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Multiple rogue wave solutions of a generalised Hietarinta-type equation

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Abstract. In this paper, multiple rogue wave solutions of a generalised Hietarinta-type fourth-order equation in (2 + 1)-dimensional dispersive waves were studied by applying the bilinear method. We obtained its 1-rogue wave, 3-rogue wave and 6-rogue wave solutions. Similarly, their corresponding maps which can finely explain their physical structure and properties were graphically shown through symbolic computation approach. It is obvious that the centre of the 3-rogue wave possesses a triangular structure while 6-rogue wave has a hexagon structure and they are made of three and six independent 1-rogue waves, respectively. Furthermore, the results obtained have immensely augmented the exact solutions of the generalised Hietarinta-type equation on the available literature and enabled us to understand the nonlinear dynamic system deeply.

Keywords. Multiple rogue wave solutions; bilinear method; Hietarinta-type equation; symbolic computation.

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1. Introduction

As a generic term of nonlinear mathematical physical partial differential equation including variate t, nonlinear evolution equations (NLEEs) can describe the evolution along with times in dynamics, physics, biology and other natural sciences. Recently, with the increasing development of technology and hard work of numerous mathematicians and physicists, the study of how to obtain exact solutions of NLEEs including travelling wave solutions [1], soliton solutions [2–6], periodic solutions [7–12], breather solutions [13,14], rational solutions [15–19], interaction solutions [7,20–22], three-wave solutions [23,24] and lump solutions [25–29] has attracted more and more attention. Meanwhile, Ma introduced N-soliton solutions and the Hirota conditions of (1 + 1)-dimensional [30] and (2 + 1)-dimensional integrable equation [31] in detail, using several equations as examples. In addition, via proper nonlinear transformation and an ansatz on the solution of the Hirota bilinear equation, the lump wave and nonlinearity-managed lump wave solutions of NLEEs were displayed finely [32–34]. The study of nonlinear waves, also called finite-amplitude waves because the ratio of amplitude to wavelength is finite, is one of the most essential issue in contemporary nonlinear mathematical physics. To some extent, nonlinear waves are like ocean waves. When the wave height is more, the crest will be sharp and shatter while the trough flatten out. Nonlinear wave's interaction and transformation revealed a colourful and unpredictable world with the structure more complex and the range more diverse.

As a key part of nonlinear local wave, the soliton wave is a limiting form of the shallow water which can keep the wave shape unchanged reflecting a strong nonlinear influence with its stability and particle property. When $\xi \to \infty$, $u(\xi)$ and its derivatives approach zero. Similarly, when function $u(\xi)$ is fixed, the absolute value of the corresponding $\pm\xi$ is inversely proportional to $\sqrt{c}$ (c represents velocity). Therefore, the larger is the velocity of the soliton wave, the smaller is the wave width and the higher are the crests. Likewise, soliton is a stable solitary wave which is a stable solution of fluid motion with constant speed, and there is no deformation and no damage in interaction. So soliton theory has been widely studied in various domains while breather and rogue waves are two typical nonlinear structures with obvious instability on the plane wave.

Rogue wave, also called giant wave or freak wave, is a unimodal wave which is a little similar to soliton

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wave, but it combines well with modulation instability. In the absence of external energy input, the wave train will evolve into a modulated wave train as long as the relationship between the constituent waves satisfies the self-focussing phenomenon. The steeper and more concentrated the frequency distribution of the constituent waves are, the more likely it will produce modulation instability and rogue waves. Similarly, abnormal wave has the characteristics of extremely high wave height, sharp crests and great destructive power, which poses a serious threat to the safety of offshore engineering facilities and ships. At present, the notion of rogue waves is not only confined to coastal engineering and marine engineering but also has permeated pulses of optical fibres, the atmosphere, finance and so on. The study of rogue waves began about twenty or thirty years ago, but in the recent decade plenty of achievements have been made. There are many methods to solve it such as extended tanh method [35], generalised $(m, N-m)$-fold Darboux transformation [36,37], binary Bell polynomials [38], Maccari nonlinear system [39] and bilinear transformation [40].

This paper is organised as follows. In §2, we shall give the main method of how to obtain multiple rogue wave solutions of a $(k+1)$-dimensional equation and introduce a generalised Hietarinta-type equation with its bilinear form and transformed equation after a travelling transformation. In §3, we shall obtain the 1-rogue wave solution of the generalised Hietarinta-type equation and plot several maps of the wave with some physical interpretations. In §4, 3-rogue wave solution will be given, through various plots and with given parameters, dynamic properties will be finely shown. In §5, 6-rogue wave solution with its corresponding three-dimensional, contour and density plots will be given. Conclusions and outlook will be given in the final section.

2. Main approach

In order to obtain multiple rogue wave solutions of $(k+1)$-dimensional equation of the form

$$N(u, u_{x_1}, u_{x_2}, \ldots, u_{x_k}, u_{x_1x_2}, u_{x_1x_3}, \ldots, u_{x_1x_k}, u_{x_2x_1}, \ldots) = 0 \tag{1}$$

first, through a travelling-wave transformation $X = m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_{k-1}x_{k-1}$, the $(k+1)$-dimensional system can be reduced to a $(1+1)$-dimensional equation and the bilinear form will be derived by applying the bilinear method.

Next, we introduce notations like this:

$$\begin{aligned} F_n(X, x_k) &= \sum_{j=0}^{n(n+1)/2} \sum_{i=0}^{j} a_{n(n+1)-2j,2i}\, x_k^{2i} X^{n(n+1)-2j}, \\ P_n(X, x_k) &= \sum_{j=0}^{n(n+1)/2} \sum_{i=0}^{j} b_{n(n+1)-2j,2i}\, X^{2i} x_k^{n(n+1)-2j}, \\ Q_n(X, x_k) &= \sum_{j=0}^{n(n+1)/2} \sum_{i=0}^{j} c_{n(n+1)-2j,2i}\, x_k^{2i} X^{n(n+1)-2j}, \end{aligned} \tag{2}$$

where $a_{m,l}, b_{m,l}, c_{m,l} (m, l \in \{0, 2, 4, \ldots, n(n+1)\})$ are arbitrary real parameters.

Then, we hypothesise the multiple rogue wave solutions of the $(k+1)$-dimensional system with the following form:

$$\begin{aligned} f &= \overline{F}_{n+1}(X, x_k, \psi, \varphi) = F_{n+1}(X, x_k) \\ &\quad + 2\psi x_k P_n(X, x_k) + 2\varphi X Q_n(X, x_k) \\ &\quad + (\psi^2 + \varphi^2) F_{n-1}(X, x_k), \end{aligned} \tag{3}$$

where $F_0 = 1$, $F_{-1} = P_0 = Q_0 = 0$ and φ, ψ are arbitrary real parameters. Substituting (3) into the reduced $(1+1)$-dimensional bilinear equation and taking all the coefficients of different terms equal to zero, we get a set of algebraic equations. With the aid of symbolic computation, we shall solve the above nonlinear algebraic equations.

Finally, substituting all the coefficients into expression (2) and through the bilinear transformation, we shall obtain multiple rogue wave solutions of the $(k+1)$-dimensional system [41].

In this paper we consider a generalised Hietarinta-type fourth-order equation in $(2+1)$-dimensional dispersive waves:

$$\begin{aligned} &\alpha_1(6u_xu_{xx} + u_{xxxx}) + \alpha_2(3u_tu_{tt} + 3u_{xt}v_{tt} + u_{xttt}) \\ &\quad + \beta_1u_{yt} + \beta_2u_{xx} + \beta_3u_{xt} + \beta_4u_{xy} + \beta_5u_{yy} = 0, \end{aligned} \tag{4}$$

where $u = u(x, y, t)$, $v = v(x, y, t)$, $v_x = u$, and $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ are real arbitrary constants which will be determined later and lump solution of this equation has been solved by Batwa and Ma [42].

According to the nonlinear terms of eq. (4), we suppose u, v with the form $u = A(\ln f(x, y, t))_x$, $v = A \ln f(x, y, t)$ where f is the solution of the bilinear equation, then substituting it into eq. (4) and balancing the coefficients of different terms, we shall derive $A = 2$. So the variable transformation is $u = 2(\ln f(x, y, t))_x$, $v = 2 \ln f(x, y, t)$.

If $X = x + mt$ in eq. (4), it can be reduced to a (1 + 1)-dimensional equation of the following form:

$$\alpha_1(6u_X u_{XX} + u_{XXXX}) + \alpha_2(3m^3 u_X u_{XX} + 3m^3 u_{XX} v_{XX} + m^3 u_{XXXX}) + \beta_1 m u_{Xy} + \beta_2 u_{XX} + \beta_3 m u_{XX} + \beta_4 u_{Xy} + \beta_5 u_{yy} = 0. \quad (5)$$

With the variable transformation

$$u(X, y) = 2(\ln f(X, y))_x + u_0,$$
$$v = 2\ln f(X, y) + v_0, \quad (6)$$

where u_0 and v_0 are integration constants, eq. (5) can be transformed into a bilinear equation:

$$B(f) = (\alpha_1 + \alpha_2 m^3) D_X^4 (f \cdot f) + (\beta_2 + \beta_3 m) D_X^2 (f \cdot f) + (\beta_4 + \beta_1 m) D_X D_y (f \cdot f) + \beta_5 D_y^2 (f \cdot f)$$
$$= (\alpha_1 + \alpha_2 m^3)(3 f_{XX}^2 - 4 f_X f_{XXX} + f f_{XXXX}) + (\beta_2 + \beta_3 m)(f_{XX} f - f_X^2) + (\beta_4 + \beta_1 m) \times (f_{Xy} f - f_y f_X) + \beta_5 (f_{yy} f - f_y^2) = 0. \quad (7)$$

3. 1-rogue wave solution

In order to find 1-rogue wave solution of eq. (4), we suppose

$$f = (X - \varphi)^2 + a_1 (y - \psi)^2 + a_0. \quad (8)$$

According to the step in §2, we can obtain a range of determining equations with respect to a_0 and a_1 as

$$2(\beta_3 m + \beta_2)\left(a_1 \psi^2 - \varphi^2 + a_0\right) + 12(\alpha_2 m^3 + \alpha_1) - 4(\beta_1 m + \beta_4)\varphi a_1 \psi + \beta_5\left(2a_1\left(a_1\psi^2 + \varphi^2 + a_0\right) - 4{a_1}^2\psi^2\right) = 0;$$
$$4(\beta_3 m + \beta_2)\varphi + 2(2\beta_1 m + 2\beta_4) a_1 \psi - 4\beta_5 a_1 \varphi = 0;$$
$$-2(2\beta_1 m + 2\beta_4) a_1 = 0;$$
$$-2(\beta_3 m + \beta_2) a_1 - 2\beta_5 {a_1}^2 = 0;$$
$$-2\beta_3 m + 2\beta_5 a_1 - 2\beta_2 = 0;$$
$$-4(\beta_3 m + \beta_2) a_1 \psi + 4(\beta_1 m + \beta_4)\varphi a_1 + 4\beta_5 {a_1}^2 \psi.$$

By solving the above equations, one has

$$a_1 = \frac{\beta_2 + \beta_3 m}{\beta_5}, \quad a_0 = \frac{-3(\alpha_2 m^3 + \alpha_1)}{\beta_2 + \beta_3 m}.$$

Thus, we can derive solution of the generalised bilinear Hietarinta-type equation as

$$f = (X - \varphi)^2 + \frac{\beta_2 + \beta_3 m}{\beta_5}(y - \psi)^2 - \frac{3(\alpha_2 m^3 + \alpha_1)}{\beta_2 + \beta_3 m}. \quad (9)$$

By using the variable transformation (6), the 1-rogue wave solution of eq. (4) reads as

$$v(x, y, t) = 2\ln\left(X^2 - 2\varphi X + \varphi^2 + \frac{(\beta_3 m + \beta_2)\left(\psi^2 - 2\psi y + y^2\right)}{\beta_5} - 3\frac{\alpha_2 m^3 + \alpha_1}{\beta_3 m + \beta_2}\right) + v_0,$$
$$u(x, y, t) = \frac{4X - 4\varphi}{X^2 - 2\varphi X + \varphi^2 + \frac{(\beta_3 m + \beta_2)(\psi^2 - 2\psi y + y^2)}{\beta_5} - 3\frac{\alpha_2 m^3 + \alpha_1}{\beta_3 m + \beta_2}} + u_0, \quad (10)$$

where $X = x + mt$ and $m, \alpha_1, \alpha_2, \beta_2, \beta_3, \beta_5, \varphi, \psi$ are arbitrary real constants.

We choose particular values to illustrate the 1-rogue wave solution of the generalised Hietarinta-type equation:

$$u_0 = v_0 = \phi = \varphi = 0, \quad m = 1, \quad \beta_2 = \beta_3 = 3,$$
$$\beta_5 = 1, \quad \alpha_1 = \alpha_2 = -2. \quad (11)$$

The dynamic characters and structures of the 1-rogue wave solution are clearly shown in figure 1 which contains a three-dimensional plot (see figure 1a) that exhibits localised structures, a density plot (see figure 1b) and contour plot (see figure 1c) in the (x, y)-plane. It is apparent that the wave (10) has a trough and a crest, one is lower than the horizontal plane, the other is higher. From the solution, we can see that the wave reaches the trough at the point

$$\left(\frac{(\beta_3 m + \beta_2)\varphi + \sqrt{3(\alpha_2 m^3 + \alpha_1)}}{\beta_3 m + \beta_2}, \psi\right)$$

and reaches the crest at the point

$$\left(\frac{(\beta_3 m + \beta_2)\varphi - \sqrt{3(\alpha_2 m^3 + \alpha_1)}}{\beta_3 m + \beta_2}, \psi\right)$$

in the plane (x, y). The maximum amplitude of the 1-rogue wave is

$$u = \frac{2\sqrt{3(\beta_3 m + \beta_2)(\alpha_2 m^3 + \alpha_1)}}{3(\alpha_2 m^3 + \alpha_1)} + u_0$$

and (φ, ψ) is its centre.

4. 3-rogue wave solution

In this section, in order to establish 3-rogue wave solution of eq. (4), taking $n = 1$ in expression (2), we shall obtain

$$f(X, y) = F_2(X, y) + 2\psi y P_1(X, y) + 2\varphi X Q_1(X, y) + (\psi^2 + \varphi^2), \tag{12}$$

where

$$F_2(X, y) = X^6 + a_{4,0}X^4 + a_{4,2}y^2X^4 + (a_{2,0} + a_{2,2}y^2 + a_{2,4}y^4)X^2 + a_{0,0} + a_{0,2}y^2 + a_{0,4}y^4 + a_{0,6}y^6,$$
$$P_1(X, y) = b_{0,0} + b_{0,2}X^2 + b_{2,0}y^2,$$
$$Q_1(X, y) = c_{0,0} + c_{0,2}y^2 + c_{2,0}X^2. \tag{13}$$

Substituting expression (12) into bilinear eq. (7) and collecting all the coefficients of $X^{i_1} y^{i_2}$, we shall obtain a set of algebraic equations about the parameters. Solving these equations, one has

$$a_{0,0} = -\frac{1}{3}\,\frac{-\psi^2 a_{4,2} {b_{0,2}}^2 + 3\psi^2 {a_{4,2}}^2 + 3\varphi^2 {a_{4,2}}^2 - 3{c_{0,2}}^2\varphi^2}{{a_{4,2}}^2},$$
$$a_{0,2} = \frac{1}{12}{a_{4,0}}^2 a_{4,2}, \quad a_{0,4} = -\frac{1}{9}\, a_{4,0}{a_{4,2}}^2,$$
$$a_{0,6} = \frac{1}{27}\,{a_{4,2}}^3, \quad a_{2,0} = \frac{1}{4}\,{a_{4,0}}^2,$$
$$a_{2,2} = 0, \quad a_{2,4} = \frac{1}{3}\,{a_{4,2}}^2, \quad a_{4,2} = \frac{3(\beta_2 + m\beta_3)}{\beta_5},$$
$$b_{0,0} = \frac{1}{6}\,a_{4,0}b_{0,2},$$
$$b_{2,0} = -\frac{1}{9}\,a_{4,2}b_{0,2}, \quad c_{0,0} = -\frac{1}{2}\,\frac{a_{4,0}c_{0,2}}{a_{4,2}},$$
$$c_{2,0} = -\frac{c_{0,2}}{a_{4,2}}. \tag{14}$$

The other parameters are arbitrary real constants.

Therefore, we get the 3-rogue wave solution of eq. (4) as

$$u(x, y, t) = \frac{M(X, y)}{N(X, y)} + u_0, \tag{15}$$

where

$$M(X, y) = 8\,b_{0,2}X\psi\, y - 8\,\frac{X^2c_{0,2}\varphi}{\frac{3(\beta_2+m\beta_3)}{\beta_5}} + 4\left(-\frac{X^2c_{0,2}}{a_{4,2}} + c_{0,2}y^2 - \frac{1}{2}\,\frac{a_{4,0}c_{0,2}}{a_{4,2}}\right)\varphi + 12\,X^5 + 8\,a_{4,0}X^3 + 8\,\frac{3(\beta_2 + m\beta_3)}{\beta_5}y^2X^3 + \left(y^4\frac{(\beta_2 + m\beta_3)^2}{\beta_5} + {a_{4,0}}^2\right)X$$

and

$$N(X, y) = 2\left(-\frac{1}{9}\,y^2a_{4,2}b_{0,2} + b_{0,2}X^2 + \frac{1}{6}\,a_{4,0}b_{0,2}\right)\psi y + 2\left(-\frac{X^2c_{0,2}}{\frac{3(\beta_2+m\beta_3)}{\beta_5}} + c_{0,2}y^2 - \frac{1}{2}\,\frac{a_{4,0}c_{0,2}}{\frac{3(\beta_2+m\beta_3)}{\beta_5}}\right)\varphi X + \psi^2 + \varphi^2 + X^6 + a_{4,0}X^4 + \frac{3(\beta_2 + m\beta_3)}{\beta_5}y^2X^4 + \left(y^4\frac{(\beta_2 + m\beta_3)^2}{\beta_5} + \frac{1}{4}\,{a_{4,0}}^2\right)X^2 + \frac{1}{4}\,{a_{4,0}}^2\frac{(\beta_2 + m\beta_3)}{\beta_5}y^2 - \frac{1}{3}\,a_{4,0}\frac{(\beta_2 + m\beta_3)^2}{\beta_5}\,y^4 + \frac{1}{9}\,\frac{(\beta_2 + m\beta_3)^3}{\beta_5}\,y^6 - \frac{-\psi^2(\beta_2 + m\beta_3){b_{0,2}}^2 + \psi^2(\beta_2 + m\beta_3)^2 + \varphi^2(\beta_2 + m\beta_3)^2 - \beta_5\varphi^2{c_{0,2}}^2}{3(\beta_2 + m\beta_3)^2},$$

where $X = x + mt$ and $m, \alpha_1, \alpha_2, \beta_2, \beta_3, \beta_5, \varphi, \psi$ are arbitrary real parameters.

We choose particular values to illustrate the 3-rogue wave solution to the generalised Hietarinta-type equation:

$$u_0 = 0, \quad \varphi = \psi = 1000, \quad m = 2.95,$$
$$b_{0,2} = c_{0,2} = 1.3, \quad a_{4,0} = \beta_3 = 1,$$
$$\beta_2 = -0.95, \quad \beta_5 = 6. \tag{16}$$

In contrast to 1-rogue wave, it is obvious that figure 2, 3-rogue wave, has three crests and three troughs which are made of three 1-rogue waves in the plane (x, y). By increasing the value of ψ and φ, a triangle was formed by the three waves' centre. Similarly, the wave's shape is not a simple sine curve but is an asymmetric curve with a steep crest and a flat trough. The bigger is the wave force, relative wave height, relative wave length, the more nonlinear and faster is the wave velocity.

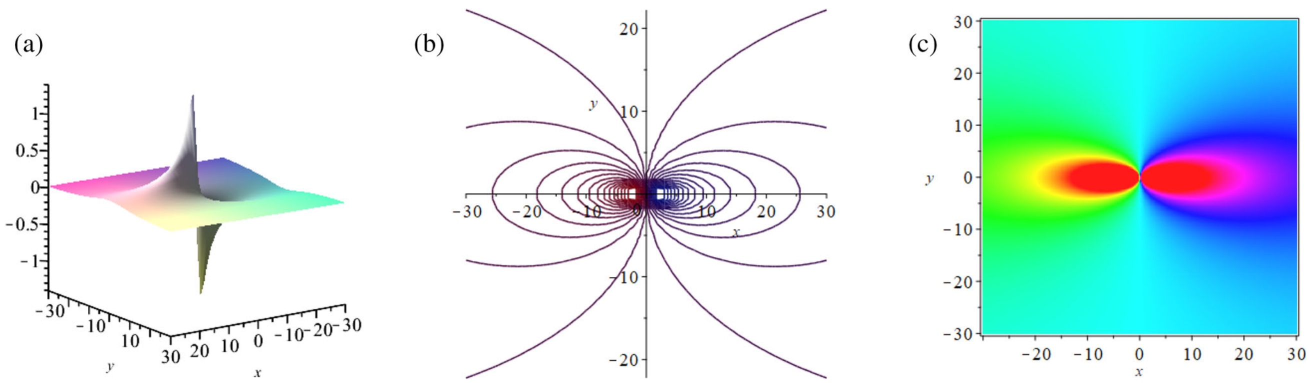

Figure 1. Three-dimensional (**a**), contour (**b**) and density (**c**) plots with the parameters (11) of the 1-rogue wave.

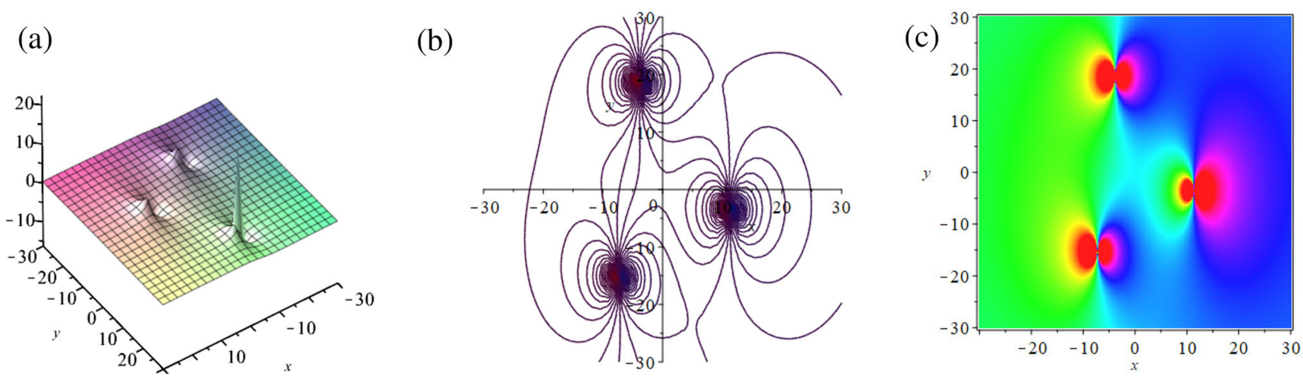

Figure 2. Three-dimensional (**a**), contour (**b**) and density (**c**) plots with the parameters (16) of the 3-rogue wave.

5. 6-rogue wave solution

In this section, in order to establish 6-rogue wave solution of eq. (4), taking $n = 2$ in expression (2), we shall obtain

$$f(X, y) = F_3(X, y) + 2\psi y P_2(X, y) + 2\varphi X Q_2(X, y) + (\psi^2 + \varphi^2) F_1(X, y), \quad (17)$$

where

$$F_1(X, y) = X^2 + (\beta_2 + \beta_3 m) y^2 - \frac{3(\alpha_2 m^3 + \alpha_1)}{\beta_2 + \beta_3 m},$$

$$\begin{aligned} F_3(X, y) = {} & X^{12} + \left(a_{10,2} y^2 + a_{10,0}\right) X^{10} + \left(a_{8,4} y^4 + a_{8,2} y^2 + a_{8,0}\right) X^8 + \left(a_{6,6} y^6 + a_{6,4} y^4 + a_{6,2} y^2 + a_{6,0}\right) X^6 \\ & + \left(a_{4,8} y^8 + a_{4,6} y^6 + a_{4,4} y^4 + a_{4,2} y^2 + a_{4,0}\right) X^4 + \left(a_{2,10} y^{10} + a_{2,8} y^8 + a_{2,6} y^6 + a_{2,4} y^4 + a_{2,2} y^2 + a_{2,0}\right) X^2 \\ & + a_{0,0} + a_{0,2} y^2 + a_{0,4} y^4 + a_{0,6} y^6 + a_{0,8} y^8 + a_{0,10} y^{10} + a_{0,12} y^{12}, \end{aligned}$$

$$P_2(X, y) = b_{0,0} + \left(b_{2,4} X^4 + b_{2,2} X^2 + b_{2,0}\right) y^2 + \left(b_{4,2} X^2 + b_{4,0}\right) y^4 + y^6 + b_{0,2} X^2 + b_{0,4} X^4 + b_{0,6} X^6,$$

$$Q_2(X, y) = c_{0,0} + c_{0,2} y^2 + c_{0,4} y^4 + c_{0,6} y^6 + \left(c_{2,4} y^4 + c_{2,2} y^2 + c_{2,0}\right) X^2 + \left(c_{4,2} y^2 + c_{4,0}\right) X^4 + X^6.$$

Substituting expression (17) into eq. (7) and collecting all the coefficients of $X^{i_1} y^{i_2}$, we shall obtain a range of algebraic equations about the parameters. Solving these equations, one has

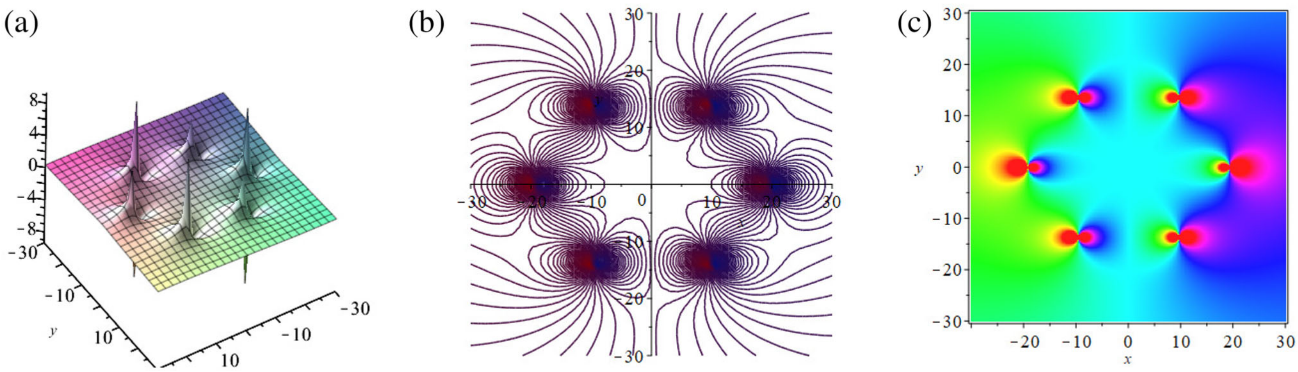

Figure 3. Three-dimensional (**a**), contour (**b**) and density (**c**) plots of 6-rogue wave.

$$a_{0,0}=\frac{{a_{6,0}}^2}{4\,\psi^2+4\,\varphi^2+4},\quad a_{0,4}=\frac{\left(6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2\right)^2}{36{a_{6,0}}^2{a_{10,2}}^{12}},$$

$$a_{0,6}=-\frac{a_{6,0}{a_{10,2}}^3}{216},\ a_{0,8}=-\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{648\,a_{6,0}{a_{10,2}}^3},$$

$$a_{0,10}=0,\quad a_{0,12}=\frac{{a_{10,2}}^6}{46656},$$

$$a_{2,0}=-\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+\psi^2{a_{10,2}}^7+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-559872\,\psi^2}{{a_{10,2}}^7},$$

$$a_{2,2}=\frac{\left(6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2\right)^2}{3{a_{6,0}}^2{a_{10,2}}^{13}},\quad a_{2,4}=\frac{5\,a_{6,0}{a_{10,2}}^2}{12},$$

$$a_{2,6}=\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{27{a_{10,2}}^4a_{6,0}},\quad a_{2,8}=0,\quad a_{2,10}=\frac{{a_{10,2}}^5}{1296},$$

$$a_{4,0}=\frac{\left(6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2\right)^2}{{a_{6,0}}^2{a_{10,2}}^{14}},\quad a_{4,2}=-\frac{5}{2}\,a_{6,0}a_{10,2},$$

$$a_{4,4}=\frac{5}{9}\,\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{a_{6,0}{a_{10,2}}^5},\quad a_{4,6}=0,\quad a_{4,8}=\frac{5\,{a_{10,2}}^4}{432},$$

$$a_{6,2}=\frac{4}{3}\,\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{a_{6,0}{a_{10,2}}^6},\quad a_{6,4}=0,$$

$$a_{6,6}=\frac{5\,{a_{10,2}}^3}{54},\quad a_{8,0}=-2\,\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{a_{6,0}{a_{10,2}}^7},$$

$$a_{8,2}=0,\quad a_{8,4}=\frac{5\,{a_{10,2}}^2}{12},\quad a_{10,0}=0,\quad a_{10,2}=6\,m\beta_3+6\,\beta_2,\quad b_{0,0}=-108\,\frac{a_{6,0}}{{a_{10,2}}^3},\quad b_{2,2}=b_{4,0}=0,$$

$$b_{0,2}=-216\,\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{a_{6,0}{a_{10,2}}^{10}},$$

$$b_{0,4}=0,\quad b_{0,6}=\frac{1080}{{a_{10,2}}^3},\quad b_{2,0}=-36\,\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{a_{6,0}{a_{10,2}}^9},$$

$$b_{2,4}=-\frac{180}{{a_{10,2}}^2},\quad b_{4,2}=-\frac{54}{a_{10,2}},\quad c_{0,0}=\frac{a_{6,0}}{2},$$

$$c_{0,2}=-\frac{6\,\psi^2a_{0,2}{a_{10,2}}^6+6\,\varphi^2a_{0,2}{a_{10,2}}^6-\varphi^2{a_{10,2}}^7+6\,a_{0,2}{a_{10,2}}^6-279936\,\psi^2}{6a_{6,0}{a_{10,2}}^6},$$

$$c_{0,4}=0,\quad c_{0,6}=\frac{5\,{a_{10,2}}^{3}}{216},\quad c_{2,0}=-\frac{6\,\psi^{2}a_{0,2}{a_{10,2}}^{6}+6\,\varphi^{2}a_{0,2}{a_{10,2}}^{6}-\varphi^{2}{a_{10,2}}^{7}+6\,a_{0,2}{a_{10,2}}^{6}-279936\,\psi^{2}}{a_{6,0}{a_{10,2}}^{7}},$$
$$c_{2,2}=0,\quad c_{2,4}=-\frac{5\,{a_{10,2}}^{2}}{36},\quad c_{4,0}=0,\, c_{4,2}=-\frac{3a_{10,2}}{2},$$

where other parameters are arbitrary real constants.

Hence, the 6-rogue wave solution of eq. (4) reads as

$$u(x,y,t)=\frac{K(X,y)}{L(X,y)}+u_0, \tag{18}$$

where

$$\begin{aligned}
K(X,y)&=24\,X^{11}+720\,y^{2}\,(m\beta_3+\beta_2)\,X^{9}-32\,\frac{\psi^{2}}{a_{6,0}\,(m\beta_3+\beta_2)^{5}}X^{3}+12a_{6,0}X^{5}\\
&+16\left(15\,y^{4}\,(m\beta_3+\beta_2)^{2}-2\,\frac{(a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)}{a_{6,0}\,(m\beta_3+\beta_2)}+2\,\frac{\varphi^{2}}{a_{6,0}}+2\,\frac{\psi^{2}}{a_{6,0}\,(m\beta_3+\beta_2)^{7}}\right)X^{7}\\
&+12\left(20\,y^{6}\,(m\beta_3+\beta_2)^{3}+8\,\frac{y^{2}\left((a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)-\varphi^{2}\,(m\beta_3+\beta_2)\right)}{a_{6,0}}-2\,\frac{\psi^{2}}{a_{6,0}\,(m\beta_3+\beta_2)^{6}}\right)X^{5}\\
&+8\left(15\,y^{8}\,(m\beta_3+\beta_2)^{4}+20\,\frac{y^{4}\,(a_{0,2})\left(\psi^{2}+\varphi^{2}-\varphi^{2}\,(6\,m\beta_3+6\,\beta_2)+6\,a_{0,2}\right)(m\beta_3+\beta_2)}{a_{6,0}}\right)X^{3}\\
&+2\left(-15\,y^{2}a_{6,0}\,(m\beta_3+\beta_2)+\left(\frac{(a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)-\varphi^{2}\,(m\beta_3+\beta_2)}{a_{6,0}\,(m\beta_3+\beta_2)}-\frac{\psi^{2}}{a_{6,0}\,(m\beta_3+\beta_2)^{7}}\right)^{2}\right)X^{3}\\
&+4\left(\frac{y^{6}\left(8\,a_{0,2}\left((\psi\,(m\beta_3+\beta_2))^{2}+(\varphi\,(m\beta_3+\beta_2))^{2}+1\right)^{2}-\varphi^{2}\,(m\beta_3+\beta_2)^{3}\right)}{a_{6,0}}\right)X\\
&+2\left(2\,\frac{1}{m\beta_3+\beta_2}\left(\frac{y\left((a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)-\varphi^{2}\,(m\beta_3+\beta_2)\right)}{a_{6,0}}-\frac{\psi^{2}}{a_{6,0}\,(m\beta_3+\beta_2)^{6}}\right)^{2}\right)X\\
&+2\left(-\frac{(a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)+\left(\psi^{2}-\varphi^{2}\right)(m\beta_3+\beta_2)}{m\beta_3+\beta_2}-2\,\frac{\psi^{2}}{(m\beta_3+\beta_2)^{7}}+12\,y^{10}\,(m\beta_3+\beta_2)^{5}\right)X\\
&-80\,\frac{\psi X^{3}y^{3}}{(m\beta_3+\beta_2)^{2}}-72\,\frac{\psi X\,y^{5}}{m\beta_3+\beta_2}-8\,\frac{\psi\left((a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)-\varphi^{2}\,(m\beta_3+\beta_2)\right)Xy}{a_{6,0}\,(m\beta_3+\beta_2)^{4}}\\
&-\frac{\psi^{3}Xy}{54\,a_{6,0}\,(m\beta_3+\beta_2)^{10}}+120\,\frac{\psi yX^{5}}{(m\beta_3+\beta_2)^{3}}+2\,\varphi\,a_{6,0}+30\,y^{4}a_{6,0}\,(m\beta_3+\beta_2)^{2}\,X\\
&+4\,\varphi\left(-\frac{y^{2}\left((a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)-\varphi^{2}\,(m\beta_3+\beta_2)\right)}{a_{6,0}}-\frac{\psi^{2}}{a_{6,0}\,(m\beta_3+\beta_2)^{6}}+5\,y^{6}\,(m\beta_3+\beta_2)^{3}\right)\\
&+4\,\varphi\left(3\left(-5\,y^{4}\,(m\beta_3+\beta_2)^{2}-\frac{(a_{0,2})\left(\psi^{2}+\varphi^{2}+1\right)-\varphi^{2}\,(m\beta_3+\beta_2)}{a_{6,0}\,(m\beta_3+\beta_2)}-\frac{\psi^{2}}{6a_{6,0}\,(m\beta_3+\beta_2)^{7}}\right)X^{2}\right)\\
&-2\varphi\left(45\,y^{2}\,(m\beta_3+\beta_2)\,X^{4}-7\,X^{6}\right)+8\,\varphi\left(\psi^{2}+\varphi^{2}\right)X^{2}-32\,\frac{\psi^{2}}{(m\beta_3+\beta_2)^{4}\,a_{6,0}}X
\end{aligned}$$

and

$$L(X,y)=2\,\psi y^{7}+2\,\varphi X^{7}-2\,\frac{\psi^{2}y^{8}}{a_{6,0}\,(m\beta_3+\beta_2)^{3}}+\frac{{a_{6,0}}^{2}}{4\,\psi^{2}+4\,\varphi^{2}+4}-8\left(\frac{\psi^{2}}{(m\beta_3+\beta_2)^{4}\,a_{6,0}}\right)X^{2}$$

$$+\left(6\,\left(y\left(m\beta_3+\beta_2\right)\right)^{50}+8\,\frac{y^6\left(a_{0,2}\left(\left(\psi\left(m\beta_3+\beta_2\right)\right)^2+\left(\varphi\left(m\beta_3+\beta_2\right)\right)^2+1\right)^2-\varphi^2\left(m\beta_3+\beta_2\right)^3\right)}{a_{6,0}}\right)X^2$$

$$+2\,\varphi\,X\left(\frac{1}{2}\,a_{6,0}-\frac{\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)y^2-\varphi^2\left(m\beta_3+\beta_2\right)y^2}{a_{6,0}}+\frac{y^2\psi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)^6}+5\,y^6\left(m\beta_3+\beta_2\right)^3\right)$$

$$+2\,\varphi\left(-5\,y^4\left(m\beta_3+\beta_2\right)^2-\frac{\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)-\varphi^2\left(m\beta_3+\beta_2\right)}{a_{6,0}\left(m\beta_3+\beta_2\right)}+\frac{\psi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)^7}\right)X^3$$

$$-2\,\psi\left(\frac{\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)-\varphi^2\left(m\beta_3+\beta_2\right)}{a_{6,0}\left(m\beta_3+\beta_2\right)^3}+1/36\,\frac{\psi^2+108\,a_{6,0}\left(m\beta_3+\beta_2\right)^7X^4}{a_{6,0}\left(m\beta_3+\beta_2\right)^9}\right)y^3+\left(m\beta_3+\beta_2\right)^6y^{12}$$

$$-\frac{\left(a_{0,2}\left(\psi^2+\varphi^2+1\right)\left(m\beta_3+\beta_2\right)\right)^3y^8-2\,\varphi^2\left(m\beta_3+\beta_2\right)^4y^8}{a_{6,0}}-18\,\varphi y^2\left(m\beta_3-\beta_2\right)X^5$$

$$+\left(\frac{y^2\left(\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)-\varphi^2\left(m\beta_3+\beta_2\right)\right)}{a_{6,0}}-\frac{y^2\psi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)^6}\right)^2-8\,\frac{\psi X^2y^5}{m\beta_3+\beta_2}-a_{6,0}\left(m\beta_3+\beta_2\right)^3y^6$$

$$+2\,\frac{X^2}{m\beta_3+\beta_2}\left(\frac{y\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)-y\varphi^2\left(m\beta_3+\beta_2\right)}{a_{6,0}}-\frac{y\psi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)^6}\right)^2+X^{12}+a_{0,2}y^2$$

$$+\left(15\,y^4a_{6,0}\left(m\beta_3+\beta_2\right)^2-\frac{\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)}{m\beta_3+\beta_2}+\psi^2-\varphi^2-559872\,\frac{\psi^2}{\left(6\,m\beta_3+6\,\beta_2\right)^7}\right)X^2$$

$$-\left(4\,\frac{\psi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)^5}+15\,y^2a_{6,0}\left(m\beta_3+\beta_2\right)-\left(\frac{\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)-\varphi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)}-\frac{\psi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)^7}\right)^2\right)X^4$$

$$+\left(15\,y^8\left(m\beta_3+\beta_2\right)^4+20\,\frac{\left(y\left(\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)\left(m\beta_3+\beta_2\right)-\left(\varphi\left(m\beta_3+\beta_2\right)\right)^4\right)\right)^4}{a_{6,0}}\right)X^4$$

$$+\left(20\,y^6\left(m\beta_3+\beta_2\right)^3+8\,\frac{y^2\left(\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)-\varphi^2\left(m\beta_3+\beta_2\right)\right)}{a_{6,0}}-\frac{2\,\psi^2+\left(\left(a_{6,0}\right)\left(m\beta_3+\beta_2\right)\right)^6}{a_{6,0}\left(m\beta_3+\beta_2\right)^6}\right)X^6$$

$$+\left(15\,y^4\left(m\beta_3+\beta_2\right)^2-2\,\frac{\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)}{a_{6,0}\left(m\beta_3+\beta_2\right)}-2\,\frac{\varphi^2}{a_{6,0}}-\frac{\psi^2}{a_{6,0}\left(m\beta_3+\beta_2\right)^7}\right)X^8$$

$$-\frac{\psi a_{6,0}y}{\left(m\beta_3+\beta_2\right)^3}-2\,\frac{\psi\,y\left(\left(a_{0,2}\right)\left(\psi^2+\varphi^2+1\right)X^2-\varphi^2\left(m\beta_3+\beta_2\right)X^2\right)}{\left(m\beta_3+\beta_2\right)^4a_{6,0}}$$

$$-\frac{\psi\,y\left(\psi^2X^2+1080\,a_{6,0}\left(m\beta_3+\beta_2\right)^7X^6\right)}{108\,a_{6,0}\left(m\beta_3+\beta_2\right)^{10}}+y^2\left(6\,m\beta_3+6\,\beta_2\right)X^{10}+\left(\psi^2+\varphi^2\right)\left(X^2+\left(m\beta_3+\beta_2\right)y^2\right),$$

where $X = x + mt$ and $m, \alpha_1, \alpha_2, \beta_2, \beta_3, \beta_5, \varphi, \psi$ are arbitrary real parameters.

Figure 3 vividly shows the structure and the form of 6-rogue wave which has six crests and six troughs with $u_0 = 0, \varphi = \psi = 5000, m = 1, a_{0,2} = 5, a_{6,0} = 1, \beta_2 = -3.5, \beta_3 = 5, \beta_5 = 1$. It is made of six 1-rogue waves in the plane (x, y). By increasing the value of ψ and φ, a hexagon is formed by the six waves' centre, at the same time, as we can see the centre of the waves is symmetric.

6. Conclusion

In this paper, multiple rogue wave solutions of a generalised Hietarinta-type fourth-order equation in (2 + 1)-dimensional dispersive waves have been systematically constructed and studied based on the bilinear method and a novel assumption. First, an expression has been given to solve the $(k+1)$-dimensional equations, in order to transform it into a $(1+1)$-dimensional equation and reduce to a bilinear equation. We have free choice to choose a proper transformation. Then, by selecting the number of n in expression (3), we successfully obtained 1-rogue wave solution, 3-rogue wave solution and 6-rogue wave solution of a generalised Hietarinta-type fourth-order equation in (2 + 1)-dimensional dispersive waves equation when $n = 0$, $n = 1$, $n = 2$. Finally, by choosing the values of φ, ψ and other parameters, a set of maps including 3D, contour and density plots have been vividly shown by the symbolic computation. It is obvious that by increasing value of φ, ψ, the number of rogue waves will grow and 3-rogue and 6-rogue waves are made of three and six independent 1-rogue waves, respectively. At the same time, the 1-rogue wave, 3-rogue wave, 6-rogue wave solutions are with the common property of $\lim_{x\to\pm\infty} u(x, y, t) = u_0$ and $\lim_{y\to\pm\infty} u(x, y, t) = u_0$. The exact solutions of a generalised Hietarinta-type equation were fairly expanded due to this method and this approach can be applied to calculate other integrable equations' multiple rogue wave solutions or more rogue wave solutions of the generalised Hietarinta-type equation such as when $n = 3, n = 4$, etc.

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