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Space–time fractional nonlinear partial differential system: Exact solution and conservation laws

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Abstract. The objective of this manuscript is to analyse space–time fractional generalised Hirota–Satsuma coupled Korteweg–de Vries (HSCKdV) system with time-dependent variable coefficients for exact solution using power series method corresponding to Lie symmetry reduction of HSCKdV system. The exact solution obtained in power series form is further analysed for convergence. Conservation laws of the HSCKdV system are constructed by using the new conservation theorem and generalised fractional Noether's operator.

Keywords. Fractional differential equations with time-dependent variable coefficients; power series solution; conservation laws.

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1. Introduction

In numerous research and development applications, the fractional partial differential equations (FPDEs) appear in certain fields including physics, biology, hydrodynamics, viscoelasticity, control theory, electrochemistry etc. [1–12]. They have recently gained great attention, and there has been relevant scientific progress that has taken place in this field. It is very well-known that the key instrument for characterising the nonlinear physical phenomena is to study the nonlinear fractional partial differential equations (NLFPDEs).

Several powerful methods for finding exact solutions of NLFPDEs have been developed in the literature recently. Some of them are: the method of Lie symmetry, the method of exp function, the method of fractional sub-equation, the method of exact power series solutions, residual power series, $(D_t^\alpha G/G)$-method, etc. For a deeper understanding of nonlinear phenomena, researchers are increasingly interested in space–time NLFPDEs rather than in integer-order NLPDEs. There are many techniques to find solution of fractional partial differential equations. In some publications, the investigation of conservation laws has been defined [13–22]. The conservation laws can be investigated for NLFPDEs which are very important mechanisms for the study of mathematical along with physical differential equations [23,24]. Noether's theorem [25] involves a structured methodology for constructing FPDE's conservation laws, using symmetries associated with Noether's operator. However, there are several other methods to obtain the conservation laws of PDEs [26–28].

After finding symmetry transformations by classical method, the conserved vectors can be found using symmetry transformations. For this, the considered problem must be investigated for the nonlinear self-adjointness. Fractional-order Noether's operators have been generalised to find conservation laws using a new conservation theorem [22].

In certain FPDEs, the power series approach is being used to discover exact solution in the form of convergent power series. This type of solution generally assumes a power series having coefficients to be determined, and the substitution that series into the FPDEs provides a recurrence relation between coefficients. Power series solutions are very important solutions of FPDEs. The power series method (PSM) [29,30] illustrates how to find the exact series solution of FPDEs. The system of space–time fractional generalised Hirota–Satsuma coupled Korteweg–de Vries (HSCKdV) equations is studied for explicit power series solution, and conservation laws in this work.

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The main aim of this article is to find exact solutions of the governing system. For this, the Lie symmetry reduction of the governing system obtained in our previous work [31] is used. The PSM is imposed on the reduced form of the governing system, involving fractional Erdèlyi–Kober differential operators. The power series solutions are obtained for reduced form and then using symmetry transformations can be obtained for the original equation. The conservation laws and symmetries of FPDEs have one to one correspondence. The new conservation theorem [22] and generalised Noether's operators are imposed to construct conservation laws.

Consider the space–time fractional generalised Hirota–Satsuma coupled Korteweg–de Vries equations [5,32] with time-dependent variable coefficients as

$$\begin{aligned}\frac{\partial^\gamma u}{\partial t^\gamma} &= A_1(t)u\frac{\partial^\beta u}{\partial x^\beta} + A_2(t)v\frac{\partial^\beta v}{\partial x^\beta} + A_3(t)\frac{\partial^\beta w}{\partial x^\beta}\\ &\quad +A_4(t)\frac{\partial^3 u}{\partial x^3},\\ \frac{\partial^\gamma v}{\partial t^\gamma} &= A_5(t)u\frac{\partial^\beta v}{\partial x^\beta} + A_6(t)\frac{\partial^3 v}{\partial x^3},\\ \frac{\partial^\gamma w}{\partial t^\gamma} &= A_7(t)u\frac{\partial^\beta w}{\partial x^\beta} + A_8(t)\frac{\partial^3 w}{\partial x^3}. \end{aligned} \tag{1}$$

The generalised HSCKdV equations with integer order and fractional order have been studied, by many researchers, using different methods. The generalised HSCKdV equations has been studied in [5] using variational iteration method. In [11], soliton solutions have been obtained. Numerical solutions have been obtained in [33]. Lie symmetry reduction of fractional HSCKdV has been obtained in [34]. We have studied the governing system using Lie symmetry method for symmetry reduction in [31].

The sections are organised as follows: Section 2 contains the preliminaries. Power series solution is provided in §3. Section 4 presents the convergence of the exact solution from §3. In §5, conservation laws are constructed. In §6, conclusion of the study is drawn.

2. Preliminaries

The governing system (1) is studied for symmetry reduction via Lie symmetry method in our previous work [31]. The present section contains the important results from [31] for further study of system (1) in this article.

Under one-parameter Lie group of transformations, the space–time fractional generalised HSCKdV with time-dependent variable coefficients (1) with $A_4(t) = K_4t^m$, $A_6(t) = K_6t^s$ and $A_8(t) = K_8t^r$ (where K_4, K_6, K_8 are arbitrary constants and m, s, r are real numbers) is invariant. Then, the following values are obtained in [31]:

$$\begin{aligned}&A_1(t) = K_1t^{\frac{1-3\gamma}{2}+\frac{\beta(\gamma+m)}{3}},\\ &A_2(t) = K_2A_1(t), \quad A_3(t) = K_3A_1(t),\\ &A_5(t) = K_5A_1(t), \quad A_7(t) = K_7A_1(t),\end{aligned}$$

where K_1, K_2, K_3, K_5, K_7 are arbitrary constants.

Further infinitesimals of system (1) has been obtained in [31]

$$\begin{aligned}&\xi = \frac{c_1x}{3}, \ \tau = \frac{c_1t}{\gamma+m}, \ \eta = \frac{\gamma-1}{2(\gamma+m)}c_1u,\\ &\phi = \frac{\gamma-1}{2(\gamma+m)}c_1v, \ \psi = \frac{\gamma-1}{2(\gamma+m)}c_1w,\end{aligned} \tag{2}$$

for some arbitrary constant c_1.

Also, symmetry variable and similarity transformations have been obtained as

$$y = xt^{-\frac{\gamma+m}{3}} \tag{3}$$

$$\begin{aligned}&u(x,t) = t^{\frac{\gamma-1}{2}}g(y),\\ &v(x,t) = t^{\frac{\gamma-1}{2}}h(y),\\ &w(x,t) = t^{\frac{\gamma-1}{2}}f(y).\end{aligned} \tag{4}$$

The symmetry reductions of system (1) have been obtained in [31] as

$$\begin{aligned}\left(\mathcal{T}_{\frac{3}{(\gamma+m)}}^{\frac{1-\gamma}{2},\gamma}g\right)(y) &= y^{-\beta}\Big(M_1g(y)\left(\mathcal{R}_1^{-\beta,\beta}g\right)(y)\\ &\quad +(M_2h(y)\left(\mathcal{R}_1^{-\beta,\beta}h\right)(y)\Big)\\ &\quad +(M_3\left(\mathcal{R}_1^{-\beta,\beta}f\right)(y))\Big)\\ &\quad +K_4g'''(y),\\ \left(\mathcal{T}_{\frac{3}{(\gamma+m)}}^{\frac{1-\gamma}{2},\gamma}h\right)(y) &= K_6h'''(y) + M_4y^{-\beta}g(y)\\ &\quad\times\left(\mathcal{R}_1^{-\beta,\beta}h\right)(y),\\ \left(\mathcal{T}_{\frac{3}{(\gamma+m)}}^{\frac{1-\gamma}{2},\gamma}f\right)(y) &= K_8f'''(y) + M_5y^{-\beta}g(y)\\ &\quad\times\left(\mathcal{R}_1^{-\beta,\beta}f\right)(y),\end{aligned} \tag{5}$$

where $\left(\mathcal{T}_{\frac{3}{(\gamma+m)}}^{\frac{1-\gamma}{2},\gamma}\right)$ is the left fractional Erdèlyi–Kober differential operator and $(\mathcal{R}_1^{-\beta,\beta})$ is the right fractional Erdèlyi–Kober differential operator [31] and $M_2 = K_2M_1$, $M_3 = K_3M_1$, $M_4 = K_5M_1$, $M_5 = K_7M_1$ are arbitrary constants.

3. Power series solutions

Here, we examine the analytic explicit solution using power series technique [29,30,35,36] of the reduced system of NLFODEs (5) of considered system (1).

Let us take the solution in the form of the following series:

$$g(y)=\sum_{n=0}^{\infty}a_n y^n,$$
$$h(y)=\sum_{n=0}^{\infty}b_n y^n,$$
$$f(y)=\sum_{n=0}^{\infty}c_n y^n, \tag{6}$$

where a_n, b_n and c_n for $n = 0, 1, 2, \ldots$ are coefficients.

Therefore, from (5) with (6) we get the following system:

$$\sum_{n=0}^{\infty}\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)a_n y^n - M_1 y^{-\beta}\times\sum_{n=0}^{\infty}\sum_{k=0}^{n}a_k a_{n-k}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)y^n - M_2 y^{-\beta}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)b_k b_{n-k}y^n - M_3 y^{-\beta}\sum_{n=0}^{\infty}\left(\frac{\Gamma(1+n)}{\Gamma(1+n-\beta)}\right)c_n y^n - K_4\sum_{n=0}^{\infty}(n+1)(n+2)(n+3)a_{n+3}y^n=0,$$

$$\sum_{n=0}^{\infty}\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)b_n y^n - M_4 y^{-\beta}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)\times b_k a_{n-k}y^n - K_6\sum_{n=0}^{\infty}(n+1)(n+2)\times(n+3)b_{n+3}y^n=0,$$

$$\sum_{n=0}^{\infty}\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)c_n y^n - \left(M_5 y^{-\beta}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)\times c_k a_{n-k}y^n - K_8\sum_{n=0}^{\infty}(n+1)(n+2)\times(n+3)c_{n+3}y^n=0. \tag{7}$$

Comparing the system of eqs (7) for $n \geq 0$, we have the coefficients as follows:

$$a_{n+3}=\frac{1}{K_4(n+1)(n+2)(n+3)}\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)a_n - M_1 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)a_k a_{n-k} - M_2 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)b_k b_{n-k} - M_3 y^{-\beta}\left(\frac{\Gamma(1+n)}{\Gamma(1+n-\beta)}\right)c_n\right\}, \tag{8}$$

$$b_{n+3}=\frac{1}{K_6(n+1)(n+2)(n+3)}\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)b_n - M_4 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)b_k a_{n-k}\right\},$$

$$c_{n+3}=\frac{1}{K_8(n+1)(n+2)(n+3)}\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)c_n - M_5 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)c_k a_{n-k}\right\}. \tag{9}$$

With arbitrary constants a_i, b_i, c_i $(i = 0, 1, 2)$, all coefficients $a_n (n \geq 3)$, $b_n (n \geq 3)$ and $c_n (n \geq 3)$ can be determined for the power series (6).

Thus, the power series (6) can be written as

$$g(y)=a_0+a_1 y+a_2 y^2+\frac{1}{6K_4}\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}\right)}{\Gamma\left(\frac{1-\gamma}{2}\right)}\right)a_0 - M_1 y^{-\beta}\left(\frac{1}{\Gamma(1-\beta)}\right)a_0^2\right.$$

$$-M_2 y^{-\beta}\left(\frac{1}{\Gamma(1-\beta)}\right)\Bigg\} y^3$$
$$+\sum_{n=1}^{\infty}\frac{1}{K_4(n+1)(n+2)(n+3)}$$
$$\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)a_n\right.$$
$$-M_1 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)a_k a_{n-k}$$
$$-M_2 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)b_k b_{n-k}$$
$$\left.-M_3 y^{-\beta}\left(\frac{\Gamma(1+n)}{\Gamma(1+n-\beta)}\right)c_n\right\} y^{n+3},$$

$$h(y)=b_0+b_1 y+b_2 y^2+\frac{1}{6K_6}\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2})\right)}{\Gamma\left(\frac{1-\gamma}{2})\right)}\right)b_0\right.$$
$$\left.-M_4 y^{-\beta}\left(\frac{1}{\Gamma(1-\beta)}\right)b_0 a_0\right\} y^3$$
$$+\sum_{n=1}^{\infty}\frac{1}{K_6(n+1)(n+2)(n+3)}$$
$$\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)b_n\right.$$
$$\left.-M_4 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)b_k a_{n-k}\right\}$$
$$\times y^{n+3},$$

$$f(y)=c_0+c_1 y+c_2 y^2$$
$$+\frac{1}{6K_8}\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2})\right)}{\Gamma\left(\frac{1-\gamma}{2})\right)}\right)c_0\right.$$
$$\left.-M_5 y^{-\beta}\left(\frac{1}{\Gamma(1-\beta)}\right)c_0 a_0\right\} y^3$$
$$+\sum_{n=1}^{\infty}\frac{1}{K_8(n+1)(n+2)(n+3)}$$
$$\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)c_n\right.$$
$$\left.-M_5 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)c_k a_{n-k}\right\}$$
$$\times y^{n+3}. \tag{10}$$

Hence,

$$u(x,t)=a_0 t^{\frac{\gamma-1}{2}}+a_1 x t^{\frac{\gamma-1}{2}-\frac{(\gamma+m)}{3}}$$
$$+a_2 x^2 t^{\frac{\gamma-1}{2}-\frac{2(\gamma+m)}{3}}$$
$$+\sum_{n=0}^{\infty}\frac{1}{K_4(n+1)(n+2)(n+3)}$$
$$\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)a_n\right.$$
$$-x^{-\beta}t^{\frac{\gamma-1}{2}-\frac{-\beta(\gamma+m)}{3}}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)$$
$$\times\Big(M_1 a_k a_{n-k}+M_2 b_k b_{n-k}\Big)$$
$$-M_3 x^{-\beta}t^{\frac{\gamma-1}{2}-\frac{-\beta(\gamma+m)}{3}}$$
$$\left.\times\left(\frac{\Gamma(1+n)}{\Gamma(1+n-\beta)}\right)c_n\right\}$$
$$\times x^{n+3}t^{\frac{\gamma-1}{2}-\frac{(n+3)(\gamma+m)}{3}},$$

$$v(x,t)=b_0 t^{\frac{\gamma-1}{2}}+b_1 x t^{\frac{\gamma-1}{2}-\frac{(\gamma+m)}{3}}+b_2 x^2 t^{\frac{\gamma-1}{2}-\frac{2(\gamma+m)}{3}}$$
$$+\sum_{n=0}^{\infty}\frac{1}{K_6(n+1)(n+2)(n+3)}$$
$$\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)b_n\right.$$
$$\left.-M_4 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)b_k a_{n-k}\right\}$$
$$\times x^{n+3}t^{\frac{\gamma-1}{2}-\frac{(n+3)(\gamma+m)}{3}},$$

$$w(x,t)=c_0 t^{\frac{\gamma-1}{2}}+c_1 x t^{\frac{\gamma-1}{2}-\frac{(\gamma+m)}{3}}+c_2 x^2 t^{\frac{\gamma-1}{2}-\frac{2(\gamma+m)}{3}}$$
$$+\sum_{n=0}^{\infty}\frac{1}{K_8(n+1)(n+2)(n+3)}$$
$$\times\left\{\left(\frac{\Gamma\left(\frac{1+\gamma}{2}-n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2}-n(\frac{\gamma+m}{3})\right)}\right)c_n\right.$$
$$\left.-M_5 y^{-\beta}\sum_{k=0}^{n}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)c_k a_{n-k}\right\}$$

$$\times x^{n+3} t^{\frac{\gamma-1}{2}-\frac{(n+3)(\gamma+m)}{3}} \tag{11}$$

are the required solutions of (1) in the form of power series.

4. Convergence analysis

In this segment, we analyse the convergence [37] of solution (11).

From (8), we have

$$|a_{n+3}| \leq J\Bigg(|a_n| + \sum_{k=0}^{n} |a_k||a_{n-k}| + \sum_{k=0}^{n} |b_k||b_{n-k}| + |c_n|\Bigg), \tag{12}$$

where

$$J = \max\left\{\frac{\Gamma\left(\frac{1+\gamma}{2} - n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2} - n(\frac{\gamma+m}{3})\right)}, M_1 y^{-\beta}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right), M_2 y^{-\beta}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right), M_3 y^{-\beta}\left(\frac{\Gamma(1+n)}{\Gamma(1+n-\beta)}\right)\right\},$$

for all $k = 0, 1, 2, \ldots, n$,

$$|b_{n+3}| \leq K\Bigg(|b_n| + \sum_{k=0}^{n} (|b_k||a_{n-k}|)\Bigg), \tag{13}$$

where

$$K = \max\left\{\frac{\Gamma\left(\frac{1+\gamma}{2} - n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2} - n(\frac{\gamma+m}{3})\right)}, M_1 y^{-\beta}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)\right\},$$

for all $k = 0, 1, 2, \ldots, n$, and

$$|c_{n+3}| \leq L\Bigg(|c_n| + \sum_{k=0}^{n} (|c_k||a_{n-k}|)\Bigg), \tag{14}$$

where

$$L = \max\left\{\frac{\Gamma\left(\frac{1+\gamma}{2} - n(\frac{\gamma+m}{3})\right)}{\Gamma\left(\frac{1-\gamma}{2} - n(\frac{\gamma+m}{3})\right)}, M_5 y^{-\beta}\left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)}\right)\right\},$$

for all $k = 0, 1, 2, \ldots, n$.

Let us choose following power series:

$$P = P(y) = \sum_{n=0}^{\infty} p_n y^n,$$

$$Q = Q(y) = \sum_{n=0}^{\infty} q_n y^n$$

and

$$R = R(y) = \sum_{n=0}^{\infty} r_n y^n,$$

having the coefficients with particular condition

$$p_i = |a_i|, \quad q_i = |b_i|, \quad r_i = |c_i|, \text{ for } \quad i = 0, 1, 2, 3$$

and

$$p_{n+3} = J\left(p_n + \sum_{k=0}^{n} p_k p_{n-k} + \sum_{k=0}^{n} q_k q_{n-k} + r_n\right),$$
$$q_{n+3} = K\left(q_n + \sum_{k=0}^{n} (q_k p_{n-k})\right),$$
$$r_{n+3} = L\left(r_n + \sum_{k=0}^{n} (r_k p_{n-k})\right), \tag{15}$$

where $n = 0, 1, 2, 3, \ldots$. Therefore,

$$|a_n| \leq p_n, \quad |b_n| \leq q_n, \quad |c_n| \leq r_n, \quad n = 0, 1, 2, \ldots. \tag{16}$$

Therefore, we have the series

$$P = P(y) = \sum_{n=0}^{\infty} p_n y^n,$$

$$Q = Q(y) = \sum_{n=0}^{\infty} q_n y^n$$

and

$$R = R(y) = \sum_{n=0}^{\infty} r_n y^n,$$

which are the majorant series of the series $g(y)$, $h(y)$ and $f(y)$ in (6) respectively. Now, we prove the convergence of the series $g(y)$, $h(y)$ and $f(y)$ by comparison test using the convergence of the majorant power series $P(y)$, $Q(y)$ and $R(y)$.

Therefore, we prove the convergence of the power series $P(y)$, $Q(y)$ and $R(y)$ by using implicit function theorem [38].

$$\begin{aligned} P =\ & p_0 + p_1 y + p_2 y^2 + p_3 y^3 + \sum_{n=1}^{\infty} p_{n+3} y^{n+3} \\ =\ & p_0 + p_1 y + p_2 y^2 + p_3 y^3 \\ & + \sum_{n=1}^{\infty} J\Bigg(p_n + \sum_{k=0}^{n} p_k p_{n-k} \\ & + \sum_{k=0}^{n} q_k q_{n-k} + r_n \Bigg) y^{n+3} \\ =\ & p_0 + p_1 y + p_2 y^2 + p_3 y^3 + J y^3((P - p_0) \\ & + (P^2 - (p_0)^2) + (Q^2 - (q_0)^2) + (R - r_0)), \end{aligned} \tag{17}$$

$$\begin{aligned} Q =\ & q_0 + q_1 y + q_2 y^2 + q_3 y^3 + \sum_{n=1}^{\infty} q_{n+3} y^{n+3} \\ =\ & q_0 + q_1 y + q_2 y^2 + q_3 y^3 \\ & + \sum_{n=1}^{\infty} K \left(q_n + \sum_{k=0}^{n} (q_k p_{n-k}) \right) y^{n+3} \\ =\ & q_0 + q_1 y + q_2 y^2 + q_3 y^3 + K y^3 [(Q - q_0) \\ & + (PQ - p_0 q_0)] \end{aligned} \tag{18}$$

and

$$\begin{aligned} R =\ & r_0 + r_1 y + r_2 y^2 + r_3 y^3 + \sum_{n=1}^{\infty} r_{n+3} y^{n+3} \\ =\ & r_0 + r_1 y + r_2 y^2 + r_3 y^3 \\ & + \sum_{n=1}^{\infty} L \left(r_n + \sum_{k=0}^{n} (r_k p_{n-k}) \right) y^{n+3} \\ =\ & r_0 + r_1 y + r_2 y^2 + r_3 y^3 + L y^3 [(R - r_0) \\ & + (PR - p_0 r_0)]. \end{aligned} \tag{19}$$

Let us assume the implicit functional system

$$\begin{aligned} & \Phi(y, P, Q, R) = P - p_0 - p_1 y - p_2 y^2 - p_3 y^3 \\ & \quad - J y^3((P - p_0) + (P^2 - (p_0)^2) \\ & \quad + (Q^2 - (q_0)^2) + (R - r_0)), \\ & \Theta(y, P, Q, R) = Q - q_0 - q_1 y - q_2 y^2 - q_3 y^3 \\ & \quad - K y^3 [(Q - q_0) + (PQ - p_0 q_0)], \\ & \Psi(y, P, Q, R) = R - r_0 - r_1 y - r_2 y^2 - r_3 y^3 \\ & \quad - L y^3 [(R - r_0) + (PR - p_0 r_0)]. \end{aligned} \tag{20}$$

It can be easily seen that the functions Φ, Θ, Ψ are analytic in the neighbourhood of $(0, p_0, q_0, r_0)$. Also, $\Phi(0, p_0, q_0, r_0) = 0$, $\Theta(0, p_0, q_0, r_0) = 0$ and $\Psi(0, p_0, q_0, r_0) = 0$, with non-zero Jacobian determinant

$$\left. \frac{\partial(\Phi, \Theta, \Psi)}{\partial(P, Q, R)} \right|_{(0, p_0, q_0, r_0)} = 1.$$

Therefore, the series $P = P(y)$, $Q = Q(y)$ and $R = R(y)$ are analytic in a neighbourhood of the point $(0, p_0, q_0, r_0)$, using implicit function theorem [38]. This proves the convergence of power series $P = P(y)$, $Q = Q(y)$ and $R = R(y)$ about the point $(0, p_0, q_0, r_0)$. Therefore, by comparison test the series (6) are convergent with positive radius of convergence.

5. Conservation laws

The present paper contains the conservation laws of fractional generalised HSCKdV (1), by using new conservation theorem and fractional Noether's operator [22,39,40].

The continuation equation

$$D_t(C^t) + D_x(C^x) = 0,$$

defines conservation laws. The vectors $C^t(x, t, u, v)$ and $C^x(x, t, u, v)$ define the conserved vectors of (1). So we have to calculate these vectors by using fractional Noether's operators. These vector fields spans vector field. In order to construct conserved vectors, the necessary definitions are given below:

DEFINITION 1

Euler–Lagrange operator for the FPDEs system

Euler–Lagrange operators denoted by $\delta/\delta u^j$ is defined as follows:

$$\begin{aligned} \frac{\delta}{\delta u^j} =\ & \frac{\partial}{\partial u^j} + (D_t^{\gamma})^* \frac{\partial}{\partial(D_t^{\gamma} u^j)} + (D_x^{\beta})^* \frac{\partial}{\partial(D_x^{\beta} u^j)} \\ & + \sum_{k=1}^{\infty} (-1)^k D_{i_1} D_{i_2}, \ldots, D_{i_k} \frac{\partial}{\partial(u^j)_{i_1, i_2, \ldots, i_k}}, \end{aligned} \tag{21}$$

where D_{i_k} is the i_k^{th} variable total derivative operator. Also $(D_t^{\gamma})^*$ and $(D_x^{\beta})^*$ are the adjoint operators of fractional derivatives D_t^{γ} and D_x^{β}, respectively.

DEFINITION 2

Lagrangian for the FPDEs system (1)

The operator defined by

$$\mathcal{L} = p\bigg(\frac{\partial^{\gamma} u}{\partial t^{\gamma}} - A_1(t) u \frac{\partial^{\beta} u}{\partial x^{\beta}} - A_2(t) v \frac{\partial^{\beta} v}{\partial x^{\beta}} - A_3(t) \frac{\partial^{\beta} w}{\partial x^{\beta}}$$

$$-A_4(t)\frac{\partial^3 u}{\partial x^3}\bigg) + q\bigg(\frac{\partial^\gamma v}{\partial t^\gamma} - A_5(t)u\frac{\partial^\beta v}{\partial x^\beta} -A_6(t)\frac{\partial^3 v}{\partial x^3}\bigg) +r\bigg(\frac{\partial^\gamma w}{\partial t^\gamma} - A_7(t)u\frac{\partial^\beta w}{\partial x^\beta} - A_8(t)\frac{\partial^3 w}{\partial x^3}\bigg), \quad (22)$$

where p, q and r are new dependent variables, is called Lagrangian [40] for the FPDEs system (1).

DEFINITION 3

Adjoint equation for the FPDEs system

The adjoint equations for FPDEs system are given by

$$F_j^* \equiv \frac{\delta\mathcal{L}}{\delta u^j} = 0, \quad j = 1, 2, 3. \quad (23)$$

Now the adjoint equations for (1), are obtained as

$$\frac{\delta\mathcal{L}}{\delta u} = F_1^* = (D_t^\gamma)^* p - A_1(t)(D_x^\beta)^*(pu) - A_1(t)p\partial_x^\beta u -A_5(t)q\partial_x^\beta v - A_7(t)r\partial_x^\beta w + A_4(t)D_x^3(p) = 0,$$
$$\frac{\delta\mathcal{L}}{\delta v} = F_2^* = (D_t^\gamma)^* q - A_2(t)(D_x^\beta)^*(pv) -A_5(t)(D_x^\beta)^*(qu) -A_2(t)p\partial_x^\beta v + A_6(t)D_x^3(q) = 0,$$
$$\frac{\delta\mathcal{L}}{\delta w} = F_3^* = (D_t^\gamma)^* r - A_3(t)(D_x^\beta)^*(p) -A_7(t)(D_x^\beta)^*(ru) + A_8(t)D_x^3(r) = 0. \quad (24)$$

The non-linearity of system (1) depends on the condition that (24) satisfies, where new dependent variables

$$p = \psi(x, t, u, v),$$
$$q = \varphi(x, t, u, v),$$
$$r = \mu(x, t, u, v) \quad (25)$$

are not simultaneously zero.

The third derivative of $p = \psi(x, t, u, v)$ with respect to x is

$$\begin{aligned} p_{xxx} &= \psi_{xxx} + 6\psi_{xuv}u_x v_x + 6\psi_{xuw}u_x w_x \\ &+6\psi_{xvw}v_x w_x + 3\psi_{uuv}u_x^2 v_x \\ &+3\psi_{uuw}u_x^2 w_x + 3\psi_{vvw}v_x^2 w_x \\ &+3\psi_{uww}u_x w_x^2 + 3\psi_{vww}v_x w_x^2 + 3\psi_{uu}u_x u_{xx} \\ &+3\psi_{vv}v_x v_{xx} + 3\psi_{ww}w_x w_{xx} + 3\psi_{uvv}u_x v_x^2 \\ &+3\psi_{xu}u_{xx} + 3\psi_{xv}v_{xx} + 3\psi_{xw}w_{xx} + \psi_v v_{xxx} \\ &+3\psi_{uv}(u_x v_{xx} + v_x u_{xx}) + 3\psi_{uw}(u_x w_{xx} + w_x u_{xx}) \\ &+3\psi_{vw}(v_x w_{xx} + w_x v_{xx}) + 3\psi_{xxv}v_x + 3\psi_{xxu}u_x \\ &+3\psi_{xxw}w_x + \psi_u u_{xxx} + \psi_w w_{xxx} + 3\psi_{xuu}u_x^2 \\ &+3\psi_{xvv}v_x^2 + 3\psi_{xww}w_x^2 + \psi_{uuu}u_x^3 \\ &+\psi_{vvv}v_x^3 + \psi_{www}w_x^3 + 2\psi_{ux}u_{xx} \\ &+2\psi_{vx}v_{xx} + 2\psi_{wx}w_{xx} + 6\psi_{uvw}u_x v_x w_x. \end{aligned} \quad (26)$$

Similarly, we can find the other required derivatives q_{xxx} and r_{xxx}. Therefore, the nonlinear self-adjointness conditions are obtained as follows:

$$\begin{aligned} \frac{\delta\mathcal{L}}{\delta u} &= \lambda_1(\partial_t^\gamma u - A_1(t)u\partial_x^\beta u - A_2(t)v\partial^\beta v_x \\ &-A_3(t)\partial_x^\beta w - A_4(t)\partial_x^3 u) \\ &+\lambda_2(\partial_t^\gamma v - A_5(t)u\partial_x^\beta v - A_6(t)\partial_x^3 v) \\ &+\lambda_3(\partial_t^\gamma w - A_7(t)u\partial_x^\beta w - A_8(t)\partial_x^3 w), \\ \frac{\delta\mathcal{L}}{\delta v} &= \lambda_4(\partial_t^\gamma u - A_1(t)u\partial_x^\beta u - A_2(t)v\partial^\beta v_x \\ &-A_3(t)\partial_x^\beta w - A_4(t)\partial_x^3 u) \\ &+\lambda_5(\partial_t^\gamma v - A_5(t)u\partial_x^\beta v - A_6(t)\partial_x^3 v) \\ &+\lambda_6(\partial_t^\gamma w - A_7(t)u\partial_x^\beta w - A_8(t)\partial_x^3 w), \\ \frac{\delta\mathcal{L}}{\delta w} &= \lambda_7(\partial_t^\gamma u - A_1(t)u\partial_x^\beta u - A_2(t)v\partial^\beta v_x \\ &-A_3(t)\partial_x^\beta w - A_4(t)\partial_x^3 u) \\ &+\lambda_8(\partial_t^\gamma v - A_5(t)u\partial_x^\beta v - A_6(t)\partial_x^3 v) \\ &+\lambda_9(\partial_t^\gamma w - A_7(t)u\partial_x^\beta w - A_8(t)\partial_x^3 w), \end{aligned} \quad (27)$$

for some undetermined coefficients λ_i ($i = 1, 2, 3, 4, 5, 6, 7, 8, 9$).

Thus,

$$\begin{aligned} &(D_t^\gamma)^*\psi - A_1(t)(D_x^\beta)^*(\psi u) - A_1(t)\psi\partial_x^\beta u \\ &-A_5(t)\varphi\partial_x^\beta v - A_7(t)\mu\partial_x^\beta w \\ &+A_4(t)(\psi_{xxx} + 6\psi_{xuv}u_x v_x + 6\psi_{xuw}u_x w_x \\ &+6\psi_{xvw}v_x w_x + 3\psi_{uuv}u_x^2 v_x + 3\psi_{uuw}u_x^2 w_x \\ &+3\psi_{vvw}v_x^2 w_x + 3\psi_{uww}u_x w_x^2 \\ &+3\psi_{vww}v_x w_x^2 + 3\psi_{uu}u_x u_{xx} + 3\psi_{vv}v_x v_{xx} \\ &+3\psi_{ww}w_x w_{xx} + 3\psi_{uvv}u_x v_x^2 + 3\psi_{xu}u_{xx} \\ &+3\psi_{xv}v_{xx} + 3\psi_{xw}w_{xx} \\ &+3\psi_{uv}(u_x v_{xx} + v_x u_{xx}) \\ &+3\psi_{uw}(u_x w_{xx} + w_x u_{xx}) \\ &+3\psi_{vw}(v_x w_{xx} + w_x v_{xx}) + 3\psi_{xxv}v_x \\ &+3\psi_{xxu}u_x + 3\psi_{xxw}w_x + \psi_u u_{xxx} \\ &+\psi_v v_{xxx} + \psi_w w_{xxx} + 3\psi_{xuu}u_x^2 \\ &+3\psi_{xvv}v_x^2 + 3\psi_{xww}w_x^2 + \psi_{uuu}u_x^3 + \psi_{vvv}v_x^3 \\ &+\psi_{www}w_x^3 + 2\psi_{ux}u_{xx} \\ &+2\psi_{vx}v_{xx} + 2\psi_{wx}w_{xx} \\ &+6\psi_{uvw}u_x v_x w_x) \\ &= \lambda_1(\partial_t^\gamma u - A_1(t)u\partial_x^\beta u - A_2(t)v\partial^\beta v_x - A_3(t)\partial_x^\beta w \\ &-A_4(t)\partial_x^3 u) + \lambda_2(\partial_t^\gamma v - A_5(t)u\partial_x^\beta v - A_6(t)\partial_x^3 v) \end{aligned}$$

$$
\begin{aligned}
&+\lambda_3(\partial_t^\gamma w - A_7(t)u\partial_x^\beta w - A_8(t)\partial_x^3 w),\\
&(D_t^\gamma)^*\varphi - A_2(t)(D_x^\beta)^*(\psi v) - A_5(t)(D_x^\beta)^*(\varphi u)\\
&-A_2(t)\psi\partial_x^\beta v + A_6(t)(\varphi_{xxx} + 6\varphi_{xuv}u_x v_x\\
&+6\varphi_{xuw}u_x w_x + 6\varphi_{xvw}v_x w_x + 3\varphi_{uuv}u_x^2 v_x\\
&+3\varphi_{uuw}u_x^2 w_x + 3\varphi_{vvw}v_x^2 w_x\\
&+3\varphi_{uww}u_x w_x^2 + 3\varphi_{vww}v_x w_x^2\\
&+3\varphi_{uu}u_x u_{xx} + 3\varphi_{vv}v_x v_{xx}\\
&+3\varphi_{ww}w_x w_{xx} + 3\varphi_{uvv}u_x v_x^2\\
&+3\varphi_{xu}u_{xx} + 3\varphi_{xv}v_{xx}\\
&+3\varphi_{xw}w_{xx} + 3\varphi_{uv}(u_x v_{xx} + v_x u_{xx})\\
&+\varphi_w w_{xxx} + 3\varphi_{uw}(u_x w_{xx} + w_x u_{xx})\\
&+3\varphi_{vw}(v_x w_{xx} + w_x v_{xx})\\
&+3\varphi_{xxv}v_x + 3\varphi_{xxu}u_x + 3\varphi_{xxw}w_x + \varphi_u u_{xxx}\\
&+\varphi_v v_{xxx} + 3\varphi_{xuu}u_x^2 + 3\varphi_{xvv}v_x^2 + 3\varphi_{xww}w_x^2\\
&+\varphi_{uuu}u_x^3 + \varphi_{vvv}v_x^3\\
&+\varphi_{www}w_x^3 + 2\varphi_{ux}u_{xx} + 2\varphi_{vx}v_{xx}\\
&+2\varphi_{wx}w_{xx} + 6\varphi_{uvw}u_x v_x w_x)\\
&= \lambda_4(\partial_t^\gamma u - A_1(t)u\partial_x^\beta u - A_2(t)v\partial^\beta v_x - A_3(t)\partial_x^\beta w\\
&-A_4(t)\partial_x^3 u) + \lambda_5(\partial_t^\gamma v - A_5(t)u\partial_x^\beta v - A_6(t)\partial_x^3 v)\\
&+\lambda_6(\partial_t^\gamma w - A_7(t)u\partial_x^\beta w - A_8(t)\partial_x^3 w),
\end{aligned}
$$

$$
\begin{aligned}
&(D_t^\gamma)^*\mu - A_3(t)(D_x^\beta)^*(\psi) - A_7(t)(D_x^\beta)^*(\mu u)\\
&+A_8(t)(\mu_{xxx} + 6\mu_{xuv}u_x v_x + 6\mu_{xuw}u_x w_x\\
&+6\mu_{xvw}v_x w_x + 3\mu_{uuv}u_x^2 v_x\\
&+3\mu_{uuw}u_x^2 w_x + 3\mu_{vvw}v_x^2 w_x + 3\mu_{uww}u_x w_x^2\\
&+3\mu_{vww}v_x w_x^2 + 3\mu_{uu}u_x u_{xx} + 3\mu_{vv}v_x v_{xx}\\
&+3\mu_{ww}w_x w_{xx} + 3\mu_{uvv}u_x v_x^2\\
&+3\mu_{xu}u_{xx} + 3\mu_{xv}v_{xx}\\
&+3\mu_{xw}w_{xx} + 3\mu_{uv}(u_x v_{xx}\\
&+v_x u_{xx}) + 3\mu_{uw}(u_x w_{xx}\\
&+w_x u_{xx}) + 3\mu_{vw}(v_x w_{xx} + w_x v_{xx})\\
&+3\mu_{xxv}v_x + 3\mu_{xxu}u_x\\
&+3\mu_{xxw}w_x + \mu_u u_{xxx} + \mu_v v_{xxx} + \mu_w w_{xxx}\\
&+3\mu_{xuu}u_x^2 + 3\mu_{xvv}v_x^2\\
&+3\mu_{xww}w_x^2 + \mu_{uuu}u_x^3 + \mu_{vvv}v_x^3\\
&+\mu_{www}w_x^3 + 2\mu_{ux}u_{xx}\\
&+2\mu_{vx}v_{xx} + 2\mu_{wx}w_{xx}\\
&+6\mu_{uvw}u_x v_x w_x)\\
&= \lambda_7(\partial_t^\gamma u - A_1(t)u\partial_x^\beta u - A_2(t)v\partial^\beta v_x\\
&-A_3(t)\partial_x^\beta w\partial_x^\beta v\\
&-A_4(t)\partial_x^3 u) + \lambda_8(\partial_t^\gamma v - A_5(t)u\\
&-A_6(t)\partial_x^3 v)\\
&+\lambda_9(\partial_t^\gamma w - A_7(t)u\partial_x^\beta w - A_8(t)\partial_x^3 w).
\end{aligned}
\tag{28}
$$

Now solving the determining equations from (28), we have the values $\lambda_i = 0$ $(1 \le i \le 9)$ and

$$
\begin{aligned}
\psi &= r_1(t)x^2 + r_2(t)x + r_3(t),\\
\varphi &= r_4(t)x^2 + r_5(t)x + r_6(t),\\
\mu &= r_7(t)x^2 + r_8(t)x + r_9(t),
\end{aligned}
\tag{29}
$$

where $r_i(t)$, $1 \le i \le 9$ are arbitrary functions of t. The Lie characteristic functions W^1, W^2 and W^3 corresponding to symmetry generators are defined by

$$
\begin{aligned}
W^1 &= \eta - \xi u_x - \tau u_t,\\
W^2 &= \phi - \xi v_x - \tau v_t,\\
W^3 &= \psi - \xi w_x - \tau w_t.
\end{aligned}
\tag{30}
$$

The fractional Noether's operator [39] for the variable t is defined by

$$
\begin{aligned}
C^t = \sum_{j=1}^{3}\Bigg[&\sum_{k=0}^{m-1}(-1)^k D_t^{\gamma-1-k}(W^j)D_t^k\left(\frac{\partial\mathcal{L}}{\partial(D_t^\gamma u_j)}\right)\\
&-(-1)^m\mathcal{J}_1\left(W^j, D_t^m\left(\frac{\partial\mathcal{L}}{\partial(D_t^\gamma u_j)}\right)\right)\Bigg],
\end{aligned}
\tag{31}
$$

where $m = [\gamma] + 1$, u_1, u_2 and u_3 are dependent variables and $\mathcal{J}_1(h_1, h_2)$ is the integral defined as

$$
\mathcal{J}_1(h_1, h_2) = \frac{1}{\Gamma(m-\gamma)}\int_0^t\int_t^q \frac{h_1(x,s)h_2(x,r)}{(r-s)^{\gamma+1-m}}\mathrm{d}r\,\mathrm{d}s.
$$

Equivalently, the fractional Noether's operator for the component x of the conserved vector is defined as

$$
\begin{aligned}
C^x = \sum_{j=1}^{3}\Bigg[&\sum_{k=0}^{n-1}(-1)^k D_x^{\beta-1-k}(W^j)D_x^k\left(\frac{\partial\mathcal{L}}{\partial(D_x^\beta u_j)}\right)\\
&-(-1)^n\mathcal{J}_2\left(W^j, D_x^n\left(\frac{\partial\mathcal{L}}{\partial(D_x^\beta u_j)}\right)\right)\Bigg],
\end{aligned}
\tag{32}
$$

where $n = [\beta] + 1$, u_1, u_2 and u_3 are dependent variables and $\mathcal{J}_2(h_1, h_2)$ is the integral defined as

$$
\mathcal{J}_2(h_1, h_2) = \frac{1}{\Gamma(n-\beta)}\int_0^x\int_x^p \frac{h_1(s,t)h_2(r,t)}{(r-s)^{\beta+1-n}}\mathrm{d}r\,\mathrm{d}s
$$

for any two functions $h_1(x,t)$ and $h_2(x,t)$. By using (30) and infinitesimals (2), the Lie characteristic functions are

$$
\begin{aligned}
W^1 &= \frac{3(\gamma-1)u}{2(\gamma+m)} - \frac{x}{3}u_x - \frac{t}{\gamma+m}u_t,\\
W^2 &= \frac{3(\gamma-1)v}{2(\gamma+m)} - \frac{x}{3}v_x - \frac{t}{\gamma+m}v_t,\\
W^3 &= \frac{(\gamma-1)w}{\gamma+m} - \frac{x}{3}w_x - \frac{t}{\gamma+m}w_t.
\end{aligned}
\tag{33}
$$

Now, the conserved vectors of system (1) can be calculated as follows:

Consider the following cases:

Case 1. For $0 < \gamma < 1$, the t-component of the conserved vector is obtained as

$$C^t = I_t^{1-\gamma}(W^1)\psi + I_t^{1-\gamma}(W^2)\varphi + I_t^{1-\gamma}(W^3)\mu + \mathcal{J}_1(W^1, \psi_t) + \mathcal{J}_1(W^2, \varphi_t) + \mathcal{J}_1(W^3, \mu_t).$$

Case 2. For $1 < \gamma < 2$, the t-component of the conserved vector is

$$C^t = D_t^{\gamma-1}(W^1)\psi + D_t^{\gamma-1}(W^2)\varphi + D_t^{\gamma-1}(W^3)\mu - I_t^{2-\gamma}(W^1)\phi_t - I_t^{2-\gamma}(W^2)\varphi_t - I_t^{2-\gamma}(W^3)\mu_t + \mathcal{J}_1(W^1, \psi_{tt}) + \mathcal{J}_1(W^2, \varphi_{tt}) + \mathcal{J}_1(W^3, \mu_{tt}).$$

Case 3. For $0 < \beta < 1$, the x-component of the conserved vector is

$$C^x = -I_x^{1-\beta}(W^1)(A_1(t)u\psi) - I_x^{1-\beta}(W^2)(A_2(t)v\psi + A_5(t)u\varphi) - I_x^{1-\beta}(W^3)(A_3(t)\psi + A_7(t)u\mu) + \mathcal{J}_2(W^1, D_x(-A_1(t)u\psi)) + \mathcal{J}_2(W^2, D_x(-A_2(t)v\psi - A_5(t)u\varphi)) + \mathcal{J}_2(W^3, D_x(A_3(t)\psi + A_7(t)u\mu)).$$

Case 4. For $1 < \beta < 2$, the x-component of the conserved vector is obtained as

$$C^x = D_x^{\beta-1}(W^1)(-A_1(t)u\psi) - D_x^{1-\beta}(W^2)(A_2(t)v\psi + A_5(t)u\varphi) + D_x^{\beta-1}(W^3)(-A_3(t)\psi - A_7(t)u\mu) - I_x^{2-\beta}(W^1)D_x(-A_1(t)u\psi) + I_x^{2-\beta}(W^2)D_x(A_2(t)v\psi + A_5(t)u\varphi) - I_x^{2-\beta}(W^3)D_x(-A_3(t)\psi - A_7(t)u\mu) - \mathcal{J}_2(W^1, D_x^2(-A_1(t)u\psi)) - \mathcal{J}_2(W^2, D_x^2(A_2(t)v\psi + A_5(t)u\varphi)) - \mathcal{J}_2(W^3, D_x^2(-A_3(t)\psi - A_7(t)u\mu)).$$

6. Conclusion

In this article, we employed the power series method to obtain exact solution of space–time fractional generalised Hirota–Satsuma coupled KdV system with time-dependent variable coefficients. We used the reduced form of governing system of eq. (1) in the form of NLFODEs with variable coefficients involving Erdèlyi–Kober operator obtained in our previous work [31]. Further, the obtained exact solution in the form of power series are analysed for convergence. The conservation laws are also successfully constructed with the help of symmetries of the governing system of equations and using new conservation theorem.

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