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Quantum phase fluctuations of coherent light coupled to a direct band-gap semiconductor

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Abstract. The coherent light interacting with a direct band-gap semiconductor is modelled as two coupled harmonic oscillators with Kerr-type nonlinearity in one of the oscillators. By considering the nonlinearity in the exciton mode, we derive approximate analytical solutions of the coupled differential equations involving two bosonic modes. For both photon and exciton modes which are in initial coherent states, we calculate the phase fluctuation parameters due to Carruthers and Nieto. We have used the Pegg–Barnett formalism for defining the useful phase operators. We report the increase and decrease of phase fluctuation parameters of input radiation field compared to its initial values. Effects of vacuum field on the phase fluctuation parameters are found distinctly different from their counterparts without vacuum. The complete analytical approach of the present investigation is substantiated by the numerical calculations based on QuTip.

Keywords. Direct band-gap semiconductor; quantum phase fluctuations; Pegg–Barnett formalism.

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1. Introduction

One of the ten most beautiful experiments in physics is the Young double-slit experiment [1]. The interference pattern obtained from the double-slit experiment clearly demonstrates the wave nature of light. The corresponding intensity pattern could only be explained with the concept of phases. Of course, the phase has no meaning in the absolute sense. It is the phase difference which we talk about. The diffraction of light is also an outcome of phases. In addition to these classical optical phenomena, we also find several nonlinear optical phenomena where the concept of phases are important. These include higher harmonic generation [2], holography and optical phase conjugation [3]. Apart from these classical phenomena, the concept of phases is also widely used for explaining non-classical phenomena. These include Hanle resonances [4], laser without population inversion [5], coherent population trapping [6], dark resonances [7], absorptionless dispersion [8], Aharonov–Bohm effects etc. [9]. It is also interesting to note that the concept of phase is useful in realising the quantum technology-based quantum radar [10,11]. The phase in classical optics is easily introduced with the help of polar decomposition of complex field amplitude [12–14]. On the other hand, to respect quantum formalism, it is essential to introduce a suitable Hermitian phase operator. With the analogy of classical introduction of phase, Dirac introduced the polar decomposed phase operator [15]. Unfortunately, the exercise of Dirac did not materialise because the Hermitian nature of the phase operator was not achieved and the uncertainty relation was not respected as well. We find several attempts to define meaningful phase operators. These include Susskind–Glowgower (SG) formalism [16] and Pegg–Barnett (PB) [17–20] formalism. An interesting investigation by Vaccaro and Ben-Aryeh [21] concluded that the antinormal ordering of the SG phase operators in infinite-dimensional Hilbert space is formally justified from the PB in truncated Hilbert space formalism. Before we go further, we would like to mention that we are talking about the dynamical phases only, not the geometrical one. In the context of quantum phases, we normally observe two different approaches towards the problem. The first one deals with the phase properties [22–30] and the second one involves the phase fluctuations of the input radiation fields [31–37]. The phase fluctuation parameters due to Carruthers and Nieto (CN)

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[38] are widely used to investigate the phase fluctuations of the input electromagnetic field. In addition to these theoretical investigation on the quantum phase, we also find lot of experiments where the investigators have reported some interesting results involving quantum phase measurements [39–41].

Of late, we find lot of interests on the subject of cavity quantum electrodynamics and hence cavity optomechanics because these concepts are of immense importance for realising fundamental laws of physics. These subjects employ the fundamental interaction between the matter and quantised electromagnetic field. Now, the interaction of quantised electromagnetic field and the semiconductor is of tremendous importance from the condensed matter physicist points of view [42–49]. These investigations takes care of the squeezing and higher-order squeezing of electromagnetic field coupled to the semiconductors. However, we do not find any investigation on the quantum phase fluctuation or the quantum phase properties of the radiation field coupled to the semiconducting medium. Considering the immense importance of the interaction between electromagnetic field and semiconductor, we investigate the phase fluctuations of the coherent light coupled to the semiconductor. In the present investigation, we follow the BP formalism for the introduction of useful phase operators. Finally, with the help of the phase fluctuation parameters of CN [38], the quantum phase fluctuation of the input coherent light coupled to the semiconductor is investigated.

2. Analytical formulation of the problem

In a seminal paper, the idea of excitons is first coined by Frankel [50] for investigating the transformation of light into heat in solids. An electron in the conduction band forms a bound state with a hole in the valence band for an insulating medium with lowest energy excited states. These bound states are termed as excitons [51]. Let us consider that a monochromatic electromagnetic field of energy ω_2 (in the units $\hbar = 1$) is coupled through a two-band semiconductor with direct band gap, which is highly excited and allows interband dipole transition. A diagrammatic representation of the interaction is exhibited in figure 1. Interestingly, the corresponding interaction could be represented by the Feynmann diagram [44]. Let the band-gap energy of the semiconductor be E_g. Then the interaction of this semiconductor with the monochromatic light with $\omega_2 < E_g$ is energetically unable to lift up the electron from the valence band to the conduction band to create free electrons and free holes. Instead, due to the Coulomb attractive interaction, bound states, each is composed of an electron and a hole, can be formed as the energy of such an electron–hole pair lies within the band gap just below the bottom of the conduction band. Such an interacting electron–hole pair is regarded as a kind of quasiparticle referred to in the literature as an exciton. Note that, although both the electron and the hole considered separately are fermions in nature, the exciton itself (i.e., the bound electron–hole pair as a whole) behaves like a boson [44]. Hence, the exciton follows the bosonic quantisation rule, and consequently the Hamiltonian of the semiconductor coupled to the monochromatic electromagnetic field can be expressed in the units $\hbar = 1$ as [45,46]

$$H = \omega_1 a^\dagger a + \omega_2 c^\dagger c - g\left(a^\dagger c + c^\dagger a\right) + \chi a^{\dagger 2} a^2, \quad (1)$$

where a ($a^\dagger$) and c ($c^\dagger$) are annihilation (creation) operators for the exciton and field mode, respectively. In the derivation of Hamiltonian (1), the effects of environment are neglected altogether [52,53]. This is to mention that the effect of environment causes decays of the quantum coherence. There are some exceptions where the noise can indeed enhance the transport efficiency [53]. The effects of environment and/or damping on the oscillator corresponding to Hamiltonian (1) may be included through the master equation. The master equation after incorporating the effects of environment, are solved by using the superoperator approach [52]. However, in the present investigation we neglect the effects of damping if any. Now, the coupling constant g takes care of the coupling between the field and the exciton modes. The nonlinear interaction between exciton–exciton pairs is contributed by the parameter χ. Interestingly, the Hamiltonian reduces to the Hamiltonian of two coupled harmonic oscillators corresponding to photon and exciton modes. In the absence of coupling (i.e $g = 0$) and nonlinearity (i.e. $\chi = 0$), Hamiltonian (1) reduces to the Hamiltonian of two independent oscillators with frequency ω_1 and ω_2. It is possible to include the lowest-order nonlinearity in field mode proportional to $c^{\dagger 2} c^2$. It will certainly complicate the problem in a large way. One more possibility is to include the exciton-assisted photon–exciton transition [44]. It is because the exciton-assisted photon–exciton transition can easily be included in our system by adding a term proportional to $\eta(a^{\dagger 2} ac + a^{\dagger 2} ac^\dagger a^\dagger a^2)$ in Hamiltonian (1). This is to mention that a few non-classical features associated with this system are already taken care off. For example, by using the so-called polariton representation of photon and exciton, the amount of squeezing of light as a function of exciton–exciton interaction is investigated [45]. The so-called secular approximation is extensively used in the calculation. In a different context, an approximate closed form analytical solution of this system

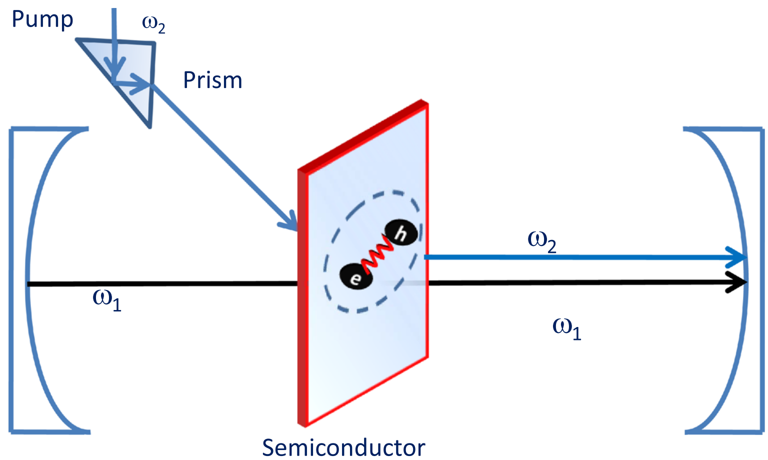

Figure 1. Block diagram of photon–exciton interaction in a cavity. The semiconductor crystal is put in a cavity filled with radiation.

for weak nonlinearity (i.e., $\chi \ll 1$) is also investigated [54]. Further, for strong nonlinearity, a numerical investigation was performed and the collapse and revival phenomena in the periodic exchange of energy between atomic oscillator and field were observed. In this numerical investigation, the relevant bosonic operators were replaced by their eigenvalues and hence c-numbered differential equations were formed instead of operator differential equations. This reduced the computational difficulty and led to an exact result at the cost of the phase information which is of great significance in the study of squeezing and entanglement.

The purpose of the present investigation is to discuss phase fluctuations of the coherent light coupled to a semiconductor corresponding to Hamiltonian (1). In order to do so, we construct the Heisenberg's equations of motion involving operators a and c. These are given by

$$\dot{a} = -i\left(\omega_1 a - gc + 2\chi a^\dagger a^2\right),$$
$$\dot{c} = -i\left(\omega_2 c - ga\right). \tag{2}$$

The overdots in eqs (2) denotes the derivative with respect to time. In order to investigate the phase fluctuations and other non-classical properties of the radiation field coupled to the semiconductor, it is essential to explore the solutions of the differential equations involving the bosonic operators a and c involving eqs (2). Unfortunately, eqs (2) are not only nonlinear but also coupled to each other. Interestingly, for $\chi = 0$, the equations are decoupled instantaneously and are solvable in closed analytical forms. The corresponding solutions are given by

$$a(t) = \mathrm{e}^{-i\omega t}\left(\left\{\cos kt + \frac{i\omega}{k}\sin kt\right\} a(0) + \frac{\sin kt}{k}\dot{a}(0)\right),$$

$$c(t) = \mathrm{e}^{-i\omega t}\left(\left\{\cos kt + \frac{i\omega}{k}\sin kt\right\} c(0) + \frac{\sin kt}{k}\dot{c}(0)\right), \tag{3}$$

where

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

and

$$k = \frac{\sqrt{(\omega_1 - \omega_2)^2 + 4g^2}}{2}.$$

For $\chi \neq 0$, eqs (2) are to be solved either numerically or by some approximate analytical method. The non-commuting nature of the bosonic operators is on the way for getting the analytical solutions of the nonlinear differential equations. Still, it is possible to use the QuTip [55,56] method for getting useful numerical solutions of the differential equations. Secondly, it is possible to explore the analytical solutions under restricted situation. In this article, we are intended to explore both the numerical and analytical methods. As a matter of fact, we use QuTip as a numerical tool for investigating the phase problems and hence the validity of the analytical solution. Before we go for the solutions of the coupled differential equations (2), we would like to mention that it is possible to obtain the short time approximated solution readily. The essence of the short time method is to get good analytical solutions for the coupled operator differential equations at the cost of short time validity of the same [57,58]. Certainly, a first-hand information about the system is available through the less complicated short-time approximation. The important drawback of the short-time solution is that they are secular in nature. Of course, the secular nature removed the condition for small t (i.e. short time). In the present investigation, however, we provide a method better than those short-time approximations. The method was devised by our group sometime back and is widely used for several physical systems involving coupled operator differential equations. The essence of this method is to propose a solution in terms of unknown parameters. These unknown parameters finally give rise to coupled differential equations involving c-numbers. These c-numbered differential equations are solved to obtain unknown parameters and hence the solutions of the operator differential equations. It is to be noted that the proposed solutions are extremely important and are based on mathematical and physical bases. Nevertheless, the solutions are valid beyond short time. Most importantly, the solutions are free from the so-called secular term which is inherent in a perturbative solution. However, it is necessary to maintain the condition of perturbative solutions, $gt < 1$ and $\chi t < 1$. We are not going through the details of the mathematical reasoning behind the proposed solutions. These are available in our earlier publications [59–63]. Therefore, we try the solutions in the following form [63]

$$\begin{aligned} a(t) &= f_1 a(0) + f_2 c(0) + f_3 a^\dagger(0) a^2(0) + f_4 a(0) \\ &\quad + f_5 a^\dagger(0) a(0) c(0) + f_6 c^\dagger(0) a^2(0) \\ &\quad + f_7 a^\dagger(0) a^2(0) + f_8 a^{\dagger 2}(0) a^3(0), \\ c(t) &= h_1 c(0) + h_2 a(0) + h_3 c(0) + h_4 a^\dagger(0) a^2(0), \end{aligned} \tag{4}$$

where f_is and h_is are unknown parameters. Now, we give a few physical reasoning for the proposed solutions (4). At the very outset, $f_1(0) = 1 = h_1(0)$ serve as the initial conditions. In the absence of interaction, f_1 and h_1 correspond to the free evolution of the operators a and c respectively. It would be relevant to discuss the physical significance of the annihilation of exciton mode a corresponding to the first one of eqs (2). The first one of the equation shows that the temporal evolution of the annihilation of exciton mode could be accounted by several processes. Apart from the free evolution contributed by f_1, the annihilation of exciton is contributed by the field mode and is contributed by f_2. It corroborates the fact that the photon and exciton are coupled to each other. Therefore, the annihilation of exciton could be accounted for by the annihilation of photon. Now, have a look at the term f_3 initiated by the nonlinear term χ in the exciton mode. The term f_3 is responsible for simultaneous creation and destruction of one and two excitons respectively. As a result, the net destruction of one exciton through the nonlinear processes is contributed by f_3. In a similar manner, the appearance of the remaining f_is and h_is could also be explained. The proposed solutions (4) do not impose the short-time restriction. However, in the choice of solutions (4), we have neglected the terms beyond third order in the dimensionless interaction time (gt and/or χt). This is in sharp contrast with those solutions under short-time approximations [57,58]. In order to obtain the closed form solutions of the evolution of operators a and c, we have to evaluate the unknown parameters f_is and h_is. After going through some algebraic steps, we obtain the coefficients f_i and h_i. These are [63]

$$\begin{aligned} f_1 &= \mathrm{e}^{-i\omega_1 t}, \\ f_2 &= \frac{g\mathrm{e}^{-i\omega_1 t}}{\Delta\omega}(-1 + \mathrm{e}^{i\Delta\omega t}), \\ f_3 &= -2i\chi t \mathrm{e}^{-i\omega_1 t}, \\ f_4 &= \frac{g^2\mathrm{e}^{-i\omega_1 t}}{(\Delta\omega)^2}(-1 + \mathrm{e}^{i\Delta\omega t} - it\Delta\omega), \\ f_5 &= \frac{4g\chi\mathrm{e}^{-i\omega_1 t}}{(\Delta\omega)^2}(1 - \mathrm{e}^{i\Delta\omega t} + it\Delta\omega), \\ f_6 &= \frac{2g\chi\mathrm{e}^{-i\omega_1 t}}{(\Delta\omega)^2}(-1 + \mathrm{e}^{-i\Delta\omega t} + it\Delta\omega), \\ f_7 &= f_8 = -2\chi^2 t^2 \mathrm{e}^{-i\omega_1 t} \end{aligned} \tag{5}$$

and

$$\begin{aligned} h_1 &= \mathrm{e}^{-i\omega_2 t}, \\ h_2 &= \frac{g\mathrm{e}^{-i\omega_1 t}}{\Delta\omega}(-1 + \mathrm{e}^{i\Delta\omega t}) = f_2, \\ h_3 &= \frac{g^2\mathrm{e}^{-i\omega_2 t}}{(\Delta\omega)^2}(-1 + \mathrm{e}^{-i\Delta\omega t} + i\Delta\omega t), \\ h_4 &= \frac{2g\chi\mathrm{e}^{-i\omega_1 t}}{(\Delta\omega)^2}(1 - \mathrm{e}^{i\Delta\omega t} + i\Delta\omega t), \end{aligned} \tag{6}$$

where $\Delta\omega = \omega_1 - \omega_2$ is the difference between the energy of the exciton and photon. The detailed algebraic calculations is available in our recent publication [63]. Interestingly, the absence of exciton–exciton interaction causes $f_3 = f_5 = f_6 = f_7 = f_8 = h_4 = 0$. The consistency of the obtained solution is further checked by confirming that the obtained solution satisfies equal time commutation relation $[a(t), a^\dagger(t)] = [c(t), c^\dagger(t)] = 1$ and $[a(t), c^\dagger(t)] = [a(t), c(t)] = 0$. The obtained solution may now be used to calculate the expectation values of various operators that are required for the present investigation on the non-classical behaviour of the exciton and field modes considering that the initial state is a composite coherent state

$$|\psi(0)\rangle = |\alpha\rangle|\beta\rangle. \tag{7}$$

This composite coherent state can be viewed as a product of two coherent states $|\alpha\rangle$ and $|\beta\rangle$, such that $a\,(0)\,|\alpha\rangle = \alpha|\alpha\rangle$ and $c\,(0)\,|\beta\rangle = \beta|\beta\rangle$. Therefore, it is clear that the parameters α and β, in general, are complex. Without loss of generality, let us consider that α is real. The parameter associated with the field mode $\beta = |\beta|\,\mathrm{e}^{-i\phi}$ is associated with phase angle ϕ. Therefore, the phase angle ϕ may be regarded as the phase of the input radiation field which is controllable externally. Now, $|\alpha|^2$ and $|\beta|^2$ are the number of excitons and photons in the composite bosonic fields. Now, we are in a position to calculate useful parameters in terms of the initial composite states (7). For example, for this initial state, we can obtain the expectation values of the number operators $\langle N_a(t)\rangle = \langle a^\dagger(t)a(t)\rangle$ and $\langle N_c(t)\rangle = \langle c^\dagger(t)c(t)\rangle$ as follows:

$$\langle N_a(t)\rangle = \bar{N}_a = |\alpha|^2 + |f_2|^2(|\beta|^2 - |\alpha|^2) + \{f_1^* f_2 \alpha^* \beta - h_1 h_4^* |\alpha|^2 \alpha\beta^* + \text{c.c.}\} \tag{8}$$

and

$$\langle N_c(t)\rangle = \bar{N}_c = |\beta|^2 + |h_2|^2(|\alpha|^2 - |\beta|^2) + \{(h_1^* h_2 \beta^* \alpha + h_1^* h_4 |\alpha|^2 \alpha\beta^*) + \text{c.c.}\}, \tag{9}$$

where c.c. stands for the complex conjugate. The exponential phase operators under BP formulations denoted by $E(E^\dagger)$ are as follows [17–20]:

$$E_i = \left(\bar{N}_i + \frac{1}{2}\right)^{-\frac{1}{2}} i(t),$$
$$E_i^\dagger = \left(\bar{N}_i + \frac{1}{2}\right)^{-\frac{1}{2}} i^\dagger(t), \tag{10}$$

where $i = a, c$ are the exciton and photon modes respectively. The corresponding cosine and sine of the phase operators for the ith mode are given by

$$C_i = \frac{1}{2}(E_i + E_i^\dagger)$$
$$S_i = \frac{1}{2i}(E_i - E_i^\dagger). \tag{11}$$

Therefore, the square of the average values of the operators C and S for both modes are given by

$$\begin{bmatrix} \langle C_a\rangle^2 \\ \langle S_a\rangle^2 \end{bmatrix} = \frac{1}{4\left(\bar{N}_a + \frac{1}{2}\right)} \big[2[|\alpha|^2 + (|\beta|^2 - |\alpha|^2)|f_2|^2 - |f_3|^2|\alpha|^4] + (2 f_1 f_2^* \alpha\beta^* + 2\left(f_1^* f_5 + f_1 f_6^* + f_2 f_3^*\right) |\alpha|^2 \alpha^*\beta + \text{c.c.}) \pm \big\{ f_1^2 + 2 f_1 f_4 \alpha^2 + 2 f_1 f_2 \alpha\beta + 2(f_1 f_3 + f_1 f_7)|\alpha|^2\alpha^2 \pm 2(f_1 f_5 + f_2 f_3)|\alpha|^2\alpha\beta + 2 f_1 f_6 \alpha^3\beta^* + f_2^2\beta^2 \pm \left(2 f_1 f_8 + f_3^2\right)|\alpha|^4\alpha^2 + \text{c.c.}\big\}\big] \tag{12}$$

and

$$\begin{bmatrix} \langle C_c\rangle^2 \\ \langle S_c\rangle^2 \end{bmatrix} = \frac{1}{4\left(\bar{N}_c + \frac{1}{2}\right)} \big[2(|\beta|^2 + (|\alpha|^2 - |\beta|^2)|h_2|^2) + \{2(h_1 h_4^* |\alpha|^2 + h_1 h_2^*)\alpha^*\beta + \text{c.c.}\} \pm \big[2(h_1 h_3 \beta^2 + h_1 h_4 |\alpha|^2 \alpha\beta + h_1 h_2 \alpha\beta) \pm (h_1^2\beta^2 + h_2^2\alpha^2) + \text{c.c.}\big]. \tag{13}$$

Now we calculate the second-order variance of the sine and cosine operators $\left(\Delta C_j\right)^2 = \langle C_j^2\rangle - \langle C_j\rangle^2$ and $\left(\Delta S_j\right)^2 = \langle S_j^2\rangle - \langle S_j\rangle^2$, where subscript j stands for the corresponding mode. Therefore, we have

$$\begin{bmatrix} (\Delta C_a)^2 \\ (\Delta S_a)^2 \end{bmatrix} = \frac{1}{4\left(\bar{N}_a + \frac{1}{2}\right)} \big[1 + 2|f_3|^2|\alpha|^4 \pm \{(f_1 f_3 + f_1 f_7)\alpha^2 + f_1 f_5 \alpha\beta + 2 f_1 f_8 |\alpha|^2 \alpha^2 + \text{c.c.}\}\big] \tag{14}$$

and

$$\begin{bmatrix} (\Delta C_c)^2 \\ (\Delta S_c)^2 \end{bmatrix} = \frac{1}{4\left(\bar{N}_c + \frac{1}{2}\right)}. \tag{15}$$

Now, we follow the prescription of CN [38] to define useful parameters for investigating the phase fluctuations of the input coherent light coupled to the semiconductor medium. These parameters are

$$U^i(\theta, gt) = (\Delta N_i)^2 \frac{[(\Delta S_i)^2 + (\Delta C_i)^2]}{[\langle S_i\rangle^2 + \langle C_i\rangle^2]} \tag{16}$$

$$S^i = (\Delta N_i)^2 (\Delta S_i)^2 \tag{17}$$

$$Q^i = \frac{S^i}{\langle C_i\rangle^2}. \tag{18}$$

The superscript i is used for the phase fluctuation parameters for the ith mode. The superscript is used to differentiate between the phase fluctuation parameter and the usual sine operator. In order to calculate the parameters U^i, S^i, Q^i, we need to calculate the second-order variances of the number operators for a and c modes. It is clear that, we are to calculate the second-order variances of the number operators corresponding to the field and exciton modes. Therefore, we have

$$(\Delta N_a)^2 = |\alpha|^2 + |f_2|^2(|\beta|^2 - |\alpha|^2) + \big[f_1^* f_2 \alpha^*\beta + \left(3 f_1^* f_5 + 3 f_1 f_6^* + 5 f_2 f_3^* + 2 f_1^{*2} f_2 f_3\right)|\alpha|^2 \alpha^*\beta + \text{c.c.}\big] \tag{19}$$

and

$$(\Delta N_c)^2 = |\beta|^2 + |h_2|^2(|\alpha|^2 - |\beta|^2) + [(h_1^* h_4 |\alpha|^2 \alpha\beta^* + h_1^* h_2 \alpha\beta^*) + \text{c.c.}]. \tag{20}$$

Now, we are ready to calculate the phase fluctuation parameters involving eqs (16)–(18).

(a) (b)

(c)

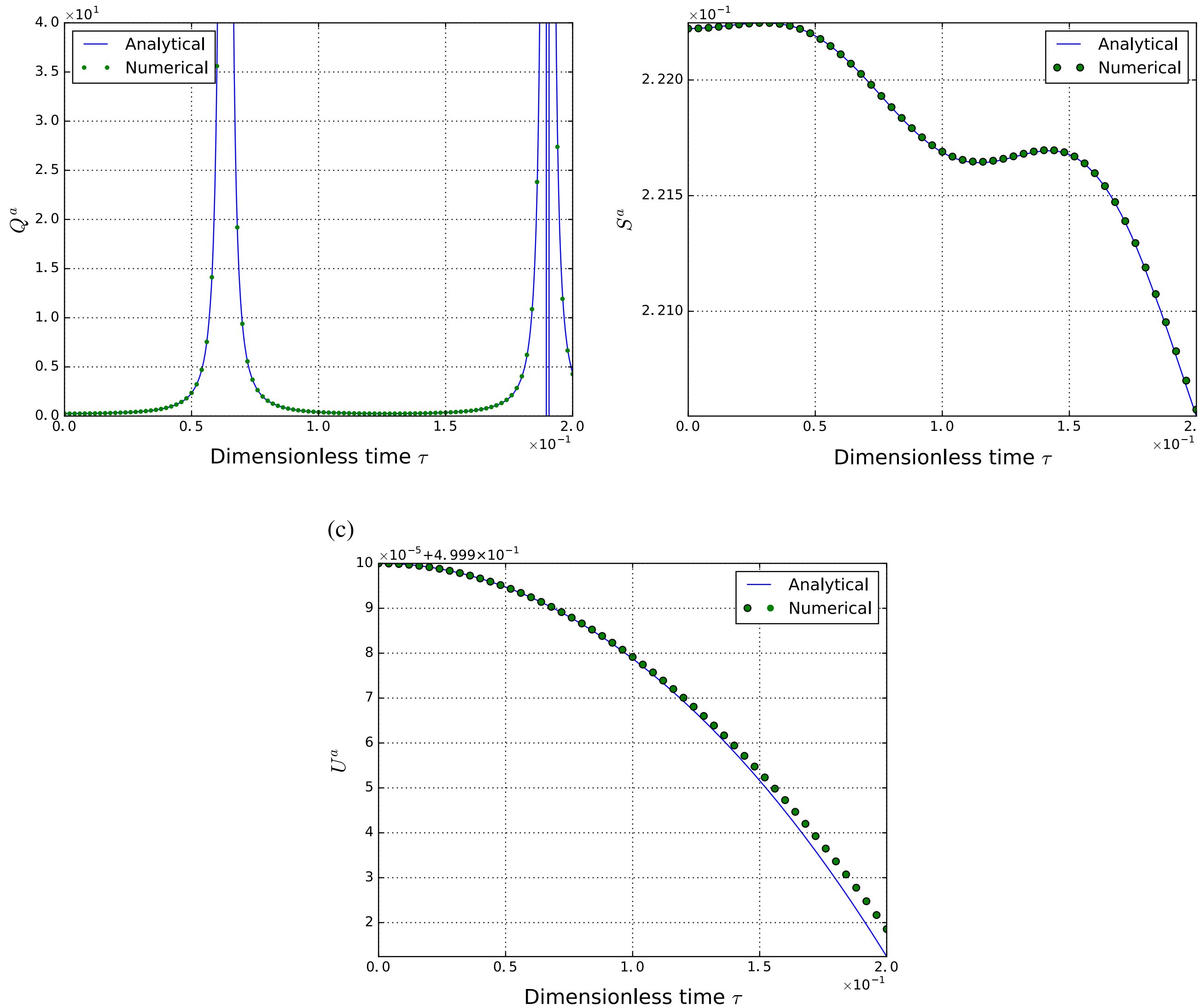

Figure 2. Plots of Q^a, S^a and U^a for the exciton mode a as a function of dimensionless interaction time $\tau = gt$ in (**a**), (**b**) and (**c**) respectively by using $(\omega_1/g) = 25.277$, $(\omega_2/g) = 24.013$, $(\chi/g) = 5.304 \times 10^{-4}$, $\alpha = 2.0$ and $\beta = 1.0$

3. Symbolic calculations of phase fluctuation parameters

In order to give some flavour of the analytical expressions for the phase fluctuation parameters of coherent light coupled to a semiconductor, we provide a few symbolic calculations by using the prescription of CN [38]. Throughout the symbolic calculation, we assume that $g = \chi$. The dimensionless interaction time is to be regarded as $gt = \tau$. Throughout the calculation the value of dimensionless interaction constant τ is to be chosen small compared to unity (i.e. $\tau < 1$). The phase of the input coherent radiation field $\phi = 0$. Therefore, the entire investigation is based on the conditions that α and β are real. The parameters Q^a, S^a and U^a are plotted against the dimensionless interaction time $gt = \tau$ in figures 2a, 2b and 2c respectively. The superscript a corresponds to the exciton mode. Interestingly, the phase fluctuation parameter Q^a remains unchanged compared to the initial ($t = 0$) counterpart except for $gt \approx 0.06$ and ≈ 0.19. The steep increase (decrease) of the parameter Q^a is exhibited in figure 2a for $gt \approx 0.06$ ($gt \approx 0.19$). The singular-like behaviour in these two occasions shown in figure 2a are attributed to the denominator $\langle C_i \rangle^2$. On the other hand, the parameters S^a and U^a are continually decreased compared to their initial values (i.e. at $t = 0$) with the increase of dimensionless interaction time gt and are shown in figures 2b and 2c respectively. Interestingly, the perfect coincidence of our analytical results with those of the

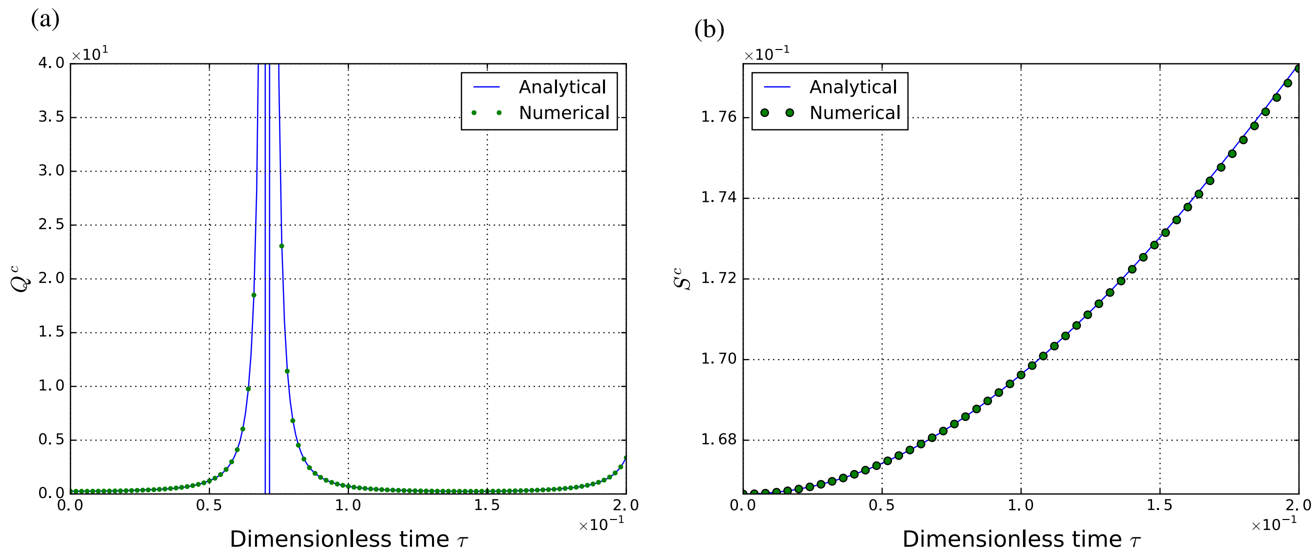

Figure 3. Plots of Q^c and S^c for the field mode c as a function of dimensionless interaction time $\tau = gt$ in figures 3a and 3b respectively by using $(\omega_1/g) = 25.277$, $(\omega_2/g) = 24.013$, $(\chi/g) = 5.304 \times 10^{-4}$, $\alpha = 2.0$ and $\beta = 1.0$.

numerically computed results are shown in figures 2a and 2b. However, a small deviation of the parameter U^a from the numerical results is found in figure 1c. The deviation becomes significant as the dimensionless interaction constant gt is increased. These are quite consistent because the analytical results are valid for small dimensionless interaction time (see figure 2c). The parameter Q^c for the exciton mode behaves almost in an identical way compared to its field mode counterpart Q^a. However, the parameter S^c increases compared to its $t = 0$ counterpart when the dimensionless interaction time is increased. These results are in sharp contrast with its field counterpart S^a. The initial values of the phase fluctuation parameters for the exciton mode are

$$U_0^a = \frac{1}{2}, \quad S_0^a = \frac{|\alpha|^2}{4\,|\alpha|^2 + 2}$$

and

$$Q_0^a = \frac{1}{4\cos^2\omega_1 t}.$$

The subscripts stand for the initial values $\tau = 0$. In a similar manner, it follows that

$$U_0^c = \frac{1}{2}, \quad S_0^c = \frac{|\beta|^2}{4\,|\beta|^2 + 2} \quad \text{and} \quad Q_0^c = \frac{1}{4\cos^2\omega_2 t}$$

are the initial values of the phase fluctuation parameters for the photon mode c. If the exciton mode is in vacuum state (i.e .$\alpha = 0$), the corresponding phase fluctuation parameters are

$$U^a = \frac{1}{2}, \quad S^a = \frac{|f_2|^2\,|\beta|^2}{4\,|f_2|^2\,|\beta|^2 + 2}$$

and

$$Q^a = \frac{|f_2|^2}{2\,|f_2|^2 + f_2^2 + f_2^{*2}}$$

respectively. These results are shown in figures 4a and 4b. Both Q^a and S^a in figures 4a and 4b exhibit distinctly different behaviour compared to its counterparts for $\alpha \neq 0$ in figures 2a and 2b respectively. Interestingly, for $\alpha = 0$, U^a remains unchanged from its $t = 0$ counterpart (not shown). For $\alpha \neq 0$, these results are in sharp contrast with U^a shown in figure 2c. Now, for $\beta = 0$, the phase fluctuation parameters for the exciton mode are

$$U^c = \frac{1}{2}, \quad S^c = \frac{|h_2|^2\,|\alpha|^2}{4\,|h_2|^2\,|\alpha|^2 + 2}$$

and

$$Q_c = \frac{|h_2|^2}{2\,|h_2|^2 + h_2^2 + h_2^{*2}}$$

respectively. Therefore, it is clear that for $\beta = 0$, the qualitative nature of Q^c, S^c and U^c are identical with the $\alpha = 0$ counterparts (not shown).

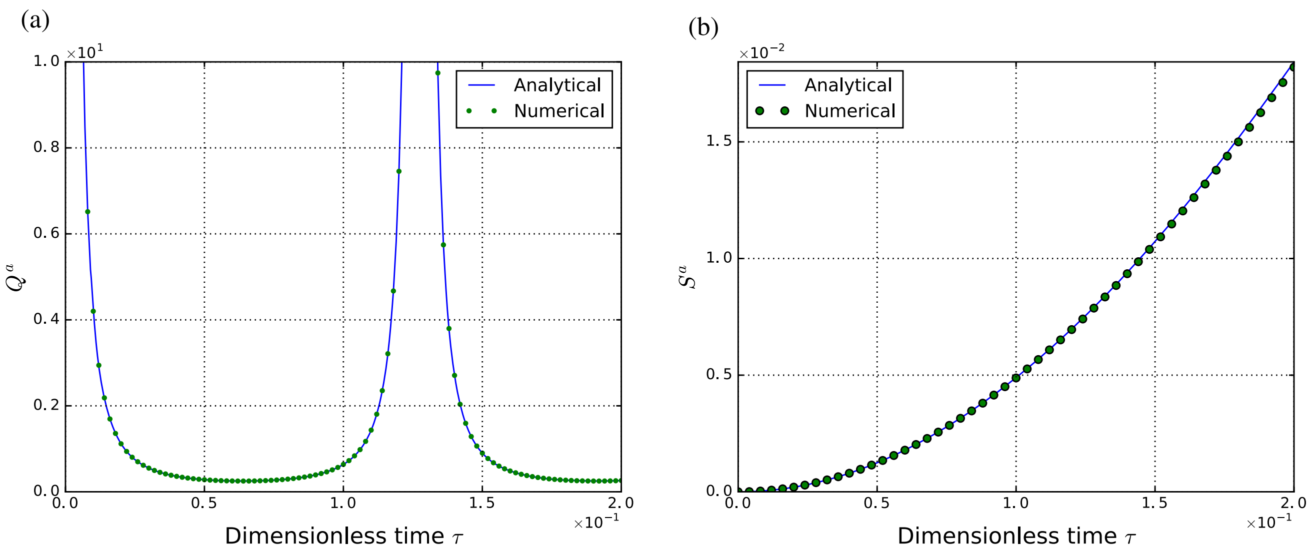

Figure 4. Plots of Q^a and S^a for the exciton mode a as a function of dimensionless interaction time $\tau = gt$ in figures 4a and 4b respectively by using $(\omega_1/g) = 25.277$, $(\omega_2/g) = 24.013$, $(\chi/g) = 5.304 \times 10^{-4}$, $\alpha = 0.0$ and $\beta = 1.0$.

4. Conclusion

The coherent light interacting with a semiconductor is regarded as two coupled harmonic oscillators. One of the oscillator is contributed by the field mode and the the other one is contributed by the exciton. Interestingly, the exciton behaves as a boson. In our investigation, we accommodated a quartic-type nonlinearity in the excitonic mode. The source of nonlinearity is inherent [44]. The Hamiltonian which is basically two coupled oscillators along with quartic nonlinearity in one of the modes (exciton mode) is unsolvable in closed analytical forms. The presence of nonlinearity poses a great difficulty in finding analytical solutions. We obtained coupled differential equations involving the field and excitonic modes. The effects of environment and/or damping are excluded in the present investigation. The damping effects could have been included by constructing the master equation. However, we do not have any intention to include the damping effects which will invite lots of mathematical intricacies. We adopted a better prescription for getting approximate analytical solutions to the present problem. The justification and the strength of the adopted approach is already tested for various physical problems. The solution is finally obtained up to the second orders in dimensionless interaction time $\tau = gt = \chi t$. Nevertheless, the solution presented here is free from secular nature which is inherent in perturbative solutions. The obtained solutions are finally used to define the phase operators. Of course, we adopt the so-called PB approach for defining the phase operators. The absolute phase is of no significance. For this reason, we rely on phase fluctuations. In order to investigate the phase fluctuations of the coherent light coupled to the semiconductor medium we adopt the prescription of CN [38]. Accordingly, we calculated three parameters introduced by the earlier investigators [38]. We compared these calculated parameters post and prior to interaction and hence the fluctuations. On the basis of analytical solution, the entire investigation is being made. The analytical results are quite complicated and are difficult to grasp. For this reason, we presented some symbolic calculations so as to give some feelings about the analytical results. Finally, the presented results are compared with the numerical one based on QuTip 3.1.0 [55,56]. Interestingly, we observe almost exact coincidence of our analytical results with those of computed results based on QuTip. Now, we have to accept the fact that we have abandoned the possibilities of nonlinearities in the field mode because the problem will be complicated to a large extent. One more point is that we computed the phase fluctuation parameters by using theoretical data. Of course, the data used for the calculation are quite logically chosen so as to fit with a possible experiment. Lastly, we mention that the parameters α and β are real. It makes the expressions and hence the calculations a little bit easier. However, it is possible to control the phase of the input coherent light, may be as non-zero and hence β as complex. As the analytical expressions are available, it is not a problem to put non-zero phase for input coherent light. Finally, we have provided the analytical results which can be tested easily after getting the data from a real experiment.

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