Pramana – J. Phys. (2021) 95:121
https://doi.org/10.1007/s12043-021-02129-6

Z-Production dependence on the identification of the scale energy parameter ($\tilde{Q}^2$) involved in the PDFs

W GONZALEZ[1], M GÓMEZ-BOCK[2], L LÓPEZ-LOZANO[3], S ROSADO-NAVARRO[4] and A ROSADO[1],*

[1]Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla, Pue.C.P. 72570, Mexico
[2]DAFM, Universidad de las Américas Puebla, Ex. Hda. Sta. Catarina Mártir s/n, San Andrés Cholula, Puebla., Mexico
[3]Universidad Autónoma del Estado de Hidalgo, Carr. Pachuca-Tulancingo Km. 4.5, C.P. 42184, Pachuca, Hidalgo, Mexico
[4]Facultad de Ciencias Físico-Matemáticas, Benemérita Universidad Autónoma de Puebla, Apartado Postal 1364, C.P. 72000, Puebla, Pue., Mexico
*Corresponding author. E-mail: rosado@ifuap.buap.mx

MS received 28 January 2020; revised 10 September 2020; accepted 18 February 2021

Abstract. We discuss Z-production in deep inelastic scattering $e + p \rightarrow e + Z + X$ using the parton model, in the context of the Standard Model at leading order. In contrast to the deep inelastic ep-scattering ($e + p \rightarrow e + X$), where $\tilde{Q}^2$, the momentum transfer square, is unambiguous, in the case of boson production it depends upon the mechanism involved, that is, it is related to the electroweak interactions. We present estimates for the total cross-section as a function of $\sqrt{s}$, the energy of the ep system, in the range $300 \leq \sqrt{s} \leq 1300$ GeV, and also for different assignements of $\tilde{Q}^2$.

Keywords. Scale energy parameter; deep inelastic scattering; Z-production.

PACS Nos **12.60.Jv; 12.10.Dm; 98.80.Cq; 11.30.Hv**

1. Introduction

LHeC at CERN will provide us with the possibility to observe ep collisions with a maximal energy $E^{\max} = 60$ GeV of the electron and $E_p^{\max} = 7$ TeV of the proton [1]. This means that the maximal total energy of the ep system will be $\sqrt{s} \approx 1300$ GeV. The high luminosity, which is planned to be reached at LHeC will make possible to increase the number of produced Z bosons via ep collisions per year up to in three orders of magnitude, the number of the Z bosons produced at HERA. This means that the LHeC will be an excellent collider for the search of physics beyond the Standard Model (SM) [2–4]. However, this can be done only if the results of the SM are well known in all details and without ambiguities. Therefore, in this work we discuss the possibility of finding a prescription to calculate Z-production unambiguously at leading order using the deep inelastic process $e + p \rightarrow e + Z + X$, in the context of the SM and by making use of the parton model (PM) [5]. We perform our numerical calculations using the QCD improved PM [6,7]. According to the PM, the final step in the evaluation of $\mathrm{d}\sigma^{ep}$ consists of putting together the parton cross-sections $\mathrm{d}\sigma^{eq}$ and the parton distribution functions (PDFs) $f_q(x', \tilde{Q}^2)$ and to make a sum over all the possible partons. In the PDFs, x' is the fraction of momentum that the incoming quark carries of the colliding proton and the parameter $\tilde{Q}^2$ stands for a scale energy, usually identified as the momentum transfer square in the collisions. At the lowest order in α, in deep inelastic ep scattering ($e + p \rightarrow e + X$), the choice of $\tilde{Q}^2$ is unambiguous (see figure 1), whereas in

Published online: 19 July 2021

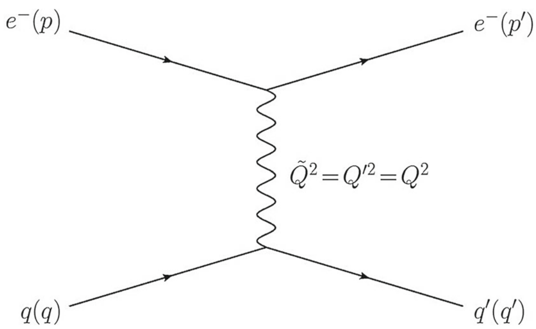

Figure 1. Feynman diagram which contributes at the lowest order in α, to the deep inelastic process $e + P \to e + X$, at the quark level.

the case of Z-production this is not so, as the momentum transfer square depends on the reaction mechanism, in other words, whether the Z is emitted at the lepton or at the quark line. Our aim in this paper is to show the dependence of the total cross-section on the prescription used for the scale parameter $\tilde{Q}^2$ and the way that one makes the convolution of the PDFs with the amplitude of the quark processes. At the lowest order in α, only two types of reaction mechanisms will contribute: Z-production at the lepton line and at the quark line at the parton level (see figure 2). Diagrams containing the exchange of Higgs bosons will be ignored because of the smallness of the Higgs-fermion coupling involved in the mentioned process. We present results for the total cross-section as a function of the total energy $\sqrt{s}$ of the system ep, in the range $300 \leq \sqrt{s} \leq 1300$ GeV. Taking $\sqrt{s} \approx 1300$ GeV, we find that the difference among the rates of the total cross-section for different prescriptions that we have adopted can reach up to 15%. This difference (15%) corresponds to $\approx 7 \times 10^3$ produced Z bosons, at the planned LHeC by taking an integrated luminosity of 0.1 ab^{-1}/ year [1]. We perform our numerical calculations using the parton distribution functions (PDFs) reported by Pumplin *et al* [8,9], and we use the CTEQ PDFs provided in the $n_f = 5$ active flavours scheme.

We end the introduction remarking the following. We study the production of the $e + Z + X$ final state in an electron–proton deep inelastic interaction, in the centre of mass energy range of 300 to 1300 GeV. The calculation is performed using leading-order expressions in the QCD-improved PM. The main purpose of the paper is to study in detail the dependence of the total cross-section rates on the value of the scale energy parameter $\tilde{Q}^2$ used in the quark distribution functions and to introduce a prescription to calculate Z via ep-DIS which take into account the kinematics of the process considered and keeping the criterion that the scale energy parameter should be identified with the momentum transfer square in the contributing diagrams.

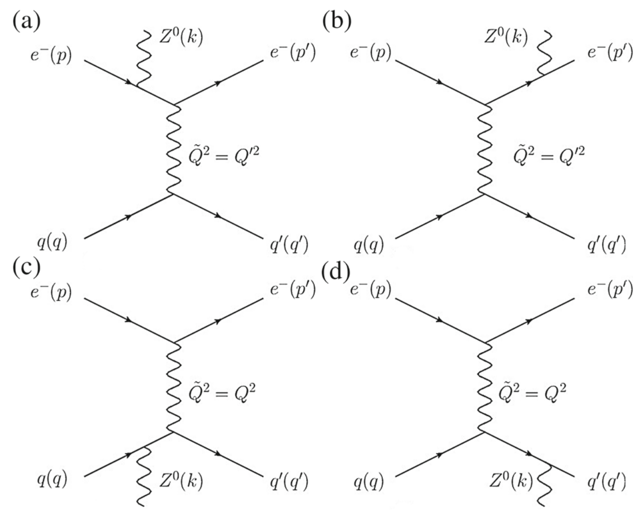

Figure 2. Feynman diagrams which contribute at the lowest order in α, to Z-production through the process $e + P \to e + Z + X$, at the quark level, emitted from the initial (**a**) and final (**b**) electron, the initial (**c**) and final (**d**) quark.

This paper is organised as follows: In §2 we describe briefly the calculation of the differential cross-section for inclusive Z-production via ep-collisions as a function of the identification of the scale energy parameter. In §3 we present and discuss our results for the total cross-section dependence on the identification of $\tilde{Q}^2$. Finally, our conclusions are summarised in §4.

2. Kinematics and formulae

Bohm and Rosado [10] presented the kinematics and the formulae required to calculate the cross-section of the production of a Z boson through the inclusive process

$$e + p \to e + Z + X. \tag{1}$$

We shall denote the four-momenta of these particles by $p,\ P_p,\ p'$ and k, respectively. X stands for *anything*. As usual, the following invariants are defined [11]:

$$\begin{aligned} s &= (p + P_p)^2, \\ Q^2 &= -(p - p')^2, \\ \nu &= P_p(p - p'), \\ s' &= (p + P_p - k)^2 \\ Q'^2 &= -(p - p' - k)^2, \\ \nu' &= P_p(p - p' - k), \\ W &= -(p + P_p - p' - k)^2, \end{aligned} \tag{2}$$

where W is the invariant mass. Now we define the dimensionless variables:

$$x = \frac{Q^2}{2\nu}, \quad y = \frac{2\nu}{s}, \quad \tau = \frac{s'}{s},$$
$$x' = \frac{Q'^2}{2\nu'}, \quad y' = \frac{2\nu'}{s}.$$

The physical region of these kinematical variables has been discussed in detail in ref. [10]. The quark cross-section is obtained from the invariant matrix element $\mathcal{M}(\mathrm{eq} \to \mathrm{eq}Z)$:

$$\mathrm{d}\sigma_{\mathrm{tot}}^{eq} = \frac{2(2\pi^{-5})}{\hat{s}} \frac{1}{4} \mid \mathcal{M}_{\mathrm{tot}}^{eq} \mid^2 \mathrm{d}\Gamma_3. \tag{3}$$

The 3-particle phase space $\mathrm{d}\Gamma_3$ can be expressed with the help of the dimensionless set of variables

$$\mathrm{d}\Gamma_3 = \frac{\pi s^3}{32}\, y\, \frac{\mathrm{d}x\, \mathrm{d}y\, \mathrm{d}y'\, \mathrm{d}\tau}{\sqrt{-\Delta_4\,(p,\; P_p,\; p',\; k)}} \tag{4}$$

with $\Delta_4(p, P_p, p', k)$ as the Jacobi determinant. The Feynman diagrams which contribute at the lowest order in α to $\mathcal{M}_{\mathrm{tot}}^{eq}$ are depicted in figure 2. As we have already said, the Z boson can be produced from the electron line (figures 2a and 2b) and the quark line (figures 2c and 2d). Then we write

$$\mathcal{M}_{\mathrm{tot}}^{eq} \;=\; \mathcal{M}_l^{eq} \;+\; \mathcal{M}_q^{eq}. \tag{5}$$

Explicit expressions for the quantities needed for the calculation of $|\mathcal{M}_{\mathrm{tot}}^{eq}|^2$ are presented in ref. [10]. Also, the sum over the polarisations of the produced boson is performed there. The final step in the evaluation of $\mathrm{d}\sigma^{ep}$ consists now of putting together the parton cross-sections $\mathrm{d}\sigma^{eq}$ and the parton distribution functions $f_q(x', \tilde{Q}^2)$. In contrast to deep inelastic ep scattering, the choice of $\tilde{Q}^2$ is not unambiguous in the case of Z-production, as the momentum transfer square to the proton depends on the reaction mechanism (in other words, whether the boson is emitted at the lepton or at the quark line). In order to make clear how strong the cross-section rates depend on the choice of the scale $\tilde{Q}^2$, we calculate in this work with the following simple prescriptions.

Prescription A

$$\mathrm{d}\sigma^{ep} = \sum_q \int \mathrm{d}x'\, f_q(x', M_Z^2) \cdot \mathrm{d}\sigma^{eq}. \tag{6}$$

Prescription B

$$\mathrm{d}\sigma^{ep} = \sum_q \int \mathrm{d}x'\, f_q(x',\; Q^2) \cdot \mathrm{d}\sigma^{eq}. \tag{7}$$

Prescription C

$$\mathrm{d}\sigma^{ep} = \sum_q \int \mathrm{d}x'\, f_q(x',\; Q'^2) \cdot \mathrm{d}\sigma^{eq}. \tag{8}$$

Prescription D

$$\mathrm{d}\sigma^{ep} = \sum_q \int \mathrm{d}x'\, f_q(x', (Q^2 + Q'^2)/2) \cdot \mathrm{d}\sigma^{eq}. \tag{9}$$

As it is known, the usual PM in which one makes the convolution of the PDFs and the amplitude square of the quark proceses is in the following form:

$$\mathrm{d}\sigma_{\mathrm{tot}}^{ep} = \sum_q \int \mathrm{d}x'\, f_q(x', \tilde{Q}^2) \cdot \mathrm{d}\sigma_{\mathrm{tot}}^{eq}, \tag{10}$$

where

$$\mathrm{d}\sigma_{\mathrm{tot}}^{eq} = \frac{2(2\pi^{-5})}{\hat{s}} \frac{1}{4} \, |\mathcal{M}_{\mathrm{tot}}^{eq}|^2 \, \mathrm{d}\Gamma_3 \tag{11}$$

and

$$|\mathcal{M}_{\mathrm{tot}}^{eq}|^2 = |\mathcal{M}_l^{eq} + \mathcal{M}_q^{eq}|^2 \,. \tag{12}$$

It has been shown in ref. [10], that one can find regions where the leptonic or the hadronic contribution is the dominating one. We can use the dependence of the cross-section as a function of the dimensionless variable y, which is related with the energy E' of the final electron and the angle θ that the momentum of the initial electron ($\vec{p}$) makes with the momentum of the final electron ($\vec{p}\,'$), define kinematical regions in which the leptonic ($0.9 \lesssim y < 1$) or the hadronic contribution ($0 < y \lesssim 0.5$) is the most important contribution to the total cross-section. Hence, in such regions for y will not be problems with the identification of the scale energy parameter $\tilde{Q}^2$. It will be equal to the momentum transfer square, associated with the most important contribution to the total cross- section: Q'^2 (Q^2), in case that the most important contribution to the total cross-section be the leptonic (hadronic) one.

3. Results

Now we present the numerical results of our calculations using the SM. We take $M_Z = 91.2$ GeV for the mass of the Z boson and $\sin^2\theta_W = 0.231$ for the electroweak mixing angle [12]. Besides the photon exchange, we have included in our computations the Z-exchange diagrams also. However, as expected, the dominant contributions to the total cross-section come from photon exchange and especially from leptonic initial state Z emission, i.e. the diagram depicted in figure 1a [10,13]. We give results for the case of unpolarised deep inelastic ep-scattering with a total energy

$\sqrt{s}$ in the range $300 \leq \sqrt{s} \leq 1300$ GeV. We take cuts of 4 GeV^2, 4 GeV^2 and 10 GeV^2 on Q^2, Q'^2 and the invariant mass W, respectively. These values are suited for the PDFs reported by Pumplin *et al* [8,9], and we use these CTEQ PDFs provided in a $n_f = 5$ active flavors scheme.

There exist already calculations for the total cross-section $\sigma(ep \to eZX)$, using different PDFs and cuts for the momentum transfer square of the exchanged boson and for the invariant mass W [10,13–16].

Our results for the total cross-section by taking the different prescriptions given in eqs (6)–(9), for $\sqrt{s} = 300$ GeV (HERA) and 1300 GeV (LHeC), are showed in table 1. We show in figure 3, the results of the total cross-section as a function of the total energy of the *ep* system $\sqrt{s}$. We can observe in this plot the evolution of the separation of the results of the cross-section rates depending on the prescription used. It is clear that differences in the rates of the total cross-section can reach up to 15% depending on the prescription used, for $\sqrt{s} \approx 1300$ GeV, which is the expected maximal total energy for the system *ep* to be reached at LHeC.

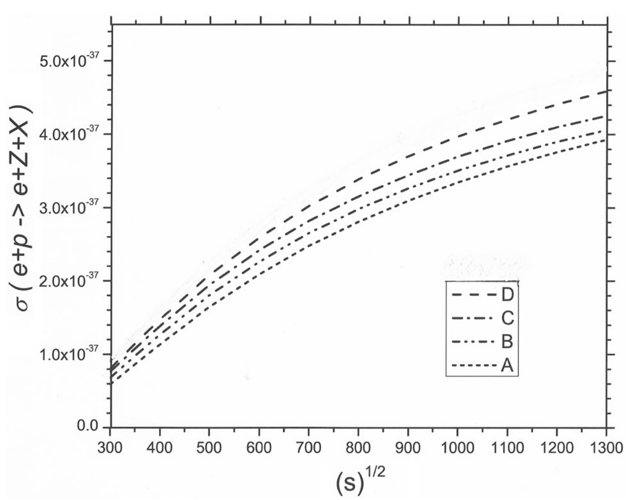

Figure 3. Cross-section rates for Z-production through the process $eP \to eZX$ with a total energy $\sqrt{s}$ in the range $300 \leq \sqrt{s} \leq 1300$ GeV, for different options that we have taken for doing the convolution of the PDFs with the amplitude of the quark processes (see eqs (17)–(21)).

We found the rates for $\sigma(ep \to eZX)$ as given in table 1. These values yield to a production of Z-bosons that we show in table 2, for the different prescriptions we have chosen for making the convolution of the amplitudes *eq* subprocesses and the PDFs. We can observe in table 1, that the differences in the rates of the total cross-section can reach up to 15% for different choices of $\tilde{Q}^2$ for the expected maximal total energy for the system *ep* to be reached at LHeC. This difference (15%) corresponds to a difference of $\approx 7 \times 10^3$ Z bosons, produced at the planned LHeC, by taking an integrated luminosity of 0.1 ab^{-1}/year [1].

The importance of finding a unique and unambiguous identification of $\tilde{Q}^2$ for Z-production via *ep* collisions at the LHeC energy is made clear with the following comment. The energy and luminosity reached at HERA predicted a production of 40–50 Z bosons per year, then the prediction that the ambiguity of $\tilde{Q}^2$ could give a difference of 11.8% between Prescriptions C and B (see table 1) was something that was hardly to be seen ($\sim$ 5–6). But the same difference (the difference between prescriptions C and B) at LHeC is 4.7%, which implies $\approx$470 Z bosons of difference between Prescriptions C and B. Even then the difference becomes smaller in percentage.

As far as we know, an analysis similar to ours has not been done, neither to the leading order nor to higher orders. It is clear to us that the inclusion of higher orders is non-trivial and will require further analysis for the proper treatment of the interplay between the weak and strong interactions [17]. We want to remark here that it

Table 1. Cross-section rates in 10^{-37} cm^2 for Z-production for $\sqrt{s} = 300$ GeV (HERA) and 1300 GeV (LHeC), for different prescriptions that we have taken for making the convolution of the PDFs with the amplitude of the quark processes.

$\sqrt{s}$ (GeV)	σ_A	σ_B	σ_C	σ_D
300	0.587	0.682	0.763	0.803
1300	3.934	4.064	4.257	4.590

Table 2. Number of Z bosons ($N_X \times 10^4$) that will be produced through $e + p \to e + Z + X$ taking $\sqrt{s} = 1300$ GeV (LHeC) and taking an integrated luminosity of 0.1 ab^{-1}/year, for different prescriptions that we have adopted for making the convolution of the PDFs with the amplitude of the quark processes.

$\sqrt{s}$ (GeV)	N_A	N_B	N_C	N_D
1300	3.93	4.06	4.26	4.59

is difficult to try to extend the analysis done in this work to higher orders, because higher orders will include box diagrams, which have no a unique momentum transfer square, but we can identify at least two in just one box diagram. Hence, it would be necessary to change the criterion that we used in our work: the identification of the scale energy $\tilde{Q}^2$ as the momentum transfer square from the diagram which is analysed.

4. Conclusions

In this work, we discuss Z-production for the kinematical conditions planned for the LHeC at CERN, where the expected centre of mass energy of $\sqrt{s} \approx 1300$ GeV is four times larger than the maximal total energy reached, $\sqrt{s} \approx 300$ GeV, at HERA. The luminosity planned to be reached at LHeC is at least two orders of magnitude larger than luminosity reached at HERA. Here we report the cross-section calculation for Z-production in *ep* deep inelastic scattering, at leading order, working in the framework of the SM and the PM. We discuss the dependence of the cross-section rates on the scale energy parameter $\tilde{Q}^2$. Although we cannot identify $\tilde{Q}^2$ unambiguously, our results show that the kinematics of the process allows the identification of a scale at leading order. We found that it is possible to find a reasonable choice of the scale energy parameter, based on the kinematics of the process at the leading order. Our paper may be seen as a first step to look for a possible unique identification of $\tilde{Q}^2$ and to make clear the necessity of finding a unique identification if we look for doing precision tests at LHeC and future *ep*-colliders.

In summary, according to our results, for the Z-production in deep inelastic scattering, it is possible to identify two scales associated with the momentum transfer square. We have two momenta transfer square: Q^2 and Q'^2. In addition, we have demonstrated that although this fact is not relevant at HERA energies, it has a strong influence at LHeC energies. A detailed comparison of the phenomenological predictions and the experimental results is required to elucidate details of how the weak and strong interactions are correlated and fix scales relevant for the description of the process under consideration. Thus, in this work we have introduced a practical and simple scale choice, based on the kinematics of the considered process at the leading order.

A detailed comparison of the phenomenological predictions and the experimental results is required to elucidate details of how the weak and strong interactions are correlated and fix scales relevant for the description of the process under consideration. We can use the dependence of the cross-section as a function of the dimensionless variable y which is related to the energy of the outgoing electron E' and the angle θ that the momentum of the incoming electron ($\vec{p}$) makes with the momentum of the final electron ($\vec{p}'$), to define kinematical regions in which the leptonic ($0.9 \lesssim y < 1$) or the hadronic contribution ($0 < y \lesssim 0.5$) is the most important contribution to the total cross-section [10]. So we could define the $\tilde{Q}^2$ as Q'^2 (Q^2) in case that the most important contribution to the total cross-section be the leptonic (hadronic) one.

Acknowledgements

This work was supported in part by the Consejo Nacional de Ciencia y Tecnología (CONACyT) and Sistema Nacional de Investigadores (SNI) de México.

References

[1] LHeC Study Group: J L Abelleira Fernandez *et al*, *J. Phys. G* **39**, 075001 (2012)
[2] S L Glashow, *Nucl. Phys.* **22**, 579 (1961); S Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); Salam, *Proc. 8th NOBEL Symposium* edited by N Svartholm (Almqvist and Wiksell, Stockholm, 1968) p. 367
[3] D Indumathi, *Pramana – J. Phys* **54**, 533 (2000)
[4] D Majumdar, A D Banik and A Biswas, *Pramana – J. Phys.* **89**: 67 (2017)
[5] J D Bjorken and E A Paschos, *Phys. Rev.* **185**, 1975 (1969)
[6] R P Feynman, *Photon-hadron interactions* (Benjamin, Reading, 1972)
[7] R P Feynman, *High Energy Collisions: Third International Conference at Stony Brook* (Gordon & Breach, 1969), p. 237
[8] J Pumplin, D R Stump, J Huston, H L Lai, P M Nadolsky and W K Tung, *JHEP* **0207**, 012 (2002)
[9] D Stump, J Huston, J Pumplin, W K Tung, H L Lai, S Kuhlmann and J F Owens, *JHEP* **0310**, 046 (2003)
[10] M Bohm and A Rosado, *Z. Phys. C* **34**, 117 (1987)
[11] E Byckling and Kajantie, *Particle kinematics* (Wiley, New York, 1972)
[12] Particle Data Group: C Patrignani *et al*, *Chin. Phys. C* **40**, 100001 (2016)
[13] C H Llewellyn-Smith and B H Wiik, Preprint: DESY-77-038, unpublished, https://doi.org/10.3204/PUBDB-2017-12235
[14] P Salati and J C Wallet, *Z. Phys. C* **16**, 155 (1982)
[15] G Altarelli, G Martinelli, B Mele and R Ruckl, *Nucl. Phys. B* **262**, 204 (1985)
[16] U Baur, J A M Vermaseren and D Zeppenfeld, *Nucl. Phys. B* **375**, 3 (1992)
[17] D K Choudhury and R C Goswami, *Pramana – J. Phys.* **33**, 555 (1989)