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Impact of global financial crisis on the complexity of emerging markets: Case study of the Nigerian Stock Exchange

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Abstract. The dynamical complexities of the Nigerian stock market have been largely uninvestigated. In this study, we analysed the return price of financial stocks on Nigerian Stock Exchange for chaotic behaviour. Our analysis was conducted for the period 2000–2015, as well as for three distinct periods covering the pre-crisis, crisis and post-crisis period of 2008. Fractal anlaysis (detrended fluctuation analysis and rescale range), entropy (Kolmogorov and permutation), recurrence quantification analysis (determinism and longest diagonal line) and Lyapunov exponent (Rosenstein and Eckmann) methods were used in the investigation. Results showed that the return prices of six financial stocks exhibit behaviour associated with random noise and chaos. The stocks were found to be more efficient post-crisis than during the pre-crisis period. Return prices post-crisis were found to be more chaotic.

Keywords. Detrended fluctuation analysis; entropy; chaos; Nigerian Stock Exchange; Lyapunov exponent.

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1. Introduction

Financial markets are complex systems that are influenced by many factors including government and international policies. Investigating the complexity in financial market data is of interest to practitioners and investors for risk analysis and management. Chaos analysis offers approaches that can differentiate noise and deterministic systems. Hence, it is suitable for the analysis of financial market data. Over the years, various tools and methods have been introduced for the investigation of chaos in financial and economic time series data. These include correlation dimension [1,2], Lyapunov exponent [3], surrogate data analysis [4], entropy methods [5,6], recurrence quantification analysis [7] and the compass rose [8]. The presence of chaos has been investigated in gross domestic products [9,10], cryptocurrencies [11], crude oil prices [3], stock markets [1,12] and exchange rates [4].

Lahmiri [3] found that volatility of West Texas Intermediate (WTI) and Brent crude oil prices before the financial crisis was not chaotic but chaotic after the financial crisis using Lyapunov exponent. However, chaos was not observed in prices and returns before and after the financial crisis. Using entropy methods, the efficiency of financial instruments during the 2008 financial crisis was lower than before and after the crisis. Lahmiri and Bekiros [6] reported that complexities in volatility of financial, exchange and commodity markets are increased during the crisis period. The dynamical properties of several financial markets were changed significantly after the crisis. Lahmiri *et al* [13] reported that the persistent nature of the markets before the crisis gave way to anti-persistence after the crisis while the entropy values also are increased after the crisis. However, the markets were not found to be chaotic during and after the crisis.

The complexity of the Nigerian Stock Exchange (NSE) has not attracted much attention from researchers. The NSE, which currently has 169 listed companies and a market capitalisation of $N14.288$ trillion, was established in 1960. The All Share Index (ASI) of the NSE has been found to show better performance with decreased minimum rediscounting rate (MRR) and exchange rate stability [14]. During the 2008 financial crisis, the NSE lost about 50% of its value. The NSE has been found

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to be independent of other African markets but related to the US market [15]. Fuwape and Ogunjo [16] found evidence of chaos in a Nigerian mutual fund.

Despite the increasing interest in the dynamical complexity of global financial markets, very little attention has been paid to markets in developing countries of Africa. It is pertinent to investigate the impact of financial crisis on the complexity of markets in developing countries for better knowledge and improved efficiency. In this research, we aim to investigate the chaotic characteristics of return prices of financial stocks in the NSE using different approaches – recurrence quantification analysis, entropy measures and Lyapunov exponents. Furthermore, the complexity of the return prices before, during and after the financial crisis was investigated. The goal of this study is to determine the impact of financial crises on the dynamical complexity of return price of financial stocks in a developing country, Nigeria, in this study. Although the NSE has great interdependence with Asian markets compared to the European markets, it is not as integrated as the South African market with the international markets [17].

2. Methods

2.1 *Data*

Daily closing prices of six banking stocks were obtained from the archives of the NSE. Stocks with data between January 3, 2000 and November 18, 2015 were considered for this study. The pre-crisis, crisis and post-crisis periods were taken as January 3, 2000 to September 15, 2005, September 16, 2005 to October 28, 2010, and October 29, 2010 to November 18, 2015 respectively. Analysis was done on the daily price return of the stocks which was computed as the first-difference of the logarithm of the prices. The time series plot of the closing prices for the selected banking stocks is shown in figure 1.

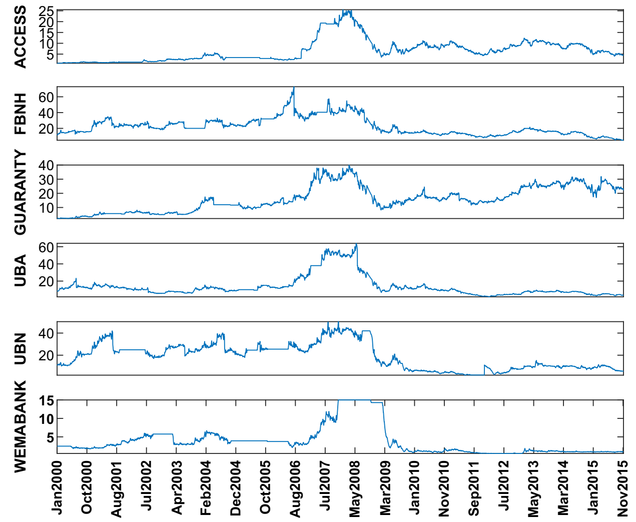

Figure 1. Time series of closing price for the Nigerian banking stocks.

2.2 *Hurst exponent*

Hurst exponent (H) is a measure of self-similarity, persistence or long-range dependence in a time series. Persistent processes have Hurst exponent in the range $0.5 < H < 1$ while antipersistent processes have Hurst exponent in the range $0 < H < 0.5$. Random walk and non-stationary processes are defined by $H = 0.5$ and $H > 1$ respectively. Rescale range (R/S) and detrended fluctuation analysis (DFA) are the two most widely used methods of Hurst exponents in the analysis of financial data [18]. Hurst exponent has been used to determine market efficiency and is an estimator for the operating regime [19].

Given a time series, y_i. To compute Hurst exponent via the DFA method, the cumulative series $x_i = \sum_{i=1}^{N}(y_i - \hat{y})$ is computed. x_i is divided into small time windows of length n and the linear trend X_n of each segment is evaluated. The root-mean-square detrended fluctuation, F, is computed as

$$F(n) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - X_n)^2}.$$

The value of H is then estimated from the log–log plot of the relationship $f(n) \propto n^{\alpha}$.

The R/S method for computing Hurst exponent was introduced by Hurst [20]. In this approach, the self-adjusted range $R(\tau)$ is

$$R(\tau) = \max_{t=1}^{\tau} X(t,\tau) - \min_{t=1}^{\tau} X(t,\tau), \quad (1)$$

where $X(t,\tau) = \sum_{u=1}^{t}[Z(u) - \langle Z\rangle_\tau]$. $\langle Z\rangle_\tau$ is the average of the sequence of observations. The standard deviation as a function of τ of the observation is given as

$$S(\tau) = \left(\frac{1}{\tau}\sum_{t=1}^{\tau}(Z(t) - \langle Z\rangle_\tau)^2\right)^{\frac{1}{2}}. \quad (2)$$

The slope of the equation $\log(R/S) = \log(\tau/2)^H$ gives the Hurst exponent [21,22].

2.3 *Entropy*

There is an increasing interest in the use of entropy measures to elucidate complexity in time series signal. The different types of entropy measures include permutation entropy, Shannon entropy, Kolmogorov entropy, sample entropy, Renyi entropy and Tsallis entropy.

In this study, the Kolmogorov entropy and permutation entropy are considered. The Kolmogorov entropy is a measure of how fast a pair of states become distinguishable to a measuring apparatus with fixed precision under forward iteration [23]. For a time series, $\tilde{x}_n = x-1, \ldots, x_n$, Kolmogorov entropy is defined as

$$K = \lim_{n\to\infty} -\sum_i P_n(\tilde{x}_n) \ln P_n\tilde{x}_n). \quad (3)$$

The larger the K value, the greater are the information loss and the degree of chaos.

The permutation entropy of order $n \geq 2$ is defined by [24] as

$$H(n) = -\sum p(\pi) \log p(\pi), \quad (4)$$

where the sum runs over all $n!$ permutations of π of order n. Permutation entropy does not consider the probability distribution of the data in detecting the linear and nonlinear properties of the time series.

2.4 *Lyapunov exponent*

Chaotic systems are characterised by at least one positive Lyapunov exponent. The magnitude of the Lyapunov exponent is related to the time scale for predictability of the system under consideration [25]. Several methods proposed for the computation of Lyapunov exponents in time series data include Wolf [26], Kantz [27], Bensaida [28], Eckmann *et al* [29], Sano and Sawada [30] and Rosenstein *et al* [31].

In this study, the Rosenstein and Eckmann methods are computed using the nonlinear measures for dynamical systems (NOLDS) Python module. While the Rosenstein approach produces the largest Lyapunov exponent, the Eckmann algorithm computes the whole spectrum of Lyapunov exponents. The Rosenstein method is fast, robust to parameter choice and easy to implement. The Eckmann method converges fast, requires only two parameters and uses common numerical techniques [32].

2.5 *Recurrence quantification analysis*

A recurrence plot (RP) is a graph that shows all those times at which a state of the dynamical system recurs [7]. It is typical for chaotic systems to have states that are arbitrary close after some time. Eckmann *et al* [33] introduced the concept of recurrence plot to represent times the state of the system recur in the phase space. Webber Jr and Marwan [34] defined the recurrence of a state as a 2D square matrix $\mathbb{R}$ with two time axes as

$$R_{i,j}^{m,\epsilon_i} = \Theta(\epsilon_i - ||x_i - x_j||), \quad i, j = 1 \cdots N, \quad (5)$$

where N, ϵ_i, $||\cdot||$ and Θ are the number of considered states, threshold distance, norm and Heaviside function respectively. To provide quantitative interpretations for recurrence plot, Zbilut and Webber [35] introduced definitions to quantify structures in recurrence plots based on diagonal and vertical line structuring. Two variables, DET and $L_{\max}$, were employed in this study.

Percent determinism (DET) is defined as the fraction of recurrence points that form diagonal lines

$$\mathrm{DET} = \frac{\sum_{l=d_{\min}}^{N} l H_D(l)}{\sum_{i,j=1}^{N} R_{i,j}}, \quad (6)$$

where H_D, the histogram of the lengths of the diagonal structure in the recurrence plot, is defined as [34]

$$H_D(l) = \sum_{i,j=1}^{N} (1 - R_{i-1,j-1})(1 - R_{i+l,j+l}) \times \prod_{k=0}^{l-1} R_{i+k,j+k}. \quad (7)$$

The second variable is the maximum line length in the diagonal direction ($L_{\max}$). It is defined as

$$L_{\max} = \arg\max_l H_D(l). \quad (8)$$

Small values of $L_{\max}$ correspond to a more divergent signal trajectory [34].

3. Results and discussion

3.1 *Extended time series analysis*

The entire time series from January 2000 to November 2015 was analysed using methods described in §2. The results of statistical and complexity analyses for the banking stocks considered are presented in table 1. The minimum and maximum mean of the raw data were found in WEMA and FBNH stocks respectively. This reflects the market capitalisation of the financial sector. FBNH and WEMA are the most and the least capitalised banking stocks in the exchange for the period under review. The highest standard deviation was observed in the stock price of UBA while the smallest standard deviation could be found in WEMA stock price. The stock price of FBNH moved from N4.98 to a peak of N72.76

Table 1. Statistical and complexity analyses of banking stocks in the Nigerian Stock Market.

ASSET	Mean	STD	MAX	MIN	MI	FNN	R/S	DFA	K	H	DET	Lmax	ROS	ECKM
ACCESS	6.56	5.18	25.50	0.80	5	11	0.56	0.52	7.66	10.95	0.98	1.4	0.0404	0.046
FBNH	22.24	10.99	72.76	4.98	3	7	0.5	0.46	9.67	10.76	0.96	4.19	0.0551	0.0512
GUARANTY	16.02	8.94	40.00	2.19	3	6	0.51	0.47	9.35	9.02	0.99	1.63	0.0521	0.0464
UBA	13.51	12.01	63.94	1.64	6	8	0.57	0.52	9.27	11.58	0.99	2.00	0.0490	0.0510
UBN	18.87	11.75	50.33	1.96	6	8	0.51	0.48	8.35	10.89	0.97	2.63	0.0454	0.0532
WEMA	3.75	3.86	15.00	0.50	7	7	0.59	0.57	5.64	9.14	0.98	2.45	0.0425	0.0458

during the stock market boom. The highest gain could be observed in the stock price of UBA which moved from a low of N1.64 to N63.94.

The mutual information and embedding dimension values for each of the stocks were also reported in table 1. The mutual information for the financial stocks were found to be in the range 3–7 while the embedding dimensions were in the range 7–11. The methods of R/S and DFA showed similar results for the Hurst exponents of the financial stocks. The values of Hurst exponents obtained using the method of DFA were generally lower than that obtained by the method of R/S. While Hurst exponents from the method of R/S showed Hurst exponent in the range 0.5–0.59, Hurst exponents from the method of DFA were found in the range 0.46–0.57. As observed from the R/S approach, FBHN can be said to be a random walk process. The other stocks are marginally antipersistent in returns. However, using the DFA approach, FBNH, GUARANTY and UBN are marginally persistent while the others show antipersistent behaviour. Hurst exponent values close to 0.5 suggest that the stock returns are efficient. Variations from the value of 0.5 indicates a less efficient market [36]. A low market efficiency is indicative of high predictability in the market. The Hurst exponent values obtained in the extended time series suggest an efficient market as the values are close to 0.5. FBHN is the most efficient stock with Hurst exponent of 0.5 while WEMA is the most inefficient stock. This implies that the return price of WEMA is more predictable than the return price of FBHN.

Entropy values computed using the Kolgomorov entropy and permutation entropy were in the range 5.64–9.67 and 9.02–11.58 respectively. These values are lower than those obtained for the Athens stock market [37] but in the range of values obtained by Mayfield and Mizrach [38] for the S & P index. Recurrence quantification analysis suggests chaos in the financial time series as seen in the DET and L_{max} values. DET values of all the stock return prices were found to be very high which is indicative of a chaotic system. The results for DET obtained in this research are in the same range with those obtained for Nifty, Hong Kong AOI and Dow Jones Industrial Average [7]. According to [39], the shorter is the L_{max} value, the more chaotic is the signal. Therefore, we can infer that FBNH is the least chaotic while ACCESS is the most chaotic of the stocks investigated. The chaotic nature of the return prices of the stocks was confirmed by the Lyapunov exponents computed using two different approaches. Both the Rosenstein and Eckmann algorithms produced similar results for the Lyapunov exponents. The existence of chaos suggests that short-term profitability of the stocks can be exploited using nonlinear models [40].

Table 2. Hurst exponents at different economic regimes computed using the R/S range and DFA methods.

	R/S			DFA		
ASSET	Pre	During	Post	Pre	During	Post
ACCESS	0.511	0.61	0.566	0.426	0.615	0.521
FBNH	0.503	0.49	0.57	0.438	0.442	0.548
GUARANTY	0.526	0.56	0.518	0.494	0.529	0.455
UBA	0.535	0.615	0.607	0.443	0.592	0.556
UBN	0.53	0.57	0.493	0.472	0.55	0.458
WEMA	0.517	0.655	0.525	0.51	0.677	0.509

Table 3. Recurrence quantification analyses for banking stocks before, during and after the banking crisis

	DET			L_{max}		
ASSET	Pre	During	Post	Pre	During	Post
ACCESS	0.69	0.44	0.19	232	55	7
FBNH	0.45	0.43	0.21	129	78	6
GUARANTY	0.55	0.27	0.21	114	35	8
UBA	0.38	0.33	0.2	103	71	8
UBN	0.54	0.49	0.51	177	141	93
WEMA	0.71	0.75	0.45	218	223	128

3.2 *Segmented time series analysis*

The data were considered in three segments (pre-crisis, crisis and post-crisis) for the analysis. Generally, the Hurst exponent computed from the DFA were found to be marginally lower than those obtained from the R/S analysis during the pre-crisis and post-crisis periods (table 2). During the crisis, the Hurst exponents obtained

Table 4. Entropy and Lyapunov.

	Kolmogorov			Permutation entropy			Rosenstein			Eckmann		
ASSET	Pre	During	Post	Pre	During	Post	Pre	During	Post	Pre	During	Post
ACCESS	4.557	7.787	8.798	2.211	9.886	2.582	0.0201	0.0407	0.0406	0.02	0.0598	0.0325
FBNH	8.462	8.262	9.317	9.687	2.536	6.83	0.0533	0.0444	0.0443	0.0476	0.0482	0.0311
GUARANTY	6.678	9.426	9.275	8.006	4.571	2.583	0.0399	0.0424	0.0402	0.0357	0.0429	0.0304
UBA	7.838	8.672	8.74	2.54	8.834	9.034	0.0489	0.0495	0.0407	0.0429	0.0446	0.0317
UBN	7.574	8	6.946	9.595	8.394	4.436	0.046	0.038	0.0352	0.041	0.0473	0.0308
WEMA	5.284	5.278	5.193	2.315	3.821	4.372	0.0385	0.0387	0.0396	0.0259	0.0557	0.0303

from DFA were marginally higher than R/S approach in ACCESS and WEMA return prices. Hurst exponent from the DFA algorithm were slightly lower than 0.5 except in WEMA. However, the R/S method produced Hurst exponents which were greater than 0.5 in the pre-crisis period. During the crisis, both methods showed the same pattern of results – antipersistence in all stocks except FBNH which showed persistence. Similar pattern was observed in the post-crisis period except for the return price of GUARANTY which showed differing results. The stocks, except for FBNH, were found to show more antipersistent trend during the crisis than at other periods in both R/S and DFA methods. Our results imply that, except the stock return price of FBHN, all the other stock returns prices investigated were less efficient during the crisis. Furthermore, GUARANTY and UBN return prices were found to be more efficient post-crisis than the pre-crisis period.

Recurrence quantification analysis was also carried out on different economic regimes. In the pre-crisis period, DET values were found to be between 38% and 71% (table 3). The range of DET values during and after the crisis were found to be in the range 27–75% and 19–51% respectively. Considering the $L_{\max}$ values, the impact of the crisis on the financial stocks can be inferred. Post-crisis return prices were observed to be the most chaotic. The chaos was less in UBN and WEMA than in the other stocks. The chaotic nature of the return prices was amplified during the crisis as the $L_{\max}$ values were very much smaller than before the crisis period. Both Kolmogorov and permutation entropy values show high level of randomness in the return prices. Chaos in the return prices before, during and after the crisis was confirmed from the Lyapunov exponent (table 4). The Lyapunov exponents from the two methods show similar values with little variations.

4. Conclusion

The Nigerian economy is one of the fastest growing economy in the African continent. This research investigated chaos in the return price of six financial stocks in the Nigerian Stock Exchange. Internal and external shocks on the economy reflects in the prices of financial stocks, and hence, their suitability for analysis. In this research, two different approaches for Hurst exponent, entropy, recurrence quantification analysis and Lyapunov exponent were used to evaluate chaos in the financial sector of the NSE. Our analysis were conducted using two time frames. In the first approach, the entire time series for the financial stocks were analysed for chaos. Results showed that the return prices exhibit behaviour consistent with chaotic systems. To investigate the impact of financial crisis on the stock market, chaos analysis was performed on the financial stocks at three different time periods: pre-crisis, crisis and post-crisis. The financial stocks were all found to be chaotic at the three different time periods. However, post-crisis return prices were found to be more chaotic than the pre-crisis return prices. This study is the first in-depth investigation into the chaotic nature of stocks in the NSE. Further studies on the external and internal drivers of chaoticity in the NSE can be done using nonlinear tools.

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