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The five-dimensional extended space model: Localisation of a plane wave and its interaction with a point charged particle

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Abstract. The process of localisation of a plane wave and its interaction with a point charged particle is considered in the framework of the five-dimensional extended space model (ESM). The equations for the plane-wave potentials describing the process of its localisation are presented. The exact solution of these equations is found. A scheme of interaction of a localised wave with a charged particle is proposed. It is shown that, within the framework of the ESM formalism introduced by us, when a point charged particle interacts with a plane electromagnetic wave incident on it, the point particle 'swells', and its spatial size is determined by the plane-wave localisation parameter.

Keywords. Plane wave localisation; $(1+4)$D space model; wave–particle interaction; photon mass variable.

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1. Introduction

Currently, in astrophysics and cosmology, as well as in elementary particle physics, there are many phenomena that do not fit into traditional concepts. Many papers are devoted to their discussion. Here are just a few recent publications related to nuclear physics [1,2] and cosmology [3,4]. We shall not analyse these phenomena and processes here, but only note that in our opinion, their explanation requires the involvement of new physical ideas. The implementation of these new ideas requires a modification of the formal mathematical apparatus.

Our hypothesis is that some physical values, that are considered constant under the traditional approach, are not, in fact, constant and may vary their values under certain conditions. We are talking about the rest mass of massive particles and the zero mass of the photon. We assume that there are some processes in nature in which a photon acquires a non-zero mass, and massive particles can vary their rest mass. For a formal description of such processes, we propose a new model, which is a generalisation of the special theory of relativity. We call it the extended space model (ESM). This name is due to the fact that it is formulated in a flat five-dimensional space $G(1,4)$ with the metric $(+----)$. The main properties of the ESM are described in our papers [5–8].

The ESM formalism is based on the following provisions. In special relativity, for a free scalar particle, there is a dispersion relation, which is satisfied by the components of the 4-vector energy–momentum of a free scalar particle.

$$\frac{E^2}{c^2} - p_x^2 - p_y^2 - p_z^2 = m^2c^2. \tag{1}$$

In the conjugate configuration space, the energy E corresponds to the time t, the momenta p_x, p_y, p_z to the coordinates x, y, z and the mass m corresponds to the interval s. These values serve as coordinates of points in the configuration space, and instead of the dispersion relation (1), we should consider the relation

$$(ct)^2 - x^2 - y^2 - z^2 = s^2. \tag{2}$$

In the special theory of relativity (STR) time t, energy E, spatial coordinates x, y, z and momenta p_x, p_y, p_z are variables. More precisely, components of 4-vectors, but the interval s and mass m are constants, scalars with respect to the Lorentz transformations.

In ESM instead of relations (1), (2), we consider the relations

$$\frac{E^2}{c^2} - p_x^2 - p_y^2 - p_z^2 - m^2c^2 = 0 \tag{3}$$

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and

$$(ct)^2 - x^2 - y^2 - z^2 - s^2 = 0. \tag{4}$$

The parameters t, x, y, z, s are coordinates of a point in extended space $G(1, 4)$. The Minkowski space $M(1, 3)$ enters it as a subspace. All values in relations (3) and (4) are components of 5-vectors. In Minkowski space $M(1, 3)$, a 4-vector of energy and momentum

$$\tilde{p} = \left(\frac{E}{c}, p_x, p_y, p_z\right) \tag{5}$$

is associated with a free scalar particle [5]. In the extended space $G(1, 4)$, we complete it to a 5-vector

$$\bar{p} = \left(\frac{E}{c}, p_x, p_y, p_z, mc\right). \tag{6}$$

For a free particle, the components of the 5-vector (6) satisfy eq. (1).

In the STR a photon with frequency ω corresponds to the 4-energy–momentum vector

$$\left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}\vec{k}\right). \tag{7}$$

Since the mass of the photon in STR is $m = 0$, the 4-vector (7) is isotropic, i.e. its length in Minkowski space is zero.

Massless fields in the extended space $G(1, 4)$ are mapped to a 5-vector

$$\left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}\vec{k}, 0\right). \tag{8}$$

In the future, we shall assume that a photon with a 4-energy–momentum vector (7) corresponds to a plane wave, which moves with a speed c in the direction given by the vector $\vec{k}$. We shall consider this wave as the photon field, and the 4-vector (7) as the energy–momentum vector of this field.

The 4-energy–momentum vector (5) characterises a free particle. It is assumed that the Lorentz transformations in the Minkowski space save the right part of relation (1) and save the length of this 4-vector. This means that only processes that do not change the rest mass are considered. We want to consider a broader class of processes that can change mass. For this purpose, instead of the ratio (1), we use the ratio (3).

The energy–momentum–mass 5-vector (6) characterises a particle for which all the parameters, energy, momentum and mass, are variables. The corresponding changes of these values can be described using transformations of the extended space $G(1, 4)$.

The interval s in Minkowski space serves as the fifth coordinate in the extended space $G(1, 4)$. The variations of mass m correspond to variations of the interval s.

We can consider the extended space $G(1, 4)$ as a set of Minkowski spaces parametrised by the value n, which we conditionally call the refractive index n. This choice of parametrisation is due to the fact that we base the physical meaning of our model on the movement of the photon and the change in its velocity. We believe that every Minkowski subspace $M(1, 3)$ of space $G(1, 4)$ is characterised by its refractive index n. This index sets the speed of the photon in this space $v = c/n$. The relationship between interval s and refractive index n will be studied in detail in our future papers.

From the point of view of ESM, the transition from a medium with one refractive index n_1 to a medium with another refractive index n_2 can be interpreted as the movement along the fifth coordinate of the expanded space.

In the ESM, the gravitational and electromagnetic fields are naturally combined into a single gravitational–electromagnetic field. This is due to the fact that in the ESM, the electromagnetic interaction generates the appearance and change of fields and particles mass.

In the ESM, it is postulated from the beginning that mass m can vary, leading to the possibility of describing new processes that cannot be explained in the STR framework. It is due to this that the gravitational interaction between elementary particles naturally occurs within the framework of the ESM.

The problem of combining the electromagnetic and gravitational fields into one single field has been discussed since the end of the nineteenth century. It is characteristic that all these attempts were made on the way of constructing geometric models of physical interactions and interpreting physics as geometry in spaces of a larger number of dimensions. German mathematician Felix Klein [9] proposed to consider the Hamilton-Jacobi theory as an optics in the space of the highest number of dimensions. However, at that time his ideas were not developed.

A new surge of interest to the problem of geometrisation of physics was stimulated by the creation of the general theory of relativity (GTR) [10]. Attempts have been made to describe electromagnetism in geometrical terms is analogy with gravity. Authors of such attempts usually did not try to create a new theory, but tried to expand the existing GTR scheme in one way or the other. The most known models were proposed by Kaluza [11] and Klein [9].

Also noteworthy are the works of Mandel [12] and Fock [13]. Characteristically, they had to use a 5D space. The problem of the physical interpretation of the fifth coordinate has not been satisfactorily solved.

Many scientists, including Einstein [14], de Broglie, Gamow and Rumer [15] tried to develop these approaches, but they failed to get any interesting results.

In our opinion, the reason is that their works were based on formal generalisations of the already existing models, without involving any new physical hypotheses.

Overview of various multidimensional theories can be found in the book of Vladimirov [16].

Wesson and his co-authors [17–20] developed a model similar to ESM model. Wesson proposed to use 'mass' as the fifth coordinate, in addition to the time and three spatial coordinates: "we ... view mass as on the same footing as time and space..." (p. 10 of [17]) and "This means that the role of the 4D uncharged mass is played in 5D geometry by the extra coordinate" (p. 191, eq. (7.40)). This choice seems illogical to us. In this case, it leads to difficulties in generalising the 4D energy–momentum tensor to the energy–momentum–mass tensor in five-dimensional space. Of course, mass can be used as the fifth coordinate, but not in the coordinate space. The mass should be considered in the pulse space, i.e. as an additional value to the energy and three components of the pulse. In coordinate space, the fifth coordinate should be another value that is associated with the mass. As a result of attribution mass as fifth coordinate in addition to time and space, it was difficult to find connections with real experiments. Recently, Overduin and Henry [21] proposed the same idea of considering the fifth coordinate as we have done before in 2000 [5].

Our approach [22–25] is fundamentally different from all these and other similar theories. First, the ESM is based on the physical hypothesis, which says the mass (rest mass) and its conjugate value interval are dynamic variables, the value of which is determined by the interaction of fields and particles. In this respect, our model is a direct generalisation of STR. In STR, the interval and the rest mass of the particles are invariants, but in the ESM they can vary.

In particular, a photon can acquire mass, both positive and negative. This mass can appear and vary as a result of electromagnetic interaction and generate gravitational forces. It is this circumstance that allows us to consider gravity and electromagnetism as a single field.

Secondly, in other five-dimensional theories, there is a problem with interpretation of the additional fifth coordinate. We do not have this problem, as in the ESM the additional fifth dimension is a well-known value – the interval. Just in the STR it is a constant value, and in the ESM it became a variable. Rumer [15] suggested using the action as the fifth coordinate. However, there are difficulties on this path, because the action is an integral of the Lagrangian and has a different form for different dynamical systems. In the following, we propose to study the relationship of an action with an interval in a five-dimensional space.

The other important aspect of the ESM is that it allows us to associate with a particle or a field certain linear parameter, which can be associated with their sizes. The value of this parameter is also determined by the external forces acting at these particles and fields. Depending on the nature of the particles and the nature of the interaction, this size can either increase or decrease. Thus, in the ESM the change of a particle mass is naturally associated with the change of its size.

One of the advantages of the ESM is that many processes and phenomena can be described only by algebraic methods using transformations in the extended space $G(1, 4)$. These transformations were studied in detail in our papers [5,8,22]. In [7,8] it was shown that all the main effects of general relativity can be obtained using only rotations in the extended space $G(1, 4)$.

2. Localisation of fields and particles

Within the framework of the ESM, one can naturally establish connection between the photon mass and a certain linear parameter. We call this parameter the localisation parameter. In a sense, it can be considered as the size of a photon. The base point for us is the analogy between the dispersion relation of a free particle

$$E^2 = (c\vec{p})^2 + m^2c^4, \tag{9}$$

and the dispersion relation of the wave mode in a hollow metal waveguide is

$$\omega^2 = \omega_{kr}^2 + (c\xi)^2. \tag{10}$$

Here ω_{kr} is the critical frequency of the waveguide mode and ξ is the wave propagation constant.

The similarity of relations (9) and (10) was noticed by Broglie [26], Feynman *et al* [27] and other scientists. The essence of the problem is that the parameter is associated with the critical frequency ω_{kr}

$$m = \hbar\omega_{kr}/c^2 \tag{11}$$

which has the dimension of mass. The question arises, can this value be interpreted as the real mass?, the mass that the electromagnetic field acquires when it enters the waveguide. In Rivlin's works, this problem was studied in a systematic way [28,29].

But here we shall not go into this question, but only note the fact that mass m is related to the geometry and size of the waveguide. In particular, if the waveguide has a circular shape with a diameter of l, then this connection has the form

$$l = \sqrt{2}\pi\hbar/mc. \tag{12}$$

It is this value that we propose to consider as a linear parameter that is associated with a massless particle when it acquires a mass m. We believe that at the same time as a massless particle gets a non-zero mass when it enters into the external field, the corresponding infinite plane wave is compressed to a finite size. This finite size is characterised by the localisation parameter l.

The form of the value (12) resembles the Compton wavelength of an electron, but its physical meaning is quite different. In the formula for the Compton wavelength of the electron, the parameter m is the rest mass of the electron, while in our formula (12) m is the mass that a photon acquires when it is exposed to external influences.

3. Interaction of a plane wave with a point charged particle

In the extended space $G(1,4)$, the potentials of the field combining electromagnetism and gravity are determined by the equations

$$\Pi_{(5)} A_0 = -4\pi\rho,\ \Pi_{(5)}\vec{A} = -\frac{4\pi}{c}\vec{j},$$
$$\Pi_{(5)} A_s = -\frac{4\pi}{c} j_s. \tag{13}$$

Here

$$\Pi_{(5)} = \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}. \tag{14}$$

On the right-hand side of the equations in system (13) are the components of the five-dimensional current vector $\bar{\rho}$.

The five-dimensional current vector $\bar{\rho}$ is a generalisation of the four-dimensional current vector $\tilde{\rho}$. To do this, it is necessary to assign an additional component. In traditional electrodynamics of the four-dimensional current vector $\tilde{\rho}$ reads as

$$\tilde{\rho} = (\rho, \vec{j}) = \left(\frac{\rho_0 c}{\sqrt{1-\beta^2}}, \frac{\rho_0 \vec{v}}{\sqrt{1-\beta^2}}\right);\ \tilde{\rho}^2 = c^2. \tag{15}$$

In order to get the five-dimensional current vector $\bar{\rho}$ we assign an additional component to the four-dimensional current vector (15). Therefor the five-dimensional current vector $\bar{\rho}$ reads as

$$\bar{\rho} = (j_0, \vec{j}, j_4) = \left(\frac{emc}{\sqrt{1-\beta^2}}, \frac{em\vec{v}}{\sqrt{1-\beta^2}}, emc\right). \tag{16}$$

Here e is the charge of a particle. This is an isotropic vector

$$\bar{\rho}^2 = 0.$$

When the variables included in system (13) have no dependence on the variable s, system (13) splits into two independent subsystems. First four equations describe the electromagnetic field in system (13), and the fifth equation in (13) defines the scalar gravitation field. But if these values depend on the variable s, then these two fields are combined into a single electrogravity field. In [6], Lienard–Wichert potentials for such fields were found.

Now we consider the scheme of interaction of a plane electromagnetic wave with a charged particle. We assume that a plane wave is an object in empty space that fills this infinite space.

In the extended space $G(1,4)$, the field potentials without sources are determined by the equations

$$\Pi_{(5)} A_0 = 0,\quad \Pi_{(5)}\vec{A} = 0,\quad \Pi_{(5)} A_s = 0. \tag{17}$$

Let us consider the equation

$$\frac{\partial^2}{\partial s^2}u + \frac{\partial^2}{\partial x^2}u + \frac{\partial^2}{\partial y^2}u + \frac{\partial^2}{\partial z^2}u - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}u = 0. \tag{18}$$

We are looking for its solutions in the form

$$U(s,x,y,z,t) = u(s,x,y,z)\mathrm{e}^{-iks}\mathrm{e}^{i\omega t},\quad k = \frac{2\pi}{\lambda}. \tag{19}$$

We assume that the function $u(s,x,y,z)$ varies slowly over the variable s, compared to the variables x, y, z, so that the second derivative $(\partial^2/\partial s^2)u$ can be neglected. Now we get the equation

$$\frac{\partial^2}{\partial x^2}u + \frac{\partial^2}{\partial y^2}u + \frac{\partial^2}{\partial z^2}u - 2ik\frac{\partial}{\partial s}u = 0. \tag{20}$$

The neglect of the second derivative and presentation of eq. (18) in the form (20) are similar to the search for a solution for the shape of an optical wave propagating in a laser along the z-axis [30].

The solution of eq. (20) has the form of a three-dimensional Gaussian beam

$$u = u_0\left(\frac{w_0}{w}\right)^{3/2}$$
$$\times\exp\left[-i(ks+\varphi) - (x^2+y^2+z^2)\left(\frac{1}{w^2} + \frac{ik}{2R}\right)\right]. \tag{21}$$

Here w_0 is the radius of the 'neck' of the beam, i.e. its minimum width at the point $s = 0$. The value of $w^2 = w_0^2[1 + (2s/kw_0^2)^2]$ is the diameter of the beam at the point z. $R(z) = z[1 + (kw_0^2/2s)^2]$ is the radius of curvature of the beam wavefront.

We see that for $s \to \infty$, both the radius and the beam width also tend to infinity, and solution (21) corresponds to a plane wave. When $s \to 0$, the plane wave is localised in a volume that looks like a ball with radius $r = w_0$. This localisation process takes place without changing the energy and is described by orthogonal rotations in the planes (SX), (SY), (SZ). Such rotations correspond to the localisation parameter (12), and we can map it to the radius of the beam neck (21).

The square of the wave modulus (21) reads as

$$|u|^2 = |u_0|^2 \left(\frac{w_0}{w}\right)^3 \exp\left[-(x^2 + y^3 + z^2)\left(\frac{2}{w^2}\right)\right]. \tag{22}$$

We see that as s decreases, the localisation of the wave (21) increases and reaches its maximum value at $s = 0$. However, this degree of localisation is not really achieved. The fact is that the process of localisation of a plane wave is generated by the presence of a charged particle in space. The wave field is affected by the charged particle, but the particle itself is affected by the field. We assume that in empty space (refractive index $n = 1$), a massive charged particle is concentrated at a point, i.e. its mass distribution described by δ is a function, and we consider this delta function as the free particle wave function. When such a charged particle enters the plane-wave field, the δ-function begins to transform into a Gaussian function.

$$|v_s(x, y, z)|^2 = K|v_0|^2 \left(\frac{1}{s^2\pi}\right)^3 \times \exp\left[-(x^2 + y^3 + z^2)\left(\frac{2}{s^2}\right)\right]. \tag{23}$$

Here K is some dimensional multiplier.

Expression (23) is structurally similar to the solution of the differential heat equation describing the temperature distribution. For an infinite body with an instantaneous point source at the origin, the temperature distribution has the following form:

$$T(x, y, z, t) = \frac{Q}{c\rho(4\pi at)^{3/2}} \exp\left(-\frac{x^2 + y^2 + z^2}{4at}\right). \tag{24}$$

Here T is the temperature at time t in coordinates x, y, z; Q is the heat emitted at the time $t = 0$ at the origin; t is the time elapsed since the introduction of heat; a is the thermal diffusivity, ρ is the density of the body and c is its specific heat. Equation (24) is the fundamental solution of the heat equation under the action of an instantaneous point source in an infinite body.

The process of localisation of a plane wave and the process of 'swelling' of a massive particle are consistent with each other. We assume that for the interaction of the field with the particle, it is necessary that their localisations coincide. Comparing expressions (22) and (23), we get an expression for the value s_0, which determines the minimum value of the plane-wave localisation and the maximum value of the point particle swelling.

$$s_0^2 = \frac{k^2 w_0^4}{k^2 w_0^2 - 4} = \frac{\pi^2 w_0^4}{\pi^2 w_0^2 - \lambda^2}. \tag{25}$$

Expression (25) imposes restrictions on the dynamics of interaction of the field with the particle.

4. Conclusion

We have considered a plane electromagnetic wave that enters the field of a point charged particle. An explicit form of potentials that describe such a wave in a five-dimensional extended space is found. The resulting solution describes the process of localisation of a plane wave and the appearance of its mass. The scheme of interaction of such a wave with a charged particle is considered. Using the explicit form of these potentials, we can calculate the field strengths and find their energy–momentum–mass tensor. In future, we propose to establish a connection between the localisation procedure proposed by us and the renormalisation procedure in quantum electrodynamics.

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