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Significance of viscous dissipation on MHD Eyring–Powell flow past a convectively heated stretching sheet

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Abstract. It is well known that there is hardly any fluid that obeys Newtonian fluid exactly. At high speed, the Newtonian law of viscosity fails to hold anymore, and the deviation from Newtonian law becomes very significant. Eyring–Powell fluid provides a better model for such fluids at high speed because it includes some plasticity. Eyring–Powell fluids have prime applications in polymer industries, squeezing of plastic sheets, etc. This study investigates the magnetohydrodynamic (MHD) Eyring–Powell flow past a stretching sheet with convective boundary conditions. The governing nonlinear partial differential equations are transformed to the system of nonlinear ordinary differential equations using similarity variables of flow quantities. The shooting technique is used with Runge–Kutta numerical scheme to numerically solve the problem and the results are presented as graphs. The results from this research indicate that it is sufficient to enhance viscous dissipations to boost primary velocity, secondary velocity and the temperature of MHD Eyring–Powell flow. More so, increasing magnetic field strength impedes the flow of Eyring–Powell fluid, increases the temperature profiles and reduces the shear drag in the boundary layer.

Keywords. Magnetohydrodynamic; Eyring–Powell fluid; convective boundary condition; stretching sheet.

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1. Introduction

Boundary layer flow of a fluid over a stretching plate (SP) with an imposed magnetic field is significant in many engineering and industrial processes such as metallurgy, production of papers, processing of plastic polymers and purification of molten metal. In addition, achieving high quality products from the industries is largely predicated on the rate at which the stretching surfaces are cooled. On the other hand, the magnetic field influences the quality of the final product. A typical example is the fact that cooling rate needs to be moderated in drawing strips in an MHD fluid to attain the best quality. Crane [1] pioneered the investigation of flows over SP. Ever since, problems involving the transfer of heat and mass over SP have been investigated. These include Newtonian flows over such sheets under the influence of electric and magnetic fields under different thermal boundary conditions and under varying stretching velocity [2–13]. All the aforementioned investigations refer to the flow within the boundary layer in a viscous Newtonian fluid.

During the last few decades, it has been noted that in numerous fluid mechanics problems, the fluids disobey Newton's viscosity law. Such fluids that disobey the Newtonian laws are referred to as non-Newtonian fluids. Examples are: paint, granular suspension, drilling muds, blood at low shear rate, ketchup, shampoos etc. Studies have shown that these non-Newtonian fluids have more uses in industrial and engineering processes. It is noteworthy to mention that it is impossible to model all non-Newtonian fluids using a single shear-stress/shear-rate relation because they exhibit different rheological behaviours. In this regard, non-Newtonian models are formulated for different fluids to imitate the non-Newtonian fluid flow behaviour. Some of these constitutive equations are based on fundamental theories of the physics of fluids, while others have been

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derived empirically, for instance the power-law fluid model. The model of Eyring–Powell fluid is formulated using the concept of kinetic theory of liquids and the model becomes that of Newtonian fluid when the shear stress rate is very low or very high. It is used in chemical engineering and very suitable to describe shear thinning fluids such as ketchup, tomatoes paste, blood, palm oil, toothpaste etc. These have aroused the curiosity of researchers to further probe into its thermophysical properties. Studies involve the stagnation point flow of Eyring–Powell over convectively heated stretching plate (CHSP) [14,15], with effects of radiation on such flow over an inclined stretching surface [16], and over a nonlinear stretching surface [5,17,18]. From the aforementioned literature, it is apparent that no investigation has focussed on the magnetic effect on the Eyring–Powell flow over a SP. The motivation of the current study is to numerically analyse MHD boundary layer flow of Eyring–Powell fluid past a CHSP.

2. Mathematical formulation of the governing equations

The steady laminar incompressible two-dimensional MHD boundary layer Eyring–Powell flow over a SP is considered. The sheet experiences a linear stretching with velocity $U_w(x) = ax$ $(a > 0)$ along the x-axis. A magnetic field is applied normal to the surface along the y-direction with constant magnetic field strength B_0. The magnetic induction equation is not considered in the present model formulation due to the low electrical conductivity of the fluid. The stress tensor for the Eyring–Powell fluid of viscosity μ by [19] is described using the equation

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1}\left(\frac{1}{E}\frac{\partial u_i}{\partial x_j}\right),$$

where β and E are the Eyring–Powell and rheological fluid parameters. According to [15], the first two terms of the expansion of the hyperbolic sine is enough to describe the fluid and so,

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta}\left(\frac{1}{E}\frac{\partial u_i}{\partial x_j} - \frac{1}{6}\left(\frac{1}{E}\frac{\partial u_i}{\partial x_j}\right)^3\right),$$

$$\left|\frac{1}{E}\frac{\partial u_i}{\partial x_j}\right| < 1.$$

Hence, the boundary layer flow of Eyring–Powell is governed by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho\beta E}\right)\frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho B E^3}\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) - \frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_P}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2 u^2}{\rho}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

and with the boundary conditions

$$\text{at } y = 0; \quad u = U_w(x), \ v = 0, \quad -k_f\frac{\partial T}{\partial y} = h_f\left(T_f - T\right), \ C = C_w, \tag{5}$$

$$\text{as } y \to \infty; \quad u = 0, \ T \to T_\infty, \ C \to C_\infty, \tag{6}$$

where u, v are the velocity components along the x- and y-axes respectively, $\nu = \mu/\rho$ is the kinematic viscosity, k is the thermal conductivity of the fluid, ρ is the fluid density, g is the acceleration due to gravity, β is the volumetric expansion coefficient of the fluid. Introduce the similarity variable η and the stream function ψ which are dimensionless variables and quantities as

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y, \ \psi = (a\nu)^{1/2} x f(\eta), \ \theta = \frac{T - T_\infty}{T_f - T_\infty},$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, \ u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x}.$$

These dimensionless quantities satisfy the continuity equation automatically and the dimensionless equations are

$$(1 + \Gamma) f''' - \epsilon\Gamma\left(f''\right)^2 f''' + f'' f - \left(f'\right)^2 + Gr\theta - Mf' = 0, \tag{7}$$

$$\theta'' + Pr\theta' f + PrEc[\left(f''\right)^2 + M\left(f'\right)^2] = 0, \tag{8}$$

$$\phi'' + Lef\phi' = 0. \tag{9}$$

The dimensionless boundary conditions

$$\text{at } \eta = 0; \quad f = 0, \ f' = 1, \ \theta' = Bi(1 - \theta), \ \phi = 1, \tag{10}$$

$$\text{as } \eta \to \infty; \quad f' = 0, \ \theta \to 0, \ \phi \to 0, \tag{11}$$

where

$$Gr = \frac{\beta g\left(T_f - T_\infty\right)}{a^2 x},\ M = \frac{\sigma B_0^2}{\rho a},\ Pr = \frac{\rho c_p}{k},$$
$$Ec = \frac{a^2 x^2 \nu}{c_p\left(T_f - T_\infty\right)},\ Le = \frac{\nu}{D},\ Bi = \frac{h}{k}\sqrt{\frac{\nu}{a}}$$

are the Grashof number, magnetic parameter, Prandtl number, Eckert number, Lewis number and Biot number respectively and

$$\Gamma = \frac{1}{\nu\rho\beta E} \quad \text{and} \quad \epsilon = \frac{a^3 x^2}{2E^2\nu}$$

are the material fluid parameters.

3. Methodology

The system of equations

$$(1+\Gamma) f''' - \epsilon\Gamma\left(f''\right)^2 f''' + f'' f - \left(f'\right)^2 + Gr\theta - Mf' = 0,$$
$$\theta'' + Pr\theta' f + PrEc[\left(f''\right)^2 + M\left(f'\right)^2] = 0,$$
$$\phi'' + Lef\phi' = 0$$

with its boundary conditions

$$\text{at } \eta = 0;\quad f = 0,\ f' = 1,\ \theta' = Bi\,(1-\theta)\,,\ \phi = 1$$
$$\text{as } \eta \to \infty;\quad f' = 0,\ \theta \to 0,\ \phi \to 0$$

is rendered into a system of ordinary differential equations

$$X_1' = X_2$$
$$X_2' = X_3$$
$$X_3' = \frac{-X_3 X_1 + X_2^2 - Gr X_4 + M X_2}{1 + \Gamma - \epsilon\Gamma X_3^2}$$
$$X_4' = X_5$$
$$X_5' = -Pr X_5 X_1 - Pr Ec(X_3^2 + M X_2^2)$$
$$X_6' = X_7$$
$$X_7' = -Le X_1 X_7$$

with the initial conditions

$$X_1 = 0, X_2 = 1, X_3 = s_1, X_4 = s_2, X_5 = Bi(1 - s_2),$$
$$X_6 = 1, X_7 = s_3 \text{ at } \eta = 0,$$

where s_1, s_2, s_3 are obtained using the shooting technique so that the conditions $X_2(\infty) = 0, X_4(\infty) = 0, X_6(\infty) = 0$ are satisfied. This new system is solved using the Runge–Kutta–Gills method (for some other methods, see [20]) with the step size $\Delta\eta = 0.01$ and absolute tolerance 10^{-6}. Results are illustrated as graphs.

4. Results and discussion

The incompressible steady laminar two-dimensional MHD Eyring–Powell flow over an SP is modelled as eqs (1)–(6). The equations are non-dimensionalised into the differential equations (7)–(11). The problem is transformed to IVP using shooting technique with the Runge–Kutta–Gills method (see [20] for a list of other methods of solution), and the solutions are depicted as graphs.

As magnetic parameter increases, the induced Lorentz force increases at the region near the surface. The consequences of increasing the magnetic parameter on the dynamics of MHD Eyring–Powell flow are depicted in figures 1–5. The induced Lorentz force opposes the flow, thereby causing a reduction in the overall velocities of the flow and figures 1 and 2 show that the secondary and the primary velocities reduce as magnetic parameter increases. Meanwhile, shear stress reduces at the boundary layer but increases far from the surface (as shown in figure 3) as magnetic parameter increases. Due to the increase in induced Lorentz force, the temperature is enhanced and the concentration is also enhanced. This gives us the reason as to why the temperature profile and concentration profile increase respectively in figures 4 and 5. The results here are in agreement with the findings in [11].

Varying the Eyring–Powell fluid parameter Γ from 0 to a more positive value changes the flow from Newtonian to Eyring–Powell flow. The fluid becomes strongly Eyring–Powell as the value of Γ increases and more heat energy is converted to kinetic energy of the fluid particle. This explains why the secondary and primary velocity profiles increase with increasing Eyring–Powell parameter Γ as shown in figures 6 and 7 respectively. The shear stress increases near the surface but decreases far away from the surface as Γ becomes more positive (see figure 8). Since the heat energy is converted to kinetic energy in the flow as Γ becomes more positive, the temperature profiles drop at the boundary layer but increases slightly at the free stream. This is shown in figure 9. More so, as Γ becomes more positive, the nanoparticle migration is enhanced (as a consequence of the increased kinetic energy), thereby reducing the concentration profiles as shown in figure 10. These results are in full agreement with the findings of [7].

Eckert number Ec is defined as the ratio of kinetic energy to heat dissipation potential. Eckert number Ec measures the amount of wasted energy in the flow as a result of viscous dissipation. Enhancement in the rate of viscous dissipation is a major factor to increase the flow velocities in all directions (as shown in figures 11 and 12). Also, enhancement of viscous dissipation increases

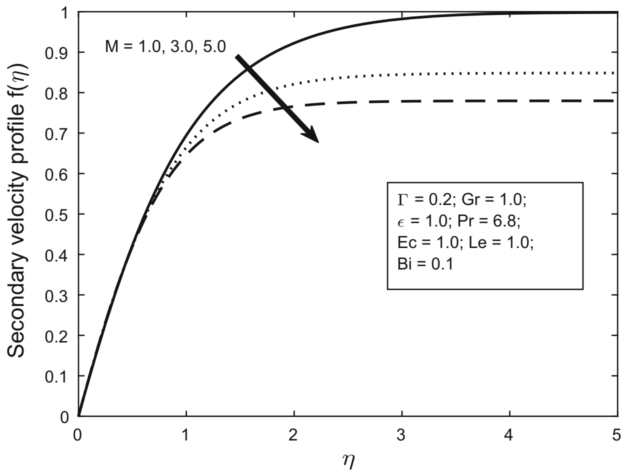

Figure 1. Variation of secondary velocity with magnetic parameter M.

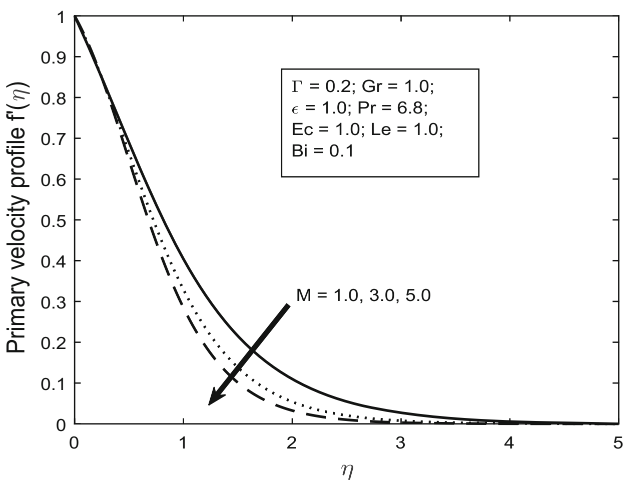

Figure 2. Variation of primary velocity with magnetic parameter M.

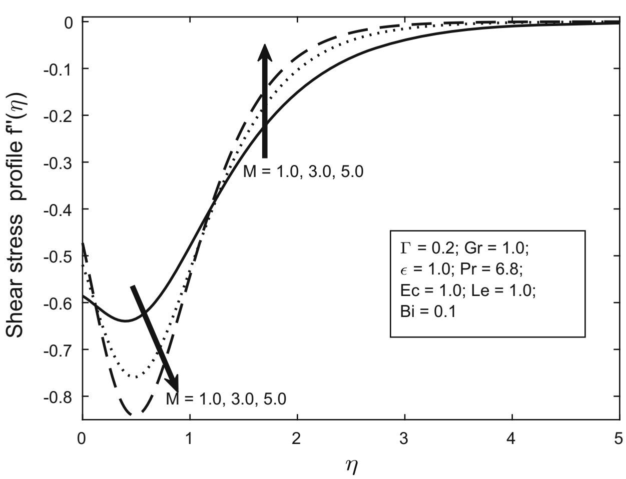

Figure 3. Variation of shear stress with magnetic parameter M.

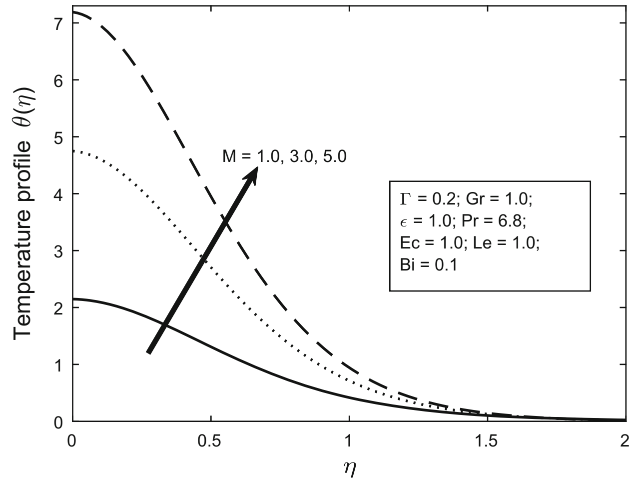

Figure 4. Variation of temperature with magnetic parameter M.

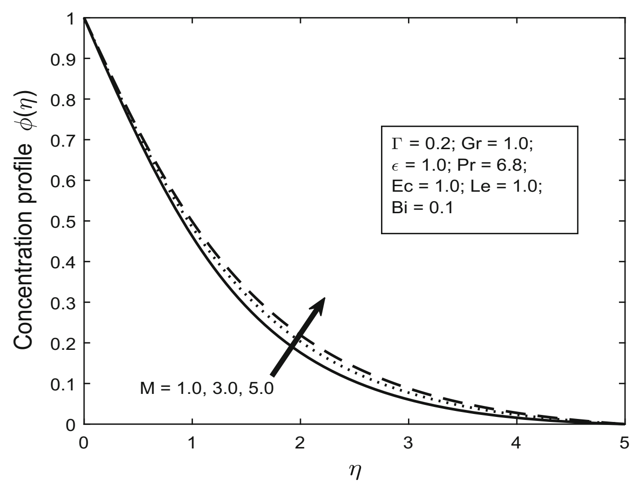

Figure 5. Variation of concentration with magnetic parameter M.

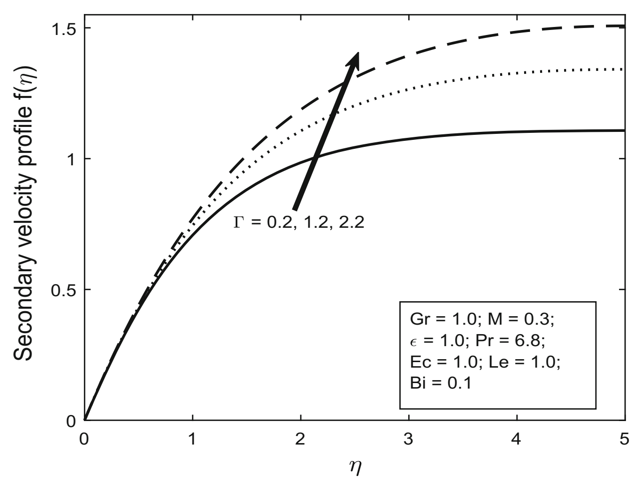

Figure 6. Variation of secondary velocity with fluid parameter Γ.

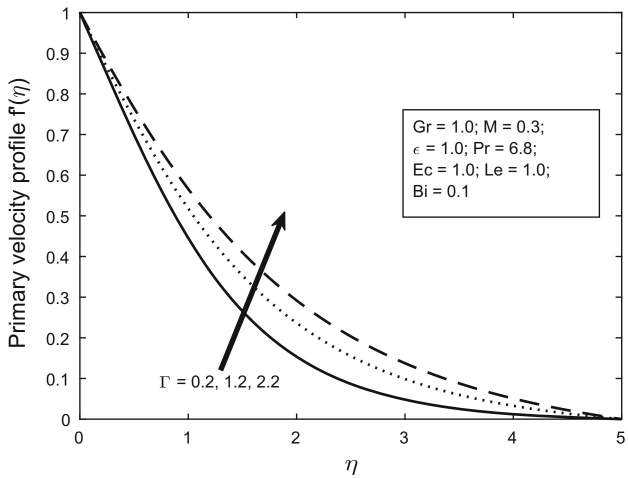

Figure 7. Variation of primary velocity with fluid parameter Γ.

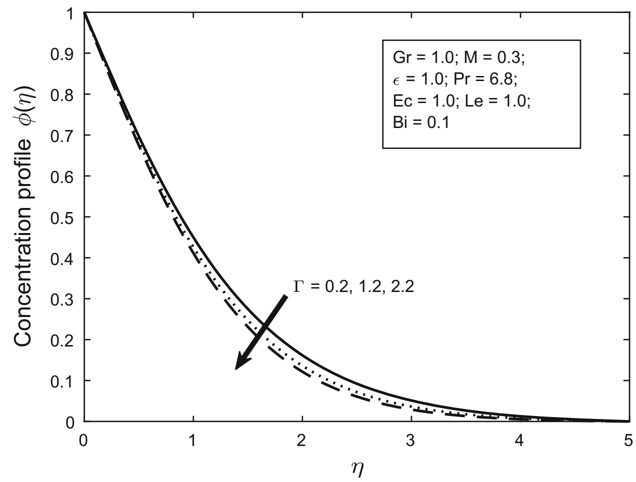

Figure 10. Variation of concentration with fluid parameter Γ.

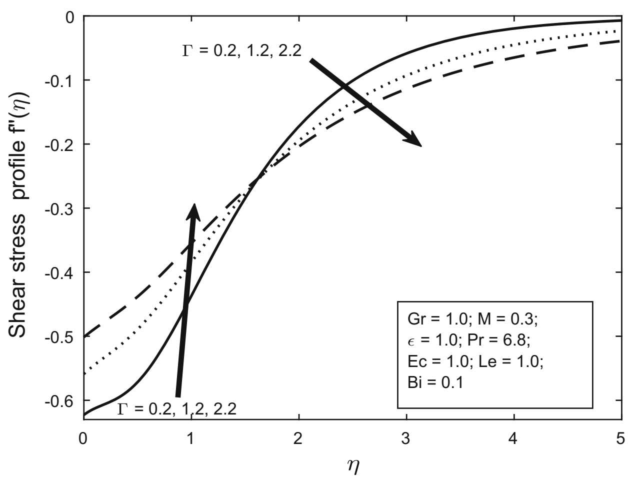

Figure 8. Variation of shear stress with fluid parameter Γ.

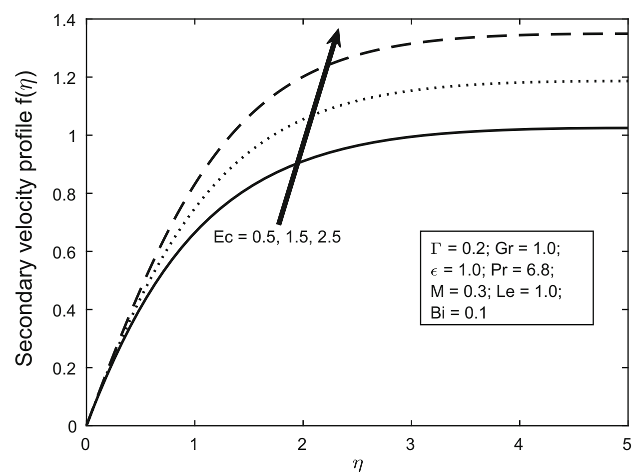

Figure 11. Variation of secondary velocity with Eckert number Ec.

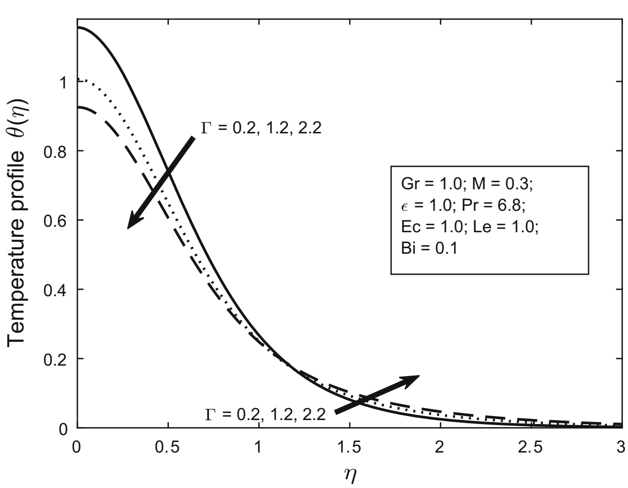

Figure 9. Variation of temperature with fluid parameter Γ.

the shear drag in the boundary layer due to the energy used without accomplishing useful work (figure 13). Meanwhile, the wasted mechanical energy from the viscous dissipation converts to heat energy which consequently increases the temperature profile (see figure 14). Consequently, the concentration profile decreases as Ec increases as seen in figure 15.

Tables 1–3 show the variations of the quantities of interest (i.e. skin friction coefficient, heat transfer rate and Sherwood number). The coefficient of skin friction responds positively to increase in magnetic parameter M (table 1), Eyring–Powell parameter Γ (table 2) and Eckert number Ec (table 3). Heat transfer rate increases as magnetic parameter M (table 1) and Eckert number Ec (table 3) increase but decreases as Eyring–Powell parameter Γ (table 2) increases. Sherwood number increases as Eyring–Powell parameter Γ (table 2) and

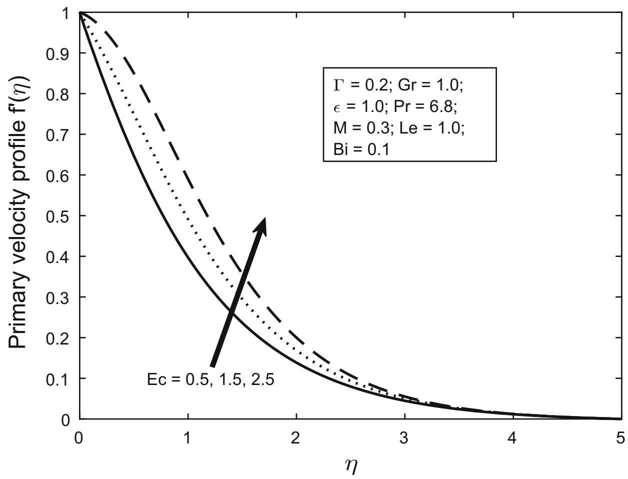

Figure 12. Variation of primary velocity with Eckert number Ec.

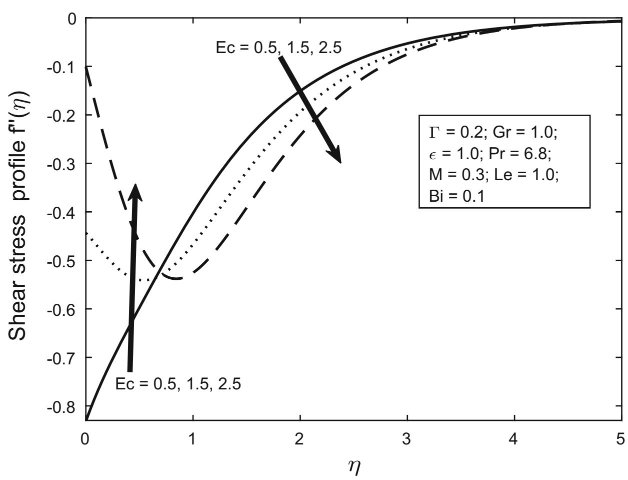

Figure 13. Variation of shear stress with Eckert number Ec.

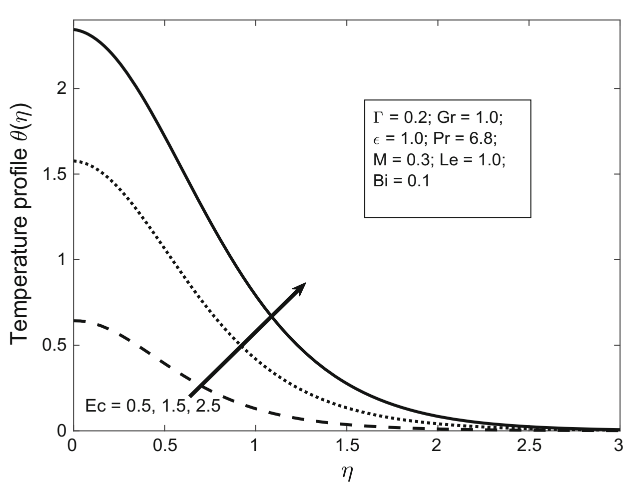

Figure 14. Variation of temperature with Eckert number Ec.

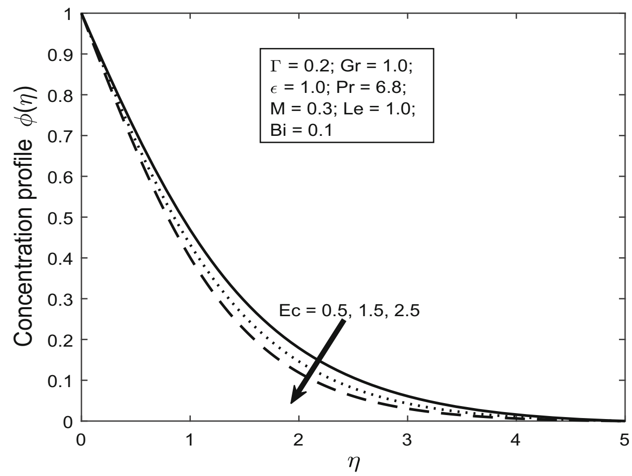

Figure 15. Variation of concentration with Eckert number Ec.

Eckert number Ec (table 3) increase but decreases as magnetic parameter M (table 1) increases.

5. Conclusion

This study focusses on the flow of electrically conducting Eyring–Powell fluid past a convectively heated stretching plate. A system of nonlinear partial differential equations that are transformed into ODEs using similarity transformations of flow variables and then solved by Runge–Kutta is formulated to model the flow. From the analysis, the effects of the magnetic field parameter, Eyring–Powell fluid parameter and Eckert number are studied. The following important results are highlighted:

i. Primary and secondary velocity profiles increase with increasing Eckert number.
ii. Shear stress decreases at the boundary layer with increasing Eckert number.
iii. Temperature and concentration profiles decrease with increasing Eckert number.
vi. Primary and secondary velocity profiles decrease with magnetic field strength.
v. Shear stress decreases at the boundary layer with increasing magnetic field strength.
vi. Temperature and concentration profiles increase with magnetic field strength.
vii. Temperature and concentration profiles decrease with Eyring–Powell fluid parameter.
viii. Primary and secondary velocity profiles increase with increase in Eyring–Powell fluid parameter.

Table 1. Variation of quantities of interest with magnetic parameter M.

M	Γ	Ec	C_{fx}	N_{ux}	Sh
1.0	0.2	1.0	−0.5859	0.1148	0.6151
3.0	0.2	1.0	−0.5192	0.3749	0.5885
5.0	0.2	1.0	−0.4726	0.6190	0.5726

Table 2. Variation of quantities of interest with Eyring–Powell parameter Γ.

M	Γ	Ec	C_{fx}	N_{ux}	Sh
0.3	0.2	1.0	−0.6228	0.0156	0.6294
0.3	1.2	1.0	−0.5589	0.0007	0.6583
0.3	2.2	1.0	−0.5014	−0.0074	0.6762

Table 3. Variation of skin friction coefficient, heat transfer rate and Sherwood number with Eckert number Ec.

M	Γ	Ec	C_{fx}	N_{ux}	Sh
0.3	0.2	0.5	−0.8318	−0.0357	0.6051
0.3	0.2	1.5	−0.4436	0.0577	0.6514
0.3	0.2	2.5	−0.1009	0.1345	0.6939

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