Pramana – J. Phys. (2021) 95:111
https://doi.org/10.1007/s12043-021-02143-8
© Indian Academy of Sciences

The effect of dust size distribution on shock wave in quantum dusty plasma

LING-LING TAO, LIN WEI, BO LIU, HENG ZHANG* and WEN-SHAN DUAN*

Department of College of Physics and Electric Engineering, Northwest Normal University, Lanzhou, China
*Corresponding authors. E-mail: zhangheng@nwnu.edu.cn; duanws@nwnu.edu.cn

MS received 7 October 2020; revised 24 February 2021; accepted 12 March 2021

Abstract. Present paper studies the contribution of both the quantum effect and the dust size distribution effect on the shock wave characters. It is concluded that the quantum effect of dust particles is negligible, while the quantum effects of both electrons and ions on the shock wave of a quantum dusty plasma cannot be neglected in certain cases. It is found that the speed and the amplitude of the shock wave, considering the dust size distribution, are larger than that of the quantum dusty plasma with the average dust size of a monosized dust plasma. Furthermore, the speed and the amplitude of the shock wave increase, while the width of the shock wave decreases as the ratios of the maximum dust size to the minimum one increases. The quantum effects may affect both the amplitude and width of the shock wave, while it has no effect on the shock wave speed. The width of the shock wave increases, while its amplitude decreases as H_e and H_i increase.

Keywords. Quantum dusty plasma; dust size distribution; shock wave.

PACS Nos 52.27.Lw; 52.35.Tc; 52.27.−h

1. Introduction

Dusty plasmas, composed of nano-micron-sized dust particles immersed in ionised gas, have attracted great attention due to their importance in space plasmas and man-made environments, such as planetary rings, the Earth's atmosphere, comets and some production processes [1–7]. The dust particles in the plasma are charged. They not only interact with each other, but also with the other charged particles such as electrons and ions. The surrounding plasma environment and the charging mechanism determine whether it is positively or negatively charged, although there are more negatively charged dust particles. The collective behaviour of the plasma can be altered by highly negatively charged dust particles, which also can stimulate new wave modes, such as dust-acoustic (DA) waves [5], dust ion-acoustic (DIA) waves [6,7], dust lattice (DL) waves [8], dust-acoustic shock waves [9–11], etc.

Over the past two decades, researchers have conducted many theoretical and experimental studies on the collective interactions of dusty plasmas [12,13]. But, most of these researches are focussed on dusty plasma composed of same size dust particles. Studies have shown that the size of dust particles varies from nanometres to micrometres and their distribution is determined by various conditions. The distribution of dust particles in space plasma can be usually described by a power-law distribution function [14–16]: $n(r)\,\mathrm{d}r = Kr^{-\beta}\,\mathrm{d}r$ in the range $(r_{\min}, r_{\max})$, while $n(r)\,\mathrm{d}r = 0$ when $r < r_{\min}$ or $r > r_{\max}$, and the distribution of dust particles in laboratory plasma is usually described by Gaussian distribution [14,17,18]: $n(r)\,\mathrm{d}r = D\mathrm{e}^{-\mu'(r-\bar{r})^2}\,\mathrm{d}r$. Many studies have shown that the dust size distribution can affect the characteristics of a dusty plasma. Duan and Parkes [19] investigated the effects of power law dust size distribution on DA waves in the magnetised dusty plasma and Duan and Shi [20] investigated the same in magnetised two-ion temperature dusty plasma. Later on, the effect of Gaussian dust size distribution on the linear phase velocity of DA waves in the dusty plasma have been studied by Duan *et al* [21]

Though there are many research works which are mainly focussed on the classical behaviour of a dusty plasma, many researchers pay their attention on the

Published online: 01 July 2021

quantum effects of a dusty plasma. They introduced quantum correction based on the classical fluid model, namely the 'Bohm potential' [22]. This additional term can properly describe the negative differential resistance in a resonant tunnel diode [23]. There are many research works related to the quantum effects of a dusty plasma [24–26].

When the temperature of a dusty plasma is so low that the de Broglie wavelength $\lambda_{\mathrm{B}}(=\hbar/mV_T$, where $\hbar$ is the Planck constant, V_T is the thermal velocity and m is the mass of particles) of the particles that make up the plasma is comparable to the size of the system ($d = n^{-1/3}$, where n is the number density of particles), the quantum effect of an ultracold dusty plasma cannot be neglected and it behaves like Fermi gas [22]. The thermal velocity is given by $V_T = K_{\mathrm{B}}T/m$. The quantum effects play significant roles when the plasma density is particularly high and the temperature is particularly low [27]. It has been reported that quantum effects are important in the research of laser plasmas [28], microelectronic devices [29] and ultradensity cosmic physics [30], such as white dwarfs and neutron stars which are the products of the last stage of stellar evolution and have very high density. Ali and Shukla [31] and El-Taibany and Wadati [32] have studied quantum dust-acoustic (QDA) waves using the QHD model via the Korteweg–de Vries (KdV) equation. Han *et al* [27] and El-Labani *et al* [33] investigated the effect of dust size distribution on QDA waves in quantum plasmas. However, a few researchers have studied how the dust size distribution affects shock waves in quantum dusty plasmas.

In this paper, we study the quantum dust-acoustic (QDA) shock wave [34,35] in an external magnetic, collisionless, ultracold quantum dusty plasma consisting of inertialess electrons, ions and negatively charged dust particles which have a power-law distribution. Then we employ the reductive perturbation method [36] (RPT) in the three-dimensional QHD model to derive a quantum ZK-Burgers [37,38] equation which describes the dynamics of QDA shock waves. We focus on both the quantum effects and the dust size distribution on the shock wave characters. This paper tries to answer this question. It is found that the quantum effect of dust particles is negligible, while the quantum effects of both electrons and ions on the shock wave of a quantum dusty plasma cannot be neglected in certain cases. It is found that the speed and the amplitude of the shock wave, considering the dust size distribution, are larger than that of the quantum dusty plasma with the averaged dust size of a monosized dusty plasma. Furthermore, it is found that the speed and the amplitude of the shock wave increase, while the width of the shock wave decreases as the ratios of the maximum dust size to the minimum one increase. The quantum effects may affect both the amplitude and width of the shock wave, while it has no effect on the shock wave speed. The width of the shock wave increases, while its amplitude decreases as H_e and H_i increase.

2. Theoretical model

We now study the quantum shock wave in a three-dimensional quantum dusty plasma consisting of inertialess electrons, inertialess ions and negatively charged dust particles which have a dust size distribution with the external magnetic field $\mathbf{B} = B_0\mathbf{x}$ where B_0 is the strength of the magnetic field. It is assumed that the pressure of the plasma particles is given by [39–41]

$$P_s = \frac{m_s V_{Fs}^2}{3n_{s0}^2} n_s^3, \tag{1}$$

where n_s is the number density with its equilibrium value n_{s0} ($s = dj$ for the jth dust particle, e for electrons, i for ions). m_s is its mass and $V_{Fs} = \sqrt{2K_{\mathrm{B}}T_{Fs}/m_s}$ is the Fermi thermal speed where K_{B} is the Boltzmann constant and T_{Fs} is the Fermi temperature.

At equilibrium, the charge neutrality condition is given by the equation

$$n_{i0} = n_{e0} + \sum_{j=1}^{N} Z_{dj} n_{dj0}, \tag{2}$$

where n_{dj0}, n_{e0} and n_{i0} are the unperturbed jth dust particles, electron and ion number densities, respectively. Z_{dj} is the number of charges on the jth dust particle surface. Therefore, the dynamics of the low phase velocity and low frequency nonlinear QDA shock wave in the quantum plasma with dust size distributions are given by [27,33,42]

$$\frac{\partial n_{dj}}{\partial t} + \nabla\cdot(n_{dj}\mathbf{u_{dj}}) = 0 \tag{3}$$

$$\frac{\partial \mathbf{u_{dj}}}{\partial t} + \mathbf{u_{dj}}(\nabla\cdot\mathbf{u_{dj}}) = \frac{Z_{dj}}{m_{dj}}\nabla\phi - \frac{\sigma_d}{m_{dj}} n_{dj} \frac{Z_{dj}}{m_{dj}}\Omega(\mathbf{u_{dj}}\times\mathbf{x}) + \frac{\eta}{m_{dj}}\nabla^2\mathbf{u_{dj}} + \frac{H_d^2}{2m_{dj}^2}\nabla\left(\frac{\nabla^2\sqrt{n_{dj}}}{\sqrt{n_{dj}}}\right) \tag{4}$$

$$\nabla^2\phi = \mu_e n_e - \mu_i n_i + \sum_{j=1}^{N} Z_{dj} n_{dj} \tag{5}$$

$$0=\nabla\phi-\sigma_e n_e\nabla n_e+\frac{H_e^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) \quad (6)$$

$$0=-\nabla\phi-\sigma_i n_i\nabla n_i+\frac{H_i^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n_i}}{\sqrt{n_i}}\right), \quad (7)$$

where

$$\mu_i=\frac{n_{i0}}{\overline{Z_{d0}}N_{\rm tot}},\quad \mu_e=\frac{n_{e0}}{\overline{Z_{d0}}N_{\rm tot}},$$

$$\sigma_e=\frac{T_e}{T_{\rm eff}},\quad \sigma_i=\frac{T_i}{T_{\rm eff}},\quad \sigma_d=\frac{T_{Fd}}{\overline{Z_{d0}}T_{\rm eff}},$$

$$H_e^2=\frac{\hbar^2\overline{Z_{d0}}\omega_{pd}^2}{m_e c_d^4},\quad H_i^2=\frac{\hbar^2\overline{Z_{d0}}\omega_{pd}^2}{m_i c_d^4},$$

and

$$H_d^2=\frac{\hbar^2\overline{Z_{d0}}\omega_{pd}^2}{\overline{m}_d c_d^4},$$

where $T_{\rm eff}$ is the effective temperature defined as

$$T_{\rm eff}=\frac{T_iT_e}{\mu_iT_i+\mu_eT_e},$$

T_i and T_e are ion and electron temperatures, $N_{\rm tot}$ is the total number density of the dust particles determined using the equation

$$N_{\rm tot}=\sum_{j=1}^{N}n_{dj},$$

$\overline{Z_{d0}}$ is the average values defined by the equation

$$\overline{Z_{d0}}N_{\rm tot}=\sum_{j=1}^{N}m_{dj}n_{dj},$$

$\overline{m_d}$ is the average mass of the dust particles defined by

$$\overline{m_d}=1/N_{\rm tot}\sum_{j=1}^{N}/m_{dj}n_{dj},$$

c_d is the dust-acoustic speed defined by

$$c_d=\sqrt{2\overline{Z_{d0}}T_{\rm eff}/\overline{m_d}},$$

inverse of dust plasma frequency

$$\omega_{pd}=\sqrt{4\pi e^2\overline{Z_{d0}}^2N_{\rm tot}/\overline{m_d}},$$

magnetic field parameter

$$\Omega=e\overline{Z_{d0}}B_0/\overline{m_d}c_d\omega_{pd}$$

and η is the viscosity coefficient of the dust grains. n_{dj}, n_e and n_i are the number densities of the jth dust particle, electrons and ions, respectively. $\mathbf{u_{dj}}$ is the velocity of the jth dust particle expressed by

$$\mathbf{u_{dj}}=u_{djx}\mathbf{i}+u_{djy}\mathbf{j}+u_{djz}\mathbf{k},$$

where **i**, **j**, **k** are the unit vectors along the x, y and z directions, respectively. ϕ is the static-electric potential.

We considered the following dimensionless variables: $t\to t'/\omega_{pd}^2$, $u_{dj}\to u'_{dj}/c_d$, $n_{dj}\to n'_{dj}/N_{\rm tot}$, $n_e\to n'_e/n_{e0}$, $n_i\to n'_i/n_{i0}$, $Z_{dj}\to Z'_{dj}/\overline{Z_{d0}}$, $m_{dj}\to m'_{dj}/m_d$, $m_e\to m'_e/\overline{m_d}$, $m_i\to m'_i/\overline{m_d}$, $\phi\to e\phi'/T_{\rm eff}$, $\nabla\to\nabla'/\lambda_{\rm D}$, $\eta\to\eta'/\omega_{pd}\lambda_{\rm D}^2\overline{m_d}N_{\rm tot}$, where $\lambda_{\rm D}$ is the Debye length.

Now, we employ the RPT to study the property of the QDA shock wave and introduce the following stretched coordinates [33,43,44]:

$$X=\epsilon(x-\lambda t),\quad Y=\epsilon y,\quad Z=\epsilon z \quad (8)$$

$$T=\epsilon^3 t. \quad (9)$$

For the stretches of the coordinate transformations, the small parameter ϵ actually represents the wave number, i.e., ϵ is in the order of the wave number, suggesting the long wavelength approximation. The other parameter of ϵ in the expansion of all the physical quantities actually stands for the ratio of the perturbed quantity to that at the equilibrium state, i.e., ϵ is in the order of the ratio of the perturbed quantity to that at equilibrium state, which suggest the small but finite-amplitude perturbed waves. λ is the phase speed of the shock wave. The dependent variables are expanded around the equilibrium value as a power series in ϵ as [20,33,45,46]:

$$\begin{aligned}
n_{dj}&=n_{dj0}+\epsilon^2n_{dj1}+\epsilon^4n_{dj2}+\cdots\\
n_e&=1+\epsilon^2n_{e1}+\epsilon^4n_{e2}+\cdots\\
n_i&=1+\epsilon^2n_{i1}+\epsilon^4n_{i2}+\cdots\\
u_{djx}&=\epsilon^2u_{djx1}+\epsilon^4u_{djx2}+\cdots\\
u_{djy}&=\epsilon^3u_{djy1}+\epsilon^4u_{djy2}+\cdots\\
u_{djz}&=\epsilon^3u_{djz1}+\epsilon^4u_{djz2}+\cdots\\
\phi&=\epsilon^2\phi_1+\epsilon^4\phi_2+\cdots
\end{aligned} \quad (10)$$

Substituting eqs (8)–(10) into eqs (3)–(7), we collect lowest order terms of ϵ to obtain

$$n_{e1}=\frac{\phi_1}{\sigma_e},\quad n_{i1}=-\frac{\phi_1}{\sigma_i}$$

$$n_{dj1}=\frac{Z_{dj}n_{dj0}}{\sigma_dn_{dj0}^2-\lambda^2m_{dj}}\phi_1,$$

$$u_{djx1}=\frac{\lambda}{n_{dj0}}\frac{Z_{dj}n_{dj0}}{\sigma_dn_{dj0}^2-\lambda^2m_{dj}}\phi_1$$

$$u_{djy1}=-(1+\frac{Z_{dj}n_{dj0}}{\sigma_dn_{dj0}^2-\lambda^2m_{dj}})\frac{1}{\Omega}\frac{\partial\phi_1}{\partial Z}$$

$$u_{djz1}=(1+\frac{Z_{dj}n_{dj0}}{\sigma_dn_{dj0}^2-\lambda^2m_{dj}})\frac{1}{\Omega}\frac{\partial\phi_1}{\partial Y}$$

$$\sum_{j=1}^{N} \frac{Z_{dj}^2 n_{dj0}}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2} = 1. \tag{11}$$

It is noted that the temperature of the dust fluid is usually low enough and the mass is large enough [47–52]. The pressure of the dust particles is $P = \sqrt{K_B T/m}$, where P is the thermal pressure of the dusty plasma, K_B is the Boltzmann constant, T is the temperature of the dusty plasma and m is the mass of the dust particle [53–55]. Therefore, the pressure of the dust particles is small enough which can be neglected in some cases. In order to simplify the above-mentioned dispersion relationship, we put $\sigma_d = 0$ to ignore the thermal effects of the dust [32].

$$\lambda^2 = \sum_{j=1}^{N} \frac{n_{dj0} Z_{dj}^2}{m_{dj}}. \tag{12}$$

Then, collecting the ϵ^4 terms of the second-order velocity perturbation in the y and z directions, we have

$$u_{djy2} = -\frac{\lambda m_{dj}}{\Omega^2 Z_{dj}} \left(1 + \frac{Z_{dj} n_{dj0}}{\sigma_d n_{dj0}^2 - \lambda^2 m_{dj}}\right) \frac{\partial^2 \phi_1}{\partial X \partial Y} \tag{13}$$

$$u_{djz2} = -\frac{\lambda m_{dj}}{\Omega^2 Z_{dj}} \left(1 + \frac{Z_{dj} n_{dj0}}{\sigma_d n_{dj0}^2 - \lambda^2 m_{dj}}\right) \frac{\partial^2 \phi_1}{\partial X \partial Z}. \tag{14}$$

Finally, we obtain ZK-Burgers equation from the next lowest order terms and eqs (11)–(14)

$$\frac{\partial \phi_1}{\partial T} + A\phi_1 \frac{\partial \phi_1}{\partial X} + B\frac{\partial^3 \phi_1}{\partial X^3} + C\frac{\partial}{\partial X}\left(\frac{\partial^2 \phi_1}{\partial Y^2} + \frac{\partial^2 \phi_1}{\partial Z^2}\right) - D\frac{\partial^2 \phi_1}{\partial X^2} = 0 \tag{15}$$

with the coefficients

$$A = -\frac{3}{2\lambda^3} \sum_{j=1}^{N} \frac{Z_{dj}^3 n_{dj0}}{m_{dj}^2} + \frac{1}{2\lambda} \sum_{j=1}^{N} \frac{\sigma_d}{m_{dj}} \frac{Z_{dj} n_{dj0}^2}{\sigma_d n_{dj0}^2 - \lambda^2 m_{dj}} + \frac{\lambda}{2}\left(\frac{\mu_e}{\sigma_e^2} - \frac{\mu_i}{\sigma_i^2}\right) \tag{16}$$

$$B = \frac{\lambda}{2} - \frac{1}{2\lambda} \sum_{j=1}^{N} \frac{H_d^2}{4m_{dj}^2} - \frac{\lambda}{2}\left(\frac{\mu_e H_e^2}{4\sigma_e^2} + \frac{\mu_i H_i^2}{4\sigma_e^i}\right) \tag{17}$$

$$C = \frac{\lambda}{2} \sum_{j=1}^{N} \frac{m_{dj} n_{dj0}}{\Omega^2} \left(1 + \frac{\sigma_d n_{dj0}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2}\right) + B \tag{18}$$

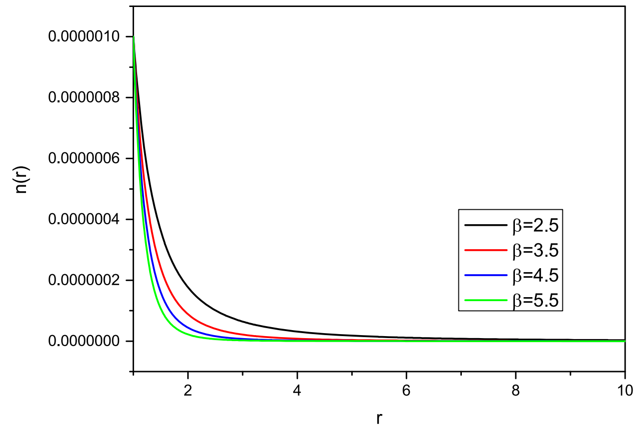

Figure 1. The dependence of $n(r)$ on r for different values of β.

$$D = \frac{\eta'}{2\lambda^2} \sum_{j=1}^{N} \frac{Z_{dj}^2 n_{dj0}}{m_{dj}^2}. \tag{19}$$

One of the solutions of the ZK-Burgers equation [56] of eq. (19) is

$$\phi_1 = \frac{6}{25} \frac{D^2}{FAl^2} \left[1 - \tanh\left(\frac{\chi}{W}\right) + \frac{1}{2}\text{sech}^2\left(\frac{\chi}{W}\right)\right], \tag{20}$$

where $\chi = lX + mY + nZ - UT$, $F = Bl^2 + C(m^2 + n^2)$ and $l^2 + m^2 + n^2 = 1$. Here l, m and n are respectively the directional cosines of the wave vector along the X-, Y- and Z-axes. $W = 10Fl/D$ is the width of the shock wave and

$$4\phi_m = \frac{9}{25} \frac{D^2}{FAl^2}$$

is the amplitude of the shock wave.

3. Results and discussions

We now consider the case that sizes of all the dust particle are smaller than the Debye length ($r \ll \lambda_D$). It is reported that the charge and mass of a dust particle are as follows [57]: $Z_{dj} = k_z r_j$ and $m_{dj} = k_m r_j^3$, where k_z and k_m are constants, r_j is the radius of the jth dust particle. We shall consider the power-law dust size distribution to understand the effect of dust size distributions on shock wave characters. The dust size distribution function is $n(r)\,\mathrm{d}r = Kr^{-\beta}$ in the range (r_1, r_2).

Figure 1 shows the density of dust particles $n(r)$ with respect to the different dust size r. Notice that the density of dust particles decreases as the radius r increases. In the power function distribution, there are more small dust than large dust.

Then we have

$$N_{\text{tot}} = \frac{K}{1-\beta} r_1^{1-\beta}(R^{1-\beta}-1), \tag{21}$$

where $R = r_2/r_1$.

$$\bar{r} = \frac{(1-\beta)r_1(R^{2-\beta}-1)}{(2-\beta)(R^{1-\beta}-1)}, \tag{22}$$

where $\bar{r}$ is the average radius of the dust particles.

$$\lambda^2 = -\frac{Kk_z^2}{\beta k_m} r_1^{-\beta}(R^{-\beta}-1) \tag{23}$$

$$\bar{\lambda}^2 = \frac{Kk_z^2}{k_m} \frac{(2-\beta)r_1^{-\beta}(R^{1-\beta}-1)^2}{(1-\beta)^2(R^{2-\beta}-1)} \tag{24}$$

$$A = -\frac{3}{2\lambda}\frac{k_z}{k_m} r_1^{-2}(R^{-2}-1) + \frac{\lambda}{2}\left(\frac{\mu_e}{\sigma_e^2} - \frac{\mu_i}{\sigma_i^2}\right) \tag{25}$$

$$\bar{A} = -\frac{3}{2\bar{\lambda}}\frac{k_z}{k_m \bar{r}^2} + \frac{\bar{\lambda}}{2}\left(\frac{\mu_e}{\sigma_e^2} - \frac{\mu_i}{\sigma_i^2}\right) \tag{26}$$

$$B = \frac{1}{2\lambda}\left[\lambda^2 + \frac{H_d^2 r_1^{-5}(R^{-5}-1)}{20k_m^2} - \lambda^2\left(\frac{\mu_e H_e^2}{4\sigma_e^2} + \frac{\mu_i H_i^2}{4\sigma_i^2}\right)\right] \tag{27}$$

$$\bar{B} = \frac{1}{2\bar{\lambda}}\left[\bar{\lambda}^2 - \frac{H_d^2}{4k_m^2 \bar{r}^6} - \bar{\lambda}^2\left(\frac{\mu_e H_e^2}{4\sigma_e^2} + \frac{\mu_i H_i^2}{4\sigma_i^2}\right)\right] \tag{28}$$

$$C = B + \frac{Kk_m\lambda}{8\Omega^2(1-\beta)} r_1^{3-\beta}(R^{1-\beta}-1)(R^2-1) \tag{29}$$

$$\bar{C} = \bar{B} + \frac{Kk_m\bar{\lambda}}{2\Omega^2(1-\beta)} r_1^{1-\beta}(R^{1-\beta}-1)\bar{r} \tag{30}$$

$$D = \frac{K\eta' k_z^2}{2(-\beta-3)\lambda^2 k_m^2} r_1^{-\beta-3}(R^{-\beta-3}-1) \tag{31}$$

$$\bar{D} = \frac{K\eta' k_z^2}{2(1-\beta)\bar{\lambda}^2 k_m^2} r_1^{1-\beta}(R^{1-\beta}-1)\bar{r}^{-4}. \tag{32}$$

Both the shock wave width and amplitude ratios considering the dust size distribution to that of the mono-sized dusty plasma with the average dust radius of $\bar{r}$ are as follows:

$$\frac{W}{\bar{W}} = \frac{[Bl^2 + C(1-l^2)]\bar{D}}{[\bar{B}l^2 + \bar{C}(1-l^2)]D} \tag{33}$$

$$\frac{\phi_m}{\bar{\phi}_m} = \frac{D^2\bar{A}[\bar{B}l^2 + \bar{C}(1-l^2)]}{\bar{D}^2 A[Bl^2 + C(1-l^2)]}. \tag{34}$$

We have taken the following parameters [27,32,33,58]: $K = 10^{-6}$, $k_z \approx 1$, $k_m \approx 4$, $\mu_e = 0.3$, $\mu_i = 1.3$, $\sigma_e = 2$, $\sigma_i = 0.2$, $r_1 = 0.1$, $H_d = 10^{-8}$, $n_{i0} = 2 \times 10^{30}\,\text{m}^{-3}$, $n_{e0} = 5 \times 10^{29}\,\text{m}^{-3}$.

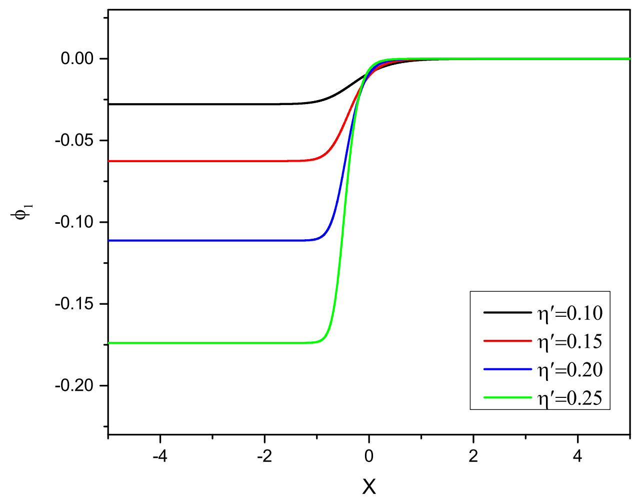

Figure 2. The dependence of ϕ, the shock wave amplitude, on position X at time $T = 0$ for different values of η', when $\Omega = 0.5$, $l = 0.9$, $\beta = 5.5$, $H_e = 0.1$, $H_i = 0.001$.

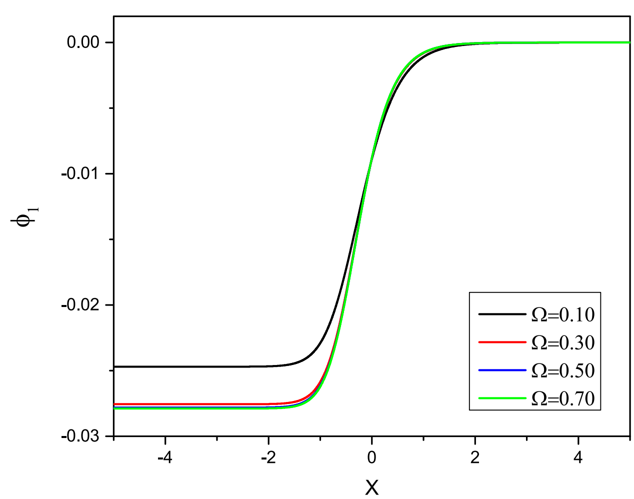

Figure 3. The dependence of ϕ, the shock wave amplitude, on position X at time $T = 0$ for different values of Ω, when $\eta = 0.1$, $l = 0.9$, $l = 0.9$, $\beta = 5.5$, $H_e = 0.1$, $H_i = 0.001$.

Figure 2 shows the waveform of a shock wave with respect to the parameter η'. Notice that the increase of η' will result in the increase (decrease) of the amplitude (width). The increase of η' leads to the increase of the collision frequency which enhances damping. Figure 3 shows the waveform of a shock wave with respect to the parameter Ω.

Figures 4–6 show how the parameter $R = r_2/r_1$ affects $\lambda^2/\bar{\lambda}^2$, $W/\bar{W}$ and $\phi_m/\bar{\phi}_m$ for different values of β. Notice that both the speed and the amplitude of shock waves increase as R increases, while they decrease as β increases. The widths of the shock waves decrease as R increases, and increase as β increases. It is found that the effect of dust size distribution on the width and

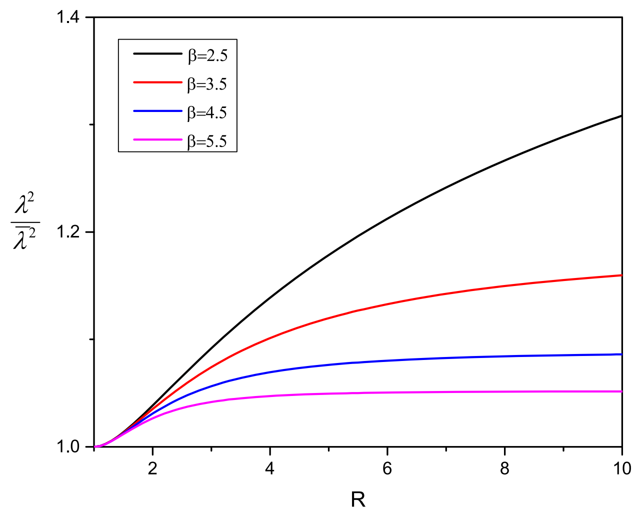

Figure 4. The dependence of $\lambda^2/\overline{\lambda}^2$ on parameter of R for different values of β, when $H_i = 10^{-3}$ and $H_e = 0.1$.

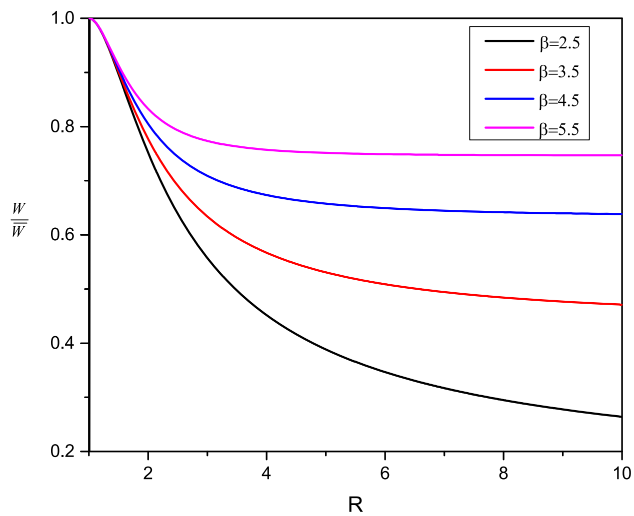

Figure 5. The dependence of $W/\overline{W}$ on parameter R for different values of β, when $H_i = 10^{-3}$, $H_e = 0.1$ and $\Omega = 0.5$.

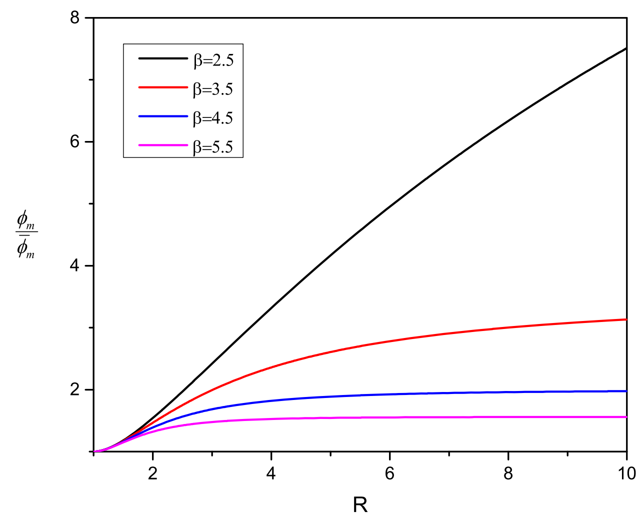

Figure 6. The dependence of $\phi_m/\overline{\phi_m}$ on parameter R for different values of β, when $H_i = 10^{-3}$, $H_e = 0.1$ and $\Omega = 0.5$.

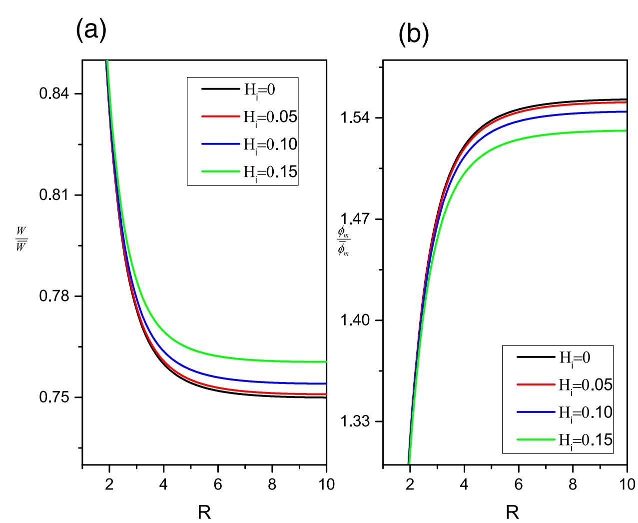

Figure 7. The variation of $W/\overline{W}$ and $\phi_m/\overline{\phi_m}$ with respect to R for different values of H_i shown in (**a**), (**b**), when $H_e = 1.95$, $\beta = 5.5$, $\eta = 0.5$ and $\Omega = 0.5$.

the amplitude of shock waves is different from that of the solitary waves [33]. It also indicates that the speed and the amplitude of the shock wave, considering the dust size distribution, are lager than that of the monosized case and they increase as the ratio of maximum to minimum dust size increases. The width of the shock waves, considering the dust size distribution, is smaller than that of the monosized dusty plasma. They decrease as the ratio of maximum to minimum dust size increases (figure 5).

We know from eqs (12), (16)–(19) that quantum parameters have no effects on the velocity of the shock wave, but they affect both the width and the amplitude of the shock wave. This conclusion is different from that of the solitary waves [33]. Notice from figure 7 that the shock wave width increases, while its amplitude decreases as H_i increases. Similarly, it is found from figure 8 that the shock wave width increases, while its amplitude decreases as H_e increases. Furthermore, it seems that the quantum effect of the dust particles H_d is so small that it is negligible.

4. Conclusion

The present study focusses on how both the quantum effect and the dust size distribution affect the shock wave characters of a quantum dusty plasma. First, by

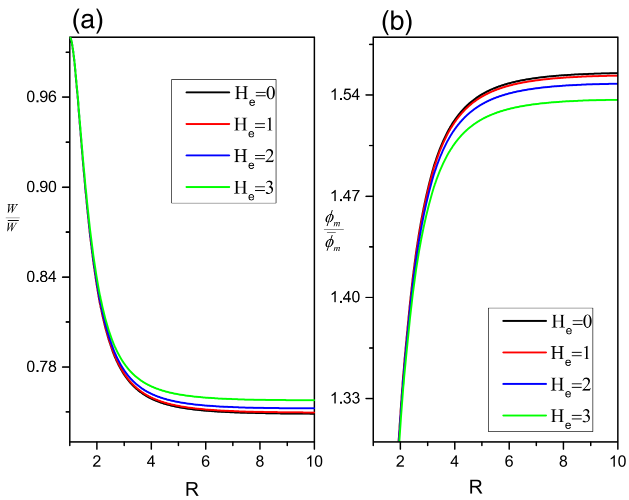

Figure 8. The variation of $W/\overline{W}$ and $\phi_m/\overline{\phi_m}$ with respect to R for different values of H_i shown in (**a**), (**b**), when $H_i = 0.05$, $\beta = 5.5$, $\eta = 0.5$ and $\Omega = 0.5$.

using the RPT method and applying the QHD model, we derive a ZK-Burgers equation to describe the shock wave in a quantum dusty plasma. Then, by using the power-law dust size distribution function, we study the shock wave in a quantum plasma. It is found that the speed and the amplitude of the shock wave, considering the dust size distribution, are larger than that of the quantum dusty plasma with the monosized dust particles. It is further noted that the speed and the amplitude of the shock wave increase as R, the ratio of the maximum dust size to the minimum one, increases. However, the width of the shock wave is less than that with monosized dust particles, and it decreases as R increases. It seems that the quantum effects may affect both the amplitude and width of the shock wave, while it has no effect on the shock wave speed. In addition, it is noted that the width of the shock wave increases, while its amplitude decreases when both H_e and H_i increase. Compared with the contributions of both H_e and H_i, the effect of H_d on both the width and amplitude of the shock wave is negligibly small.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grants Nos 11965019 and 11847142).

Appendix A

We have taken the following parameters [27,32,33,58]: $Z_d = 10^3$, $m_d = 2 \times 10^{-17}$ kg, $m_e = 0.91 \times 10^{-30}$ kg, $m_i = 2 \times 10^{-26}$ kg, $n_d = 1.5 \times 10^{27}$ m^{-3}, $n_i = 2 \times 10^{30}$ m^{-3}, $\hbar = 1.05 \times 10^{-34}$J·s, $T_{\text{eff}} = 3.1 \times 10^{-13}$ J, $\mu_i = 1.3$, $\mu_e = 0.3$. We have

$$H_e = \sqrt{\frac{\hbar^2 \overline{Z_{d0}} \omega_{pd}^2}{m_e c_d^4}} \approx 3.4 \tag{A.1}$$

$$H_i = \sqrt{\frac{\hbar^2 \overline{Z_{d0}} \omega_{pd}^2}{m_i c_d^4}} \approx 0.02 \tag{A.2}$$

$$H_d = \sqrt{\frac{\hbar^2 \overline{Z_{d0}} \omega_{pd}^2}{\overline{m}_d c_d^4}} \approx 7 \times 10^{-7}. \tag{A.3}$$

Hence, the contributions of dust and ion are much lower than the electron.

Appendix B

We have model fluid equations

$$\frac{\partial n_{dj}}{\partial t} + \nabla \cdot (n_{dj} \mathbf{u_{dj}}) = 0 \tag{B.1}$$

$$\frac{\partial \mathbf{u_{dj}}}{\partial t} + \mathbf{u_{dj}} (\nabla \cdot \mathbf{u_{dj}}) = \frac{Z_{dj}}{m_{dj}} \nabla \phi - \frac{\sigma_d}{m_{dj}} n_{dj} \frac{Z_{dj}}{m_{dj}} \Omega (\mathbf{u_{dj}} \times \mathbf{x}) + \frac{\eta}{m_{dj}} \nabla^2 \mathbf{u_{dj}} + \frac{H_d^2}{2m_{dj}^2} \nabla \left(\frac{\nabla^2 \sqrt{n_{dj}}}{\sqrt{n_{dj}}} \right) \tag{B.2}$$

$$\nabla^2 \phi = \mu_e n_e - \mu_i n_i + \sum_{j=1}^{N} Z_{dj} n_{dj} \tag{B.3}$$

$$0 = \nabla \phi - \sigma_e n_e \nabla n_e + \frac{H_e^2}{2} \nabla \left(\frac{\nabla^2 \sqrt{n_e}}{\sqrt{n_e}} \right) \tag{B.4}$$

$$0 = -\nabla \phi - \sigma_i n_i \nabla n_i + \frac{H_i^2}{2} \nabla \left(\frac{\nabla^2 \sqrt{n_i}}{\sqrt{n_i}} \right). \tag{B.5}$$

We introduce the stretched coordinates

$$X = \epsilon (x - \lambda t), \quad Y = \epsilon y, \quad = \epsilon z \tag{B.6}$$

$$T = \epsilon^3 t \tag{B.7}$$

and

$$\frac{\partial}{\partial x} = \epsilon \frac{\partial}{\partial X} \tag{B.8}$$

$$\frac{\partial}{\partial y} = \epsilon \frac{\partial}{\partial Y} \tag{B.9}$$

$$\frac{\partial}{\partial z} = \epsilon \frac{\partial}{\partial Z} \tag{B.10}$$

$$\frac{\partial^2}{\partial x^2} = \epsilon^2 \frac{\partial^2}{\partial X^2} \tag{B.11}$$

$$\frac{\partial}{\partial t} = \epsilon^3 \frac{\partial}{\partial T} - \epsilon\lambda \frac{\partial}{\partial X}. \tag{B.12}$$

The asymptotic expansions of the perturbed quantities are given as

$$\begin{aligned} n_{dj} &= n_{dj0} + \epsilon^2 n_{dj1} + \epsilon^4 n_{dj2} + \cdots \\ n_e &= 1 + \epsilon^2 n_{e1} + \epsilon^4 n_{e2} + \cdots \\ n_i &= 1 + \epsilon^2 n_{i1} + \epsilon^4 n_{i2} + \cdots \\ u_{djx} &= \epsilon^2 u_{djx1} + \epsilon^4 u_{djx2} + \cdots \\ u_{djy} &= \epsilon^3 u_{djy1} + \epsilon^4 u_{djy2} + \cdots \\ u_{djz} &= \epsilon^3 u_{djz1} + \epsilon^4 u_{djz2} + \cdots \\ \phi &= \epsilon^2 \phi_1 + \epsilon^4 \phi_2 + \cdots . \end{aligned} \tag{B.13}$$

Substituting eqs (B.8)–(B.13) into eqs (B.1)–(B.5), and collecting the powers of ϵ^3, we assume $\eta = \epsilon\eta'$

$$u_{djx1} = \frac{\lambda}{n_{dj0}} n_{dj1} \tag{B.14}$$

$$n_{dj1} = \frac{Z_{dj} n_{dj0}}{\sigma_d n_{dj0}^2 - \lambda^2 m_{dj}} \phi_1 \tag{B.15}$$

$$u_{djz1} = \left(1 + \frac{\sigma_d n_{dj0}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2}\right) \frac{1}{\Omega} \frac{\partial \phi_1}{\partial Y} \tag{B.16}$$

$$u_{djy1} = -\left(1 + \frac{\sigma_d n_{dj0}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2}\right) \frac{1}{\Omega} \frac{\partial \phi_1}{\partial Y} \tag{B.17}$$

$$0 = \mu_e n_{e1} - \mu_i n_{i1} + \sum_{j=1}^{N} Z_{di} n_{dj1} \tag{B.18}$$

$$n_{e1} = \frac{\phi_1}{\sigma_e} \tag{B.19}$$

$$n_{i1} = -\frac{\phi_1}{\sigma_i}. \tag{B.20}$$

Collecting terms of the same powers of ϵ^4, the y- and z-components of the second-order perturbed velocity are calculated as

$$u_{djz2} = \frac{\lambda m_d}{Z_{dj}\Omega} \frac{\partial u_{djy1}}{\partial X} \tag{B.21}$$

$$u_{djy2} = -\frac{\lambda m_d}{Z_{dj}\Omega} \frac{\partial u_{djz1}}{\partial X}. \tag{B.22}$$

From eq. (B.15) and (B.18)–(B.20), we get

$$\sum_{j=1}^{N} \frac{n_{dj0} Z_{dj}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2} = 1. \tag{B.23}$$

We put $\sigma_d = 0$ leading to a simplified linear dispersion relation

$$\lambda^2 = \sum_{j=1}^{N} \frac{n_{dj0} Z_{dj}^2}{m_{dj}}. \tag{B.24}$$

Collecting the powers of ϵ^5, we have

$$\begin{aligned} &\frac{\partial n_{dj1}}{\partial T} - \lambda \frac{\partial n_{dj2}}{\partial X} + \frac{\partial (n_{dj1} u_{djx1})}{\partial X} + n_{dj0} \frac{\partial u_{djx2}}{\partial X} \\ &- n_{dj0} \frac{\lambda m_{dj}}{Z_{dj}\Omega^2} \left(1 + \frac{\sigma_d n_{dj0}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2}\right) \frac{\partial}{\partial X} \frac{\partial^2 \phi_1}{\partial Y^2} \\ &- n_{dj0} \frac{\lambda m_{dj}}{Z_{dj}\Omega^2} \left(1 + \frac{\sigma_d Z_{dj} n_{dj0}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2}\right) \frac{\partial}{\partial X} \frac{\partial^2 \phi_1}{\partial Z^2} \\ &= 0 \end{aligned} \tag{B.25}$$

$$\begin{aligned} &\frac{\partial u_{djx1}}{\partial T} - \lambda \frac{\partial u_{djx2}}{\partial X} + u_{djx1} \frac{\partial u_{djx1}}{\partial X} = \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi_2}{\partial X} \\ &- \frac{\sigma_d}{m_{dj}} n_{dj0} \frac{\partial n_{dj2}}{\partial X} - \frac{\sigma_d}{m_{dj}} n_{dj1} \frac{\partial n_{dj1}}{\partial X} \\ &+ \frac{\eta'}{m_{dj}} \frac{\partial^2 u_{djx1}}{\partial X^2} + \frac{H_d^2}{4 n_{dj0} m_{dj}^2} \frac{\partial (\nabla^2 n_{dj1})}{\partial X} \end{aligned} \tag{B.26}$$

$$\begin{aligned} &\frac{\lambda^2 m_{dj}}{Z_{dj}\Omega^2} \left(1 + \frac{\sigma_d n_{dj0}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2}\right) \frac{\partial}{\partial Y} \frac{\partial^2 \phi_1}{\partial X^2} \\ &= \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi_2}{\partial Y} - \frac{\sigma_d}{m_{dj}} n_{dj0} \frac{\partial n_{dj2}}{\partial Y} \\ &- \frac{\sigma_d}{m_{dj}} n_{dj1} \frac{\partial n_{dj1}}{\partial Y} + \frac{H_d^2}{4 n_{dj0} m_{dj}^2} \frac{\partial (\nabla^2 n_{dj1})}{\partial Y} \end{aligned} \tag{B.27}$$

$$\begin{aligned} &\frac{\lambda^2 m_{dj}}{Z_{dj}\Omega^2} \left(1 + \frac{\sigma_d n_{dj0}^2}{\lambda^2 m_{dj} - \sigma_d n_{dj0}^2}\right) \frac{\partial}{\partial Z} \frac{\partial^2 \phi_1}{\partial X^2} \\ &= \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi_2}{\partial Z} - \frac{\sigma_d}{m_{dj}} n_{dj0} \frac{\partial n_{dj2}}{\partial Z} \\ &- \frac{\sigma_d}{m_{dj}} n_{dj1} \frac{\partial n_{dj1}}{\partial Z} + \frac{H_d^2}{4 n_{dj0} m_{dj}^2} \frac{\partial (\nabla^2 n_{dj1})}{\partial Z} \end{aligned} \tag{B.28}$$

$$\nabla^2 \phi_1 = \mu_e n_{e2} - \mu_i n_{i2} + \sum_{j=1}^{N} Z_{dj} n_{dj2} \tag{B.29}$$

$$0 = \nabla \phi_2 - \sigma_e n_{e1} \nabla n_{e1} - \sigma_e \nabla n_{e2} + \frac{H_e^2}{4} \nabla(\nabla^2 n_{e1}) \tag{B.30}$$

$$0 = -\nabla \phi_2 - \sigma_i n_{i1} \nabla n_{i1} - \sigma_i \nabla n_{i2} + \frac{H_i^2}{4} \nabla(\nabla^2 n_{i1}). \tag{B.31}$$

Solving eqs (B.25)–(B.31) with the aid of eqs (B.14)–(B.24), we get ZK-Burgers equation

$$\begin{aligned} &\frac{\partial \phi_1}{\partial T} + A\phi_1 \frac{\partial \phi_1}{\partial X} + B \frac{\partial^3 \phi_1}{\partial X^3} \\ &+ C \frac{\partial}{\partial X} \left(\frac{\partial^2 \phi_1}{\partial Y^2} + \frac{\partial^2 \phi_1}{\partial Z^2}\right) - D \frac{\partial^2 \phi_1}{\partial X^2} = 0, \end{aligned} \tag{B.32}$$

where the values of A, B, C and D are consistent with eqs (16)–(19).

References

[1] M Horanyi, *Annu. Rev. Astron. Astrophys.* **34**, 383 (1996)
[2] O Havnes, F Melandso, C la Hoz, T K Aslaksen and T Hartquist, *Phys. Scr.* **45**, 535 (1992)
[3] D A Mendis and M Rosenberg, *Annu. Rev. Astron. Astrophys.* **32**, 419 (1994)
[4] S V Vladimirov and K Ostrikov, *Phys. Rep.* **393**, 175 (2004)
[5] N N Rao, P K Shukla and M Y Yu, *Planet. Space Sci.* **38**, 543 (1990)
[6] P K Shukla and V P Silin, *Phys. Scr.* **45**, 508 (1992)
[7] A Barkan, R L Merlino and N DAngelo, *Phys. Plasmas* **2**, 10 (1995)
[8] F Melandso, *Phys. Plasmas* **3**, 3890 (1996)
[9] F Li and O Havnes, *Phys. Rev. E* **64**, 066407 (2001)
[10] W Lin, M S Murillo and Y Feng, *Phys. Rev. E* **100**, 4 (2019)
[11] M Marciante and M S Murillo, *Phys. Rev. Lett.* **118**, 2 (2017)
[12] P K Shukla and A A Mamun, *Introduction to dusty plasma physics* (IOP Publishing Ltd., England, 2002)
[13] V E Fortov, A V Ivlev, S A Khrapak and G E Morfill, *Phys. Rep.* **421**, 1 (2005)
[14] A Brattll, Q Havens and F Melandso, *J. Plasma Phys.* **58**, 691 (1997)
[15] V W Chow, D A Mendis and M J Rosenberg, *J. Geophys. Res.* **98**, 19056 (1993)
[16] M Horanyi and C K Goertz, *Astrophys. J.* **361**, 1 (1990)
[17] W S Duan, *Phys. Plasmas* **8**, 3583 (2001)
[18] P Meuris, F Verheest and G S Lakhina, *Planet. Space Sci.* **45**, 4 (1997)
[19] W S Duan and J Parkes, *Phys. Rev. E* **68**, 067402 (2003)
[20] W S Duan and Y R Shi, *Chaos Solitons Fractals* **18**, 321 (2003)
[21] W S Duan, H J Yang, Y R Shi and K P Lu, *Phys. Lett. A* **361**, 368 (2007)
[22] G Manfredi and F Haas, *Phys. Rev. B* **64**, 075316 (2001)
[23] C Gardner, *SIAM Soc. Ind. Appl. Math. J. Appl. Math.* **54**, 409 (1994)
[24] F Haas, G Manfredi and M Feix, *Phys. Rev. E* **62**, 2763 (2000)
[25] F Haas, L G Garcia, J Goedert and G Manfredi, *Phys. Plasmas* **10**, 3858 (2003)
[26] F Haas, *Phys. Plasmas* **12**, 062117 (2005)
[27] J F Han, D N Gao, H Zhang, X Y Wang and W S Duan, *Front. Phys.* **10**, 105201 (2015)
[28] D Kremp, T Bornath, M Bonitz and M Schlanges, *Phys. Rev. E* **60**, 4725 (1999)
[29] P A Makowich, C A Ringhofer and C Schmeiser, *Semiconductor equations* (Springer, Vienna, 1990)
[30] Y D Jung, *Phys. Plasmas* **8**, 3842 (2001)
[31] S Ali and P K Shukla, *Phys. Plasmas* **13**, 022313 (2006)
[32] W F El-Taibany and M Wadati, *Phys. Plasmas* **14**, 042302 (2007)
[33] S K El-Labany, N M El-Siragy, W F El-Taibany and E E Behery, *Phys. Plasmas* **16**, 093701 (2009)
[34] J Goswami, S Chandra and B Ghosh, *Astrophys. Space Sci.* **364**, 4 (2019)
[35] J Goswami, S Chandra, J Sarkar, S Chaudhuri and B Ghosh, *Breast Cancer Online* **38**, 1 (2020)
[36] X Qi, Y Xu, W S Duan and L Yang, *Phys. Plasmas* **21**, 1 (2014)
[37] S Sarkar, S Ghosh, M Khan and M R Gupta, *Phys. Plasmas* **18**, 093703 (2011)
[38] H W Yang, Z H Xu, D Z Yang, X R Feng, B S Yin and H H Dong, *Adv. Diff. Equ.* **2016**, 1 (2016)
[39] G Manfredi, *Fields Inst. Commun.* **46**, 263 (2005)
[40] F Haas, L G Garcia, J Goedert and G Manfriedi, *Phys. Plasmas* **10**, 3858 (2003)
[41] P K Shukla and S Ali, *Phys. Plasmas* **12**, 114502 (2005)
[42] S A El-Tantawy, *Astrophys. Space Sci.* **361**, 249 (2016)
[43] H W Yang *et al*, *Adv. Diff. Equ.* **2016**(**1**), 1 (2016)
[44] G J He, W S Duan and D X Tian, *Phys. Plasmas* **15**, 043702 (2008)
[45] E F EL-Shamy and A M Al-Asbali, *Phys. Plasmas* **21**, 093701 (2014).
[46] J Zhang, Y Yang, Y X Xu, L Yang, X Qi and W S Duan, *Phys. Plasmas* **21**, 103706 (2014)
[47] E C Whipple, T G Northrop and D A Mendis, *J. Geophys. Res. Space Phys.* **90**, A8 (1985)
[48] U D Angelis, V Formisano and M Giordano, *J. Plasma Phys.* **40**, 3 (1988)
[49] A A Mamun and M N Alam, *Indian J. Phys.* **72**(**5**), 559 (1998)
[50] N N Rao and P K Shukla, *Planet. Space Sci.* **42**(**3**), 221 (1994)
[51] T S Gill, H Kaur and N S Saini, *J. Plasma Phys.* **70**(**4**), 481–495 (2004)
[52] H R Pakzad and K Javidan, *Chaos Solitons Fractals* **42**(**5**), 2904 (2009)
[53] S V Singh, N N Rao and P K Shukla, *J. Plasma Phys.* **60**(**3**), 551 (1998)
[54] J K Xue and L P Zhang, *Chaos Solitons Fractals* **32**(**2**), 592 (2007)
[55] S K El-Labany *et al*, *Phys. Plasmas* **16**(**9**), 175 (2009)
[56] N A El-Bedwehy and W M Moslem, *Astrophys. Space Sci.* **335**, 435 (2011)
[57] P Meuris, *Planet. Space Sci.* **45**, 1171 (1997)
[58] H J Ren, Z W Wu, J T Cao and P K Chu, *Phys. Plasmas* **16**, 103705 (2009)