \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+}, \quad k_2^- = p^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}$$

and

$$k_3^- = p'^- - \frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)}.$$

For $k^+ < 0$, all the three poles lie above the real axis. Thus, by closing the contour in the lower half-plane, the integral vanishes. Similarly, for $k^+ > p^+$, as all the three poles lie below the real axis, the integral vanishes on closing the contour in the upper half-plane. For $0 < k^+ < p^+$, we close the contour below the real axis. k_2^- and k_3^- do not contribute as they fall outside the contour. The only contribution to I_1 for $0 < k^+ < p^+$ comes from the pole at k_1^- and using the residue theorem one obtains

which leads to

$$\begin{aligned} \Lambda_{1,p,p',q}^\mu &= e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+k'^+k''^+} \\ &\times \frac{\gamma^\alpha(\mathcal{H}'_{\text{on}} + m)\gamma^\mu(\mathcal{H}'_{\text{on}} + m)\gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p^- - k_{\text{on}}^- - k'_{\text{on}}^-)(p^- - q^- - k_{\text{on}}^- - k''_{\text{on}}^-)} \\ &- e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{k^+k'^+k''^+} \\ &\times \frac{\gamma^\alpha(\mathcal{H}'_{\text{on}} + m)\gamma^\mu(\mathcal{H}'_{\text{on}} + m)\gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p^- - k_{\text{on}}^- - k'_{\text{on}}^-)(p^- - p'^- - k'_{\text{on}}^- + k''_{\text{on}}^-)} \\ &- 2e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{(k^+)^2 k'^+ k''^+} \\ &\times \frac{\gamma^+(\mathcal{H}'_{\text{on}} + m)\gamma^\mu(\mathcal{H}'_{\text{on}} + m)\gamma^+}{(p^- - p'^- - k'_{\text{on}}^- + k''_{\text{on}}^-)}. \end{aligned} \quad (102)$$

Next, we consider eq. (93), which can be written as

$$I_1 = \frac{-2\pi i d_{\alpha\beta}(k_{\text{on}})\theta(k^+)\theta(p'^+ - k^+)}{\left[p^- - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] \right] \left[p'^- - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] - \left[\frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)} \right] \right]}. \quad (98)$$

For $p'^+ < k^+ < p^+$, only k_2^- contributes to I_1 on closing the contour above the real axis because k_1^- and k_3^- lie below the real axis. This contribution is equal to

$$I_1 = \frac{-2\pi i d_{\alpha\beta}(k^+, k_2^-, k^\perp)\theta(k^+ - p'^+)\theta(p^+ - k^+)}{\left[p^- - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] \right] \left[p'^- - p^- + \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] - \left[\frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)} \right] \right]}. \quad (99)$$

Thus,

$$\begin{aligned} \Lambda_{1,p,p',q}^\mu &= e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+k'^+k''^+} \frac{\gamma^\alpha(\mathcal{H}'_{\text{on}} + m)\gamma^\mu(\mathcal{H}'_{\text{on}} + m)\gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p^- - k_{\text{on}}^- - k'_{\text{on}}^-)(p^- - q^- - k_{\text{on}}^- - k''_{\text{on}}^-)} \\ &- e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{k^+k'^+k''^+} \frac{\gamma^\alpha(\mathcal{H}'_{\text{on}} + m)\gamma^\mu(\mathcal{H}'_{\text{on}} + m)\gamma^\beta d_{\alpha\beta}(k^+, k_2^-, k^\perp)}{(p^- - k_{\text{on}}^- - k'_{\text{on}}^-)(p^- - p'^- - k'_{\text{on}}^- + k''_{\text{on}}^-)}. \end{aligned} \quad (100)$$

However,

$$d_{\alpha\beta}(k_2^-) = d_{\alpha\beta}(k_{\text{on}}^-) + \frac{2(p^- - k'_{\text{on}}^- - k_{\text{on}}^-)\delta_\alpha + \delta_{\beta+}}{k^+} \quad (101)$$

$$\begin{aligned} \Lambda_{2,p,p',q}^\mu &= -ie^3 \int \frac{d^2\mathbf{k}_\perp dk^+}{(2\pi)^4} \frac{\gamma^+(\mathcal{H}'_{\text{on}} + m)\gamma^\mu(\mathcal{H}'_{\text{on}} + m)\gamma^+}{(k^+)^2 2(p^+ - k^+)2(p'^+ - k^+)} I_2, \end{aligned} \quad (103)$$

where

$$I_2 = \int \frac{dk^-}{\left[p^- - k^- - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] \right] \left[p'^- - k^- - \left[\frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)} \right] \right]} \quad (104)$$

I_2 has a pole at

$$k_1^- = p^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}$$

and

$$k_2^- = p'^- - \frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)}.$$

For $k^+ < p^+$, both the poles lie above the real axis and the integral vanishes on closing the contour below it whereas for $k^+ > p^+$, they lie below the real axis and hence the integral goes to zero when the contour is closed above. For $p^+ > k^+ > p'^+$, we close the contour below the real axis. Thus, the only contribution to I_2 comes from the residue at k_2^- and is equal to

$$I_2 = \frac{2\pi i \theta(k^+ - p'^+) \theta(p^+ - k^+)}{\left[p^- - p'^- - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] + \left[\frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)} \right] \right]} \quad (105)$$

Thus,

$$\Lambda_{2,p,p',q}^\mu = 2e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \times \int_{p'^+}^{p^+} \frac{dk^+}{(k^+)^2 k'^+ k''^+} \frac{\gamma^+ (\mathcal{W}'_{\text{on}} + m) \gamma^\mu (\mathcal{W}'_{\text{on}} + m) \gamma^+}{(p^- - p'^- - k'^-_{\text{on}} + k''^+_{\text{on}})} \quad (106)$$

$$I_3 = \int \frac{dk^- d_{\alpha\beta}(k)}{\left[k^- - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] \right] \left[p'^- - k^- - \left[\frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)} \right] \right]} \quad (111)$$

It is to be noted that the last term of eq. (102) is cancelled by $\Lambda_{2,p,p',q}^\mu$, which actually has arisen from the third term of the photon propagator.

The numerator of the integrand in $\Lambda_{3,p,p',q}^\mu$ of eq. (94) can be written as

$$\gamma^\alpha (\mathcal{W}'_{\text{on}} + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k) = 2\gamma^+ \gamma^\mu \mathcal{W}'_{\text{on}} + k''^+ \left(2\gamma^\mu \gamma^+ \gamma^- - \frac{2\gamma^\mu \gamma^+ \gamma^\perp \cdot \mathbf{k}^\perp}{k^+} \right) - 4mg^{\mu+}$$

$$-2g^{+\mu} (\mathcal{W}'_{\text{on}} - m) \left(\gamma^+ \gamma^- - \frac{\gamma^+ \gamma^\perp \cdot \mathbf{k}^\perp}{k^+} \right) \quad (107)$$

using the identities

$$\begin{aligned} &\gamma^\alpha \gamma^\mu \gamma^\nu \gamma^\beta d_{\alpha\beta}(k) \\ &= -4g^{\mu\nu} + \frac{2k_\rho}{k^+} [g^{\mu\rho} \gamma^\nu \gamma^+ - g^{\rho\nu} \gamma^\mu \gamma^+ \\ &\quad + g^{\rho+} \gamma^\mu \gamma^\nu - g^{+\nu} \gamma^\mu \gamma^\rho + g^{+\mu} \gamma^\nu \gamma^\rho] \end{aligned} \quad (108)$$

and

$$\begin{aligned} &\gamma^\alpha \gamma^\sigma \gamma^\mu \gamma^\nu \gamma^\beta d_{\alpha\beta}(k) \\ &= \frac{2k_\rho}{k^+} [g^{\rho+} \gamma^\nu \gamma^\mu \gamma^\sigma + g^{+\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \\ &\quad + g^{\rho\sigma} \gamma^\mu \gamma^\nu \gamma^+ - g^{\rho\mu} \gamma^\sigma \gamma^\nu \gamma^+ \\ &\quad + g^{\rho\nu} \gamma^\sigma \gamma^\mu \gamma^+ + g^{+\nu} \gamma^\sigma \gamma^\mu \gamma^\rho \\ &\quad - g^{+\mu} \gamma^\sigma \gamma^\nu \gamma^\rho - g^{\rho+} \gamma^\sigma \gamma^\mu \gamma^\nu] \end{aligned} \quad (109)$$

which can easily be derived using the anticommutation relations of γ -matrices. As there are no terms involving k^- in the numerator, there are no arc contributions to the contour integral. Equation (94) can thus be written as

$$\Lambda_{3,p,p',q}^\mu = ie^3 \int \frac{d^2\mathbf{k}_\perp dk^+}{(2\pi)^4} \frac{\gamma^\alpha (\mathcal{W}'_{\text{on}} + m) \gamma^\mu \gamma^+ \gamma^\beta}{2k^+ 2(p^+ - k^+) 2(p'^+ - k^+)} I_3, \quad (110)$$

where

which has poles at

$$k_1^- = \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+}$$

and

$$k_2^- = p'^- - \frac{(\mathbf{p}'_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)}.$$

For $k^+ < 0$, both k_1^- and k_2^- lie above while for $k^+ > p'^+$, both lie below the real axis. Hence, $I_3 = 0$ for these

ranges of k^+ . Thus, I_3 is non-zero only for $0 < k^+ < p'^+$ and is equal, on closing the contour below the real axis, to the residue calculated at the pole k_1^- . Hence,

$$I_3 = \frac{-2\pi i \theta(k^+) \theta(p'^+ - k^+) d_{\alpha\beta}(k_{\text{on}})}{\left[p'^- - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p'^+ - k^+)} \right] \right]} \quad (112)$$

Therefore,

$$\Lambda_{3,p,p',q}^\mu = e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \times \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \frac{\gamma^\alpha (\mathcal{H}'_{\text{on}} + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p'^- - k_{\text{on}}^- - k''_{\text{on}}^-)} \quad (113)$$

Similarly, the numerator of the integrand in $\Lambda_{4,p,p',q}^\mu$ of eq. (95) can be written as

$$\begin{aligned} & \gamma^\alpha \gamma^+ \gamma^\mu (\mathcal{H}'_{\text{on}} + m) \gamma^\beta d_{\alpha\beta}(k) \\ &= 2 \left[\mathcal{H}'_{\text{on}} \gamma^\mu \gamma^+ + \gamma^\mu \mathcal{H}'_{\text{on}} \gamma^+ - g^{-\mu} \gamma^+ \mathcal{H}'_{\text{on}} \gamma^+ \right. \\ & \quad + \frac{k^\perp \cdot g^{\perp\mu} \gamma^+ \mathcal{H}'_{\text{on}} \gamma^+}{k^+} \\ & \quad + k_{\text{on}}'^- \gamma^+ \gamma^\mu \gamma^+ - \frac{k'^\perp \cdot k^\perp \gamma^+ \gamma^\mu \gamma^+}{k^+} \\ & \quad + 2k'^+ \gamma^+ \gamma^\mu \gamma^- - \frac{k'^+ \gamma^+ \gamma^\mu \gamma^\perp \cdot k^\perp}{k^+} \\ & \quad - g^{+\mu} \gamma^+ \mathcal{H}'_{\text{on}} \gamma^- + \frac{g^{+\mu} \gamma^+ \mathcal{H}'_{\text{on}} \gamma^\perp \cdot k^\perp}{k^+} \\ & \quad \left. - \gamma^+ \gamma^\mu \mathcal{H}'_{\text{on}} - m g^{+\mu} \gamma^+ \gamma^- - \frac{m g^{+\mu} \gamma^+ \gamma^\perp \cdot k^\perp}{k^+} \right] \\ & \quad + 4k^- \left[\frac{k'^+ \gamma^+ \gamma^\mu \gamma^+}{k^+} - \frac{g^{+\mu} \gamma^+ \mathcal{H}'_{\text{on}} \gamma^+}{k^+} \right]. \quad (114) \end{aligned}$$

Here, the coefficient of k^- is

$$4 \left[\frac{k'^+ \gamma^+ \gamma^\mu \gamma^+}{k^+} - \frac{g^{+\mu} \gamma^+ \mathcal{H}'_{\text{on}} \gamma^+}{k^+} \right]$$

and the two terms in the bracket cancel. Thus, in the case of $\Lambda_{4,p,p',q}^\mu$ too, there are no terms involving k^- in the numerator. As a result, arc contributions to the contour integral are absent. Thus, eq. (95) is written as

$$\Lambda_{4,p,p',q}^\mu = ie^3 \int \frac{d^2\mathbf{k}_\perp dk^+}{(2\pi)^4} \frac{\gamma^\alpha \gamma^+ \gamma^\mu (\mathcal{H}'_{\text{on}} + m) \gamma^\beta}{2k^+ 2(p^+ - k^+) 2(p'^+ - k^+)} I_4, \quad (115)$$

where

$$I_4 = \int \left[k^- - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] \right] \left[p^- - k^- - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] \right] dk^- d_{\alpha\beta}(k) \quad (116)$$

which has poles at

$$k_1^- = \frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+}$$

and

$$k_2^- = p^- - \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}.$$

Same arguments as for I_3 follow with p' replaced by p . Hence,

$$I_4 = \frac{-2\pi i \theta(k^+) \theta(p^+ - k^+) d_{\alpha\beta}(k_{\text{on}})}{\left[p^- - \left[\frac{\mathbf{k}_\perp^2 + \mu^2 - i\epsilon}{2k^+} \right] - \left[\frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)} \right] \right]} \quad (117)$$

leading to

$$\Lambda_{4,p,p',q}^\mu = e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \times \int_0^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \frac{\gamma^\alpha \gamma^+ \gamma^\mu (\mathcal{H}'_{\text{on}} + m) \gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p^- - k_{\text{on}}^- - k''_{\text{on}}^-)} \quad (118)$$

Substituting eqs (102), (106), (113) and (118) in eq. (91), we see that

$$\begin{aligned} \Lambda_{p,p',q}^\mu &= e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \\ & \quad \times \frac{\gamma^\alpha (\mathcal{H}'_{\text{on}} + m) \gamma^\mu (\mathcal{H}'_{\text{on}} + m) \gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p^- - k_{\text{on}}^- - k''_{\text{on}}^-) (p^- - q^- - k_{\text{on}}^- - k''_{\text{on}}^-)} \\ & - e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \\ & \quad \times \frac{\gamma^\alpha (\mathcal{H}'_{\text{on}} + m) \gamma^\mu (\mathcal{H}'_{\text{on}} + m) \gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p^- - k_{\text{on}}^- - k''_{\text{on}}^-) (p^- - p'^- - k''_{\text{on}}^- + k'_{\text{on}}^-)} \\ & + e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \\ & \quad \times \frac{\gamma^\alpha (\mathcal{H}'_{\text{on}} + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p'^- - k_{\text{on}}^- - k''_{\text{on}}^-)} \\ & + e^3 \int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \end{aligned}$$

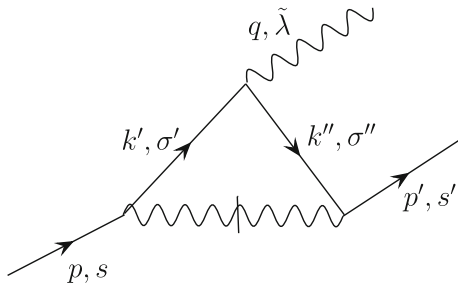


Figure 5. Instantaneous photon exchange diagram.

$$\times \frac{\gamma^\alpha \gamma^+ \gamma^\mu (\not{k}'_{\text{on}} + m) \gamma^\beta d_{\alpha\beta}(k_{\text{on}})}{(p^- - k_{\text{on}}^- - k'_{\text{on}}^-)}. \quad (119)$$

Taking into account the normalisation factor λ given by

$$\lambda^{-1} = (2\pi)^{3/2} \sqrt{2p^+} \sqrt{2p'^+} \sqrt{2q^+},$$

each term in eq. (119) is equal to the expressions for diagrams in figures 4a through 4d, viz. eqs (75), (76), (85), (87) respectively. The following inferences can thus be drawn:

- (i) A general component of the vertex correction receives non-zero contributions from $\Lambda_{3p,p',q}^\mu$ and $\Lambda_{4p,p',q}^\mu$ which produce instantaneous fermion diagrams. These contributions were absent in ref. [12] and were not evaluated in ref. [8] as only the ‘+’ component of the vertex correction was considered in both the works. These contributions arise from the off-shell part of fermion propagator.
- (ii) The on-shell part of the fermion propagator when considered with the three-term photon propagator in the covariant theory corresponds to the regular vertex correction diagrams in LFTOPT.
- (iii) Had we started with the two-term photon propagator in eq. (88) instead of the three-term propagator used here, eq. (93) and in turn, eq. (106) would have been absent, thus retaining the last term in eq. (102). This term corresponds to the instantaneous photon exchange diagram given in figure 5.
- (iv) No contribution to vertex correction is received when we consider the off-shell parts of both propagators simultaneously.

7. Summary

We have re-visited the issue of equivalence of covariant QED and LFQED with special emphasis on which form

of the photon propagator should be used in the proof of equivalence. We observe that in covariant formulation of QED, the three-term propagator is derived from the Lagrangian in eq. (3) wherein both Lorenz condition as well as the gauge fixing condition $A^+ = 0$ have been taken into account in the form of Lagrange’s multiplier. In contrast, the LFQED Hamiltonian in ref. [8], which has been the reference point for the earlier work on this subject of equivalence at one-loop level, is actually derived by eliminating the dependent degrees of freedom using only the LF gauge fixing condition. We, therefore, derive the LFQED Hamiltonian (eq. (10)) following the procedure in ref. [8] but now also taking into account the Lorenz condition. We find that this Hamiltonian does not have the instantaneous photon exchange interaction and therefore the set of one-loop graphs in this theory does not contain the diagrams involving instantaneous photon exchange. We consider this theory and show that indeed the one-loop graphs of this theory can be obtained from the covariant expressions containing the three-term propagator by integrating over the LF energy k^- . We compare our results with the work of Mantovani *et al* who have established equivalence of one-loop expressions with the expressions in ref. [8] using the two-term photon propagator. Justification for using the two-term propagator, as given by Mantovani *et al*, is that the contribution of the third term in the propagator cancels the contribution of the instantaneous interaction (the last term in eq. (4)) and therefore, it is sufficient to work with the two-term propagator. Thus, it is clear that the issue of equivalence as addressed by Matovani *et al* and by us is at different levels. Our aim in this work is to establish the equivalence of covariant formulation of QED in LF gauge in instant form (eq. (3)) with the Hamiltonian formulation of LFQED in LF gauge at one-loop level. Mantovani *et al* [14], on the other hand, have compared the Lagrangian formulation of LFQED in LF gauge (eq. (4)) with the corresponding Hamiltonian version. As we start with the theory based on manifestly covariant Lagrangian in eq. (3), the photon propagator will have the third term also and the LFQED Hamiltonian to be used for deriving LF Feynman rules for corresponding theory will be given by the Hamiltonian in eq. (10). On the other hand, if one starts with the interaction Lagrangian in eq. (4), there is no need to add the Lagrange’s multiplier (as one has already used the condition to eliminate the unphysical degrees of freedom) and hence it is sufficient to use the two-term propagator only.

We stress on the fact that the off-shell propagator used in ref. [14] in the covariant expression is the two-term propagator which on being split into off-shell and on-shell parts gives the third term. What Mantovani *et al*

have done in §III of ref. [14] is a splitting of the essentially two-term propagator at a general momentum into on-shell and off-shell parts (which is not the same as the three-term propagator at a general momentum used by us) and it indeed does lead to equivalence with the results of Mustaki *et al* (ref. [8]). This is because Mustaki *et al* use only the LF gauge condition and hence results from Mustaki *et al* can be consistent with the results of covariant formulation only if the two-term propagator (which is obtained by using only LF gauge condition as a Lagrange multiplier, avoiding the Lorenz condition) is used. This is different from using the three-term ‘off-shell’ propagator as a starting point in covariant expression as done by us in §3. The primary purpose of the work is to show equivalence between covariant and LF approaches and in this regard, we have shown that either the two-term or the three-term photon propagator can be used as long as consistency of gauge choice is maintained in deriving the photon propagator and the interactions in the LF Hamiltonian. As shown by us, if one uses the three-term propagator (which results from using both the LF gauge condition and the Lorenz condition as Lagrange multipliers) without putting on-shell condition, the instantaneous photon exchange diagram in Mustaki *et al* is not reproduced. However, this in no way hampers the proof of equivalence between the covariant and LF formulation.

After clarifying the issue of the form of the photon propagator, we have established equivalence between the equal-time covariant QED and LF time-ordered Hamiltonian QED at the level of one-loop Feynman diagrams using two methods. In §3, we used the method of splitting the photon propagator in on-shell and off-shell parts [14] to establish equivalence for fermion self-energy and vacuum polarisation graphs. In §4, we introduced an alternative method called the asymptotic method and verified the results of §3.1 using this method. In order to establish equivalence for a general component of one-loop vertex correction, we first calculated the instantaneous fermion exchange graphs contributing to one-loop vertex correction in §5 which were not calculated in earlier works. We have then extended our earlier proof of equivalence of vertex correction graphs to a general component of Λ^μ . The asymptotic method was used by us in ref. [13] to show that the covariant expression for one-loop vacuum polarisation reproduces the corresponding LFQED diagrams on performing the k^- -integration. In the present work, we have shown that all the one-loop self-energy and vertex correction diagrams of LFQED can also be reproduced starting from the covariant expressions using the asymptotic method. We establish this within both our approach as well as in the approach of ref. [14].

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Appendix A: Basics and conventions

The 4-vector x^μ , in LF coordinates, has the components $(x^+, x^-, \mathbf{x}^\perp)$ where

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^3}{\sqrt{2}}, \quad \mathbf{x}^\perp = (x^1, x^2).$$

We use the following metric tensor:

$$g_{\alpha\beta} = g^{\alpha\beta} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

The following representation is used for the γ -matrices:

$$\begin{aligned} \gamma^0 &= \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \quad \gamma^k = \begin{bmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{bmatrix}, \\ \gamma^+ &= \frac{\gamma^0 + \gamma^3}{\sqrt{2}}, \quad \gamma^- = \frac{\gamma^0 - \gamma^3}{\sqrt{2}}. \end{aligned} \quad (\text{A.1})$$

The γ -matrices satisfy

$$\begin{aligned} \{\gamma^\alpha, \gamma^\beta\} &= 2g^{\alpha\beta} \\ (\gamma^+)^2 &= (\gamma^-)^2 = 0 \\ (\gamma^0)^\dagger &= \gamma^0 \\ (\gamma^k)^\dagger &= -\gamma^k \quad \text{for } k = 1, 2, 3 \\ \gamma^+ \gamma^- \gamma^+ &= 2\gamma^+, \quad \gamma^- \gamma^+ \gamma^- = 2\gamma^- \\ \gamma^\alpha \gamma^\mu \gamma^\beta d_{\alpha\beta}(k) &= \frac{2}{k^+} (\gamma^+ k^\mu + g^{+\mu} \not{k}). \end{aligned} \quad (\text{A.2})$$

The Dirac spinors satisfy the following properties:

$$\begin{aligned} \bar{u}_{p,s} u_{p,s'} &= -\bar{v}_{p,s} v_{p,s'} = 2m \delta_{ss'} \\ \bar{u}_{p,s} \gamma^\mu u_{p,s'} &= -\bar{v}_{p,s} \gamma^\mu v_{p,s'} = 2p^\mu \delta_{ss'} \end{aligned} \quad (\text{A.3})$$

and the completeness relations

$$\begin{aligned} \sum_{s=\pm 1/2} u_{p,s} \bar{u}_{p,s} &= \not{p} + m \\ \sum_{s=\pm 1/2} v_{p,s} \bar{v}_{p,s} &= \not{p} - m. \end{aligned} \quad (\text{A.4})$$

For photon polarisations, we choose

$$\epsilon_{\mu}^1 = \left(\frac{p^1}{p^+}, 0, -1, 0 \right), \quad \epsilon_{\mu}^2 = \left(\frac{p^2}{p^+}, 0, 0, -1 \right). \quad (\text{A.5})$$

The null-plane Hamiltonian is

$$P^- = H_0 + V_1 + V_2 + V_3$$

when the gauge field satisfies the LF gauge condition and

$$P^- = H_0 + V_1 + V_2$$

when the gauge field satisfies the LF gauge condition as well as the Lorenz condition.

Here, in addition to the free Hamiltonian H_0 and the standard three-point order- e interaction

$$V_1 = e \int d^2\mathbf{x}_{\perp} dx \bar{\xi} \gamma^{\mu} \xi a_{\mu}, \quad (\text{A.6})$$

there exist additional order- e^2 non-local interactions

$$V_2 = -\frac{i}{4} e^2 \int d^2\mathbf{x}_{\perp} dx^- dy^- \epsilon(x^- - y^-) \times (\bar{\xi} a_k \gamma^k)(x) \gamma^+ (a_j \gamma^j \xi)(y) \quad (\text{A.7})$$

corresponding to an instantaneous fermion exchange and

$$V_3 = -\frac{e^2}{4} \int d^2\mathbf{x}_{\perp} dx^- dy^- (\bar{\xi} \gamma^+ \xi)(x) \times |x^- - y^-| (\bar{\xi} \gamma^+ \xi)(y) \quad (\text{A.8})$$

corresponding to an instantaneous photon exchange.

Appendix B

B.1: LFTOPT diagram calculations for vertex correction

In this appendix, we present the details of the calculation of the expression for the diagram of figure 4c. The transition amplitude that contributes to one-loop correction arising from figure 4c is

$$\begin{aligned} T_{p,p',q}^{(c)} &= \left\langle p', s'; q, \tilde{\lambda} \left| V_1 \frac{1}{p^- - H_0} V_2 \right| p, s \right\rangle \\ &= \int_{-\infty}^{+\infty} d^2\mathbf{k}_{\perp}'' d^2\mathbf{k}_{\perp} d^2\mathbf{k}_{1\perp}'' d^2\mathbf{k}_{1\perp} \\ &\quad \times \int_0^{\infty} dk''^+ dk^+ dk_1''^+ dk_1^+ \end{aligned}$$

$$\begin{aligned} &\times \sum_{\sigma'', \lambda, \sigma_1'', \lambda_1} \left\langle p', s'; q, \tilde{\lambda} \left| V_1 \right| k'', \sigma''; k, \lambda; q, \tilde{\lambda} \right\rangle \\ &\times \left\langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} \left| \frac{1}{p^- - H_0} \right| k_1'', \sigma_1''; k_1, \lambda_1; q, \tilde{\lambda} \right\rangle \\ &\times \left\langle k_1'', \sigma_1''; k_1, \lambda_1; q, \tilde{\lambda} \left| V_2 \right| p, s \right\rangle \\ &= \int \frac{d^3\mathbf{k}'' d^3\mathbf{k} d^3\mathbf{k}_1'' d^3\mathbf{k}_1 \theta(k''^+) \theta(k^+) \theta(k_1''^+) \theta(k_1^+)}{p^- - k_1''^- - k_1^- - q^-} \\ &\times \sum_{\sigma'', \lambda, \sigma_1'', \lambda_1} \left\langle p', s'; q, \tilde{\lambda} \left| V_1 \right| k'', \sigma''; k, \lambda; q, \tilde{\lambda} \right\rangle \\ &\times \left\langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} \left| k_1'', \sigma_1''; k_1, \lambda_1; q, \tilde{\lambda} \right\rangle \right. \\ &\times \left. \left\langle k_1'', \sigma_1''; k_1, \lambda_1; q, \tilde{\lambda} \left| V_2 \right| p, s \right\rangle \right. \\ &= \int \frac{d^3\mathbf{k}'' d^3\mathbf{k} \theta(k''^+) \theta(k^+)}{p^- - k''^- - k^- - q^-} \\ &\times \sum_{\sigma'', \lambda} \left\langle p', s'; q, \tilde{\lambda} \left| V_1 \right| k'', \sigma''; k, \lambda; q, \tilde{\lambda} \right\rangle \\ &\times \left\langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} \left| V_2 \right| p, s \right\rangle, \quad (\text{B.1}) \end{aligned}$$

where the orthonormality of states is used to arrive at the final step. Using eqs (A.6) and (A.7), the matrix elements in the above expression for transition amplitude, on Fourier expanding the fields, are written as

$$\begin{aligned} &\left\langle p', s'; q, \tilde{\lambda} \left| V_1 \right| k'', \sigma''; k, \lambda; q, \tilde{\lambda} \right\rangle \\ &= e \int d^2\mathbf{x}_{\perp} dx^- \int_{-\infty}^{+\infty} \frac{d^2\mathbf{p}_{1\perp} d^2\mathbf{p}_{2\perp} d^2\mathbf{q}_{1\perp}}{(2\pi)^{9/2} \sqrt{8}} \\ &\times \int_0^{\infty} \frac{dp_1^+ dp_2^+ dq_1^+}{\sqrt{p_1^+ p_2^+ q_1^+}} \\ &\times \sum_{s_1, s_2, \lambda_1} \left\langle p', s'; q, \tilde{\lambda} \left| \left[\bar{u}_{p_1, s_1} e^{ip_1 \cdot x} b_{p_1, s_1, x}^{\dagger} \right. \right. \right. \\ &\quad \left. \left. \left. + \bar{v}_{p_1, s_1} e^{-ip_1 \cdot x} d_{p_1, s_1, x} \right] \gamma^{\mu} \right. \right. \\ &\quad \times \left. \left[u_{p_2, s_2} e^{-ip_2 \cdot x} b_{p_2, s_2, x} + v_{p_2, s_2} e^{ip_2 \cdot x} d_{p_2, s_2, x}^{\dagger} \right] \epsilon_{\mu}^{\lambda_1}(q_1) \right. \\ &\quad \left. \times \left[e^{-iq_1 \cdot x} a_{q_1, \lambda_1, x} + e^{iq_1 \cdot x} a_{q_1, \lambda_1, x}^{\dagger} \right] \right| k'', \sigma''; k, \lambda; q, \tilde{\lambda} \right\rangle \end{aligned}$$

where $e^{ip_1 \cdot x} = e^{i[p_1^+ x^- - \mathbf{p}_{1\perp} \cdot \mathbf{x}_{\perp}]}$ etc. Using

$$\begin{aligned} &\left\langle p', s'; q, \tilde{\lambda} \left| b_{p_1, s_1, x}^{\dagger} b_{p_2, s_2, x} a_{q_1, \lambda_1, x} \right| k'', \sigma''; k, \lambda; q, \tilde{\lambda} \right\rangle \\ &= \delta^3(\mathbf{q}_1 - \mathbf{k}) \delta_{\lambda_1 \lambda} \delta^3(\mathbf{p}_2 - \mathbf{k}'') \delta_{s_2 \sigma''} \delta^3(\mathbf{p}_1 - \mathbf{p}') \delta_{s_1 s'}, \end{aligned}$$

where $\delta^3(\mathbf{q}_1 - \mathbf{k}) = \delta^2(\mathbf{q}_{1\perp} - \mathbf{k}_\perp)\delta(q_1^+ - k^+)$ etc., we obtain

$$\begin{aligned} & \langle p', s'; q, \tilde{\lambda} | V_1 | k'', \sigma''; k, \lambda; q, \tilde{\lambda} \rangle \\ &= e \int d^2\mathbf{x}_\perp dx^- \\ & \times \int_{-\infty}^{+\infty} \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2 d^3\mathbf{q}_1 \theta(p_1^+) \theta(p_2^+) \theta(q_1^+)}{(2\pi)^{9/2} \sqrt{8} \sqrt{p_1^+ p_2^+ q_1^+}} \\ & \times \sum_{s_1, s_2, \lambda_1} \bar{u}_{p_1, s_1} e^{ip_1 \cdot x} \gamma^\mu u_{p_2, s_2} e^{-ip_2 \cdot x} \epsilon_\mu^{\lambda_1}(q_1) e^{-iq_1 \cdot x} \\ & \times \delta^3(\mathbf{q}_1 - \mathbf{k}) \delta_{\lambda_1 \lambda} \delta^3(\mathbf{p}_2 - \mathbf{k}'') \delta_{s_2 \sigma''} \delta^3(\mathbf{p}_1 - \mathbf{p}') \delta_{s_1 s'} \\ &= e \int \frac{d^2\mathbf{x}_\perp dx^- \theta(p'^+) \theta(k''^+) \theta(k^+)}{(2\pi)^{9/2} \sqrt{8} \sqrt{p'^+ k''^+ k^+}} \\ & \times \bar{u}_{p', s'} \gamma^\mu u_{k'', \sigma''} \epsilon_\mu^\lambda(k) e^{i(p' - k'' - k) \cdot x} \\ &= \frac{e}{(2\pi)^{3/2}} \frac{1}{\sqrt{8}} \frac{1}{\sqrt{p'^+ k''^+ k^+}} \bar{u}_{p', s'} \gamma^\mu u_{k'', \sigma''} \epsilon_\mu^\lambda(k) \\ & \times \delta^3[\mathbf{k}'' - (\mathbf{p}' - \mathbf{k})] \theta(p'^+) \theta(k''^+) \theta(k^+). \quad (\text{B.2}) \end{aligned}$$

Similarly,

$$\begin{aligned} & \langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} | V_2 | p, s \rangle \\ &= \frac{-ie^2}{4} \int d^2\mathbf{y}_\perp dy^- dz^- \epsilon(y^- - z^-) \\ & \times \int_{-\infty}^{+\infty} \frac{d^2\mathbf{p}_3 \perp d^2\mathbf{p}_4 \perp d^2\mathbf{q}_2 \perp d^2\mathbf{q}_3 \perp}{(2\pi)^6 4} \\ & \times \int_0^\infty \frac{dp_3^+ dp_4^+ dq_2^+ dq_3^+}{\sqrt{p_3^+ p_4^+ q_2^+ q_3^+}} \\ & \times \sum_{s_3, s_4, \lambda_2, \lambda_3} \langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} | [\bar{u}_{p_3, s_3} e^{ip_3 \cdot y} b_{p_3, s_3, y}^\dagger \\ & + \bar{v}_{p_3, s_3} e^{-ip_3 \cdot y} d_{p_3, s_3, y}] \epsilon_k^{\lambda_2}(q_2) [e^{-iq_2 \cdot y} a_{q_2, \lambda_2, y} \\ & + e^{iq_2 \cdot y} a_{q_2, \lambda_2, y}^\dagger] \gamma^k \gamma^+ \gamma^j \\ & \times \epsilon_j^{\lambda_3}(q_3) [e^{-iq_3 \cdot z} a_{q_3, \lambda_3, z} + e^{iq_3 \cdot z} a_{q_3, \lambda_3, z}^\dagger] \\ & \times [u_{p_4, s_4} e^{-ip_4 \cdot z} b_{p_4, s_4, z} + v_{p_4, s_4} e^{ip_4 \cdot z} d_{p_4, s_4, z}^\dagger] | p, s \rangle. \end{aligned}$$

Again using

$$\begin{aligned} & \langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} | b_{p_3, s_3, y}^\dagger a_{q_2, \lambda_2, y}^\dagger a_{q_3, \lambda_3, z}^\dagger b_{p_4, s_4, z} | p, s \rangle \\ &= \delta^3(\mathbf{p}_4 - \mathbf{p}) \delta_{s_4 s} \delta^3(\mathbf{q}_3 - \mathbf{k}) \delta_{\lambda_3 \lambda} \\ & \times \delta^3(\mathbf{q}_2 - \mathbf{q}) \delta_{\lambda_2 \tilde{\lambda}} \delta^3(\mathbf{p}_3 - \mathbf{k}'') \delta_{s_3 \sigma''}, \end{aligned}$$

we obtain

$$\begin{aligned} & \langle k'', \sigma''; k, \lambda; q, \tilde{\lambda} | V_2 | p, s \rangle \\ &= \frac{-ie^2}{4} \int d^2\mathbf{y}_\perp dy^- dz^- \end{aligned}$$

$$\begin{aligned} & \times \int_{-\infty}^{+\infty} \frac{d^3\mathbf{p}_3 d^3\mathbf{p}_4 d^3\mathbf{q}_2 d^3\mathbf{q}_3 \theta(p_3^+) \theta(p_4^+) \theta(q_2^+) \theta(q_3^+)}{(2\pi)^6 4 \sqrt{p_3^+ p_4^+ q_2^+ q_3^+}} \\ & \times \epsilon(y^- - z^-) \sum_{s_3, s_4, \lambda_2, \lambda_3} \bar{u}_{p_3, s_3} e^{ip_3 \cdot y} \epsilon_k^{\lambda_2}(q_2) e^{iq_2 \cdot y} \\ & \times \gamma^k \gamma^+ \gamma^j \epsilon_j^{\lambda_3}(q_3) e^{iq_3 \cdot z} u_{p_4, s_4} \\ & \times e^{-ip_4 \cdot z} \delta^3(\mathbf{p}_4 - \mathbf{p}) \delta_{s_4 s} \delta^3(\mathbf{q}_3 - \mathbf{k}) \delta_{\lambda_3 \lambda} \delta^3 \\ & \times (\mathbf{q}_2 - \mathbf{q}) \delta_{\lambda_2 \tilde{\lambda}} \delta^3(\mathbf{p}_3 - \mathbf{k}'') \delta_{s_3 \sigma''} \\ &= \frac{-ie^2}{4} \int \frac{d^2\mathbf{y}_\perp dy^- dz^- \theta(k''^+) \theta(q^+) \theta(k^+) \theta(p^+)}{(2\pi)^6 4 \sqrt{k''^+ q^+ k^+ p^+}} \\ & \times \epsilon(y^- - z^-) \bar{u}_{k'', \sigma''} \\ & \times e^{ik'' \cdot y} \epsilon_k^{\tilde{\lambda}}(q) e^{iq \cdot y} \gamma^k \gamma^+ \gamma^j \epsilon_j^\lambda(k) e^{ik \cdot z} u_{p, s} e^{-ip \cdot z} \\ &= \frac{-e^2}{8} \int \frac{d^2\mathbf{y}_\perp dy^- \theta(k''^+) \theta(q^+) \theta(k^+) \theta(p^+)}{(2\pi)^6 \sqrt{k''^+ q^+ k^+ p^+} (k^+ - p^+)} \bar{u}_{k'', \sigma''} \\ & \times \gamma^k \gamma^+ \gamma^j u_{p, s} \epsilon_j^\lambda(k) \epsilon_k^{\tilde{\lambda}}(q) e^{i(k'' + q + k - p) \cdot y} \\ &= \frac{e^2}{(2\pi)^3 8} \frac{1}{\sqrt{k''^+ q^+ k^+ p^+}} \frac{\theta(k''^+) \theta(q^+) \theta(k^+) \theta(p^+)}{(p^+ - k^+)} \\ & \times \bar{u}_{k'', \sigma''} \gamma^k \gamma^+ \gamma^j u_{p, s} \epsilon_j^\lambda(k) \epsilon_k^{\tilde{\lambda}}(q) \\ & \times \delta^3(\mathbf{k}'' + \mathbf{q} + \mathbf{k} - \mathbf{p}), \quad (\text{B.3}) \end{aligned}$$

where the identity

$$\int dz^- f(z^-) \epsilon(y^- - z^-) = \frac{2}{\partial_-} f(y)$$

is used for arriving at the above result. Substituting eqs (B.2) and (B.3) in eq. (B.1), we get

$$\begin{aligned} & T_{p, p', q}^{(c)} \\ &= e^3 \lambda \int \frac{d^3\mathbf{k}'' d^3\mathbf{k} \theta(k''^+) \theta(k^+) \theta(p^+) \theta(p'^+) \theta(q^+)}{(4\pi)^3 k^+ k''^+ (p^+ - k^+)} \\ & \times \sum_{\sigma'', \lambda} \frac{\bar{u}_{p', s'} \gamma^\mu u_{k'', \sigma''} \bar{u}_{k'', \sigma''} \gamma^k \gamma^+ \gamma^j u_{p, s}}{(p^- - k''^- - k^- - q^-)} \\ & \times \epsilon_\mu^\lambda(k) \epsilon_j^\lambda(k) \epsilon_k^{\tilde{\lambda}}(q) \delta^3(\mathbf{p}' - \mathbf{k} - \mathbf{k}'') \delta^3 \\ & \times (\mathbf{k}'' + \mathbf{q} + \mathbf{k} - \mathbf{p}), \end{aligned}$$

where

$$\lambda = \frac{1}{(2\pi)^{3/2} \sqrt{2p^+} \sqrt{2p'^+} \sqrt{2q^+}}.$$

Using the completeness relations

$$\sum_{s=\pm 1/2} u_{p, s} \bar{u}_{p, s} = \not{p} + m$$

and

$$\sum_{\lambda=1, 2} \epsilon_\mu^\lambda(p) \epsilon_\nu^\lambda(p) = d_{\mu\nu}(p)$$

$$= -g_{\mu\nu} + \frac{\delta_{\mu+P\nu} + \delta_{\nu+P\mu}}{p^+}$$

and performing the k'' -integral using the delta functions, the amplitude for the diagram in figure 4c reduces to eq. (82). A similar calculation leads to eq. (83) for the amplitude of figure 4d.

B.2 Calculation of numerators of $T_{p,p',q}^{(c)}$ and $T_{p,p',q}^{(d)}$

Here we present the steps used for simplifying the numerator of eqs (82) and (83) to obtain eqs (84) and (86) respectively. First we observe that as $(\gamma^+)^2 = 0$ and $\epsilon_- = 0$, the ‘+’ and ‘-’ components of μ do not contribute. Next we consider

$$\begin{aligned} & \gamma^\alpha (\mathcal{K}'' + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k) \epsilon_{\mu}^{\tilde{\lambda}}(q) \\ &= \gamma^\alpha (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^\beta d_{\alpha\beta}(k) \epsilon_k^{\tilde{\lambda}}(q) \\ &= [\gamma^\alpha (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^- d_{\alpha-}(k) \\ & \quad + \gamma^\alpha (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^j d_{\alpha j}(k)] \epsilon_k^{\tilde{\lambda}}(q). \end{aligned}$$

Now,

$$\begin{aligned} & \gamma^\alpha (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^- d_{\alpha-}(k) \\ &= \gamma^\alpha (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^- \left[-g_{\alpha-} + \frac{\delta_{\alpha+k-} + \delta_{-+k\alpha}}{k^+} \right] \\ &= -\gamma^+ (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^- \\ & \quad + \gamma^+ (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^- \left(\frac{k_-}{k^+} \right) \\ &= 0. \end{aligned}$$

Therefore,

$$\begin{aligned} & \gamma^\alpha (\mathcal{K}'' + m) \gamma^\mu \gamma^+ \gamma^\beta d_{\alpha\beta}(k) \epsilon_{\mu}^{\tilde{\lambda}}(q) \\ &= \gamma^\alpha (\mathcal{K}'' + m) \gamma^k \gamma^+ \gamma^j d_{\alpha j}(k) \epsilon_k^{\tilde{\lambda}}(q). \end{aligned} \tag{B.4}$$

Similarly,

$$\begin{aligned} & \gamma^\alpha \gamma^+ \gamma^\mu (\mathcal{K}' + m) \gamma^\beta d_{\alpha\beta}(k) \epsilon_{\mu}^{\tilde{\lambda}}(q) \\ &= \gamma^\alpha \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^\beta d_{\alpha\beta}(k) \epsilon_j^{\tilde{\lambda}}(q) \\ &= [\gamma^- \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^\beta d_{-\beta}(k) \\ & \quad + \gamma^k \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^\beta d_{k\beta}(k)] \epsilon_j^{\tilde{\lambda}}(q). \end{aligned}$$

Using

$$\begin{aligned} & \gamma^- \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^\beta d_{-\beta}(k) \\ &= \gamma^- \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^\beta \left[-g_{-\beta} + \frac{\delta_{-+k\beta} + \delta_{\beta+k-}}{k^+} \right] \\ &= -\gamma^- \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^+ \\ & \quad + \gamma^- \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^+ \left(\frac{k_-}{k^+} \right) \\ &= 0 \end{aligned}$$

we obtain

$$\begin{aligned} & \gamma^\alpha \gamma^+ \gamma^\mu (\mathcal{K}' + m) \gamma^\beta d_{\alpha\beta}(k) \epsilon_{\mu}^{\tilde{\lambda}}(q) \\ &= \gamma^k \gamma^+ \gamma^j (\mathcal{K}' + m) \gamma^\beta d_{k\beta}(k) \epsilon_j^{\tilde{\lambda}}(q). \end{aligned} \tag{B.5}$$

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