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# Evaluation of bulk thermodynamical properties of QGP with two-loop corrections in the potential

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**Abstract.** We extend the work of two-loop correction in the mean-field potential to study the thermodynamical properties of quark–gluon plasma (QGP). In the study, we look forward to determine bulk thermodynamic properties like pressure, entropy density, specific heat, quark number density, quark number fluctuations and speed of sound. The determinations are shown in figures and the figures are found almost similarly enhanced to our earlier results of one-loop correction. It means that the model with the inclusion of two-loop corrections can provide the entire thermodynamic information about the phase structure of QCD and the parameter used in the loop correction plays a pivotal role in forming stable droplets having less effect to calculate all the thermodynamical parameters. Still, the result can be considered in presenting QGP phase structure as it shows its representation within the range of standard thermodynamic properties and enhance the energy density and pressure from the lattice result.

**Keywords.** Quark–gluon plasma; quantum chromodynamics; heavy-ion collision.

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## 1. Introduction

The theory of strong interactions predicts quark–hadron phase transition under extremely high nuclear density and very high temperature. In the transition [1–4] phenomena, matter consists of free quarks and gluons called quark–gluon plasma (QGP) which is transformed to a confined phase of bound quarks in hadrons. The system is believed to exist for a short time and this short span of transition of the system makes it difficult to search for the transition phenomena. It is believed that the beginning of the expansion of the early Universe, which is described by Big Bang theory, was very hot and subsequently it cooled down with the expansion of the Universe. One way to explain the phase transition is by studying very high temperature system obtained in the laboratory. Another route is the phase transition of very high nuclear density matter. These very high nuclear density matters are normally obtained during the formation of compact objects, neutron star and boson stars [5,6]. These objects are believed to be found as remnants of supernova explosion. For the investigation and search about the nature of the Universe, a number of experimental facilities like relativistic heavy-ion collision (RHIC) at BNL and large hadron collider (LHC) at CERN are active around the globe. These two experiments have examined the conditions of early Universe by colliding head on with very energetic ion beams. These experiments have also claimed the creation of a mini Universe called quark–gluon plasma (QGP) [7,8]. So these two experiments give information about matter at very high temperature. On the other hand, some experimental facilities like FAIR at Darmstadt and NICA at Dubna, where the study is focussed on dense baryonic matter and the baryonic matter at Nuclotron (BM@N) experiments which extracted ion beams from modernised Nuclotron, will provide future information about the formation of QGP under the influence of compressed dense nuclear matter. All the facilities available so far are trying to detect the existence of the critical point in the phase structure, the early Universe phase transition, formation of QGP and quantum chromodynamics (QCD) phase structure at very high nuclear density [9–12]. So, the investigation on QGP through the ultrarelativistic heavy-ion collisions has become an exciting field in the current scenario of heavy-ion collider physics [13–15]. In this

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extended article, we focus on the calculation of bulk thermodynamic properties of matter at very high temperature in continuation of our earlier works of non-loop and one-loop corrections by incorporating two-loop correction [16,17]. We have a re-look at pressure, entropy density, specific heat, quark number density and quark number fluctuations which indicate the susceptibility of the matter. Obviously, by the addition of two-loop correction, more stable droplet formation of QGP was reported and parametrisation value used in the two loop has largely affected the droplet size formation so that the bulk thermodynamic properties are slightly changed from the one-loop correction. So, the droplet evolution of two-loop correction was definitely able to predict the changes in the evaluation of thermodynamic properties. The introduction of two-loop correction factor also shows almost similar pattern with a little increase from the lattice and lesser amplitude in the evaluation of the bulk thermodynamic parameters from our earlier result of one-loop correction. Even though there is a decrease in amplitude, two-loop correction increases stability with slightly lesser values for other properties of QGP thermodynamics.

## 2. One- and two-loop correction potentials

The interacting potentials among the quarks and antiquarks through one-loop and two-loop corrections are subsequently defined in the following expressions:

$$U_{\text{one}}(p) = C\left[1 + \frac{\alpha_s(p)a_1}{4\pi}\right] - \frac{m_0^2}{2p}, \tag{1}$$

where

$$C = \frac{8\pi}{p}\gamma\alpha_s(p)T^2$$

in which $\gamma$ is the root mean square value of the quark and gluon parametrisation factors. The value is different depending on non-loop to one-loop and two-loop corrections, which are taken from the earlier values of non-loop and one-loop corrections [18,19]. In the non-loop case, the values of quark and gluon parametrisation are $\gamma_q = 1/6$ and $\gamma_g = (6\text{–}8)\gamma_q$ for stable droplet formation whereas they are $\gamma_q = 1/8$ and $\gamma_g = (8\text{–}10)\gamma_q$ for one-loop correction. These values are only used when we are calculating the non-loop and one-loop corrections. Now the value is required to be modified as used in calculating the two-loop case. It is found to be different in two-loop correction in order to obtain stable droplet. It is found to be $\gamma_q = 1/14$ and $\gamma_g = (48\text{–}52)\gamma_q$ after ad-hoc search, manually for forming stable droplet. So the value, $\gamma$ is different when it is calculating for different loop expressions. However, in this two-loop calculation, the value is replaced by $\gamma_q = 1/14$ and $\gamma_g = (48\text{–}52)\gamma_q$ as the parameter is entirely used for the two-loop calculation in the free energy and it can make the stable droplet for two loop with it. Besides the stable droplet formation, the factors determine the dynamics of QGP flow and enhance the process of transformation to hadrons with the formation of stable droplets. It implies that in the route to transformation, the droplets are carried forward with the stable droplet formation by changing the parametrisation value with respect to the loop correction factor.

Then, $\alpha_s$ is the momentum-dependent coupling constant normally in QCD decreasing function of large momentum factor. In addition to these parameters, an additional constant parameter is required to define the one-loop correction factor called $a_1$. This is used for one loop in the potential equation and it is obtained through the interacting potential among the particles. So the coefficient $a_1$ is given as [20–22]

$$a_1 = 2.5833 - 0.2778n_l, \tag{2}$$

where $n_l$ is the number of light quark elements [23–26]. Now, if we extend our calculation to further interactions up to the factor of two-loop correction, then interaction potential with correction factor is obtained as

$$U_{\text{two}}(p) = U_{\text{one}}(p) + \left[\frac{\alpha_s^2(p)a_2}{16\pi^2}C\right], \tag{3}$$

where

$$a_2 = 28.5468 - 4.1471n_l + 0.0772n_l^2 \tag{4}$$

is a two-loop coefficient and $C$ is the factor which can be obtained in terms of one and two loops with the same value of $\gamma$ as it is used in the entire calculation of two loop. Now, we get the modified mass of the system after the incorporation of these loop corrections in the system. So, the thermal mass of the one-loop correction is defined as

$$m_{\text{one}}^2(T) = 2\gamma^2 g^2(p)T^2[1 + g^2(p)a_1], \tag{5}$$

whereas in the case of two-loop correction it is

$$m_{\text{two}}^2(T) = m_{\text{one}}^2(T) + 2\gamma^2T^2g^6(p)a_2. \tag{6}$$

These masses are obtained after one and two loop corrections and they are then introduced in the potential. Really the interaction potential is probably created due to the effect of thermal mass of quarks and antiquarks. The thermal mass changes linearly with the loop correction coefficients. Now to obtain our aim, we further look at the partition function through grand canonical ensemble of these loop corrections.

## 3. Partition function and free energy

We evaluate partition function through the grand canonical ensemble of two-loop correction in the interaction potential of quarks and antiquarks by the exchange of the colour particle called gluon. So the partition function obtained through the ensemble is defined in the following by incorporating the density of state of the system. Density of state is defined in such way that mean-field potential through the two-loop correction is introduced in the calculation. The free energies of quarks, gluons and hadrons can be now obtained through the partition function of the system. The general partition function of the system defined by many researchers is given [27,28] as

$$Z(T,\mu,V) = \mathrm{Tr}\{\exp[-\beta(\hat{H}-\mu\hat{N})]\}, \quad (7)$$

where $\mu$ is the chemical potential of the system, $\hat{N}$ is the quark number and $\beta = 1/T$. Using this partition function, we correlate the free energy through the density of state incorporating the two-loop correction factor and it is calculated by [29,30]

$$F_i = T\ln Z(T,\mu,V) \quad (8)$$

$$F_i = \eta T g_i \int \rho_{\mathrm{two}}(p)p^2 w(p)\mathrm{d}p, \quad (9)$$

where

$$w(p) = \ln[1+\eta \mathrm{e}^{-(\sqrt{m_i^2+p^2}-\mu)/T}], \quad \eta = -1$$

for the bosonic particle and $\eta = +1$ for fermionic particles. $g_i$ is the degree of freedom for quarks and hadronic particles. The values of this degree of freedom (dof) for two and three flavours are different. The value is contributed by spin, flavour, colour and charge for two- and three-flavour quarks and helicity and colour for gluons. It is not a number in the case of hadronic particles which is defined as $g_i = hV/(2\pi^2)$ where $h$ is the number factor like degree of freedom of quark and gluon depending on the particular hadronic particle and $V = \frac{4}{3}\pi R^3$ represents the corresponding volume of the droplet. The density of state $\rho_{\mathrm{two}}$ for the hadronic particles is unity with its momentum factor $p^2$ whereas it is defined with unit momentum factor when the two-loop correction is incorporated for the case of quarks as [31]

$$\rho_{\mathrm{two}}(p) = \frac{V}{3\pi^2}\frac{\mathrm{d}U^3_{\mathrm{two}}(p)}{\mathrm{d}p}, \quad (10)$$

or

$$\rho_{\mathrm{two}}(p) = \frac{V}{\pi^2}\frac{\gamma^9 T^6}{8}g^6(p)B, \quad (11)$$

where

$$B = \left[1+\frac{\alpha_s(p)a_1}{\pi}+\frac{\alpha_s^2(p)a_2}{\pi^2}\right]^2 \times\left[\frac{(1+\alpha_s(p)a_1/\pi+\alpha_s(p)^2a_2/\pi^2)}{p^4} + \frac{2(1+2\alpha_s(p)a_1/\pi+3\alpha_s(p)^2a_2/\pi^2)}{p^2(p^2+\Lambda^2)\ln(1+\frac{p^2}{\Lambda^2})}\right]. \quad (12)$$

Similarly, $V$ is the volume occupied by the corresponding quarks or the hadron droplets and $g^2(p) = 4\pi\alpha_s(p)$. Again, if the two-loop correction is removed in the interacting potential, then the value of $B$ is reduced to only one-loop correction and become $A$, which is found in terms of $a_1$ as

$$A = \left\{1+\frac{\alpha_s(p)a_1}{\pi}\right\}^2\left[\frac{(1+\alpha_s(p)a_1/\pi)}{p^4} + \frac{2(1+2\alpha_s(p)a_1/\pi)}{p^2(p^2+\Lambda^2)\ln(1+(p^2/\Lambda^2))}\right]. \quad (13)$$

It implies that due to the correction factor, the density of state is perturbed by a small factor which is seen in the plot of potential vs. momentum. In the expression, the parameter $\Lambda$ is considered in the scale of QCD as 0.15 GeV. So, we can set up the free energy of the system by finding the energies of quarks, antiquarks, gluons and all the light and medium light hadrons. In the formalism of these free energies, we use the density of states obtained through the Thomas and Fermi model in which one- and two-loop corrections are incorporated in the interacting mean-field potential. The integral is evaluated from the least value of momentum which approximately tends to zero. Taking and considering all the light and medium mass hadrons we calculate the total free energy by adding the interfacial energy of the fireball [32].

$$F_{\mathrm{total}} = \sum_i F_i + \frac{\gamma T R^2}{4}\int p^2\delta(p-T)\mathrm{d}p. \quad (14)$$

In the first term of the total free energy, $i$ in the summation stands for $u$, $d$, $s$ quarks and all the hadronic particles with gluon and it symbolises the individual contributions of free energies of these particles, say quarks, antiquarks, gluons and other hadronic particles whereas in the second term, it is interfacial energy which replaces the role of bag energy of MIT bag model in which bag energy was in the scale of $B^{1/4} = T_c$. Taking the interfacial energy in place of MIT bag energy, it can reduce the drawback produced by bag energy to the maximum effects in comparison to MIT model calculation. So the interfacial energy is dependent on temperature

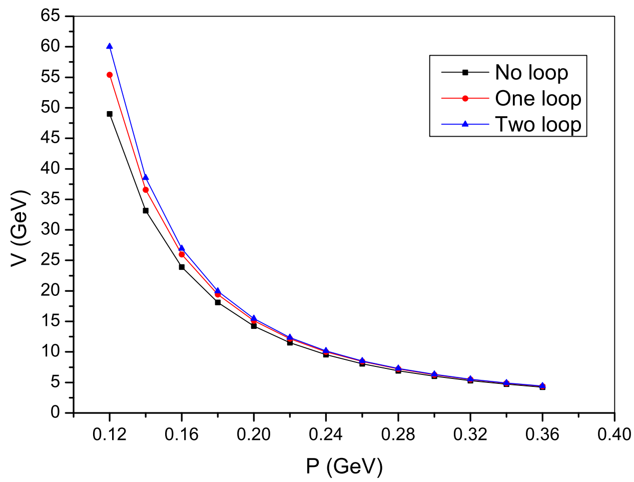


**Figure 1.** Potential vs. momentum with and without loop.

and some parametric factor, where $R$ is the size of the contributed QGP and hadron droplet with the parametrisation factor (figure 1).

## 4. Bulk thermodynamic properties

By bulk thermodynamic properties, we mean the following calculations of the system, viz., pressure, entropy density, specific heat, number density, susceptibility and speed of sound. These thermodynamic properties provide the entire information of QCD phase structure under the correction of loop factors in the phenomenological potential. All these thermodynamic properties are only calculated at finite temperature without the chemical potential $\mu$. To evaluate these bulk properties, we find the partition function using the grand canonical ensemble of the whole system. The partition function obtained through canonical ensemble is correlated to the Gibbs free energy so that we are able to evaluate all the properties of the corresponding thermodynamic relations obtained from the free energy. Obviously, there are changes in the free energy expansion of QGP fireball with loop corrections, and due to the change in free energy it impacts in the stability of droplet formation with the variation of dynamical quark and gluon flow parameters. So the flow parameters take the role of stability in forming droplet size with the differing temperatures. Besides, it gives the details of evolution of QGP fireball with different flow parametrisation values. However, the pressure is evaluated through the free energy using the standard thermodynamic relation and it is obtained in terms of density of state [33–35]

$$P(T,\mu) = -\eta T^2 \sum_i g_i \int \rho'_{\text{two}}(p) p^2 w(p) \mathrm{d}p + \frac{3\gamma T}{16\pi R} \int p^2 \delta(p-T) \mathrm{d}p, \tag{15}$$

where

$$w(p) = \ln[1 + \eta \mathrm{e}^{-(\sqrt{m_i^2+p^2}-\mu)/T}]$$

and $\rho'_{\text{two}}$ is the density of state independent of volume as pressure is calculated as first derivative of free energy with respect to volume at constant temperature. $g_i$ is the degree of freedom for the corresponding quarks, gluons and hadrons. They are defined earlier and $\eta$ is $+1$ and $-1$ depending on the particle with respect to Fermi and bosonic particles. We further calculate the entropy density through the partition function. So, the entropy density calculated through the ensemble is obtained as follows:

$$S = \eta \sum_i g_i \int \rho_{\text{two}}(p) p^2 w(p) \mathrm{d}p + \frac{\eta}{T} \sum_i g_i \int \frac{\rho_{\text{two}}(p) p^2 \sqrt{(m^2+p^2-\mu)} \mathrm{d}p}{[1 + \eta \mathrm{e}^{-(\sqrt{m_i^2+p^2}-\mu)/T}]} - \frac{3V\gamma T^3}{16\pi R}, \tag{16}$$

where

$$w(p) = \ln[1 + \eta \mathrm{e}^{-(\sqrt{m_i^2+p^2}-\mu)/T}].$$

Similarly, we also calculate specific heat using the standard thermodynamic relation and obtain specific heat as

$$C_v = \frac{1}{T^2} \sum_i g_i \int \frac{\rho_{\text{two}}(p) p^2 (m^2+p^2-\mu) x(p) \mathrm{d}p}{[1 + \eta \mathrm{e}^{-(\sqrt{m_i^2+p^2}-\mu)/T}]^2} - \frac{V\gamma T^3}{16\pi R} \tag{17}$$

where

$$x(p) = \left[ \mathrm{e}^{-(\sqrt{m_i^2+p^2}-\mu)/T} \right].$$

The entropy and specific heat are calculated for a suitable flow parameter of $\gamma_q$ and gluon parameter $\gamma_g$, particularly chosen as $\gamma_q = 1/14$ and $\gamma_g = 48\gamma_q$ taken as the most stable droplet formation for two-loop correction. The calculations of these parameters are important to determine the equilibrating state and disordered condensate states of the system. So the behaviour of entropy and specific heat with temperature indicates the nature of phase transition. Moreover, some researchers reported about determining the order of phase transition on the basis of first and second derivatives of the Gibbs potential energy. However, entropy contribution is mainly dependent on temperature. So we define entropy as a function of temperature. In addition to these thermodynamic properties, we also calculate the number density and susceptibility using relations obtained from the free energy. These relations of number density and susceptibility are found with first- and second-order derivatives of the free energy with respect to chemical potential. Thus, the quark number density and susceptibility of the system are obtained after the analytical calculation using the standard formulae [37–40].

$$n = -\eta \sum_i g_i \int \frac{\rho_{\text{two}}(p) p^2 \, \mathrm{d}p}{[1+\eta \mathrm{e}^{(\sqrt{m_i^2+p^2}-\mu)/T}]} \tag{18}$$

$$\chi = -\frac{1}{T} \sum_i g_i \int \frac{\rho_{\text{two}}(p) p^2 \mathrm{e}^{(\sqrt{m_i^2+p^2}-\mu)/T} \mathrm{d}p}{[1+\eta \mathrm{e}^{(\sqrt{m_i^2+p^2}-\mu)/T}]^2}. \tag{19}$$

Using the above definition, we evaluate number density and susceptibility. The evaluated values are represented by their behaviour with temperature in figures. The values are analysed from the figures by varying the temperature. Again, we calculate the speed of sound using the standard ratio of entropy density and specific heat. The ratio of these two thermodynamic properties of entropy and specific heat is defined by many researchers and the relation is used as [36,41]

$$C_s^2 = \frac{S/T^3}{C_v/T^3}. \tag{20}$$

The value of speed of sound is also shown in the following figure with the corresponding temperature.

## 5. Results

The analytical calculations of free energy of QGP-hadron fireball evolution without and with one- and two-loop correction factors in the interacting mean-field potential are performed by computing the variation of the interacting potential and momentum. The potential function is slightly perturbed from the unloop factor. By the loop correction it is slightly increased at the lower momentum region and with increasing momentum the perturbation is negligible indicating that the perturbations of the one- and two-loop corrections are very small in the high momentum transfer and this type of asymptotic behaviour is obtained in QGP phenomena and the approaching nature is good phenomena of the model. Due to the similarity in the asymptotic behaviour, we look forward to the bulk thermodynamic properties in terms of $P/T^4$ with the variation of temperature for one-loop and two-loop correction factors.

Now we look at the behaviour of $P/T^4$ for one-loop correction factor. It is shown in figure 2 and the plot indicates a very good output. There is a rapid rise in pressure at the initial stage of temperature and a constancy of $P/T^4$ is obtained at the temperature starting from 200 MeV. It is higher in comparison to lattice at the initial temperature. The result is very much similar to the two-flavour and three-flavour quarks. The amplitude of $P/T^4$ for three-flavour quarks is slightly more at the initial stage than that for the two-flavour quarks and there is more enhancement at higher temperature which is predicted by the lattice. This indicates that the contribution of one loop in the calculation is a little higher and almost in agreement with the lattice at higher temperature. So the effect of one loop has significant contribution in the calculation of the thermodynamic parameters of QGP

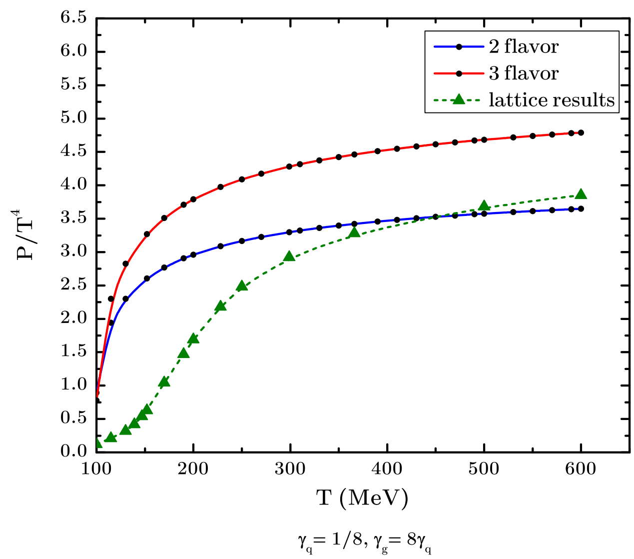


**Figure 2.** Pressure/$T^4$ vs. $T$ when $\gamma_q = 1/8$, $\gamma_g = 8\gamma_q$. It is with one-loop correction.

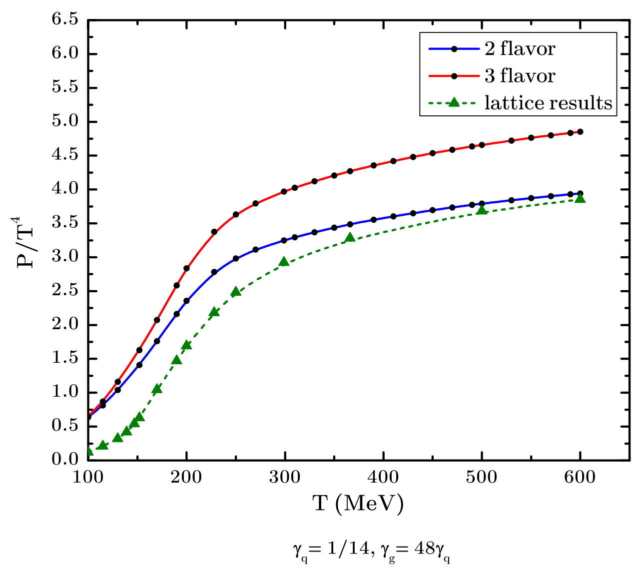


**Figure 3.** Pressure/$T^4$ vs. $T$ when $\gamma_q = 1/14$, $\gamma_g = 48\gamma_q$. It is with two-loop correction.

evolution. Now we look for the case of two-loop correction factor shown in figure 3. In the figure, a plot of $P/T^4$ vs. $T$ is shown. The plot is absolutely fine with the inclusion of two loops and slightly different at the initial stage upto the temperature, $T = 300$ MeV and there is a constancy of $P/T^4$ with the increase of temperature beyond $T = 300$. The result is comparatively a little less matching with the behaviour of our earlier result of one-loop correction. This implies that there is a slight deviation in all thermodynamic parameters comparatively with one-loop results. Yet, in comparison with the lattice, the result of two-loop correction completely is enhanced with similar pattern of the lattice in the whole range of temperature. With the inclusion of two-loop factors in the calculation, the thermodynamics parameters are closer and these results are predicted from the lattice. It implies that the result is almost in conformity with the lattice at higher temperature and have small difference in lower temperature. So, the plots of pressure vs. temperature indicated that contribution of two-loop factors in the present model can explain the free energy evolution and other relevant thermodynamic parameters. Now, in addition to the pressure, we look forward to other thermodynamic properties with the two-loop factors (figures 4 and 5).

In the case of entropy calculation, similar pattern is observed for the entire change of temperature. The result is almost similar with one-loop correction at the lower temperature, slightly less entropy density is produced by one-loop correction at higher temperatures. In the same way, when we look at the specific heat of the two-loop correction with the change of temperature, the figure shows almost identical pattern with the works of other researchers and follows the same trajectory with our earlier result of one-loop correction at lower temperatures and slightly lesser at the higher temperature.

From these information, the details of pressure, entropy and specific heat are obtained and these results indicate very significant thermodynamic properties about the evolution of QGP fireball with different flow parametrisation values. The information also reveals a basic picture about the characteristic features of the equation of state and phase transition. In addition to these features, we show the results of number density, susceptibility and speed of sound. The calculation of the quark number density explains the particle fluctuations inside the QGP with the change of chemical potential and it is shown in figure 6. This figure shows the increasing function of number density for a certain temperature and constancy is obtained at higher temperatures implying that the number density is independent of high temperature. The figure also shows that the system with the modification of two-loop correction in the mean-field potential agrees with the results

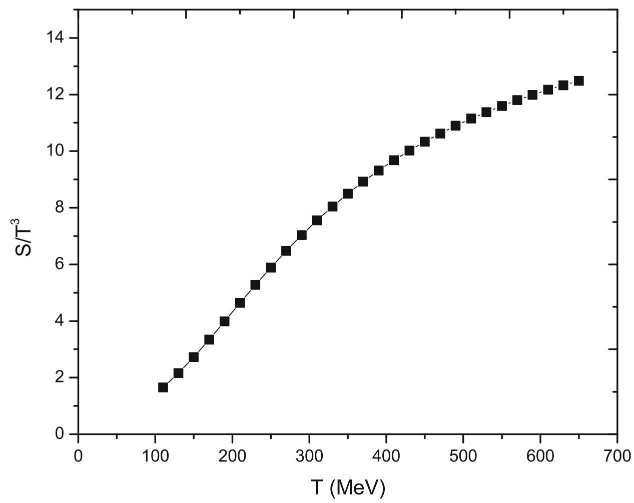


**Figure 4.** Entropy density vs. $T$ when $\gamma_q = 1/14$, $\gamma_g = 48\gamma_q$.

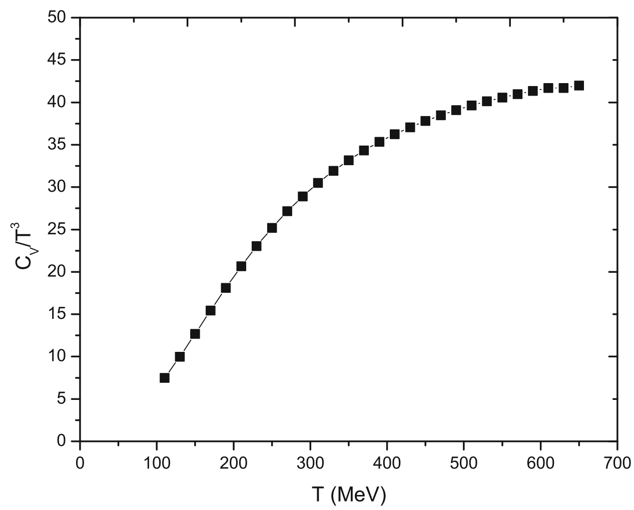


**Figure 5.** The specific heat at constant volume vs. $T$ when $\gamma_q = 1/14$, $\gamma_g = 48\gamma_q$.

of many other works [36,41–43]. In addition to number density, we study susceptibility to obtain more information about the QCD structure and plot the susceptibility of two-loop correction in figure 7. Similar information is also obtained in the case of susceptibility. The susceptibility effect is comparatively lower at lower temperature whereas it is larger at high temperature. This is the corresponding effect of fluctuation in the number density. As number density increases, the contribution of susceptibility is also increased due to the increase in the number of charge particles. It implies that the susceptibility follows similar outputs with other theoretical works [31,36] and the choice of parameter of two-loop correction in the potential shows a very important and significant role to stabilise the droplet during the time of QGP formation and further the parameter boosts up in finding the

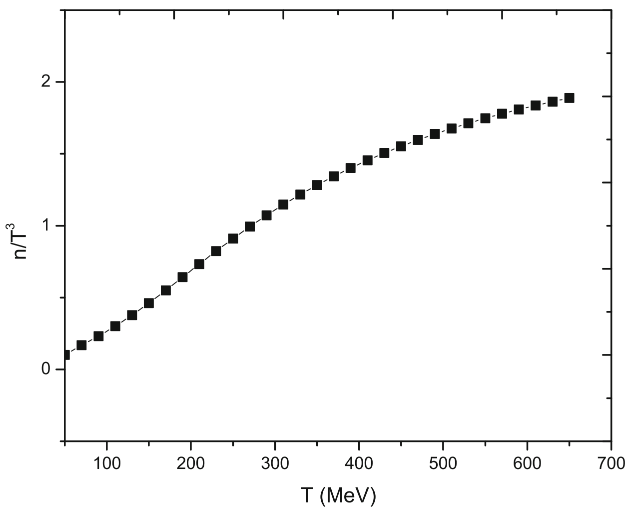


**Figure 6.** Quark number density vs. $T^3$ when $\gamma_q = 1/14$, $\gamma_g = 48\gamma_q$.

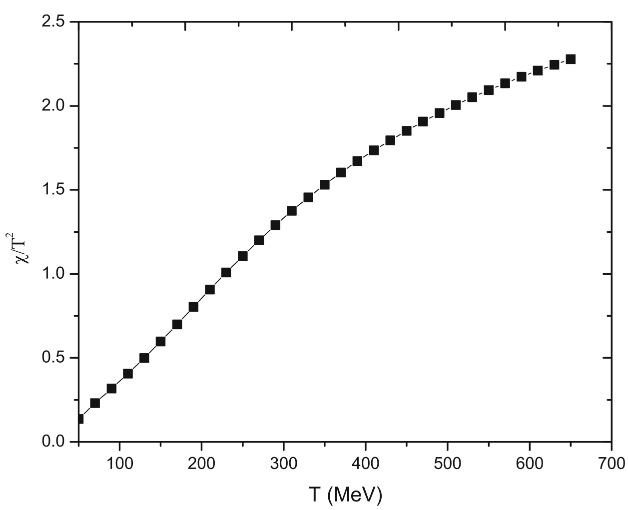


**Figure 7.** Susceptibility vs. $T^2$ when $\gamma_q = 1/14$, $\gamma_g = 48\gamma_q$.

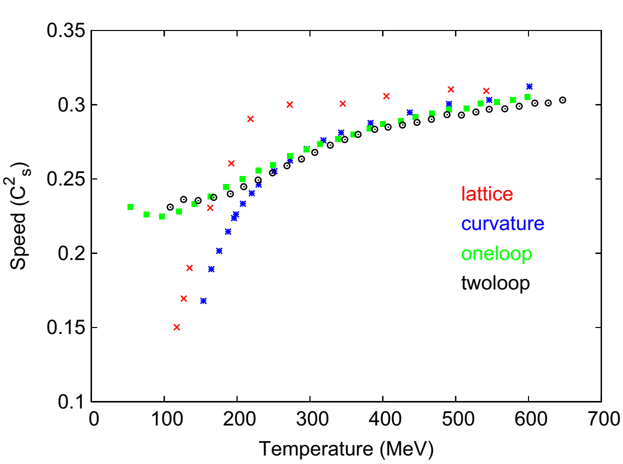


**Figure 8.** Speed of sound vs. $T$ when $\gamma_q$=1/14, $\gamma_g$=48$\gamma_q$.

bulk thermodynamic properties of the QGP. In addition to these, there is increase in its magnetic property due to the presence of large baryonic particles.

Obviously, if we see the speed of sound using the ratio of entropy and specific heat, the result is well behaved and found to be almost similar to our earlier plot of one-loop correction [44]. At temperatures below the critical temperature, the speed of sound of two-loop correction shows values larger than the lattice result and it is slightly lower to the lattice when it reaches temperature around 450 MeV. The speed of sound is negligibly and unpredictably less than the result produced using curvature correction in the free energy, the lattice and one-loop correction at very high temperature. However, the result can be assumed to be significant in representing the speed of sound under extreme conditions of temperature. These arguments can be seen in figure 8 in which all the trajectories of two-loop, one-loop, curvature effect in the free energy and lattice are plotted comparatively. The unmatched result with the lattice in the speed of sound at lower temperature is due to the effect of neglecting the contribution of massive hadrons and baryons in the calculation whereas the matching output is due to the dominance of higher temperature over the contribution of all masses, assuming as the contributing factor in the expansion of early Universe. So the model excludes the larger components of hadronic and baryonic particles and only takes care of a few light hadronic particles as they have provided free energies of QGP fireball with stable droplets. In addition to the formation of the stable droplet, the overall results are in agreement with our earlier works and almost in the range of other theoretical works [44]. It indicates that the results of the thermodynamic properties almost approach the standard lattice results and earlier results. Thus, the model with the parametrisation value as a type of Reynold number can be considered as a suitable fitting value to represent the expansion of dense nuclear fluid and to propose the phase structure of the QCD phase diagram. Hence, the simple model is able to describe the idea of the expansion of QGP fluid and can at least explain the QCD phase structure using the thermodynamic properties.

## 6. Conclusion

We can conclude from these results that due to the presence of loop corrections in the mean-field potential, the pressure changes with temperature in the case of one-loop correction is more than the lattice at lower temperature and the same at higher temperature. However, the changes of pressure in two loop is low in comparison with the standard lattice result. Our result is enhanced with similar behaviour in the whole range of temperature with lattice. Moreover, other results of the thermodynamic parameters, viz., entropy, specific heat, quark number fluctuation, susceptibility and speed of sound, are well satisfied with the lattice data at high temperature and the results are still slightly higher at the low temperature than the lattice. In the case of the stability droplet formation, it is found to be much better in two-loop correction by the smaller size of droplet which is not comparable in terms of one-loop correction and without loop correction where sizes of droplets are found to be bigger. Smaller size of droplet obviously increases the surface tension. Henceforth, it is more and more stable in the droplet. So correction leads to a good prospect in terms of finding the thermodynamic properties and in presenting the QCD phase structure.

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