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Generalised charged anisotropic quark star models

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Abstract. We find new stellar models to the field equations for charged anisotropic spheres. We use linear quark equation of state for strange quark matter. We choose a new form of pressure anisotropy as a rational function. In our model, we regain previous isotropic and anisotropic stellar models as specific cases. Isotropic models regained are those found by Komathiraj and Maharaj, Mak and Harko, and Misner and Zapolsky. Anisotropic models regained include the performance by Maharaj, Sunzu and Ray; and Sunzu and Danford. We indicate that our model meets the stability and energy conditions. We also generate stellar masses consistent with observations.

Keywords. Field equations; quark stars; anisotropy; energy conditions; stellar masses.

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1. Introduction

By applying Einstein–Maxwell field equations, properties and features of stellar objects are described. Field equations are applied to model compact spheres like quark stars, dark energy stars, black holes, gravastars, dwarfs and neutron stars. Researchers are attracted to use Einstein–Maxwell field equations in modelling gravitating spheres. Anisotropic quark star models with electric fields are found by Sunzu *et al* [1]. In the work by Mak and Harko [2], compact isotropic models for the inter-relativistic compact sphere in hydrostatic equilibrium are generated. Within the framework of general relativity, Schwarzchild [3] found a model with astrophysical significance where exact solutions to Einstein's field equations were generated with the stress tensor energy momentum for a perfect fluid. Chaisi [4] found solutions for anisotropic matter using Einstein's field equations. These solutions had astrophysical significance. Using Einstein's field equations, Chaisi and Maharaj [5] found the solution for dense spheres with masses and surface redshifts consistent with relativistic stellar objects like Vela X-1 and Her X-1. Other models with astrophysical significance were found in the work by Sunzu and Danford [6] and Sunzu *et al* [7]. We note that, Einstein's field equations are significant in describing properties and features of stellar objects.

Charged stellar objects are described using the Einstein–Maxwell field equations in a static and spherically symmetric space–time. The presence of electric field plays a great role in analysing the behaviour of relativistic compact objects like quark stars. Models, in the presence of electric field, were studied by Mak and Harko [2] and Komathiraj and Maharaj [8]. It was observed by Esculpi and Aloma [9] and Ivanov [10] that the presence of electric field affects the red-shift, mass, luminosity and stability of the stellar objects. Recently, various approaches have been used to find solutions for the system of field equations. Ngubelanga *et al* [11] found a class of models for charged matter in spherically symmetric space–time in the presence of pressure anisotropy. Maurya and Govender [12] found solutions to the quark charged star of the family Her X-1. The solutions for charged anisotropic compact stars were generated by Maurya *et al* [13]. Mafa Takisa *et al* [14] found anisotropic charged models in the linear regime using Einstein–Maxwell field equations. Other models generated in the presence of electric field include the solutions given by Sunzu and Danford [6], Matondo and Maharaj [15], Maharaj *et al* [16] and Sunzu *et al* [7].

The pressure anisotropy is a very important aspect in modelling stellar objects. The pressure anisotropy affects stability, structure and physical properties of relativistic spheres. Sharma *et al* [17] suggested that pressure anisotropy is crucial for determining stability and other physical properties of relativistic stellar models. Gleiser and Dev [18] generated stellar

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models which show that pressure anisotropy significantly affects the physical structures of the relativistic spheres which may cause observational effects. It was observed that anisotropic pressure existing near the core of stellar objects may increase stability. It was also found that, positive measure of anisotropy improves the stability of stellar bodies. Dev and Gleiser [19] showed that in charged stellar objects, the presence of pressure anisotropy under radial adiabatic perturbations improves stability compared to isotropic spheres. Recent models, in the presence of anisotropy, are given by Ngubelanga *et al* [11], Sunzu and Danford [6], Sunzu *et al* [1], Maurya and Govender [12], Matondo and Maharaj [15] and Sunzu and Thomas [20].

Several types of equations of state can be used for modelling compact stellar objects. These include linear, quadratic, Van der Waals and polytropic equation of state. Maharaj and Mafa Takisa [21] found exact anisotropic models in the presence of electromagnetic field using polytropic equation of state. Thirukkanesh and Ragel [22], for particular choices of polytropic index, generated exact anisotropic models for uncharged sphere using polytropic equation of state. New solutions were found by Mafa Takisa and Maharaj [23] using polytropic equation of state. Ngubelanga and Maharaj [24] found new solutions for relativistic stars using different indices from polytropic equation of state.

A system of field equations formulated by Thirukkanesh and Ragel [25] using Van der Waals equation of state described anisotropic compact objects in static and spherically symmetric space–time. Malaver [26] generated a model for charged objects using Van der Waals equation of state. Feroze and Siddiqui [27] and Maharaj and Mafa Takisa [21] found exact models for anisotropic matter in the presence of electric fields using equation of state in quadratic form. The solutions to the system of field equations given by Ngubelanga *et al* [11] utilised quadratic equation of state. Maharaj and Mafa Takisa [21] found regular compact stellar models using quadratic equation of state.

There are many studies for charged and anisotropic matter with linear equation of state. These include models given by Thirukkanesh and Maharaj [28] for quark star and dark energy stars. Mafa Takisa and Maharaj [23] generated exact compact regular models using linear equation of state. Other models generated using linear equation of state include the studies performed by Maharaj *et al* [16], Sunzu *et al* [1], Ngubelanga *et al* [11] and Sharma and Maharaj [29].

In the studies conducted by Sunzu and Danford [6], Sunzu *et al* [7] and Maharaj *et al* [16], new solutions to the system of field equations were obtained using anisotropy in a polynomial form. It is therefore necessary to develop new models for anisotropic stars with new choice of anisotropy in a generalised form which is missing in other findings.

The main objective of this paper is therefore to find new models to the Einstein–Maxwell field equations for anisotropic charged matter in spherically static symmetry space–time using linear bag equation of state consistent with quark stars. We choose a generalised form of anisotropy as a rational function which contains choices made in the past as special cases. This paper is organised as follows: In §2, we give and transform the field equations. We then formulate the metric function and the measure of anisotropy to obtain the master differential equation that governs our models. In §3, we generate a singular quark star model that generalises several models found in the past. In §4, we generate a non-singular quark star model that regains some other models obtained in the past. In §5, we match the interior and exterior solutions at the surface of the star. We give physical analysis on the energy conditions, regularity, casuality and stability of the model in §6. The discussion is given in §7, and finally, the conclusion of our study is given in §8.

2. The basic model

In formulating the model, we consider the interior space–time to be static and spherically symmetric with the line element given as

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

where $\lambda(r)$ and $\nu(r)$ are the corresponding variables for gravitational potentials. The Reissner–Nordstrom line element for a charged object which describes the exterior space–time is expressed as

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{2}$$

where Q and M are charge and total mass respectively. For a charged anisotropic matter, the energy–momentum tensor is given by

$$\tau_{ij} = \text{diag}\left(-\rho - \frac{1}{2}E^2,\ p_r - \frac{1}{2}E^2,\ p_t + \frac{1}{2}E^2,\ p_t + \frac{1}{2}E^2\right), \tag{3}$$

where ρ is the energy density, E is the electric field, p_r is the radial pressure and p_t is the tangential pressure.

Nonlinear Einstein–Maxwell field equations for charged anisotropic fluid sphere in general relativity are given as

$$\frac{1}{r^2}\left(1-\mathrm{e}^{-2\lambda}\right)+\frac{2\lambda'}{r}\mathrm{e}^{-2\lambda}=\rho+\frac{1}{2}E^2, \tag{4a}$$

$$\frac{-1}{r^2}\left(1-\mathrm{e}^{-2\lambda}\right)+\frac{2\nu'}{r}\mathrm{e}^{-2\lambda}=p_r-\frac{1}{2}E^2, \tag{4b}$$

$$\mathrm{e}^{-2\lambda}\left(\nu''+\nu'^2-\nu'\lambda'+\frac{\nu'}{r}-\frac{\lambda'}{r}\right)=p_t+\frac{1}{2}E^2, \tag{4c}$$

$$\sigma=\frac{1}{r^2}\mathrm{e}^{-\lambda}\left(r^2E\right)', \tag{4d}$$

where σ is the proper charge density and the primes denote derivatives with respect to radial coordinate r.

In this paper, we consider a linear equation of state that relates energy density and the radial pressure as

$$p_r=\frac{1}{3}(\rho-4B), \tag{5}$$

where B is an arbitrary constant known as the Bag constant. It can be noted that this linear equation of state is consistent with quark stars and other strange matter given by Maharaj *et al* [16] in simple model for quark stars, models found by Sunzu *et al* [1], Sunzu and Danford [6], Komathiraj and Maharaj [8]. The mass contained inside the sphere of the stellar object is expressed as

$$m(r)=\frac{1}{2}\int_0^r \omega^2\rho(\omega)\mathrm{d}\omega. \tag{6}$$

The speed of sound for relativistic matter is given by

$$v=\frac{\mathrm{d}p_r}{\mathrm{d}\rho}. \tag{7}$$

The adiabatic index which determines the stability of the model is given by

$$\Gamma=\left(\frac{p_r+\rho}{p_r}\right)v. \tag{8}$$

By introducing the new functions, the field equations (4) and the line element (1) can simply be transformed into another form as

$$x=Cr^2,\quad Z(x)=\mathrm{e}^{-2\lambda(r)},\quad A^2y^2(x)=\mathrm{e}^{2\nu(r)}, \tag{9}$$

where A is an arbitrary real constant and $C>0$. Hence, from system (4), the linear equation of state, together with the Einstein–Maxwell field equations, can be expressed in the new transformed system as

$$\rho=3p_r+4B, \tag{10a}$$

$$\frac{p_r}{C}=Z\frac{\dot{y}}{y}-\frac{\dot{Z}}{2}-\frac{B}{C}, \tag{10b}$$

$$\Delta=4CxZ\frac{\ddot{y}}{y}+C\left(6Z+2x\dot{Z}\right)\frac{\dot{y}}{y}+C\left(2\left(\dot{Z}+\frac{B}{C}\right)+\frac{(Z-1)}{x}\right), \tag{10c}$$

$$p_t=p_r+\Delta, \tag{10d}$$

$$\frac{E^2}{2C}=\frac{(1-Z)}{x}-\frac{\dot{Z}}{2}-3Z\frac{\dot{y}}{y}-\frac{B}{C}, \tag{10e}$$

$$\sigma=2\sqrt{\frac{CZ}{x}}\left(x\dot{E}+E\right). \tag{10f}$$

From system (10) which is a transformed Einstein–Maxwell field equation incorporated with linear equation of state, we have eight variables $(\rho,\sigma,p_r,p_t,\Delta,Z,E,y)$. By specifying two of the quantities, exact solutions to the field equations (4) can be found by generating an ordinary differential equation. In our models, we are going to specify the measure of anisotropy Δ and the potential variable y. After introducing the transformation, the mass function (6) becomes

$$M(x)=\frac{1}{4C^{3/2}}\int_0^x\sqrt{\omega}\rho(\omega)\mathrm{d}\omega. \tag{11}$$

The choice for metric function that specify the potential variable is of the form

$$y=\left(a+x^u\right)^v, \tag{12}$$

where u, v and a are arbitrary constants. In modelling relativistic objects, the metric function above is significant as the potential variable y is continuous within the interior of stellar objects, regular and finite. This choice of metric function was also adopted by Komathiraj and Maharaj [8], Sunzu *et al* [1], Sunzu and Danford [6] and Maharaj *et al* [16]. We use the same metric function in our study in order to regain models generated in the past. Since u and v in eq. (12) are arbitrary real constants, we are going to find exact solutions of our models by examining the two cases; the first one is when $u=\frac{1}{2}$ and $v=1$. This will regain first models for quark stars with charge and anisotropy generated by Sunzu [30], Maharaj *et al* [16], relativistic anisotropic charged matter generated by Sunzu and Danford [6] and singular models found by Mak and Harko [2]. The second case is when $u=1$ and $v=2$, where we regain the second models obtained by Komathiraj and Maharaj [8], Sunzu *et al* [1], Maharaj *et al* [16] and Sunzu and Danford [6].

In this study, we choose the expression for measure of anisotropy to be a function of the form

$$\Delta=\frac{\sum_{i=1}^n\alpha_i x^i}{(1+qx)^j}\quad\text{for}\quad n\in\mathbb{N}\quad\text{and}\quad j\in\mathbb{Z}, \tag{13}$$

where α_i and q are arbitrary real constants. There are some motivations behind choosing this form of measure of anisotropy. It is continuous throughout the stellar interior. At the centre ($x=0$), the measure of anisotropy vanishes. This is physical for realistic stellar models as we expect the radial and tangential pressure to be equal at the centre of the star. This form of measure of anisotropy generalises some existing models including isotropic models obtained by Komathiraj and Maharaj [8]. When $q = 0$ or $j = 0$, we have $\Delta = \sum_{i=1}^{n} \alpha_i x^i$, which is the choice of anisotropy adopted by Sunzu *et al* [7]. Using our choice of measure of anisotropy, we can also regain previous models obtained by Sunzu *et al* [7] for $i = 1, 2, \ldots, 5$. When $j = 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = 0$, we regain models given by Sunzu and Danford [6]. If we set $j = 0$ and $\alpha_4 = \alpha_5 = 0$, we obtain the measure of anisotropy used by Sunzu and Thomas [20] and Sunzu *et al* [1]. Hence, the new solutions to the Einstein–Maxwell field equation (10) is likely to give new model for charged anisotropic objects. The important point is that, the anisotropy can be set to vanish so as to generate isotropic models. Therefore, our choice of the metric function and measure of anisotropy enables us to regain the isotropic and anisotropic models obtained by other researchers in the past.

We formulate the differential equation governing the model. The solution of this differential equation will enable us to obtain solutions of Einstein–Maxwell field equations. Substituting eq. (12) into eq. (10c), we obtain

$$\dot{Z} + \frac{\left[\left(4vu^2 + 2auv + 2a\right) x^u + \left(2uv + 4u^2v^2 + 1\right) x^{2u} + a^2\right] Z}{x\,(a + x^u)\,(2uvx^u + 2\,(a + x^u))} = \frac{\frac{x\Delta}{C}\,(a + x^u) + \left(1 - \frac{2xB}{C}\right)(a + x^u)}{x\,(2uvx^u + 2\,(a + x^u))}. \tag{14}$$

Generally, eq. (14) is a nonlinear ordinary differential equation which governs the model for charged quark star. Solving eq. (14), we obtain the metric function Z in terms of independent variable x. We choose values of u and v which enable us to regain results generated in the past. We then find the expressions for tangential pressure p_t, energy density ρ, radial pressure p_r and electric field intensity E in system (10). This can be done by using the specified metric function and measure of anisotropy in eq. (12) and eq. (13) respectively. The important aspect is that, for the case $\Delta = 0$ our solution will have isotropic property for stellar objects.

3. Generalised singular quark star model

In this section, we find the exact model corresponding to system (10). If we set the values of $u = \frac{1}{2}$ and $v = 1$, we regain the model given by Maharaj *et al* [16] and the singular model found by Mak and Harko [2]. Substituting these values in metric function (12), we obtain

$$y = a + \sqrt{x}. \tag{15}$$

Then, eq. (14) becomes

$$\dot{Z} + \left(\frac{1}{2x} + \frac{3}{2\sqrt{x}\,\left(2a + 3\sqrt{x}\right)}\right) Z = \frac{\left[1 - \frac{2xB}{C} + \frac{\Delta x}{C}\right]\left(a + \sqrt{x}\right)}{x\,\left(2a + 3\sqrt{x}\right)}. \tag{16}$$

From eq. (13), we set $n = 5$ and $j = -1$. Then, substituting eq. (13) into eq. (16) yields the solution

$$Z = \frac{3\left(2a + \sqrt{x}\right) - \frac{B}{C}\left(4ax + 3x^{3/2}\right) + \frac{3G(x)}{C} + \frac{3k}{C}}{3\left(2a + 3\sqrt{x}\right)}, \tag{17}$$

where $G(x)$ is given as

$$\begin{aligned} G(x) = {} & \alpha_1 \left(\frac{2}{5}ax^2 + \frac{1}{3}x^{5/2} + \frac{2}{7}aqx^3 + \frac{1}{4}qx^{7/2}\right) \\ & + \alpha_2 \left(\frac{2}{7}ax^3 + \frac{1}{4}x^{7/2} + \frac{2}{9}aqx^4 + \frac{1}{5}qx^{9/2}\right) \\ & + \alpha_3 \left(\frac{2}{9}ax^4 + \frac{1}{5}x^{9/2} + \frac{2}{11}aqx^5 + \frac{1}{6}qx^{11/2}\right) \\ & + \alpha_4 \left(\frac{2}{11}ax^5 + \frac{1}{6}x^{11/2} + \frac{2}{13}aqx^6 + \frac{1}{7}qx^{13/2}\right) \\ & + \alpha_5 \left(\frac{2}{13}ax^6 + \frac{1}{7}x^{13/2} + \frac{2}{15}aqx^7 + \frac{1}{8}qx^{15/2}\right), \end{aligned}$$

where k is an arbitrary real constant of integration. So, we let $k = 0$ for non-singularity in the potential Z. We observe that, $G(x) = 0$ at the centre of the star, which is also satisfied for isotropic pressure when $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$.

Finally, the matter variables and the gravitational potentials from system (10) corresponding to this model are given by

$$e^{2\nu} = A^2\left(a + \sqrt{x}\right)^2, \tag{18a}$$

$$e^{2\lambda} = \frac{3\left(2a + 3\sqrt{x}\right)}{3\left(2a + \sqrt{x}\right) - \frac{B}{C}\left(4ax + 3x^{3/2}\right) + \frac{3G(x)}{C}}, \tag{18b}$$

$$p_r = C\left[\frac{6a^2 + 10a\sqrt{x} + 3x}{2\sqrt{x}\left(a + \sqrt{x}\right)\left(2a + 3\sqrt{x}\right)^2}\right] - B\left[\frac{16a^3 + 81a^2\sqrt{x} + 120ax + 54x^{3/2}}{6\left(a + \sqrt{x}\right)\left(2a + 3\sqrt{x}\right)^2}\right]$$

$$-\left[\frac{K(x)}{2\sqrt{x}\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right],$$

$$\rho = 3C\left[\frac{6a^2+10a\sqrt{x}+3x}{2\sqrt{x}\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right] + B\left[\frac{16a^3+47a^2\sqrt{x}+48ax+18x^{3/2}}{2\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right] - 3\left[\frac{K(x)}{2\sqrt{x}\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right], \tag{18c}$$

$$p_t = C\left[\frac{6a^2+10a\sqrt{x}+3x}{2\sqrt{x}\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)}\right] - B\left[\frac{16a^3+81a^2\sqrt{x}+120ax+54x^{3/2}}{6\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right] + \frac{L(x)}{2\sqrt{x}\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}, \tag{18d}$$

$$\Delta = \frac{\alpha_1 x+\alpha_2 x^2+\alpha_3 x^3+\alpha_4 x^4+\alpha_5 x^5}{(1+qx)^{-1}}, \tag{18e}$$

$$E^2 = C\left[\frac{-2a^2-2a\sqrt{x}+3x}{2\sqrt{x}\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right] + B\left[\frac{a^2\sqrt{x}+2ax}{2\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right] - \frac{N(x)}{\sqrt{x}\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}, \tag{18f}$$

where

$$\begin{aligned}
K(x) &= \alpha_1\left[\frac{8}{5}a^3x+\frac{64}{15}a^2x^{3/2}+a\left(\frac{18}{5}+\frac{12}{7}a^2q\right)x^2 + \left(1+\frac{141}{28}a^2q\right)x^{5/2}+\frac{67}{14}aqx^3+\frac{3}{2}qx^{7/2}\right]\\
&+\alpha_2\left[\frac{12}{7}a^3x^2+\frac{141}{28}a^2x^{5/2} +a\left(\frac{67}{14}+\frac{16}{9}a^2q\right)x^3+\left(\frac{3}{2}+\frac{82}{15}a^2q\right)x^{7/2} +\frac{82}{15}aqx^4+\frac{9}{5}qx^{9/2}\right]\\
&+\alpha_3\left[\frac{16}{9}a^3x^3+\frac{82}{15}a^2x^{7/2} +a\left(\frac{82}{15}+\frac{20}{11}a^2q\right)x^4+\left(\frac{9}{5}+\frac{379}{66}a^2q\right)x^{9/2} +\frac{65}{11}aqx^5+2qx^{11/2}\right]\\
&+\alpha_4\left[\frac{20}{11}a^3x^4+\frac{379}{66}a^2x^{9/2} +a\left(\frac{65}{11}+\frac{24}{13}a^2q\right)x^5+\left(2+\frac{540}{91}a^2q\right)x^{11/2} +\frac{566}{91}aqx^6+\frac{15}{7}qx^{13/2}\right]\\
&+\alpha_5\left[\frac{24}{13}a^3x^5-\frac{48}{91}a^2x^{11/2} -a\left(\frac{190}{91}-\frac{28}{15}a^2q\right)x^6+\left(\frac{15}{7}+\frac{243}{40}a^2q\right)x^{13/2} +\frac{129}{20}aqx^7+\frac{9}{4}qx^{15/2}\right],
\end{aligned}$$

$$\begin{aligned}
L(x) &= \alpha_1\left[\frac{32}{5}a^3x+\frac{416}{15}a^2x^{3/2} +a\left(\frac{192}{5}+\frac{44}{7}a^2q\right)x^2+\left(17+\frac{755}{28}a^2q\right)x^{5/2} +\frac{521}{14}aqx^3+\frac{33}{2}qx^{7/2}\right]\\
&+\alpha_2\left[\frac{44}{7}a^3x^2+\frac{755}{28}a^2x^{5/2} +a\left(\frac{521}{14}+\frac{56}{9}a^2q\right)x^3+\left(\frac{32}{2}+\frac{398}{15}a^2q\right)x^{7/2} +\frac{548}{14}aqx^4+\frac{81}{5}qx^{9/2}\right]\\
&+\alpha_3\left[\frac{56}{9}a^3x^3+\frac{398}{15}a^2x^{7/2} +a\left(\frac{548}{15}+\frac{68}{11}a^2q\right)x^4+\left(\frac{81}{5}+\frac{1733}{66}a^2q\right)x^{9/2} +\frac{397}{11}aqx^5+16qx^{11/2}\right]\\
&+\alpha_4\left[\frac{68}{11}a^3x^4+\frac{1733}{66}a^2x^{9/2} +a\left(\frac{397}{11}+\frac{80}{13}a^2q\right)x^5+\left(16+\frac{2372}{91}a^2q\right)x^{11/2} +\frac{3256}{91}aqx^6+\frac{111}{7}qx^{13/2}\right]\\
&+\alpha_5\left[\frac{80}{13}a^3x^5+\frac{2960}{91}a^2x^{11/2} +a\left(\frac{4012}{91}+\frac{92}{15}a^2q\right)x^6+\left(\frac{111}{7}+\frac{1037}{40}a^2q\right)x^{13/2} +\frac{711}{20}aqx^7+\frac{63}{4}qx^{15/2}\right],
\end{aligned}$$

$$
\begin{aligned}
N(x) &= \alpha_1\left[\frac{16}{5}a^3x^{3/2}+\frac{64}{5}a^2x^2 + a\left(\frac{84}{5}+\frac{20}{7}a^2q\right)x^{5/2}+\left(7+\frac{313}{28}a^2q\right)x^3 + \frac{101}{7}aqx^{7/2}+6qx^4\right]\\
&+\alpha_2\left[\frac{20}{7}a^3x^{5/2}+\frac{313}{28}a^2x^3 + a\left(\frac{101}{7}+\frac{8}{3}a^2q\right)x^{7/2}+\left(6+\frac{154}{15}a^2q\right)x^4 + \frac{196}{15}aqx^{9/2}+\frac{27}{5}qx^5\right]\\
&+\alpha_3\left[\frac{8}{3}a^3x^{7/2}+\frac{154}{15}a^2x^4 + a\left(\frac{196}{15}+\frac{28}{11}a^2q\right)x^{9/2}+\left(\frac{27}{5}+\frac{213}{22}a^2q\right)x^5 + \frac{134}{11}aqx^{11/2}+5qx^6\right]\\
&+\alpha_4\left[\frac{28}{11}a^3x^{9/2}+\frac{213}{22}a^2x^5 + a\left(\frac{134}{11}+\frac{32}{13}a^2q\right)x^{11/2}+\left(5+\frac{844}{91}a^2q\right)x^6 + \frac{1052}{91}aqx^{13/2}+\frac{33}{7}qx^7\right]\\
&+\alpha_5\left[\frac{128}{13}a^3x^{11/2}+\frac{3280}{91}a^2x^6 + a\left(\frac{3320}{91}+\frac{12}{5}a^2q\right)x^{13/2}+\left(\frac{33}{7}+\frac{35}{40}a^2q\right)x^7 + \frac{111}{10}aqx^{15/2}+\frac{9}{2}qx^8\right].
\end{aligned}
$$

From system (18), the corresponding line element is

$$
\mathrm{d}s^2 = -A^2\left(a+\sqrt{x}\right)^2\mathrm{d}t^2 + \left[\frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right)-\frac{B}{C}\left(4ax+3x^{3/2}\right)+\frac{3G(x)}{C}}\right]\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right). \tag{20}
$$

Using the system of equations (18), we get

$$
\begin{aligned}
\frac{\mathrm{d}p_r}{\mathrm{d}x} &= -C\left[\frac{12a^4+78a^3\sqrt{x}+146a^2x+99ax^{\frac{3}{2}}+18x^2}{4\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3x^{\frac{3}{2}}}\right]\\
&-B\left[\frac{34a^4+93a^3\sqrt{x}+78a^2x+18ax^{\frac{3}{2}}}{12\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3\sqrt{x}}\right]\\
&-\left[\frac{\left(8a^3\sqrt{x}+32a^2x+42ax^{\frac{3}{2}}+18x^2\right)\dot{K}(x)}{4x\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3}\right]\\
&+\left[\frac{\left(32a^2+4a^3xx^{\frac{-1}{2}}+63a\sqrt{x}+36x\right)K(x)+}{4x\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3}\right],
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}x} &= -3C\left[\frac{12a^4+78a^3\sqrt{x}+146a^2x+99ax^{3/2}+18x^2}{4\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3x^{\frac{3}{2}}}\right]\\
&-3B\left[\frac{34a^4+93a^3\sqrt{x}+78a^2x+18ax^{3/2}}{12\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3\sqrt{x}}\right]\\
&-3\left[\frac{\left(8a^3\sqrt{x}+32a^2x+42ax^{3/2}+18x^2\right)\dot{K}(x)}{4x\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3}\right]\\
&+3\left[\frac{\left(32a^2+4a^3xx^{-1/2}+63a\sqrt{x}+36x\right)K(x)}{4x\left(a+\sqrt{x}\right)^2\left(2a+3\sqrt{x}\right)^3}\right].
\end{aligned} \tag{22}
$$

Clearly, from eqs (21) and (22), the speed of sound $v = \frac{1}{3}$. This agrees with eq. (5). Using eq. (8) and system (18), the adiabatic index for this model becomes

$$
\Gamma = \frac{C\left(216a^2+360a\sqrt{x}+108x+108x\right)+B(32a^3\sqrt{x}+60a^2x+24ax^{3/2})-36K(x)}{C\left(54a^2+90a\sqrt{x}+27x\right)-B(48a^3\sqrt{x}+243a^2x+360ax^{3/2}+162x^2)-9K(x)}. \tag{23}
$$

If we set $q = 0$ and $\alpha_4 = \alpha_5 = 0$, we regain the anisotropic model obtained by Maharaj *et al* [16] with the line element

$$\mathrm{d}s^2 = -A^2\left(a+\sqrt{x}\right)^2\mathrm{d}t^2 + \left[\frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right)-\frac{B}{C}\left(4ax+3x^{3/2}\right)+3\frac{G_*(x)}{C}}\right]\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right), \quad (24)$$

where

$$G_*(x) = \alpha_1\left(\frac{2}{5}ax^2+\frac{1}{3}x^{5/2}\right)+\alpha_2\left(\frac{2}{7}ax^3+\frac{1}{4}x^{7/2}\right) +\alpha_3\left(\frac{2}{9}ax^4+\frac{1}{5}x^{9/2}\right).$$

If we set $q = 0$ and $\alpha_1 = \alpha_2 = \alpha_5 = 0$ in system (18) and eq. (20), we regain the results obtained by Sunzu and Danford [6]. The line element (1) for this case is given as

$$\mathrm{d}s^2 = -A^2\left(a+\sqrt{x}\right)^2\mathrm{d}t^2 + \left[\frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right)-\frac{B}{C}\left(4ax+3x^{3/2}\right)+3\frac{N_*(x)}{C}}\right]\mathrm{d}r^2 +r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right),$$

where

$$N_*(x)=\alpha_3\left(\frac{2}{9}ax^4+\frac{1}{5}x^{9/2}\right)+\alpha_4\left(\frac{2}{11}ax^5+\frac{1}{6}x^{11/2}\right).$$

Moreover, if we let the measure of anisotropy to be zero, that is, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$, we regain the isotropic models for quark stars obtained by Komathiraj and Maharaj [8]. The line element (1) for this case is given as

$$\mathrm{d}s^2 = -A^2\left(a+\sqrt{x}\right)^2\mathrm{d}t^2 + \left[\frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right)-\frac{B}{C}\left(4ax+3x^{3/2}\right)}\right]\mathrm{d}r^2 +r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right).$$

If we set $q = 0$, $\Delta = 0$ and $a = 0$ in system (18), we regain the model found by Mak and Harko [31] with the line element

$$\mathrm{d}s^2 = -A^2Cr^2\,\mathrm{d}t^2+\left[\frac{3}{1-Br^2}\right]\mathrm{d}r^2 +r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right). \quad (25)$$

Furthermore, when we set $q = 0$, $\Delta = 0$, $a = 0$ and $B = 0$ in system (18), we obtain the solutions given by Misner and Zapolsky [32], which is a particular solution obtained using a linear equation of state $p = \frac{1}{3}\rho$. This indicates that with this choice of u and v, we regain several isotropic and anisotropic models generated in the past.

4. Generalised non-singular quark star model

In this section, we set $u = 1$ and $v = 2$. With this choice, we regain the second model generated by Maharaj *et al* [16], Sunzu and Danford [6] and Sunzu *et al* [1]. The metric function (12) becomes

$$y = (a+x)^2. \quad (26)$$

Using these values of u and v, eq. (14) becomes

$$\dot{Z}+\left(\frac{1}{2x}+\frac{2}{a+x}+\frac{3}{a+3x}\right)Z = \frac{\left(1-\frac{2xB}{C}+\frac{\Delta x}{C}\right)(a+x)}{2x\,(a+3x)}. \quad (27)$$

Substituting eq. (13) into eq. (27), we obtain the solution

$$Z = \frac{35a^3+35a^2x+21ax^2+5x^3}{35\,(a+x)^2\,(a+3x)} + \frac{\frac{H(x)}{C}}{(a+x)^2\,(a+3x)} - \frac{2B}{C}\left(\frac{105a^3x+189a^2x^2+135ax^3+35x^4}{315\,(a+x)^2\,(a+3x)}\right) + \frac{k}{2\sqrt{x}\,(a+x)^2\,(a+3x)}, \quad (28)$$

where $H(x)$ can be expressed as

$$\begin{aligned} H(x) = {} & \alpha_1\left(\frac{1}{5}a^3x^2+\frac{1}{7}a^2(3+aq)x^3+\frac{3}{9}a(1+aq)x^4 +\frac{1}{11}(1+3aq)x^5+\frac{1}{13}qx^6\right)\\ & +\alpha_2\left(\frac{1}{7}a^3x^3+\frac{1}{9}a^2(3+aq)x^4 +\frac{3}{11}a(1+aq)x^5+\frac{1}{13}(1+3aq)x^6+\frac{1}{15}qx^7\right)\\ & +\alpha_3\left(\frac{1}{9}a^3x^4+\frac{1}{11}a^2(3+aq)x^5 +\frac{3}{13}a(1+aq)x^6+\frac{1}{15}(1+3aq)x^7+\frac{1}{17}qx^8\right)\\ & +\alpha_4\left(\frac{1}{11}a^3x^5+\frac{1}{13}a^2(3+aq)x^6 +\frac{3}{15}a(1+aq)x^7+\frac{1}{17}(1+3aq)x^8+\frac{1}{19}qx^9\right) \end{aligned}$$

$$+\alpha_5\left(\frac{1}{13}a^3x^6+\frac{1}{15}a^2(3+aq)x^7+\frac{3}{17}a(1+aq)x^8+\frac{1}{19}(1+3aq)x^9+\frac{1}{21}qx^{10}\right).$$

We set $k=0$ in order to avoid singularity in the gravitational potential Z.

The matter variables and the gravitational potentials under this model are summarised as

The corresponding line element (1) for system (29) is given as

$$\mathrm{d}s^2=-A^2(a+x)^4\,\mathrm{d}t^2+\left[\frac{35a^3+35a^2x+21ax^2+5x^3}{35(a+x)^2(a+3x)}+\frac{\frac{H(x)}{C}}{(a+x)^2(a+3x)}\right.$$

$$\mathrm{e}^{2\nu}=A^2(a+x)^4, \tag{29a}$$

$$\mathrm{e}^{2\lambda}=\left[\frac{35a^3+35a^2x+21ax^2+5x^3}{35(a+x)^2(a+3x)}+\frac{\frac{H(x)}{C}}{(a+x)^2(a+3x)}-\frac{2B}{C}\left(\frac{105a^3x+189a^2x^2+135ax^3+35x^4}{315(a+x)^2(a+3x)}\right)\right]^{-1}, \tag{29b}$$

$$p_r=C\left[\frac{140a^4+434a^3x+318a^2x^2+150ax^3+30x^4}{35(a+x)^3(a+3x)^2}\right]-B\left[\frac{210a^5+2982a^4x+11124a^3x^2+16780a^2x^3+11770ax^4+3150x^5}{315(a+x)^3(a+3x)^2}\right]+\frac{\Pi(x)}{105(a+x)^3(a+3x)^2}, \tag{29c}$$

$$\rho=C\left[\frac{420a^4+1302a^3x+954a^2x^2+450ax^3+90x^4}{35(a+x)^3(a+3x)^2}\right]+B\left[\frac{210a^5+798a^4x+1476a^3x^2+2540a^2x^3+2090ax^4+630x^5}{105(a+x)^3(a+3x)^2}\right]+\frac{3\Pi(x)}{105(a+x)^3(a+3x)^2}, \tag{29d}$$

$$p_t=C\left[\frac{140a^4+434a^3x+318a^2x^2+150ax^3+30x^4}{35(a+x)^3(a+3x)^2}\right]-B\left[\frac{210a^5+2982a^4x+11124a^3x^2+16780a^2x^3+11770ax^4+3150x^5}{315(a+x)^3(a+3x)^2}\right]+\frac{\Sigma(x)}{105(a+x)^3(a+3x)^2}, \tag{29e}$$

$$\Delta=\frac{\alpha_1x+\alpha_2x^2+\alpha_3x^3+\alpha_4x^4+\alpha_5x^5}{(1+qx)^{-1}}, \tag{29f}$$

$$E^2=C\left[\frac{196a^3x^2+1452a^2x^3+1356ax^4+420x^5}{35x(a+x)^3(a+3x)^2}\right]-B\left[\frac{168a^4x^2+1296a^3x^3+6528a^2x^4+7280ax^5+2520x^6}{315x(a+x)^3(a+3x)^2}\right]-\frac{\delta(x)}{315(a+x)^3(a+3x)^2}. \tag{29g}$$

$$-\frac{2B}{C}\left(\frac{105a^3x+189a^2x^2+135ax^3+35x^4}{315\,(a+x)^2\,(a+3x)}\right)\Bigg]^{-1}\mathrm{d}r^2$$
$$+\,r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right). \qquad (30)$$

For simplicity, we have set

$$\begin{aligned}H(x)=\alpha_1&\left(\frac{1}{5}a^3x^2+\frac{1}{7}a^2\,(3+aq)\,x^3+\frac{3}{9}a\,(1+aq)\,x^4\right.\\&\left.+\frac{1}{11}\,(1+3aq)\,x^5+\frac{1}{13}qx^6\right)\\&+\alpha_2\left(\frac{1}{7}a^3x^3+\frac{1}{9}a^2\,(3+aq)\,x^4\right.\\&\left.+\frac{3}{11}a\,(1+aq)\,x^5+\frac{1}{13}\,(1+3aq)\,x^6+\frac{1}{15}qx^7\right)\\&+\alpha_3\left(\frac{1}{9}a^3x^4+\frac{1}{11}a^2\,(3+aq)\,x^5\right.\\&\left.+\frac{3}{13}a\,(1+aq)\,x^6+\frac{1}{15}\,(1+3aq)\,x^7+\frac{1}{17}qx^8\right)\\&+\alpha_4\left(\frac{1}{11}a^3x^5+\frac{1}{13}a^2\,(3+aq)\,x^6\right.\\&\left.+\frac{3}{15}a\,(1+aq)\,x^7+\frac{1}{17}\,(1+3aq)\,x^8+\frac{1}{19}qx^9\right)\\&+\alpha_5\left(\frac{1}{13}a^3x^6+\frac{1}{15}a^2\,(3+aq)\,x^7\right.\\&\left.+\frac{3}{17}a\,(1+aq)\,x^8+\frac{1}{19}\,(1+3aq)\,x^9+\frac{1}{21}qx^{10}\right),\end{aligned}$$

$$\begin{aligned}\Pi(x)=\alpha_1&\left[-21a^5x-a^4\left(57+\frac{45}{2}aq\right)x^2\right.\\&+a^3\left(20-\frac{185}{2}aq\right)x^3\\&+a^2\left(\frac{1360}{11}-\frac{1145}{11}aq\right)x^4\\&+a\left(105-\frac{315}{13}aq\right)x^5\\&\left.+\left(\frac{315}{11}+\frac{7245}{286}aq\right)x^6+\frac{315}{26}qx^7\right]\\&+\alpha_2\left[-\frac{45}{2}a^5x^2-a^4\left(\frac{185}{2}+\frac{70}{3}aq\right)x^3\right.\\&-a^3\left(\frac{1145}{11}+\frac{3710}{33}aq\right)x^4\\&-a^2\left(\frac{315}{13}+\frac{2310}{13}aq\right)x^5\\&+a\left(\frac{7245}{286}-\frac{17206}{143}aq\right)x^6\\&\left.+\left(\frac{315}{26}-\frac{392}{13}aq\right)x^7\right]\\&+\alpha_3\left[-\frac{70}{3}a^5x^3-a^4\left(\frac{3710}{33}+\frac{525}{22}aq\right)x^4\right.\\&-a^3\left(\frac{2310}{13}+\frac{3255}{26}aq\right)x^5\\&-a^2\left(\frac{17206}{143}+\frac{32403}{143}aq\right)x^6\\&-a\left(\frac{392}{13}+\frac{41517}{221}aq\right)x^7\\&\left.-\frac{2415}{34}aqx^8-\frac{315}{34}qx^9\right)\\&+\alpha_4\left[-\frac{525}{22}a^5x^4-a^4\left(\frac{3255}{26}+\frac{315}{13}aq\right)x^5\right.\\&-a^3\left(\frac{32403}{143}+\frac{1743}{13}aq\right)x^6\\&-a^2\left(\frac{41517}{221}+\frac{57792}{221}aq\right)x^7\\&-a\left(\frac{2415}{34}+\frac{76860}{323}aq\right)x^8\\&\left.-\left(\frac{315}{34}+\frac{33075}{323}aq\right)x^9-\frac{315}{19}qx^{10}\right]\\&+\alpha_5\left[-\frac{315}{13}a^5x^5-a^4\left(\frac{1743}{13}+\frac{49}{2}aq\right)x^6\right.\\&-a^3\left(\frac{57792}{221}+\frac{4781}{34}aq\right)x^7\\&-a^2\left(\frac{76860}{321}+\frac{92925}{323}aq\right)x^8\\&-a\left(\frac{33075}{323}+\frac{89345}{323}aq\right)x^9\\&\left.-\left(\frac{315}{19}+\frac{4835}{38}aq\right)x^{10}-\frac{45}{2}qx^{11}\right],\end{aligned}$$

$$\begin{aligned}\Sigma(x)=\alpha_1&\left[84a^5x+a^4\left(888+\frac{165}{2}aq\right)x^2\right.\\&+a^3\left(3170+\frac{1705}{2}aq\right)x^3\\&+a^2\left(\frac{54490}{11}+\frac{33505}{11}aq\right)x^4\\&+a\left(3570-\frac{62474}{13}aq\right)x^5\\&\left.+\left(\frac{10710}{11}+\frac{998235}{286}aq\right)x^6+\frac{24885}{26}qx^7\right]\\&+\alpha_2\left[-\frac{165}{2}a^5x^2-a^4\left(\frac{1705}{2}+\frac{245}{3}aq\right)x^3\right.\end{aligned}$$

$$
\begin{aligned}
&-a^3\left(\frac{33505}{11}+\frac{27475}{33}aq\right)x^4\\
&+a^2\left(\frac{62475}{13}+\frac{38640}{13}aq\right)x^5\\
&+a\left(\frac{998235}{286}+\frac{673484}{143}aq\right)x^6\\
&+\left(\frac{24885}{26}+\frac{44653}{13}aq\right)x^7+945qx^8\Bigg]\\
&+\alpha_3\left[\frac{245}{3}a^5x^3+a^4\left(\frac{27475}{33}+\frac{1785}{22}aq\right)x^4\right.\\
&+a^3\left(\frac{38640}{13}+\frac{21315}{26}aq\right)x^5\\
&+a^2\left(\frac{673484}{143}+\frac{418047}{143}aq\right)x^6\\
&+a\left(\frac{44653}{13}+\frac{1025913}{221}aq\right)x^7\\
&+\left(945+\frac{115395}{34}aq\right)x^8+\frac{31815}{34}qx^9\Bigg]\\
&+\alpha_4\left[-\frac{1785}{22}a^5x^4+a^4\left(\frac{21315}{26}+\frac{1050}{13}aq\right)x^5\right.\\
&+a^3\left(\frac{418047}{143}+\frac{10542}{13}aq\right)x^6\\
&+a^2\left(\frac{1025913}{221}+\frac{638358}{221}aq\right)x^7\\
&+a\left(\frac{115395}{34}+\frac{1483230}{323}aq\right)x^8\\
&+\left(\frac{31815}{34}+\frac{1086120}{323}aq\right)x^9+\frac{17640}{19}qx^{10}\Bigg]\\
&+\alpha_5\left[\frac{1050}{13}a^5x^5+a^4\left(\frac{10542}{13}+\frac{161}{2}aq\right)x^6\right.\\
&+a^3\left(\frac{638358}{221}+\frac{27349}{34}aq\right)x^7\\
&+a^2\left(\frac{1483230}{323}+\frac{924525}{323}aq\right)x^8\\
&+a\left(\frac{1086120}{323}+\frac{1470745}{123}aq\right)x^9\\
&+\left(\frac{17640}{19}+\frac{126835}{38}aq\right)x^{10}+\frac{1845}{2}qx^{11}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\delta(x)=\;&\alpha_1\Big[252a^5x+a^4\left(2124+225aq\right)x^2\\
&+a^3\left(6732+1845aq\right)x^3\\
&+a^2\left(\frac{100380}{11}+\frac{63210}{11}aq\right)x^4\\
&+a\left(\frac{63000}{11}+\frac{1133370}{143}aq\right)x^5\\
&+\left(\frac{15120}{11}+\frac{55755}{11}aq\right)x^6+\frac{16065}{13}qx^7\Bigg]\\
&+\alpha_2\Big[225a^5x^2+a^4\left(1845+210aq\right)x^3\\
&+a^3\left(\frac{63210}{11}+\frac{18550}{11}aq\right)x^4\\
&+a^2\left(\frac{1133370}{143}+\frac{738360}{143}aq\right)x^5\\
&+a\left(\frac{55755}{11}+\frac{78624}{11}aq\right)x^6\\
&+\left(\frac{16065}{13}+\frac{59934}{13}aq\right)x^7+1134qx^8\Bigg]\\
&+\alpha_3\left[210a^5x^3+a^4\left(\frac{18550}{11}+\frac{2205}{11}aq\right)x^4\right.\\
&+a^3\left(\frac{738360}{143}+\frac{226485}{143}aq\right)x^5\\
&+a^2\left(\frac{78624}{11}+\frac{52542}{11}aq\right)x^6\\
&+a\left(\frac{59934}{13}+\frac{1458954}{221}aq\right)x^7\\
&+\left(1134+\frac{72639}{17}aq\right)x^8+\frac{17955}{17}qx^9\Bigg]\\
&+\alpha_4\left[\frac{2205}{11}a^5x^4+a^4\left(\frac{226485}{143}+\frac{2520}{13}aq\right)x^5\right.\\
&+a^3\left(\frac{52542}{11}+1512aq\right)x^6\\
&+a^2\left(\frac{1458954}{221}+\frac{994644}{221}aq\right)x^7\\
&+a\left(\frac{72639}{17}+\frac{2001636}{323}aq\right)x^8\\
&+\left(\frac{17955}{17}+\frac{1296540}{323}aq\right)x^9+\frac{18900}{19}qx^{10}\Bigg]\\
&+\alpha_5\Big[\frac{2520}{13}a^5x^5+a^4\left(1512+189aq\right)x^6\\
&+a^3\left(\frac{994644}{221}+\frac{24801}{17}aq\right)x^7\\
&+a^2\left(\frac{2001636}{323}+\frac{1386882}{323}aq\right)x^8\\
&+a\left(\frac{1296540}{323}+\frac{1900890}{323}aq\right)x^9\\
&+\left(\frac{18900}{19}+\frac{72375}{19}aq\right)x^{10}+945qx^{11}\Bigg].
\end{aligned}
$$

From eq. (29c) we have

$$\frac{\mathrm{d}p_r}{\mathrm{d}x} = -C\left[\frac{826a^5+3634a^4x+5076a^3x^2+2292a^2x^3+690ax^4+90x^5}{35\,(a+x)^4\,(a+3x)^3}\right] - B\left[\frac{1092a^6+4188a^5x+3432a^4x^2-2696a^3x^3-2540a^2x^4-660x^5}{315\,(a+x)^4\,(a+3x)^3}\right] + \left[\frac{\left(105\,(a+x)^3\left(a+3x\right)^2\right)\dot{\Pi}(x)-315\,(a+x)^2\left(3a^2+14ax+15x^2\right)\Pi(x)}{105^2\,(a+x)^6\,(a+3x)^4}\right] \quad (31)$$

and from eq. (29d) we get

$$\frac{\mathrm{d}\rho}{\mathrm{d}x} = -3C\left[\frac{826a^5+3634a^4x+5076a^3x^2+2292a^2x^3+690ax^4+90x^5}{35\,(a+x)^4\,(a+3x)^3}\right] - 3B\left[\frac{1092a^6+4188a^5x+3432a^4x^2-2696a^3x^3-2540a^2x^4-660x^5}{315\,(a+x)^4\,(a+3x)^3}\right] + 3\left[\frac{\left(105\,(a+x)^3\left(a+3x\right)^2\right)\dot{\Pi}(x)-315\,(a+x)^2\left(3a^2+14ax+15x^2\right)\Pi(x)}{105^2\,(a+x)^6\,(a+3x)^4}\right]. \quad (32)$$

From eqs (21) and (32), the speed of sound for this model is $v=\frac{1}{3}$. This result satisfies eq. (5). The adiabatic index for this model is given as

$$\Gamma = \left[\frac{\begin{array}{c}B\left(420a^5-588a^4x-669a^3x^2-9160a^2x^3-5500ax^4-1260x^5\right)\\ +C\left(5040a^4+15624a^3x+11448a^2x^2+5400ax^3+1080x^4\right)+12\Pi\end{array}}{\begin{array}{c}3B\left(210a^5+2982a^4x+11124a^3x^2+16780a^2x^3+11770ax^4+3150x^5\right)\\ -3C\left(1260a^4+3906a^3x+2862a^2x^2+1350ax^3+270x^4\right)-9\Pi\end{array}}\right]. \quad (33)$$

From our model, if we set $q=0$ and $\alpha_4=\alpha_5=0$, we regain the anisotropic model given by Maharaj *et al* [16] with the line element given as

$$\mathrm{d}s^2 = -A^2\,(a+x)^4\,\mathrm{d}t^2 + \left[\frac{35a^3+35a^2x+21ax^2+5x^3}{35\,(a+x)^2\,(a+3x)} + \frac{\frac{H_*(x)}{C}}{(a+x)^2\,(a+3x)} - \frac{2B}{C}\left(\frac{105a^3x+189a^2x^2+135ax^3+35x^4}{315\,(a+x)^2\,(a+3x)}\right)\right]^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right), \quad (34)$$

where

$$H_*(x) = \alpha_1\left(\frac{1}{5}a^3x^2+\frac{3}{7}a^2x^3+\frac{3}{9}ax^4+\frac{1}{11}x^5\right) + \alpha_2\left(\frac{1}{7}a^3x^3+\frac{3}{9}a^2x^4+\frac{3}{11}ax^5+\frac{1}{13}x^6\right) + \alpha_3\left(\frac{1}{9}a^3x^4+\frac{3}{11}a^2x^5+\frac{3}{13}ax^6+\frac{1}{15}x^7\right).$$

If we set $q=0$ and $\alpha_1=\alpha_2=\alpha_5=0$ in system (18) and eq. (20), we regain the results obtained by Sunzu and Danford [6] with the line element given as

$$\mathrm{d}s^2 = -A^2\,(a+x)^4\,\mathrm{d}t^2 + \left[\frac{35a^3+35a^2x+21ax^2+5x^3}{35\,(a+x)^2\,(a+3x)}\right.$$

$$+\frac{\frac{H_{**}(x)}{C}}{(a+x)^2\,(a+3x)} - \frac{2B}{C}\left(\frac{105a^3x+189a^2x^2+135ax^3+35x^4}{315\,(a+x)^2\,(a+3x)}\right)\Bigg]^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2), \quad (35)$$

where

$$H_{**}(x) = \alpha_3\left(\frac{1}{9}a^3x^4+\frac{3}{11}a^2x^5+\frac{1}{13}ax^6+\frac{1}{15}x^7\right) + \alpha_4\left(\frac{1}{11}a^3x^5+\frac{3}{13}a^2x^6+\frac{3}{15}ax^7+\frac{1}{17}x^8\right).$$

Moreover, if we set the measure of anisotropy to be zero, that is, $\alpha_1=\alpha_2=\alpha_3=\alpha_4=\alpha_5=0$, we regain the isotropic models for quark stars obtained by Komathiraj and Maharaj [8]. The line element (1) for this case is given as

$$\mathrm{d}s^2 = -A^2\,(a+x)^4\,\mathrm{d}t^2 + \left[\frac{35a^3+35a^2x+21ax^2+5x^3}{35\,(a+x)^2\,(a+3x)} - \frac{2B}{C}\left(\frac{105a^3x+189a^2x^2+135ax^3+35x^4}{315\,(a+x)^2\,(a+3x)}\right)\right]^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2). \quad (37)$$

The mass function (11) using $\rho(x)$ obtained in eq. (29d) becomes

$$+\alpha_2x^{1/2}\Bigg[a^3\left(\frac{529}{27027}-\frac{1873}{173745}aq\right) - a^2\left(\frac{1}{117}-\frac{34}{5265}aq\right)x - a\left(\frac{1}{165}+\frac{23}{7425}aq\right)x^2 + \left(\frac{1}{364}-\frac{4}{585}aq\right)x^3\Bigg]$$

$$+\alpha_3x^{1/2}\Bigg[-a^4\left(\frac{1873}{173745}-\frac{18287}{2953665}aq\right) + a^3\left(\frac{34}{5265}-\frac{371}{89505}aq\right)x - a^2\left(\frac{23}{7425}-\frac{427}{126225}aq\right)x^2 - a\left(\frac{4}{585}+\frac{37}{69615}aq\right)x^3 - \frac{1}{153}aqx^4 - \frac{1}{748}qx^5\Bigg]$$

$$+\alpha_4x^{1/2}\Bigg[a^5\left(\frac{18287}{2953665}-\frac{57071}{15305355}aq\right) - a^4\left(\frac{371}{89505}-\frac{13348}{5101785}aq\right)x + a^3\left(\frac{427}{126225}-\frac{1621}{654075}aq\right)x^2$$

$$M(x) = \frac{1}{C^{1/2}}\left[\frac{3\sqrt{x}\left(6a^3+37a^2x+30ax^2+5x^3\right)}{35\,(a+x)^2\,(a+3x)} + \frac{93}{70}\sqrt{a}\arctan\left(\sqrt{\frac{x}{a}}\right) - \frac{129\sqrt{a}}{70\sqrt{3}}\arctan\left(\sqrt{\frac{3x}{a}}\right)\right] + \frac{B}{C^{3/2}}\left[\frac{\sqrt{x}\left(a^4+47a^3x+40a^2x^2+135ax^3+105x^4\right)}{315\,(a+x)^2\,(a+3x)} + \frac{31}{105}a^{3/2}\arctan\left(\sqrt{\frac{x}{a}}\right) - \frac{94a^{3/2}}{315\sqrt{3}}\arctan\left(\sqrt{\frac{3x}{a}}\right)\right] + \frac{J(x)}{C^{3/2}}, \quad (38)$$

where

$$J(x) = J1(x)+J2(x)+J3(x),$$

such that

$$J1(x) = \alpha_1x^{1/2}\Bigg[-a^2\left(\frac{25}{693}-\frac{529}{27027}aq\right) - a^2\frac{1}{11}qx + \left(\frac{1}{110}-\frac{1}{165}aq\right)x^2\Bigg]$$

$$-a^2\left(\frac{73}{69615}-\frac{7934}{3968055}aq\right)x^3 - a\left(\frac{1}{153}+\frac{1}{8721}aq\right)x^4 - \left(\frac{1}{748}+\frac{64}{10659}\right)x^5 - \frac{1}{494}qx^6\Bigg]$$

$$+\alpha_5 x^{1/2}\Bigg[-a^6\left(\frac{57071}{15305355}-\frac{174443}{74172105}aq\right)$$
$$+a^5\left(\frac{13348}{5101785}-\frac{41659}{24724035}aq\right)x$$
$$-a^4\left(\frac{1621}{654075}-\frac{69679}{41206725}aq\right)x^2$$
$$+a^3\left(\frac{7934}{3968055}-\frac{31877}{19229805}aq\right)x^3$$
$$-a^2\left(\frac{1}{8721}-\frac{685}{549423}aq\right)x^4$$
$$-a\left(\frac{64}{10659}-\frac{235}{671517}\right)x^5$$
$$-\left(\frac{1}{494}+\frac{85}{15561}aq\right)x^6-\frac{1}{420}qx^7\Bigg],$$

$$J2(x)=\alpha_1\sqrt{x}\Bigg[\frac{a^4}{(a+x)^2}\left(\frac{3}{385}-\frac{3}{1001}aq\right)$$
$$+\frac{a^3}{a+x}\left(\frac{17}{770}-\frac{17}{2002}aq\right)$$
$$-\frac{a^3}{a+3x}\left(\frac{59}{6930}-\frac{50}{27027}aq\right)\Bigg]$$
$$+\alpha_2\sqrt{x}\Bigg[-\frac{a^5}{(a+x)^2}\left(\frac{3}{1001}-\frac{1}{715}aq\right)$$
$$-\frac{a^4}{a+x}\left(\frac{17}{2002}-\frac{17}{4290}aq\right)$$
$$+\frac{a^4}{a+3x}\left(\frac{50}{27027}-\frac{157}{347490}aq\right)\Bigg]$$
$$+\alpha_3\sqrt{x}\Bigg[\frac{a^6}{(a+x)^2}\left(\frac{1}{715}-\frac{9}{12155}aq\right)$$
$$+\frac{a^5}{a+x}\left(\frac{17}{4290}-\frac{3}{1430}aq\right)$$
$$-\frac{a^5}{a+3x}\left(\frac{157}{347490}-\frac{349}{2953665}aq\right)\Bigg]$$
$$+\alpha_4\sqrt{x}\Bigg[-\frac{a^7}{(a+x)^2}\left(\frac{9}{12155}-\frac{9}{20995}aq\right)$$
$$-\frac{a^6}{a+x}\left(\frac{3}{1430}-\frac{3}{2470}aq\right)$$
$$+\frac{a^6}{a+3x}\left(\frac{349}{2953665}-\frac{989}{30610710}aq\right)\Bigg]$$
$$+\alpha_5\sqrt{x}\Bigg[\frac{a^8}{(a+x)^2}\left(\frac{9}{20995}-\frac{3}{11305}aq\right)$$
$$+\frac{a^7}{a+x}\left(\frac{3}{2470}-\frac{1}{1330}aq\right)$$
$$-\frac{a^7}{a+3x}\left(\frac{989}{30610710}+\frac{676}{74172105}aq\right)\Bigg],$$

$$J3(x)=\alpha_1 a^{5/2}\Bigg[\left(\frac{31}{770}-\frac{31}{2002}aq\right)\arctan\left(\sqrt{\frac{x}{a}}\right)$$
$$-\left(\frac{59}{2310\sqrt{3}}-\frac{50}{9009\sqrt{3}}aq\right)\arctan\left(\sqrt{\frac{3x}{a}}\right)\Bigg]$$
$$+\alpha_2 a^{7/2}\Bigg[-\left(\frac{31}{2002}-\frac{31}{4290}aq\right)\arctan\left(\sqrt{\frac{x}{a}}\right)$$
$$+\left(\frac{50}{9009\sqrt{3}}-\frac{157}{115830\sqrt{3}}aq\right)$$
$$\times\arctan\left(\sqrt{\frac{3x}{a}}\right)\Bigg]$$
$$+\alpha_3 a^{9/2}\Bigg[\left(\frac{31}{4290}-\frac{93}{24310}aq\right)\arctan\left(\sqrt{\frac{x}{a}}\right)$$
$$-\left(\frac{157}{115830\sqrt{3}}-\frac{349}{984555\sqrt{3}}aq\right)$$
$$\times\arctan\left(\sqrt{\frac{3x}{a}}\right)\Bigg]$$
$$+\alpha_4 a^{11/2}\Bigg[-\left(\frac{93}{24310}-\frac{93}{41990}aq\right)$$
$$\times\arctan\left(\sqrt{\frac{x}{a}}\right)$$
$$+\left(\frac{349}{984555\sqrt{3}}-\frac{989}{10203570\sqrt{3}}aq\right)$$
$$\times\arctan\left(\sqrt{\frac{3x}{a}}\right)\Bigg]$$
$$+\alpha_5 a^{13/2}\Bigg[\left(\frac{93}{41990}-\frac{31}{22510}aq\right)$$
$$\times\arctan\left(\sqrt{\frac{x}{a}}\right)$$
$$-\left(\frac{989}{10203570\sqrt{3}}-\frac{676}{24724035\sqrt{3}}aq\right)$$
$$\times\arctan\left(\sqrt{\frac{3x}{a}}\right)\Bigg].$$

5. Matching conditions

We need to match the interior and exterior solutions at the boundary using eqs (1) and (2). We have

$$e^{2\nu(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \tag{39}$$

and

$$e^{2\lambda(r)} = \left[1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right]^{-1}. \tag{40}$$

From system (29), we use the transformation $x = CR^2$ for $r = R$ at the boundary. The solution of the interior and exterior of exact models match at the boundary with the condition $p_r(r = R) = 0$ being satisfied. Then,

$$1 - \frac{2M}{R} - \frac{Q^2}{R^2} = A^2(a + CR^2)^4. \tag{41a}$$

$$\left[1 - \frac{2M}{R} - \frac{Q^2}{R^2}\right]^{-1} = \left[\frac{35a^3 + 35a^2CR^2 + 21aC^2R^4 + 5C^3R^6 + 35\frac{H}{C}}{35\left(a + CR^2\right)^2\left(a + 3CR^2\right)} - \frac{B}{C}\left(\frac{105a^3CR^2 + 189a^2C^2R^4 + 135aC^3R^6 + 35C^4R^8}{315\left(a + CR^2\right)^2\left(a + 3CR^2\right)}\right)\right]^{-1}.$$

$$0 = C\left[\frac{140a^4 + 434a^3CR^2 + 318a^2C^2R^4 + 150aC^3R^6 + 30C^4R^8}{35\left(a + CR^2\right)^3\left(a + 3CR^2\right)^2}\right] - B\left[\frac{210a^5 + 2982a^4CR^2 + 11124a^3C^2R^4}{315\left(a + CR^2\right)^3\left(a + 3CR^2\right)^2}\frac{+16780a^2C^3R^6 + 11770aC^4R^8 + 3150C^5R^{10}}{315\left(a + CR^2\right)^3\left(a + 3CR^2\right)^2}\right] + \left[\frac{H\left(9a + 21CR^2\right) - \dot{H}\left(a^2 + 4aCR^2 + 3C^2R^4\right)}{2\left(a + CR^2\right)^3\left(a + 3CR^2\right)^2}\right], \tag{41b}$$

where

$$H = \alpha_1\left(\frac{1}{5}a^3C^2R^4 + \frac{1}{7}a^2\left(3 + aq\right)C^3R^6 + \frac{3}{9}a\left(1 + aq\right)C^4R^8 + \frac{1}{11}\left(1 + 3aq\right)C^5R^{10} + \frac{1}{13}qC^6R^{12}\right) + \alpha_2\left(\frac{1}{7}a^3C^3R^6 + \frac{1}{9}a^2\left(3 + aq\right)C^4R^8 + \frac{3}{11}a\left(1 + aq\right)C^5R^{10} + \frac{1}{13}\left(1 + 3aq\right)C^6R^{12} + \frac{1}{15}qC^7R^{14}\right) + \alpha_3\left(\frac{1}{9}a^3C^4R^8 + \frac{1}{11}a^2\left(3 + aq\right)C^5R^{10} + \frac{3}{13}a\left(1 + aq\right)C^6R^{12} + \frac{1}{15}\left(1 + 3aq\right)C^7R^{14} + \frac{1}{17}qC^8R^{16}\right) + \alpha_4\left(\frac{1}{11}a^3C^5R^{10} + \frac{1}{13}a^2\left(3 + aq\right)C^6R^{12} + \frac{3}{15}a\left(1 + aq\right)C^7R^{14} + \frac{1}{17}\left(1 + 3aq\right)C^8R^{16} + \frac{1}{19}qC^9R^{18}\right) + \alpha_5\left(\frac{1}{13}a^3C^6R^{12} + \frac{1}{15}a^2\left(3 + aq\right)C^7R^{14} + \frac{3}{17}a\left(1 + aq\right)C^8R^{16} + \frac{1}{19}\left(1 + 3aq\right)C^9R^{18} + \frac{1}{21}qC^{10}R^{20}\right),$$

$$\dot{H} = \alpha_1\left(\frac{2}{5}a^3CR^2 + \frac{3}{7}a^2\left(3 + aq\right)C^2R^4 + \frac{12}{9}a\left(1 + aq\right)C^3R^6 + \frac{5}{11}\left(1 + 3aq\right)C^4R^8 + \frac{6}{17}qC^5R^{10}\right) + \alpha_2\left(\frac{3}{7}a^3C^2R^4 + \frac{4}{9}a^2\left(3 + aq\right)C^3R^6 + \frac{15}{11}a\left(1 + aq\right)C^4R^8 + \frac{6}{13}\left(1 + 3aq\right)C^5R^{10}\right.$$

$$+\frac{7}{15}qC^6R^{12}\Big)$$
$$+\alpha_3\left(\frac{4}{9}a^3C^3R^6+\frac{5}{11}a^2\left(3+aq\right)C^4R^8\right.$$
$$+\frac{18}{13}a\left(1+aq\right)C^5R^{10}$$
$$\left.+\frac{7}{15}\left(1+3aq\right)C^6R^{12}+\frac{8}{17}qC^7R^{14}\right)$$
$$+\alpha_4\left(\frac{5}{11}a^3C^4R^8+\frac{6}{13}a^2\left(3+aq\right)C^5R^{10}\right.$$
$$+\frac{21}{15}a\left(1+aq\right)C^6R^{12}+\frac{8}{17}\left(1+3aq\right)C^7R^{14}$$
$$\left.+\frac{9}{19}qC^8R^{16}\right)$$
$$+\alpha_5\left(\frac{6}{13}a^3C^5R^{10}+\frac{7}{15}a^2\left(3+aq\right)C^6R^{12}\right.$$
$$+\frac{24}{17}a\left(1+aq\right)C^7R^{14}$$
$$\left.+\frac{9}{19}\left(1+3aq\right)C^8R^{16}+\frac{10}{21}qC^9R^{18}\right).$$

As the electric field is given by $E=\frac{Q}{r^2}$, for $r=R$, we have

$$\begin{aligned}\frac{Q^2}{R^2}&=E^2R^2\\
&=CR^2\left[\frac{197a^3C^2R^4+1452a^2C^3R^6+1356aC^4R^8+420C^5R^{10}}{35CR^2\left(a+CR^2\right)^3\left(a+3CR^2\right)^2}\right]\\
&\quad-BR^2\left[\frac{168a^4C^2R^4+1296a^3C^3R^6+6528a^2C^4R^8+7280aC^5R^{10}+2520C^6R^{12}}{315CR^2\left(a+CR^2\right)^3\left(a+3CR^2\right)^2}\right]\\
&\quad-R^2\left[\frac{\frac{H}{CR^2}\left(2a^2+15aCR^2+23C^2R^4\right)+\dot{H}\left(a^2CR^2+4aC^2R^4+3C^3R^6\right)}{CR^2\left(a+CR^2\right)^3\left(a+3CR^2\right)^2}\right].\end{aligned}\tag{43}$$

Therefore, from system (41) and eq. (43), we obtain

$$\begin{aligned}A^2\left(a+CR^2\right)^4&=\frac{35a^3+35a^2CR^2+21aC^2R^4+5C^3R^6+35\frac{H}{C}}{35\left(a+CR^2\right)^2\left(a+3CR^2\right)}\\
&\quad-\frac{B}{C}\left(\frac{105a^3CR^2+189a^2C^2R^4+135aC^3R^6+35C^4R^8}{315\left(a+CR^2\right)^2\left(a+3CR^2\right)}\right),\end{aligned}\tag{44a}$$

$$\begin{aligned}\frac{2M}{R}&=1-A^2\left(a+CR^2\right)^4\\
&\quad-CR^2\left[\frac{197a^3C^2R^4+1452a^2C^3R^6+1356aC^4R^8+420C^5R^{10}}{35CR^2\left(a+CR^2\right)^3\left(a+3CR^2\right)^2}\right]\\
&\quad+BR^2\left[\frac{168a^4C^2R^4+1296a^3C^3R^6+6528a^2C^4R^8+7280aC^5R^{10}+2520C^6R^{12}}{315CR^2\left(a+CR^2\right)^3\left(a+3CR^2\right)^2}\right]\\
&\quad+R^2\left[\frac{\frac{H}{CR^2}\left(2a^2+15aCR^2+23C^2R^4\right)+\dot{H}\left(a^2CR^2+4aC^2R^4+3C^3R^6\right)}{CR^2\left(a+CR^2\right)^3\left(a+3CR^2\right)^2}\right],\end{aligned}\tag{44b}$$

where M is given by eq. (38). There are sufficient free parameters to satisfy this condition.

6. Physical analysis

In this section, we indicate that the exact model generated in §4 satisfies the physical properties of the relativistic objects. Such analysis is missing in other models that were generated in the past using the same approach, including [1,6,7,16].

6.1 *Energy conditions*

Any realistic stellar model should satisfy the following energy conditions:

(i) Null energy conditions: $\rho \geq 0$.
(ii) Weak dominant energy conditions: $\rho - p_r \geq 0$ and $\rho - p_t \geq 0$.
(iii) Strong dominant energy conditions: $\rho - 3p_r \geq 0$ and $\rho - 3p_t \geq 0$.
(iv) Strong energy conditions: $\rho - p_r - 2p_t \geq 0$.

From system (29) and figures 1–6, it can be observed that all the energy conditions are satisfied by our model. The strong energy decreases slowly near the centre and then sharply decreases to the minimum value where it increases sharply towards the surface. This is a new feature in the model.

6.2 *Regularity*

It can be observed from eqs (29a) and (29b) that the gravitational potentials $e^{2\nu}$ and $e^{2\lambda}$ are increasing functions with the radial distance, r with $e^{2\nu} = A^2a^4 > 0$ and $e^{2\lambda} = 1$ at the centre of the star. They are regular, finite and continuous functions throughout the stellar interior. These properties are physical for realistic stellar models. From system (29), the energy density, radial pressure and tangential pressure are decreasing functions with maximum values at the centre given by

$$\rho_0 = \frac{12C}{a} + 2B,$$

$$p_r(x=0) = p_t(x=0) = \frac{4C}{a} - \frac{2}{3}B.$$

Physically, for a realistic stellar model, the radial pressure and tangential pressure are equal at the centre of the stellar object. This makes the core of the stellar objects always isotropic in nature.

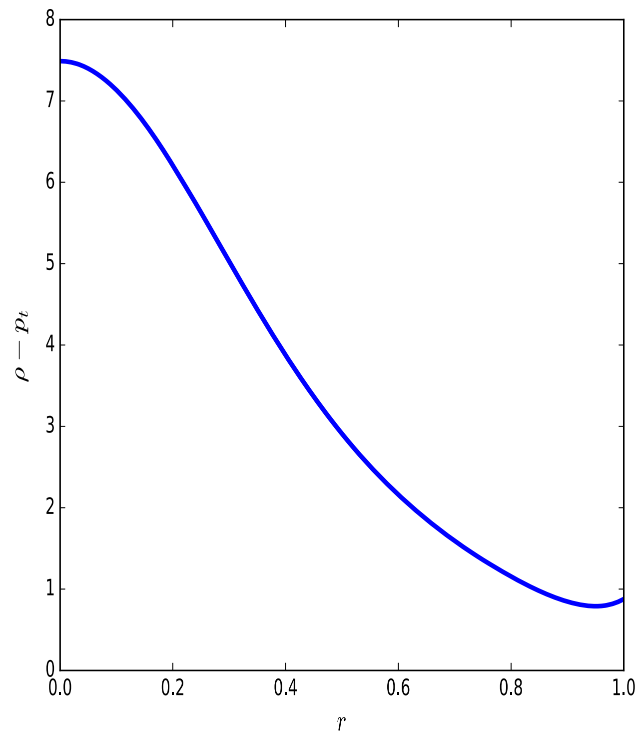

Figure 1. The plot generated for weak dominant energy condition $\rho - p_t$.

6.3 *Casuality*

The speed of sound, v, of anisotropic compact star must be less than the speed of light, c. From eqs (21) and (22), the speed of sound, $v = \frac{1}{3}$. Also, from eqs (31) and (32), $v = \frac{1}{3}$. These results show that our model satisfies the casuality condition. This is also observed in figure 10.

6.4 *Stability*

We use adiabatic index, Γ, for anisotropic relativistic stellar object as the measure of stability of anisotropic sphere. An object is said to be stable from gravitational collapse if the condition $\Gamma \geq \frac{4}{3}$ is satisfied. From eqs (29c) and (29d), the minimum value of the adiabatic index is at the centre of anisotropic object, which is

$$\Gamma = \frac{1}{3} + \frac{6C + aB}{6C - aB}. \tag{45}$$

From eq. (45), $\Gamma \geq \frac{4}{3}$ for $aB \geq 0$. Since the Bag constant for quark stars, $B > 0$, then $\Gamma \geq \frac{4}{3}$ for $a \geq 0$. It can also be observed from figure 7 that $\Gamma \geq \frac{4}{3}$. This therefore indicates that the model is stable from gravitational collapse. This is also indicated in figure 7.

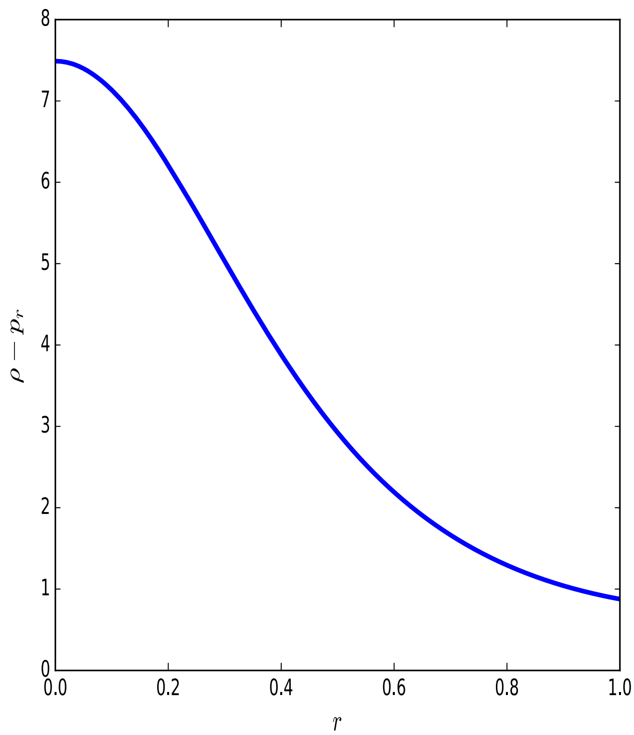

Figure 2. The plot generated for weak dominant energy condition $\rho - p_r$.

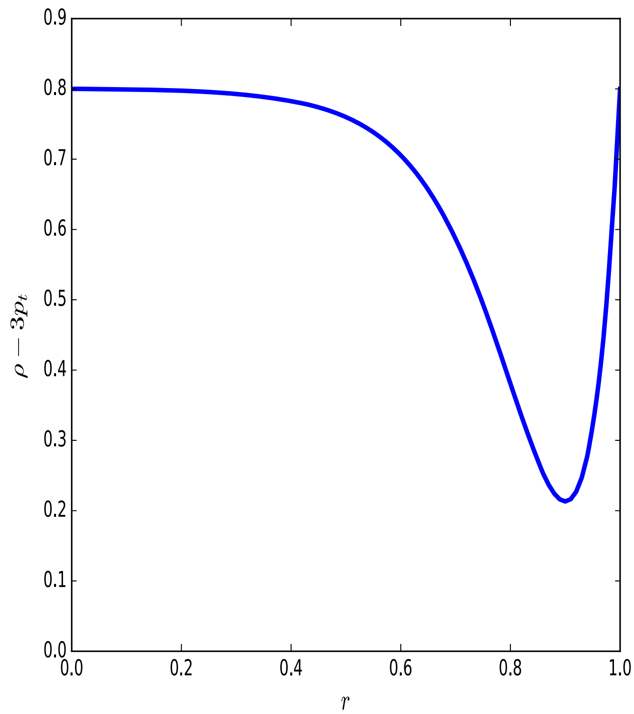

Figure 4. The plot generated for strong dominant energy condition $\rho - 3p_t$.

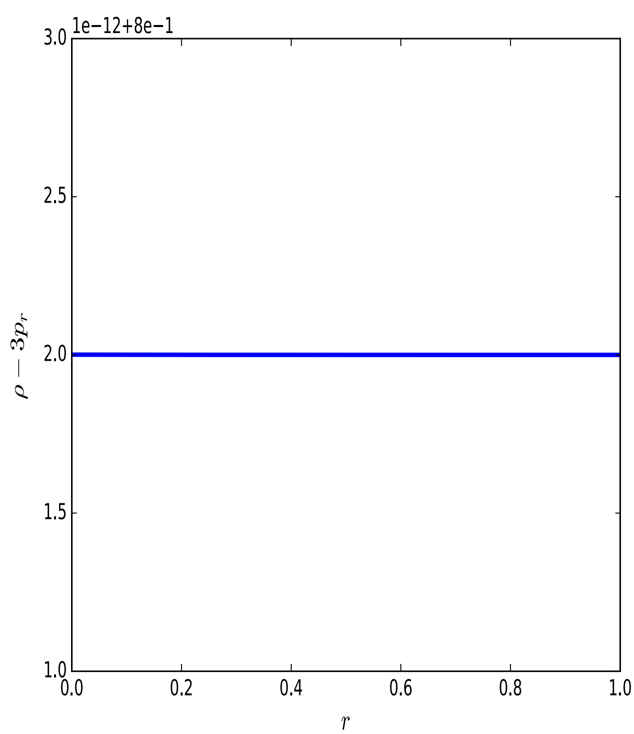

Figure 3. The plot generated for strong dominant energy condition $\rho - 3p_r$.

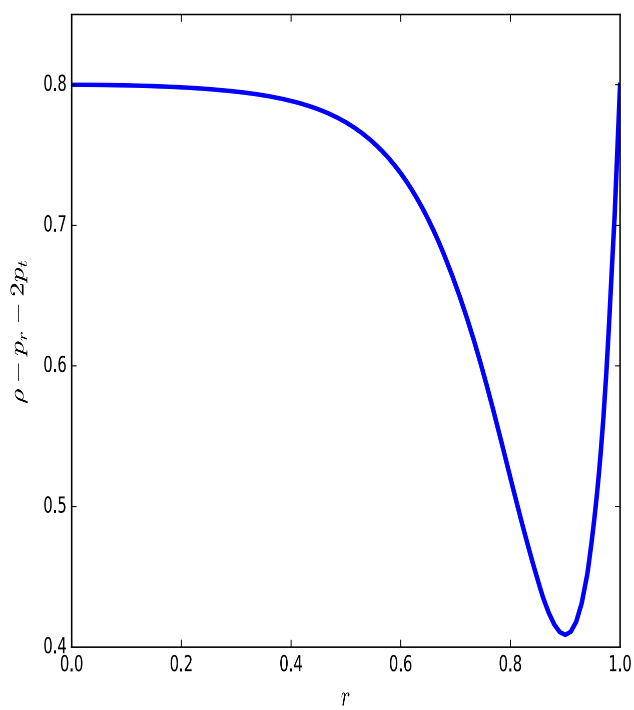

Figure 5. The plot generated for strong energy condition $\rho - p_r - 2p_t$.

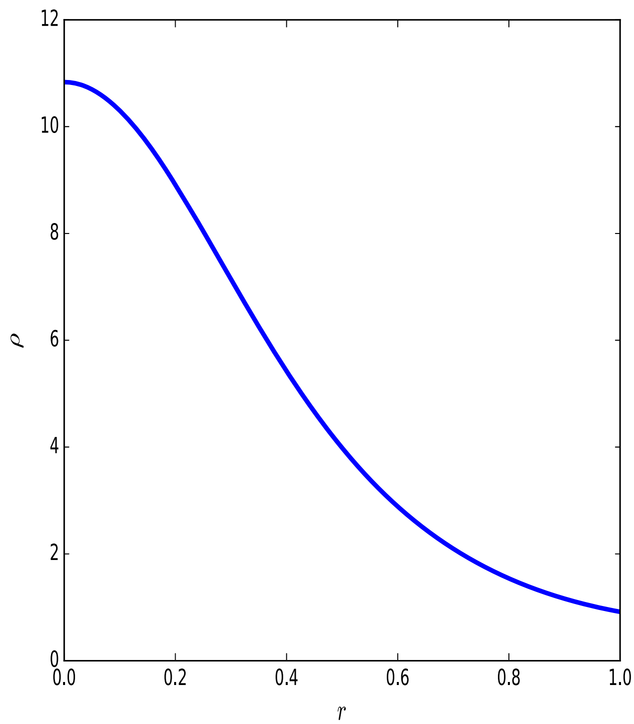

Figure 6. The plot generated for energy density ρ.

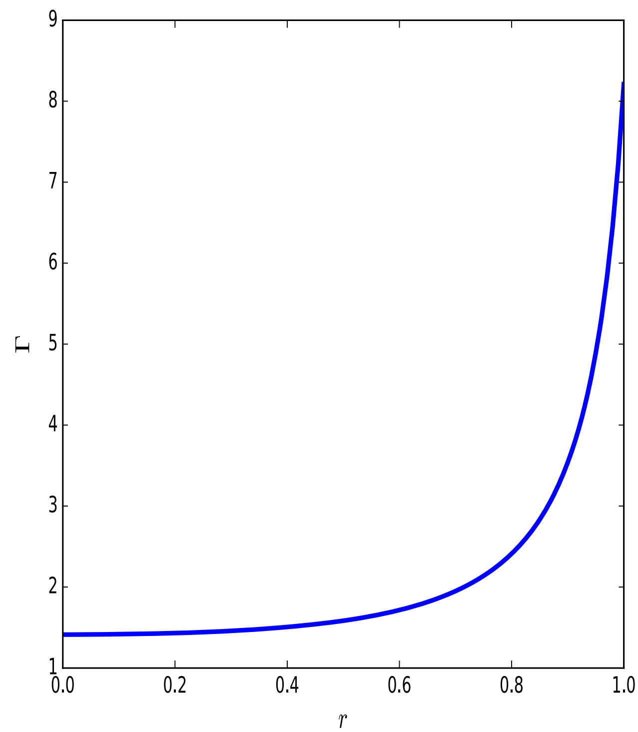

Figure 7. The plot generated for adiabatic index.

7. Discussion

We show that the exact model in system (29) is well behaved throughout the interior of the stellar object. We use Python programming language to generate plots for $q = -0.5, a = 2.3, A = 1, B = 0.2, C = 2, \alpha_1 = 0.01$, $\alpha_2 = 0.02$, $\alpha_3 = 0.03$, $\alpha_4 = 0.04$ and $\alpha_5 = 0.05$. The plots generated are for the energy conditions, namely weak dominant energy condition, $\rho - p_t$ (figure 1) and $\rho - p_r$ (figure 2); strong dominant energy condition, $\rho - 3p_r$ (figure 3) and $\rho - 3p_t$ (figure 4); and strong energy condition, $\rho - p_r - 2p_t$ (figure 5), the energy density ρ (figure 6), adiabatic index (figure 7). Gravitational potential, $e^{2\nu}$ (figure 8), gravitational potential $e^{2\lambda}$ (figure 9), the speed of sound, $v = dp_r/d\rho$ (figure 10). We also plot the graphs for the radial pressure, p_r (figure 11), the tangential pressure, p_t (figure 12), the measure of pressure anisotropy (figure 13), the electric field, E^2 (figure 14) and the graph for mass, M (figure 15). All the graphs are plotted against the radial coordinate r.

The gravitational potentials $e^{2\nu}$ and $e^{2\lambda}$ in figures 8 and 9 respectively, are increasing functions with radial distance r. They are regular, finite and continuous functions throughout the stellar interior. This is physical for realistic stellar models. The energy density ρ, the tangential pressure p_t and the radial pressure p_r in figures 11 and 12 are maximum at the core of the star. They decrease sharply near the centre and then decrease slowly from the centre to the surface. These profiles are similar to those found by Sunzu and Danford [6], Maharaj *et al* [16], Mafa Takisa and Maharaj [23] and Feroze and Siddiqui [27]. The measure of anisotropy Δ in figure 13 increases from the centre to the region near

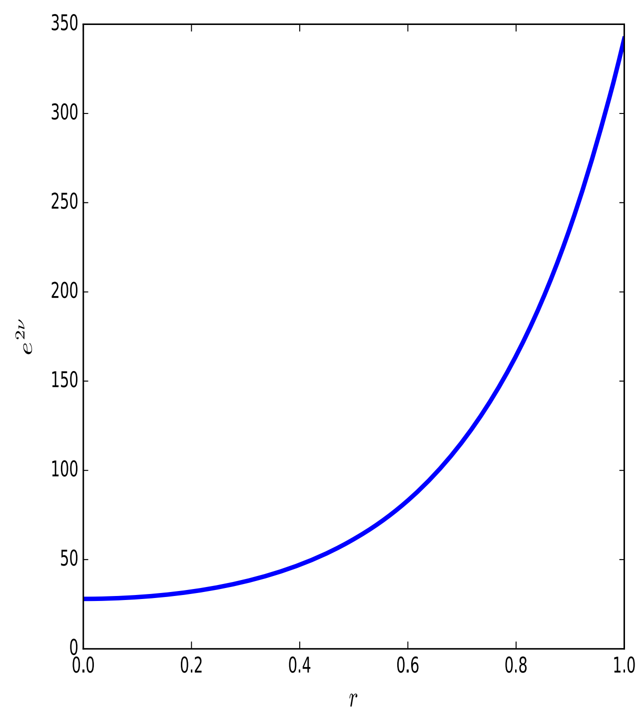

Figure 8. The plot generated for gravitational potential $e^{2\nu}$.

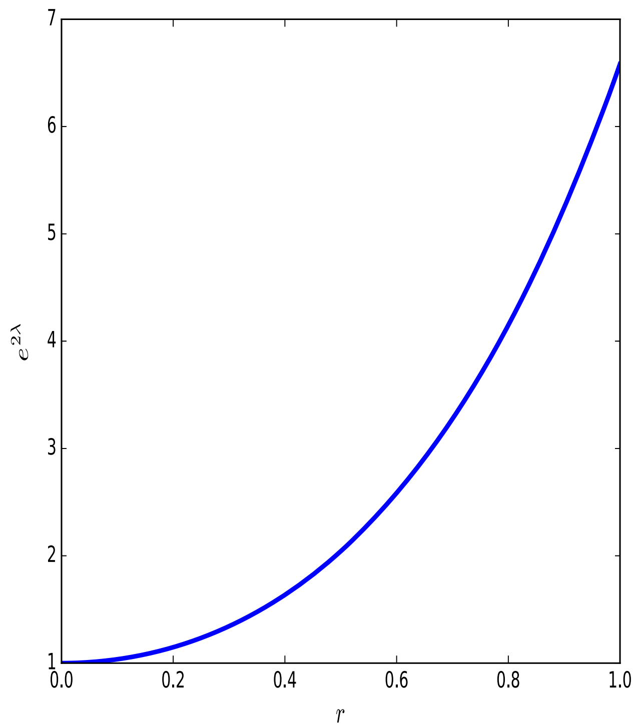

Figure 9. The plot generated for gravitational potential $e^{2\lambda}$.

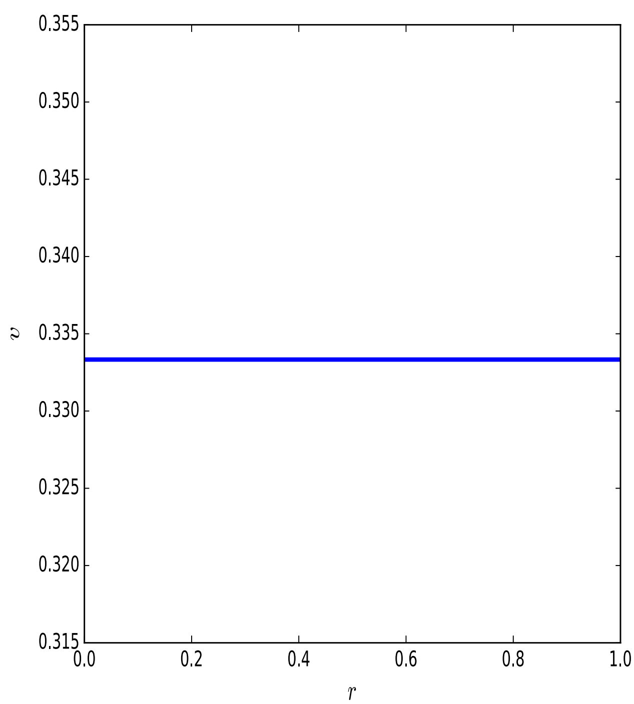

Figure 10. The plot generated for radial speed of sound v.

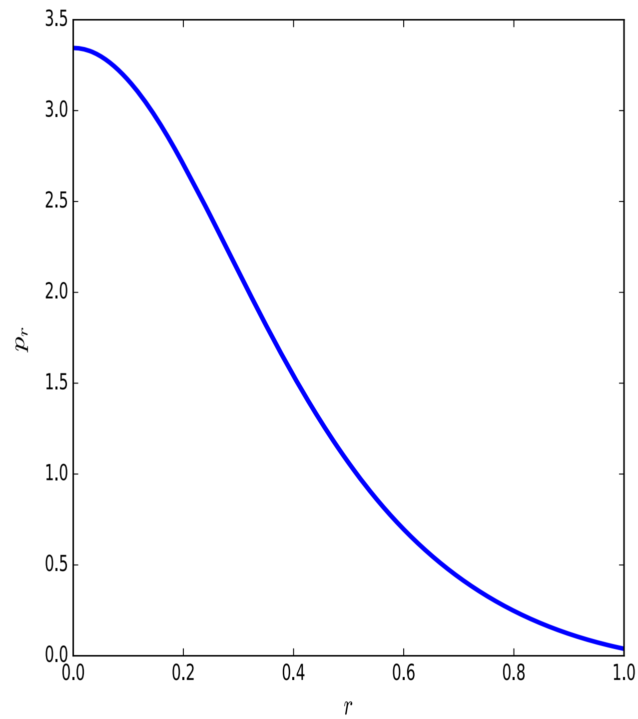

Figure 11. The plot generated for radial pressure p_r.

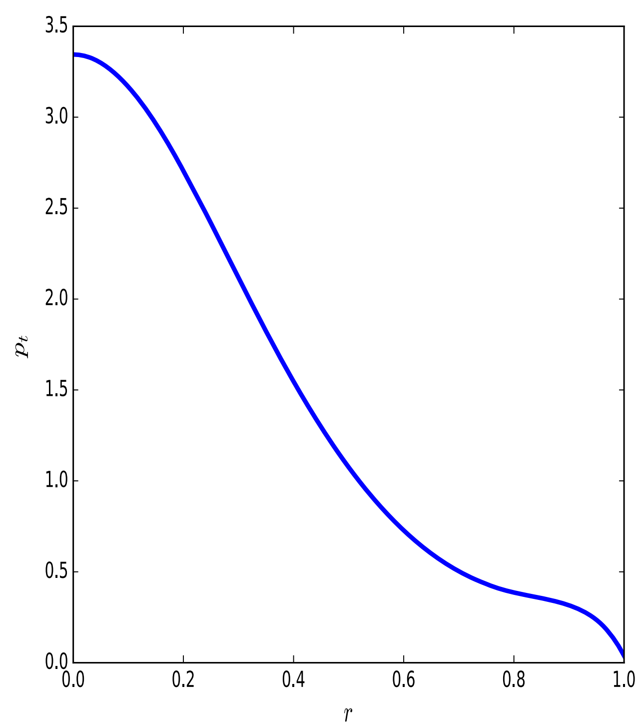

Figure 12. The plot generated for tangential pressure p_t.

the surface where it attains the maximum value and then it decreases slowly to the surface. It is important to note that $\Delta = 0$ at the centre, which is physically reasonable for any realistic model as we expect $p_r = p_t$ at the centre of the star. This profile of pressure anisotropy is similar to the findings of Sunzu and Danford [6], Sunzu *et al* [7], Maharaj *et al* [16], Mafa Takisa and Maharaj [23] and Sharma and Maharaj [29]. The electric field E^2 in figure 14 is regular, finite and increases to the maximum value away from the centre. Then, it decreases slowly

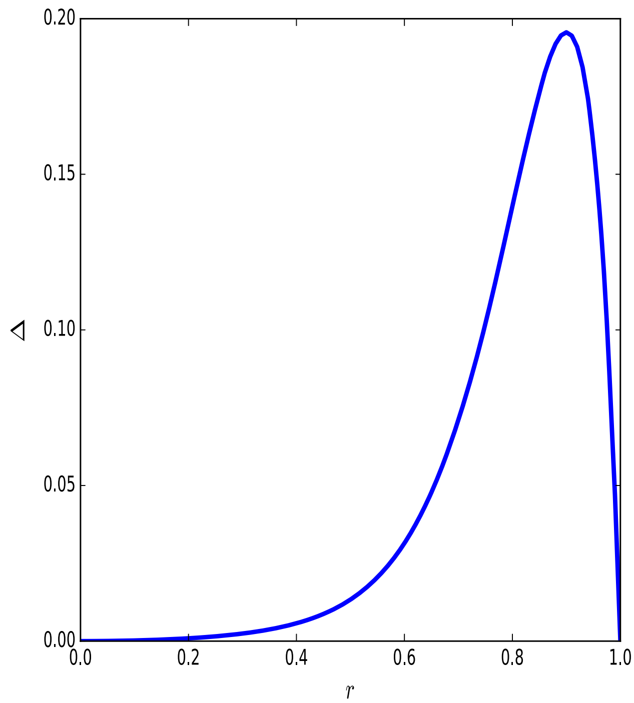

Figure 13. The plot generated for the measure of anisotropy Δ.

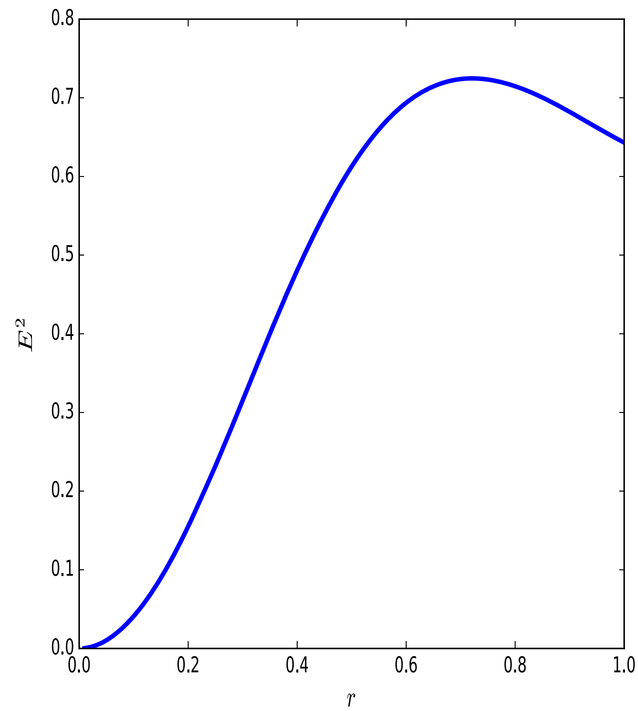

Figure 14. The plot generated for electric field E^2.

towards the surface. It is important to note that $E^2 = 0$ at the centre as we expect the electric field to vanish at the centre of a charged stellar sphere. This profile is similar to the findings obtained by Sunzu and Danford [6] and Maharaj *et al* [16]. The mass of the star in figure 15 increases slowly in the region near the centre and then sharply increases with radial distance towards the surface. This feature indicates that the quark star may constitute materials with non-uniform density. This feature proves the existence of core-envelope models. This model therefore gives insights that is missing in other studies of the same approach.

It is clear that the speed of sound v obtained using eq. (7) is constant (i.e. $v = \frac{1}{3} < 1$) and less than the speed of light $c = 1$. The adiabatic index Γ in figure 7 is greater than $\frac{4}{3}$ in the interior of the anisotropic stellar object. This condition ensures the stability of our star with linear equation of state. This profile for adiabatic index is similar to that found by Dayandan *et al* [33].

We use the mass function in eq. (38) to generate stellar masses and radii for several compact stellar objects consistent with observations. In computing stellar masses, we use the transformations $\tilde{a} = aR^2$, $\tilde{B} = BR^2$, $\tilde{C} = CR^2$, $\tilde{a} = aR^2$, $\tilde{q} = qR^2$, $\tilde{\alpha_1} = \alpha_1 R^2$, $\tilde{\alpha_2} = \alpha_2 R^2$, $\tilde{\alpha_3} = \alpha_3 R^2$, $\tilde{\alpha_4} = \alpha_4 R^2$ and $\tilde{\alpha_5} = \alpha_5 R^2$. In our case, we set $R = 13$ for computational purpose. Table 1 summarises masses and radii for the stars. We obtain different relativistic masses of stars which include $M = 2.86 M_\odot$ with $r = 9.46$ km for the star PSR J1614-2230

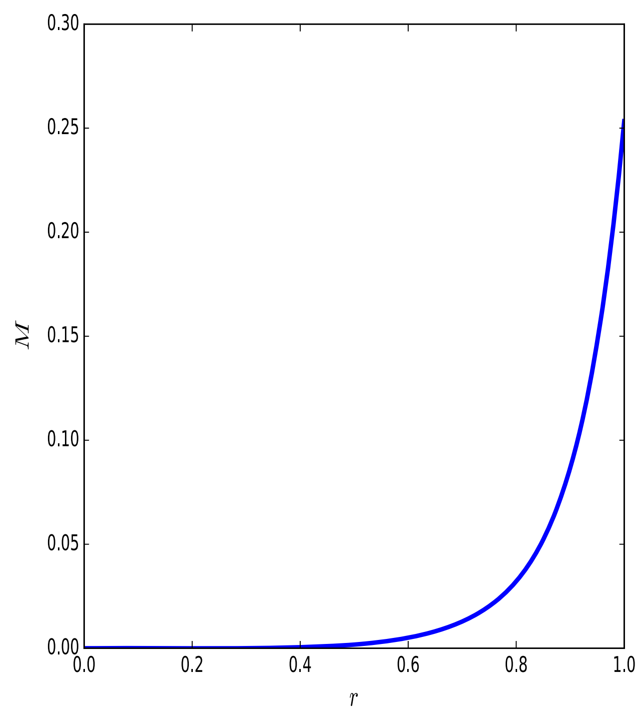

Figure 15. The plot generated for mass M.

which was found by Mak and Harko [2]; $M = 1.97 M_\odot$ with $r = 9.69$ km and $r = 10.30$ km for the star PSR J1614-2230 obtained by Demorest *et al* [34] and Mafa Takisa and Maharaj [35] respectively. We also obtain the star SAX J1808.4-3658 of mass $M = 1.6 M_\odot$ with

Table 1. Stellar masses consistent with observations.

$\tilde{B}$	$\tilde{C}$	$\tilde{\alpha_1}$	$\tilde{\alpha_2}$	$\tilde{\alpha_3}$	$\tilde{\alpha_4}$	$\tilde{\alpha_5}$	$\tilde{q}$	$\tilde{a}$	r (km)	$M(M_\odot)$	Model
1.73	1.2	7.9	8.6	10.5	9.6	5.1	0.5	7.75	10.3	1.970	Mafa Takisa and Maharaj [35]
36.8	1.9	4.9	7,9	1.5	3.6	5.1	0.5	0.3	7.61	1.600	Sunzu *et al* [1]
34.08	1.55	5.3	2.9	15.1	9.3	13.0	0.5	10.33	9.46	2.86	Mak and Harko [2]
19.6	2.11	8.1	6.8	12	3.3	3.4	0.5	18.35	8.301	1.04	Rawls *et al* [36]
38.59	3.99	6.2	2.7	10.1	16.5	13.0	0.5	0.42	5.78	1.73	Sunzu *et al* [1]
10.63	1.74	5.2	7.1	13.7	4.6	7.9	0.5	16.0	9.1	1.58	Güver *et al* [38]
3.44	2.0	5.2	7.1	10.1	16.5	17.0	0.5	11.0	8.1	0.85	Abubekerov *et al* [37]
30.86	1.49	9.8	17.1	11.3	10.6	2.4	0.5	5.20	8.849	1.30	Özel *et al* [39]
2.95	1.41	9.0	7.1	13.7	10.0	7.9	0.5	9.2	9.69	1.97	Demorest *et al* [34]

$r = 7.61$ km and the star Her-X-1 of mass $M = 1.73M_\odot$ with $r = 5.78$ km found by Sunzu *et al* [1]. We also generate mass $M = 1.04M_\odot$ and $r = 8.301$ km for the star LMC X-4 which was observed by Rawls *et al* [36]. We obtain the masses for stars similar to those generated by studies that used different approaches. For the star Her X-1, we obtain $M = 0.85M_\odot$ and $r = 8.1$ km consistent with Abubekerov *et al* [37]. We also generate mass $M = 1.58M_\odot$ with radius $r = 9.1$ km for the star SMC X-4 found by Güver *et al* [38] and the star EXO 1785-248 with mass $M = 1.30M_\odot$ and radius $r = 8.849$ km found by Özel *et al* [39]. Therefore, our model produces stellar objects consistent with observations.

8. Conclusion

We generated new exact solutions to the system of field equations. We considered Einstein–Maxwell field equations to describe relativistic stellar matter with electric field for space–time which is spherically symmetric and static. In our model, the anisotropy was formulated to determine the physical structure of the stellar object for a specified metric function. We used a linear equation of state in our model. The solutions obtained for matter variables, mass, gravitational potential and electric field are well behaved throughout the interior of the compact charged stellar object. In our model, we formulated a new form of anisotropy as a rational function. We have regained the model given by Sunzu and Danford [6], Maharaj *et al* [16] and Sunzu *et al* [1]. For $\Delta = 0$ we regained the isotropic models generated by Komathiraj and Maharaj [8], Mak and Harko [2] and Misner and Zapolsky [32]. We found that all the energy conditions are satisfied by the model and the speed of sound was shown to be constant and less then the speed of light. The adiabatic index $\Gamma \geq \frac{4}{3}$ satisfied the stability condition of the model. We also generated the stellar masses and radii which are consistent with the stars PSR J1614-2230, SAX J1808.4-3658, Her X-1, LMC X-4, SMC X-4 and EXO 1785-248. We believe that models generated in this paper will enable further investigation on behaviours and properties of charged relativistic stellar objects. In future, we expect to investigate the physical behaviour of the stellar objects by choosing different form of anisotropy.

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