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A novel approach in heating phenomena of the drift plasmas in the presence of rotating magnetic field: Appearance of anti-Hermitian part in dielectric tensor

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Abstract. Employing the dielectric tensor elements and taking into account the time variations of a rotating magnetic field (RMF) leading to the electric field generation responsible for particle acceleration, we develop a closed mathematical form for the radiated power absorbed by a long column containing multilayer inhomogeneous drift plasma with elliptical cross-section. To this end, cold-fluid equations are employed to drive explicit expressions for the dielectric tensor elements by taking into account the effect of RMF. Results reveal that the dielectric tensor has an anti-Hermitian part responsible for dissipation which comes from the rotation of external magnetic field. In order to describe the effective force exerted on the plasma, a general expression is obtained for the ponderomotive force by utilising the electric field components presented in this work. To check the accuracy of the obtained results, some limiting cases are discussed.

Keywords. Dielectric tensor; rotating magnetic field; anti-Hermitian part; ponderomotive force; elliptical cross-section; hybrid modes.

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1. Introduction

One of the most promising methods for plasma heating is to use the magnetic shock and the electric field resulting from it for accelerating charged particles [1–4]. Controlling the lifespan of the magnetic field in the plasma heating process has been the subject of many investigations [5,6]. The results obtained from the theory of general relativity or Maxwell's equations lead to the production of electric field by the rotation of magnetic field. In other words, by changing the rotation angle of the magnetic field, we have been able to deduce the presence of electric field resulted from this rotation by using Maxwell's equations. Such electric field is sufficient to allow particle acceleration given by the ponderomotive forces acting on a system containing free charge carriers [7–9]. Knowing the overall effective ponderomotive force is more important due to the practical application in both plasma heating and isotope or mass separation based on Newton's second law [10–12].

The force affecting each element in an electromagnetic (EM) active medium arises from nonlinearities in the system. This force, known as ponderomotive force, has attracted much attention due to its broad applications such as isotope or mass separation (in particular, separation of uranium isotope), modulated surface wave structures, particle acceleration, inertial confinement fusion and self-focussing of an intense laser beam [10–20]. In order to investigate ponderomotive force, the fluid theory including the momentum transfer equation by taking into account the nonlinearity effects can be used. Another method, which requires the use of dielectric permittivity tensor, is to assess the EM features of the system. Knowing the dielectric permittivity tensor of the media is the base for understanding the electric field components and consequently, the force imposed onto the particles. So, the corresponding ponderomotive force per unit value can be expressed as [21]

$$\vec{F}_V = \frac{N_\alpha}{4\pi}\left(\frac{\partial}{\partial N_\alpha}\varepsilon_{ij}(\omega,k)\right) \times\vec{\nabla}(E_i^*(\omega,k,r)E_j(\omega,k,r))\mathrm{d}^3x. \tag{1}$$

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In this method, the inhomogeneous electric field distribution indicated by the gradient term gives rise to a ponderomotive force. Nonlinear effects resulting from the inhomogeneity in material properties have received much interest because of its potential applications in laser–plasma interaction, optical harmonic generation, wake-field acceleration, fibre optics communications and materials process [22–27]. Making use of Maxwell's equation for a general inhomogeneous medium whose dielectric tensor elements have been determined, can provide a proper formalism for extracting ponderomotive force per unit volume acting on a nonlinear system. For example, an inhomogeneous collisionless drift plasma environment can be mentioned which is embedded in a rotating external magnetic field. This medium can therefore be expected to give two aspects of ponderomotive force per unit volume acting on a fluid. The first one is the effects of nonlinearity due to the inhomogeneously random media. The latter will result in the spatial variations of the electric field components in the presence of a rotating magnetic field (RMF).

Three general methods have been proposed in literature for investigating wave propagation in an EM medium. One method used to account for this is to use the fundamental equations such as Lenard–Balescu equation (for a completely ionised plasma) or Bogoliubov–Born–Green–Kirkwood–Yvon (BBGKY) (sometimes called Bogoliubov) equations of kinetic theory equation (for a weakly ionised plasma) for a distribution function according to the conditions imposed on the system [21]. This method, which is known as kinetic theory, is based on the well-known fact that the propagation pattern is obtained by making use of suitable boundary conditions and taking into account the distribution function of charged particles [21]. The latter method yields a solvable differential equation with regard to the electric and magnetic field components from the standard boundary conditions for Maxwell's equations involving the continuity equation and momentum transport equation. This method, which allow an accurate evaluation of the dispersion relations of the system, is called fluid theory and obtained by taking moments of kinetic equations [28,29]. On the contrary, some of the phenomena such as correlation between particles, have been considered negligible in this method with respect to the first method [21,28–30]. In the last method, like the previous one, the dielectric permittivity tensor of the system is acquired by eliminating all the parameters and factors indicating the response of the medium to the presence of EM waves (for example, current density and charge density in the medium) in the Maxwells equations [30,31]. Nonetheless, this method is precise enough to give a clear overall impression of letting the emitted EM waves travel in the medium. All mentioned methods can provide a description to illustrate the importance of the role of external factors such as constant electric and magnetic fields in the EM properties of the system [21,28–30]. For example, the typical relaxation times for the correlated processes, regardless of the thermal and collisional effects, may be denoted by the kinetic description [21]. The method for investigating the net force acting on the macroscopic elements (for example ponderomotve force) is based on the second method. However, the most useful method for the evaluation of EM field profiles and presence or absence of specific propagation pattern (for example TE or TM modes) may be the third method [30,31]. Many investigations have been carried out to examine the ponderomotive force and its effect by using fluid theory owing to its fascinating physical properties [32–34]. Nowadays, many scientists and engineers have made efforts in finding the elements of the dielectric permittivity tensor with coupled equations based on the third method [32–34]. These researches have been performed for a variety of structures having circular, elliptical or rectangular cross-sections by utilising the fluid formalism to achieve the coupled equations in the presence of a constantly applied magnetic field [32–38].

The study of waves in magnetised plasmas is an active area of research due to its potential applications and the importance of external magnetic field in the plasma dynamics. These days, some investigations have been devoted to examine the properties of waves in a magnetised plasma [39,40]. Ponderomotive force and nonlinear process in the interaction between plasma and a RMF are the main approach for plasma instability control in magnetic confinement fusion experiment devices [41]. The external magnetic fields represented by RMF can be used in the mass transfer processes in the gas–liquid system, to control the hydrodynamics and heat-mass transfer because it can consume less power compared to the steady magnetic fields (SMF). For example, bulk semiconductor single crystal growth, in horizontal continuous casting of $CuNi_{10}Fe_1Mn$ alloy hollow billets, and as a non-instructive mixing device in various areas of chemical engineering [41–46], can be mentioned. It is worth noting that employing dielectric permittivity tensor computed here is an efficient method for the evaluation of wave propagation and penetration.

The present paper is aimed at considering a primary system in which the dielectric permittivity tensor elements are slowly changed over time. For this reason, we consider inhomogeneous cold collisionless drift plasma columns with elliptical cross-section embedded in an external magnetic field rotating with uniform angular velocity and amplitude. In the present work, we derive explicit expressions for the effective dielectric tensor elements of such plasma. We show that the effective dielectric tensor contains a Hermitian part and an anti-Hermitian part that is responsible for dissipation and absorption. We then use this dielectric tensor as a starting point for providing an engineering evaluation of the power absorption. We present a set of coupled equations which enable us to resolve several problems in the theory of wave propagation and absorption. The discussion is motivated by current interest in the study of absorption of radiation related to the anti-Hermitian part of the dielectric tensor. Future work should focus on the calculation of the ponderomotive force acting on each element of plasma by using a set of arguments presented in this article to examine numerically a specific configuration as the master case and obtain effective forces.

The manuscript is organised as follows: In §2, the basic equations, inhomogeneous perturbed quantities and their equations are presented. Also, an estimation of the energy absorption is obtained in this section. The dielectric permittivity tensor of the system is introduced in §3 and some limiting cases are discussed to verify the accuracy of the obtained results. In §4 the fundamental field equations and their coupling are offered. Section 5 is devoted for summary and conclusion.

2. Motivation of system innovation and basic equations

What is evident from this study is that when the RMF is switched on, the dielectric tensor is divided into two terms including Hermitian and anti-Hermitian parts. One interesting feature of a system in which the dielectric tensor consists of anti-Hermitian part is that the use of this dielectric tensor correctly describes the feature of the power and energy absorption of EM waves. Here, a mathematical formalism is presented which allows closed form expressions for the total energy absorbed by electron and ion upon passing through the RMF. We suppose the length Δl of the plasma column lies within the RMF. The purpose of this study is to conceive the amount of power absorbed by this plasma column. To calculate this work, we present an expression for the ratio of the average power dissipation in a weakly inhomogeneous plasma [21]

$$\mathrm{d}p = \frac{i\omega}{4\pi}[\varepsilon^*_{ij}(\omega, k, \mathbf{r}) - \varepsilon_{ji}(\omega, k, \mathbf{r})]E_i E^*_j \mathrm{d}\xi\, \mathrm{d}\eta\, \mathrm{d}z. \quad (2)$$

From the above equation, it follows directly that the quantity of heat delivered per unit time or the average power absorbed by the length Δl of the plasma column lying within the RMF is

$$P = \frac{i\omega}{4\pi}\int\int\int[\varepsilon^*_{ij}(\omega, k, \mathbf{r}) - \varepsilon_{ji}(\omega, k, \mathbf{r})]E_i E^*_j h^2 \mathrm{d}\xi\, \mathrm{d}\eta\, \mathrm{d}z. \quad (3)$$

As in this problem the electron and ion have different velocities, the average power absorbed by the electron and the ion can be found as follows. The amount of energy absorbed by the electrons situated in length Δl of the plasma column lying within the RMF is equal to $P(\Delta l/v_{0e})$ while for the ion it is $P(\Delta l/v_{0i})$ where P is the total absorbed power and v_{0e}, v_{0i} are the initial finite drift velocities of the electron and the ion, respectively. It is obvious that knowing the dielectric tensor elements of a candidate medium is critical to calculate various properties of wave propagation and absorption. For this purpose, we continue this line of investigation by calculating the dielectric tensor of such systems. We first consider a long column of multilayer cold collisionless drift plasma with sharp confocal elliptical boundaries, as shown in figure 1. We assume that the lengths of semi-major and the semi-minor axes of the boundary of the Ith region are $a^{(I)}$ and $b^{(I)}$, respectively. This plasma cylinder is confined by means of a uniform magnetic field rotating about the axis of the cylinder. The external uniform induction magnetic field can be described by

$$\mathbf{B}_0(t) = B_0\cos(\omega' t)\hat{x} + B_0\sin(\omega' t)\hat{y}$$

that rotates in a plane perpendicular to the elliptical cylinder axis and B_0, ω' are magnetic flux density and angular frequency of RMF, respectively. The induction field $\mathbf{B}_0(t)$ has been expressed in a Cartesian coordinate system. Generally, multiphase currents with an amplitude of upto several kiloamperes at a frequency ranging from a hundred hertz to several kilohertz are required to obtain the desired RMF with energising magnetic field coils. As is well known, the magnetic field in some magnetic confinement schemes can be generated by currents flowing in the plasma and this plasma current is produced by magnetic induction [47]. Also, a RMF can be generated in an induction motor. In the new type of method, the rotating transverse magnetic field is added to the plasma by magnetic coils set around the chamber which is commonly referred to as the active control coil system (ACCS) [48]. Here, we suppose that a transverse RMF completely penetrates an elliptical plasma cylinder. In addition, we assume that the coefficients of viscosity are constant and that the interaction between the plasma and the cylinder is negligible, i.e., the radial component of the fluid velocity is zero, whereas the axial term does not necessarily vanish. In more detail, no specific boundary conditions are imposed upon the axial velocity.

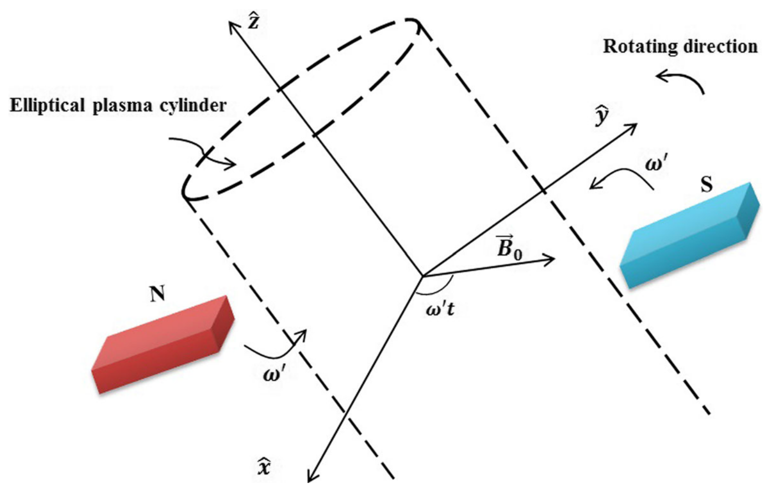

Figure 1. Sketch of the elliptical cylinder filled with multilayer cold collisionless drift inhomogeneous plasmas in RMF

In this configuration, it is assumed that all the elliptic boundaries are confocal. For explicit solutions of the problems, it is necessary to choose an appropriate coordinate system. For this reason, we consider the elliptical coordinate system (ξ, η, z), which is related to the rectangular coordinate system (x, y, z) as $x = l \cosh \xi \cos \eta$, $y = l \sinh \xi \sin \eta$; l being the semi-focal length of the (cross-sectional) ellipse. The elliptic boundary of the Ith region is defined by $\xi = \xi_I$, where $\xi_I = \tanh^{-1}(b^{(I)}/a^{(I)})$. It is assumed that all the regions have the same stationary inhomogeneous charged particle density

$$N_{0\alpha}^{(I)} = N_{0\alpha}^{(I)}(\xi, \eta),$$

where the notations (I) and α refer to the Ith region and charged particles of type of α, respectively. As starting equations, we use a two-fluid model to describe the problem. In this model, the dynamics of the plasma particles are described by continuity and momentum equations as follows [21]:

$$\frac{\partial}{\partial t} N_\alpha^{(I)}(\xi, \eta, z, t) + \nabla \cdot (N_\alpha^{(I)}(\xi, \eta, z, t)\mathbf{V}_\alpha^{(I)}(\xi, \eta, z, t)) = 0, \quad (4)$$

$$\left[\frac{\partial}{\partial t} + \mathbf{V}_\alpha^{(I)}(\xi, \eta, z, t)) \cdot \nabla\right] \frac{\mathbf{V}_\alpha^{(I)}(\xi, \eta, z, t))}{\sqrt{1-(V_\alpha^{(I)}/c)^2}} = \frac{e_\alpha}{m_\alpha}[\mathbf{E}^{(I)}(\xi, \eta, z, t) + \frac{1}{c}\mathbf{V}_\alpha^{(I)}(\xi, \eta, z, t) \times \mathbf{B}^{(I)}(\xi, \eta, z, t)]. \quad (5)$$

We suppose that each variable in the fluid description can be written as a superposition of a large-scale quantity, which represents averaged background behaviour varying slowly in time and space and a small-scale fluctuating quantity. That is in the form

$$N_\alpha^{(I)}(\xi, \eta, z, t) = N_{0\alpha}^{(I)}(\xi, \eta) + N_{1\alpha}^{(I)}(\xi, \eta)\mathrm{e}^{-i(\omega t - k_z z)} \quad (6)$$

$$\mathbf{V}_\alpha^{(I)}(\xi, \eta, z, t) = [V_{1\xi\alpha}^{(I)}(\xi, \eta)\hat{e}_\xi + V_{1\eta\alpha}^{(I)}(\xi, \eta)\hat{e}_\eta + V_{1z\alpha}^{(I)}(\xi, \eta)\hat{e}_z] \times \mathrm{e}^{-i(\omega t - k_z z)} + v_{0\alpha}^{(I)}\hat{e}_z \quad (7)$$

$$\mathbf{E}^{(I)}(\xi, \eta, z, t) = [E_{1\xi}^{(I)}(\xi, \eta)\hat{e}_\xi + E_{1\eta}^{(I)}(\xi, \eta)\hat{e}_\eta + E_{1z}^{(I)}(\xi, \eta)\hat{e}_z]\mathrm{e}^{-i(\omega t - k_z z)} \quad (8)$$

$$\mathbf{B}^{(I)}(\xi, \eta, z, t) = [B_{1\xi}^{(I)}(\xi, \eta)\hat{e}_\xi + B_{1\eta}^{(I)}(\xi, \eta)\hat{e}_\eta + B_{1z}^{(I)}(\xi, \eta)\hat{e}_z] \times \mathrm{e}^{-i(\omega t - k_z z)} + \mathbf{B}_0(t), \quad (9)$$

where $N_{1\alpha}^{(I)}(\xi, \eta)$, $V_{1\alpha}^{(I)}(\xi, \eta)$, $E_{1\alpha}^{(I)}(\xi, \eta)$ and $B_{1\alpha}^{(I)}(\xi, \eta)$ represent the perturbations upto the first order in the physical quantities in the Ith region in elliptic coordinates and $\hat{e}_\xi, \hat{e}_\eta$ and $\hat{e}_z$ are unit vectors along ξ, η and z in elliptic coordinates. Furthermore, $v_{0\alpha}^{(I)}, m_\alpha$ and e_α denote, respectively, drift velocity along z-axis in the Ith region, the rest mass and the electric charge of charged particles of type of α. It should be mentioned here that the direction of propagation has been assumed along z-axis with the wave number k_z. Substituting eqs (6)–(9) into eqs (4) and (5) and using RMF form in elliptic coordinate as

$$\mathbf{B}_0(t) = \frac{l}{h}B_0[(\cos(\omega' t) \sinh \xi \cos \eta + \sin(\omega' t) \cosh \xi \sin \eta)\hat{\xi} - (\cos(\omega' t) \cosh \xi \sin \eta$$

$$- \sin(\omega' t) \sinh \xi \cos \eta)\hat{\eta}],$$

and using linear approximation at the first step, one can find the following relations between perturbed quantities:

$$N_{1\alpha}^{(I)}(\xi,\eta) = \frac{k_z N_{0\alpha}^{(I)} V_{1z\alpha}^{(I)}(\xi,\eta,t)}{\omega - k_z v_0} - \frac{i}{\omega - k_z v_0}\frac{1}{h^2}\left(\frac{\partial}{\partial \xi}\,(h N_{0\alpha}^{(I)} V_{1\xi\alpha}^{(I)}(\xi,\eta,t)) + \frac{\partial}{\partial \eta}(h N_{0\alpha}^{(I)} V_{1\eta\alpha}^{(I)}(\xi,\eta,t))\right), \tag{10}$$

$$V_{1\xi\alpha}^{(I)}(\xi,\eta,t) = \frac{i e_\alpha}{\omega \gamma_\alpha^{(I)} m_\alpha}(1 + f_\alpha^{(I)} x_2^2) E_{1\xi}^{(I)}(\xi,\eta) - \kappa_\alpha^{(I)} x_2 E_{1z}^{(I)}(\xi,\eta) + \frac{i e_\alpha f_\alpha^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha} x_1 x_2 E_{1\eta}^{(I)}(\xi,\eta) - \frac{i e_\alpha v_{0\alpha}^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha c}(1 + f_\alpha^{(I)} x_2^2) B_{1\eta}^{(I)}(\xi,\eta) + \frac{i e_\alpha f_\alpha^{(I)} v_{0\alpha}^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha c} x_1 x_2 B_{1\xi}^{(I)}(\xi,\eta), \tag{11}$$

$$V_{1\eta\alpha}^{(I)}(\xi,\eta,t) = \frac{i e_\alpha}{\omega \gamma_\alpha^{(I)} m_\alpha} \times (1 + f_\alpha^{(I)} x_1^2) E_{1\eta}^{(I)}(\xi,\eta) - \kappa_\alpha^{(I)} x_1 E_{1z}^{(I)}(\xi,\eta) + \frac{i e_\alpha f_\alpha^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha} x_1 x_2 E_{1\xi}^{(I)}(\xi,\eta) + \frac{i e_\alpha v_{0\alpha}^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha c}(1 + f_\alpha^{(I)} x_2^2) B_{1\xi}^{(I)}(\xi,\eta) - \frac{i e_\alpha f_\alpha^{(I)} v_{0\alpha}^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha c} x_1 x_2 B_{1\eta}^{(I)}(\xi,\eta), \tag{12}$$

$$V_{1z\alpha}^{(I)}(\xi,\eta,t) = \kappa_\alpha^{(I)} x_2 E_{1\xi}^{(I)}(\xi,\eta) + \kappa_\alpha^{(I)} x_1 E_{1\eta}^{(I)}(\xi,\eta) - \frac{\kappa_\alpha^{(I)} x_2 v_{0\alpha}^{(I)}}{c} B_{1\eta}^{(I)}(\xi,\eta) + \frac{\kappa_\alpha^{(I)} x_1 v_{0\alpha}^{(I)}}{c} B_{1\xi}^{(I)}(\xi,\eta) + \frac{i e_\alpha}{\tau_\alpha^{(I)} \gamma_\alpha^{(I)} (\omega - k_z v_{0\alpha}^{(I)}) m_\alpha} E_{1z}^{(I)}(\xi,\eta), \tag{13}$$

where in the above equations we have introduced

$$x_1 = \cos(\omega' t)\sinh(\xi)\cos(\eta) + \sin(\omega' t)\cosh(\xi)\sin(\eta)$$

$$x_2 = \cos(\omega' t)\cosh(\xi)\sin(\eta) - \sin(\omega' t)\sinh(\xi)\cos(\eta)$$

$$f_\alpha^{(I)} = \frac{\Omega_\alpha^2 l^2}{\tau_\alpha^{(I)} \gamma_\alpha^{(I)2} \omega(\omega - k_z v_{0\alpha}^{(I)}) h^2}$$

$$\kappa_\alpha^{(I)} = \frac{\Omega_\alpha l e_\alpha}{\tau_\alpha^{(I)} \gamma_\alpha^{(I)2} \omega(\omega - k_z v_{0\alpha}^{(I)}) m_\alpha h}$$

$$\tau_\alpha^{(I)} = 1 - \frac{\Omega_\alpha^2}{\gamma_\alpha^{(I)2} \omega(\omega - k_z v_{0\alpha}^{(I)})}$$

$$\gamma_\alpha^{(I)} = \frac{1}{\sqrt{1 - (v_{0\alpha}^{(I)}/c)^2}}$$

$$h = l\sqrt{\cosh^2(\xi) - \cos^2(\eta)}\ , \quad \Omega_\alpha = \frac{e_\alpha B_0}{m_\alpha c}.$$

Using the differential form of Faraday's law

$$\nabla \times \mathbf{E}^{(I)}(\xi,\eta,z,t) = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}^{(I)}(\xi,\eta,z,t), \tag{14}$$

the perturbed values of magnetic fields in eqs (11)–(13) can be substituted by terms of perturbed values of electric field in the Ith region. Therefore, by computing the perturbed values of magnetic field, eqs (11)–(13) will be converted to

$$V_{1\xi\alpha}^{(I)}(\xi,\eta,t) = \frac{i e_\alpha}{\omega \gamma_\alpha^{(I)} m_\alpha}(1 + f_\alpha^{(I)} x_2^2) \times \left[1 - \frac{k_z v_{0\alpha}^{(I)}}{\omega}\right] E_{1\xi}^{(I)}(\xi,\eta) + \frac{i e_\alpha f_\alpha^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha} x_1 x_2 \times \left[1 - \frac{k_z v_{0\alpha}^{(I)}}{\omega}\right] E_{1\eta}^{(I)}(\xi,\eta) + \left[-\kappa_\alpha^{(I)} x_2 + \frac{e_\alpha v_{0\alpha}^{(I)}}{\omega^2 \gamma_\alpha^{(I)} m_\alpha h} \times \left((1 + f_\alpha^{(I)} x_2^2)\frac{\partial}{\partial \xi} + f_\alpha^{(I)} x_1 x_2 \frac{\partial}{\partial \eta}\right)\right] \times E_{1z}^{(I)}(\xi,\eta) \tag{15}$$

$$V_{1\eta\alpha}^{(I)}(\xi,\eta,t) = \frac{i e_\alpha}{\omega \gamma_\alpha^{(I)} m_\alpha}(1 + f_\alpha^{(I)} x_1^2) \times \left[1 - \frac{k_z v_{0\alpha}^{(I)}}{\omega}\right] E_{1\eta}^{(I)}(\xi,\eta) + \frac{i e_\alpha f_\alpha^{(I)}}{\omega \gamma_\alpha^{(I)} m_\alpha} x_1 x_2 \times \left[1 - \frac{k_z v_{0\alpha}^{(I)}}{\omega}\right] E_{1\xi}^{(I)}(\xi,\eta) \times \left[-\kappa_\alpha^{(I)} x_1 + \frac{e_\alpha v_{0\alpha}^{(I)}}{\omega^2 \gamma_\alpha^{(I)} m_\alpha h} \times \left((1 + f_\alpha^{(I)} x_2^2)\frac{\partial}{\partial \xi} + f_\alpha^{(I)} x_1 x_2 \frac{\partial}{\partial \eta}\right)\right]$$

$$\times E_{1z}^{(I)}(\xi,\eta) \tag{16}$$

$$V_{1z\alpha}^{(I)}(\xi,\eta,t)=\left[\frac{ie_\alpha}{\tau_\alpha^{(I)}\gamma_\alpha^{(I)}(\omega-k_z v_{0\alpha}^{(I)})m_\alpha}-\frac{ik_z v_{0\alpha}^{(I)}}{\omega h}\left(x_1\frac{\partial}{\partial\eta}+x_2\frac{\partial}{\partial\xi}\right)\right]E_{1z}^{(I)}(\xi,\eta)+\kappa_\alpha^{(I)}x_2\left[1-\frac{k_z v_{0\alpha}^{(I)}}{\omega}\right]E_{1\eta}^{(I)}(\xi,\eta)+\kappa_\alpha^{(I)}x_1\left[1-\frac{k_z v_{0\alpha}^{(I)}}{\omega}\right]E_{1\xi}^{(I)}(\xi,\eta). \tag{17}$$

3. The dielectric tensor of an inhomogeneous drift plasma with elliptical cross-section in the presence of RMF

The EM properties of inhomogeneous materials have attracted a great deal of attention due to their wide relevance in astronomy and atmospheric physics [49]. Such information can be realised by employing the dielectric permittivity of the materials. However, a real plasma is always inhomogeneous and as a wave propagates, it can change its characteristics according to the local environment. The dielectric permittivity of such a plasma as its fundamental macroscopic EM characteristic enables us to write down many EM properties of this medium. In this section, we shall particularize the general expression for the dielectric permittivity tensor of inhomogeneous drift plasma with elliptical cross-section embedded in RMF. Then, as a check on the accuracy of our calculation, some limiting situations are analysed.

3.1 *Derivation of the dielectric tensor*

For the purpose of obtaining the dielectric tensor, we require the differential form of Ampere's Law

$$\nabla\times\mathbf{B}^{(I)}(\xi,\eta,z,t)=\frac{1}{c}\frac{\partial}{\partial t}[\tilde{\varepsilon}^{(I)}\mathbf{E}^{(I)}(\xi,\eta,z,t)]=\frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}^{(I)}(\xi,\eta,z,t)+\frac{4\pi}{c}\Sigma_\alpha e_\alpha N_\alpha^{(I)}\times(\xi,\eta,z,t)\mathbf{V}_\alpha^{(I)}(\xi,\eta,z,t) \tag{18}$$

by taking the Fourier transform obtained by (6)–(9). The effective dielectric tensor can be obtained from eqs (10) and (15)–(18) as

$$\tilde{\varepsilon}^{(I)}(\xi,\eta,k_z,t)=\begin{pmatrix}\varepsilon_{\xi\xi}^{(I)}(\xi,\eta,k_z,t) & \varepsilon_{\xi\eta}^{(I)}(\xi,\eta,k_z,t) & \varepsilon_{\xi z}^{(I)}(\xi,\eta,k_z,t)\\ \varepsilon_{\eta\xi}^{(I)}(\xi,\eta,k_z,t) & \varepsilon_{\eta\eta}^{(I)}(\xi,\eta,k_z,t) & \varepsilon_{\eta z}^{(I)}(\xi,\eta,k_z,t)\\ \varepsilon_{z\xi}^{(I)}(\xi,\eta,k_z,t) & \varepsilon_{z\eta}^{(I)}(\xi,\eta,k_z,t) & \varepsilon_{zz}^{(I)}(\xi,\eta,k_z,t)\end{pmatrix}.$$

Explicit expressions for the components of this effective dielectric tensor are

$$\varepsilon_{\xi\xi}^{(I)}(\xi,\eta,k_z,t)=1-\sum_\alpha\frac{\omega_{p\alpha}^{(I)2}(\xi,\eta)}{\gamma_\alpha^{(I)}\omega^2}(1+f_\alpha^{(I)}x_2^2)\times\left[1-\frac{k_z v_{0\alpha}^{(I)}}{\omega}\right] \tag{19}$$

$$\varepsilon_{\xi\eta}^{(I)}(\xi,\eta,k_z,t)=-\sum_\alpha\frac{\omega_{p\alpha}^{(I)2}(\xi,\eta)}{\gamma_\alpha^{(I)}\omega^2}f_\alpha^{(I)}x_1x_2\left[1-\frac{k_z v_{0\alpha}^{(I)}}{\omega}\right] \tag{20}$$

$$\varepsilon_{\xi z}^{(I)}(\xi,\eta,k_z,t)=-\sum_\alpha\frac{i\omega_{p\alpha}^{(I)2}(\xi,\eta)}{\omega}\times\left[\frac{m_\alpha\kappa_\alpha^{(I)}x_2}{e_\alpha}-\frac{v_{0\alpha}^{(I)}}{\gamma_\alpha^{(I)}\omega^2 h}\times\left((1+f_\alpha^{(I)}x_2^2)\frac{\partial}{\partial\xi}+f_\alpha^{(I)}x_1x_2\frac{\partial}{\partial\eta}\right)\right] \tag{21}$$

$$\varepsilon_{\eta\xi}^{(I)}(\xi,\eta,k_z,t)=-\sum_\alpha\frac{\omega_{p\alpha}^{(I)2}(\xi,\eta)}{\gamma_\alpha^{(I)}\omega^2}(1+f_\alpha^{(I)}x_1^2)\left[1-\frac{k_z v_{0\alpha}^{(I)}}{\omega}\right]=\varepsilon_{\xi\eta}^{(I)}(\xi,\eta,k_z,t) \tag{22}$$

$$\varepsilon_{\eta\eta}^{(I)}(\xi,\eta,k_z,t)=1-\sum_\alpha\frac{\omega_{p\alpha}^{(I)2}(\xi,\eta)}{\gamma_\alpha^{(I)}\omega^2}(1+f_\alpha^{(I)}x_1^2)\left[1-\frac{k_z v_{0\alpha}^{(I)}}{\omega}\right] \tag{23}$$

$$\varepsilon_{\eta z}^{(I)}(\xi,\eta,k_z,t)=-\sum_\alpha\frac{i\omega_{p\alpha}^{(I)2}(\xi,\eta)}{\omega}\times\left[\frac{m_\alpha\kappa_\alpha^{(I)}x_1}{e_\alpha}-\frac{v_{0\alpha}^{(I)}}{\gamma_\alpha^{(I)}\omega^2 h}\times\left((1+f_\alpha^{(I)}x_2^2)\frac{\partial}{\partial\eta}+f_\alpha^{(I)}x_1x_2\frac{\partial}{\partial\xi}\right)\right] \tag{24}$$

$$\varepsilon_{z\xi}^{(I)}(\xi,\eta,k_z,t)=\sum_\alpha\frac{im_\alpha\omega_{p\alpha}^{(I)2}(\xi,\eta)}{e_\alpha\omega}\kappa_\alpha^{(I)}x_2+\frac{iv_{0\alpha}^{(I)}}{\omega^3\gamma_\alpha^{(I)}h}$$

$$\times\left[\frac{\partial}{\partial\xi}h\omega_{p\alpha}^{(I)2}(\xi,\eta)(1+f_\alpha^{(I)}x_2^2) +\frac{\partial}{\partial\eta}h\omega_{p\alpha}^{(I)2}(\xi,\eta)f_\alpha^{(I)}x_1x_2\right] \quad (25)$$

$$\varepsilon_{z\eta}^{(I)}(\xi,\eta,k_z,t) =\sum_\alpha \frac{im_\alpha\omega_{p\alpha}^{(I)2}(\xi,\eta)}{e_\alpha\omega}\kappa_\alpha^{(I)}x_1+\frac{iv_{0\alpha}^{(I)}}{\omega^3\gamma_\alpha^{(I)}h} \times\left[\frac{\partial}{\partial\xi}h\omega_{p\alpha}^{(I)2}(\xi,\eta)f_\alpha^{(I)}x_1x_2 +\frac{\partial}{\partial\eta}h\omega_{p\alpha}^{(I)2}(\xi,\eta)(1+f_\alpha^{(I)}x_1^2)\right] \quad (26)$$

$$\varepsilon_{zz}^{(I)}(\xi,\eta,k_z,t)=1 -\sum_\alpha\left(\frac{\omega_{p\alpha}^{(I)2}(\xi,\eta)}{\tau_\alpha^{(I)}\gamma_\alpha^{(I)}(\omega-k_zv_{0\alpha}^{(I)})^2} -\frac{m_\alpha v_{0\alpha}^{(I)}\omega_{p\alpha}^{(I)2}(\xi,\eta)}{e_\alpha h(\omega-k_zv_{0\alpha}^{(I)})}\kappa_\alpha^{(I)} \times(x_1\frac{\partial}{\partial\eta}+x_2\frac{\partial}{\partial\xi})\right)-\frac{m_\alpha v_{0\alpha}^{(I)}}{\omega(\omega-k_zv_{0\alpha}^{(I)})e_\alpha h^2} \times\left(\frac{\partial}{\partial\xi}h\omega_{p\alpha}^{(I)2}(\xi,\eta)\kappa_\alpha^{(I)}x_2+\frac{\partial}{\partial\eta}h\omega_{p\alpha}^{(I)2}(\xi,\eta)\kappa_\alpha^{(I)}x_1\right) +\frac{v_{0\alpha}^{(I)2}}{\omega^3(\omega-k_zv_{0\alpha}^{(I)})h^3\gamma_\alpha^{(I)}} \times\left(\frac{\partial}{\partial\xi}h\omega_{p\alpha}^{(I)2}(\xi,\eta)\left[(1+f_\alpha^{(I)}x_2^2)\frac{\partial}{\partial\xi}+f_\alpha^{(I)}x_1x_2\frac{\partial}{\partial\eta}\right] +\frac{\partial}{\partial\eta}h\omega_{p\alpha}^{(I)2}(\xi,\eta) \times\left[(1+f_\alpha^{(I)}x_2^2)\frac{\partial}{\partial\eta}+f_\alpha^{(I)}x_1x_2\frac{\partial}{\partial\xi}\right]\right), \quad (27)$$

where

$$\omega_{p\alpha}^{(I)2}(\xi,\eta)=\frac{4\pi N_{0\alpha}^{(I)}(\xi,\eta)e_\alpha^2}{m_\alpha}$$

is the square of inhomogeneous plasma frequency of the α-type particles in the Ith region. It is important to remark at this point that the dielectric tensor can be separated into an inhomogeneous non-operational Hermitian part ($\tilde{\varepsilon}_{\mathrm{H}}^{(I)}(\xi,\eta,k_z,t)$) part and an operator part ($\tilde{\varepsilon}_{\mathrm{op}}^{(I)}(\xi,\eta,k_z,t)$), as follows:

$$\tilde{\varepsilon}^{(I)}(\xi,\eta,k_z,t)=\tilde{\varepsilon}_{\mathrm{H}}^{(I)}(\xi,\eta,k_z,t) +\tilde{\varepsilon}_{\mathrm{op}}^{(I)}(\xi,\eta,k_z,t). \quad (28)$$

Clearly, the inhomogeneous operator part of the dielectric tensor can be written as the sum of the Hermitian and anti-Hermitian parts. The anti-Hermitian parts of the effective dielectric tensor given by $\varepsilon_{ij}^{aH}=(\varepsilon_{ij}-\varepsilon_{ji}^*)/2i$ is due to the presence of RMF. The anti-Hermitian part of the dielectric tensor represents not only true damping but also is responsible for the refraction and absorption of radiation. It must be mentioned that the operators $(\partial/\partial\xi,\partial/\partial\eta)$ act on all the functions located in their right-hand side. The results reveal that the elements of the dielectric tensors are, in general, complex, time-dependent and are functions of k_z (spatial dispersive) and ω (time dispersive). From eqs (19)–(27), the operator part of the dielectric tensor will be vanished only for standing plasmas ($v_{0\alpha}^{(I)}=0$) irrespective of whether plasma is homogeneous or inhomogeneous.

3.2 *Analysis of some limiting situations*

First we check the internal consistency of the formalism by verifying that our calculated dielectric tensor elements drop coincides with the results presented in the literature.

(i) In the limit of cold collisionless unmagnetised drift inhomogeneous plasma ($B_0\to 0$), we have $f_\alpha^{(I)}\to 0$, $\kappa_\alpha^{(I)}\to 0$, $\tau_\alpha^{(I)}\to 1$ and the dielectric tensor elements will be converted to

$$\varepsilon_{\xi\xi}^{(I)}(\xi,\eta,k_z) =1-\sum_\alpha\frac{\omega_{p\alpha}^{(I)2}(\xi,\eta)(\omega-k_zv_{0\alpha}^{(I)})}{\gamma_\alpha^{(I)}\omega^3} \quad (29)$$

$$\varepsilon_{\xi\eta}^{(I)}(\xi,\eta,k_z)=0 \quad (30)$$

$$\varepsilon_{\xi z}^{(I)}(\xi,\eta,k_z)=\sum_\alpha\frac{i\omega_{p\alpha}^{(I)2}(\xi,\eta)v_{0\alpha}^{(I)}}{\omega^3\gamma_\alpha^{(I)}h}\left(\frac{\partial}{\partial\xi}\right) \quad (31)$$

$$\varepsilon_{\eta\xi}^{(I)}(\xi,\eta,k_z)=0=\varepsilon_{\xi\eta}^{(I)}(\xi,\eta,k_z) \quad (32)$$

$$\varepsilon_{\eta\eta}^{(I)}(\xi,\eta,k_z)=\varepsilon_{\xi\xi}^{(I)}(\xi,\eta,k_z) =1-\sum_\alpha\frac{\omega_{p\alpha}^{(I)2}(\xi,\eta)(\omega-k_zv_{0\alpha}^{(I)})}{\gamma_\alpha^{(I)}\omega^3} \quad (33)$$

$$\varepsilon_{\eta z}^{(I)}(\xi,\eta,k_z)=\sum_\alpha\frac{i\omega_{p\alpha}^{(I)2}(\xi,\eta)v_{0\alpha}^{(I)}}{\omega^3\gamma_\alpha^{(I)}h}\left(\frac{\partial}{\partial\eta}\right) \quad (34)$$

$$\varepsilon_{z\xi}^{(I)}(\xi,\eta,k_z)=\sum_\alpha\frac{iv_{0\alpha}^{(I)}}{\omega^3\gamma_\alpha^{(I)}h^2}\left[\frac{\partial}{\partial\xi}h\omega_{p\alpha}^{(I)2}(\xi,\eta)\right] \quad (35)$$

$$\varepsilon_{z\eta}^{(I)}(\xi,\eta,k_z) =\sum_\alpha\frac{iv_{0\alpha}^{(I)}}{\omega^3\gamma_\alpha^{(I)}h^2}\left[\frac{\partial}{\partial\eta}h\omega_{p\alpha}^{(I)2}(\xi,\eta)\right] \quad (36)$$

$\varepsilon_{zz}^{(I)}(\xi,\eta,k_z)$

$$= 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^{(I)2}(\xi, \eta)}{(\omega - k_z v_{0\alpha}^{(I)})^2 \gamma_\alpha^{(I)}} + \frac{v_{0\alpha}^{(I)2}}{\omega^3(\omega - k_z v_{0\alpha}^{(I)}) h^3 \gamma_\alpha^{(I)}} \times \left(\frac{\partial}{\partial \xi} h \omega_{p\alpha}^{(I)2}(\xi, \eta) \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} h \omega_{p\alpha}^{(I)2}(\xi, \eta) \frac{\partial}{\partial \eta} \right) \quad (37)$$

which are the same as the dielectric tensor elements presented in ref. [35] when $B_0 = 0$.

(ii) In the limit of cold collisionless unmagnetised homogeneous standing plasma ($B_0 \to 0$ and $v_{0\alpha}^{(I)} \to 0$), the dielectric tensor elements will be converted to

$$\varepsilon_p = 1 - \frac{\omega_{p\alpha}^2}{\omega^2} \quad (38)$$

that is the spatially non-dispersive relative permittivity of the plasma [21,30]. These results are consistent with those in books on electromagnetism, thereby indirectly testifying to the correctness of these results.

4. Coupled equation and hybrid waves

On separating the Hermitian and anti-Hermitian parts of the dielectric tensor that arise from the use of the RMF, EM wave absorption and/or amplification can be described with the objective of finding forms of the wave equations that are appropriate for correctly describing wave properties with the inclusion of inhomogeneity effects. Some properties of wave propagation and absorption in an inhomogeneous plasma are then discussed. Due to the lack of circular symmetry, the propagation of EM waves are the combination of the TE and TM modes (hybrid modes) adopted for elliptical structures that, as might be expected, provides a closed set of coupled equations. In the course of obtaining our solutions to this problem, the dielectric tensor elements should be inserted into eq. (18) by taking into account eqs (8) and (9). Then, the components E_ξ, E_η, B_ξ, B_η in the Ith region can be obtained in terms of E_z and B_z as

$$E_\xi^{(I)}(\xi, \eta, t) = \frac{ic}{\omega \beta^{(I)} h} \left[\chi \psi_{22}^{(I)} \frac{\partial E_z^{(I)}}{\partial \xi} - \chi \psi_{12}^{(I)} \frac{\partial E_z^{(I)}}{\partial \eta} + \psi_{22}^{(I)} \frac{\partial B_z^{(I)}}{\partial \eta} + \psi_{12}^{(I)} \frac{\partial B_z^{(I)}}{\partial \xi} + \frac{i\omega h}{c} \Gamma_1^{(I)} E_z^{(I)} \right], \quad (39)$$

$$E_\eta^{(I)}(\xi, \eta, t) = \frac{ic}{\omega \beta^{(I)} h} \times \left[\chi \psi_{11}^{(I)} \frac{\partial E_z^{(I)}}{\partial \eta} - \chi \psi_{12}^{(I)} \frac{\partial E_z^{(I)}}{\partial \xi} - \psi_{12}^{(I)} \frac{\partial B_z^{(I)}}{\partial \eta} - \psi_{11}^{(I)} \frac{\partial B_z^{(I)}}{\partial \xi} + \frac{i\omega h}{c} \Gamma_2^{(I)} E_z^{(I)} \right], \quad (40)$$

$$B_\xi^{(I)}(\xi, \eta, t) = -\frac{ic}{\omega \beta^{(I)} h} \times \left[(\beta^{(I)} + \chi^2 \psi_{11}^{(I)}) \frac{\partial E_z^{(I)}}{\partial \eta} - \chi^2 \psi_{12}^{(I)} \frac{\partial E_z^{(I)}}{\partial \xi} - \chi \psi_{12}^{(I)} \frac{\partial B_z^{(I)}}{\partial \eta} - \chi \psi_{11}^{(I)} \frac{\partial B_z^{(I)}}{\partial \xi} + \frac{i\omega h}{c} \chi \Gamma_2^{(I)} E_z^{(I)} \right], \quad (41)$$

$$B_\eta^{(I)}(\xi, \eta, t) = \frac{ic}{\omega \beta^{(I)} h} \times \left[(\beta^{(I)} + \chi^2 \psi_{22}^{(I)}) \frac{\partial E_z^{(I)}}{\partial \xi} - \chi^2 \psi_{12}^{(I)} \frac{\partial E_z^{(I)}}{\partial \eta} + \chi \psi_{22}^{(I)} \frac{\partial B_z^{(I)}}{\partial \eta} + \chi \psi_{12}^{(I)} \frac{\partial B_z^{(I)}}{\partial \xi} + \frac{i\omega h}{c} \chi \Gamma_1^{(I)} E_z^{(I)} \right], \quad (42)$$

where

$$\beta^{(I)} = \psi_{11}^{(I)} \psi_{22}^{(I)} - \left(\varepsilon_{12}^{(I)2} - \frac{1}{\omega^2} \frac{\partial}{\partial t} (\varepsilon_{12}^{(I)2} - 2i\omega \varepsilon_{12}^{(I)}) \right)$$

$$\psi_{ij}^{(I)} = \begin{cases} \varepsilon_{ii}^{(I)} - \chi^2 + \dfrac{i}{\omega} \dfrac{\partial \varepsilon_{ii}^{(I)}}{\partial t} & \text{if } i = j \\ \varepsilon_{ij}^{(I)} + \dfrac{i}{\omega} \dfrac{\partial \varepsilon_{ij}^{(I)}}{\partial t} & \text{if } i \neq j \end{cases}, \quad \chi = \frac{ck_z}{\omega}$$

$$\Gamma_1^{(I)} = \psi_{22}^{(I)} \psi_{13}^{(I)} - \psi_{23}^{(I)} \psi_{12}^{(I)},$$
$$\Gamma_2^{(I)} = \psi_{11}^{(I)} \psi_{23}^{(I)} - \psi_{13}^{(I)} \psi_{12}^{(I)},$$
$$\Gamma_3^{(I)} = \psi_{22}^{(I)} \psi_{31}^{(I)} - \psi_{32}^{(I)} \psi_{12}^{(I)},$$
$$\Gamma_4^{(I)} = \psi_{11}^{(I)} \psi_{32}^{(I)} - \psi_{31}^{(I)} \psi_{12}^{(I)}.$$

For brevity, the notations (ξ, η, t) and (ξ, η, k_z, t) have been omitted. To calculate the component of the ponderomotive force influencing particles of type of α (according to eq. (1)), the electric field components and the dielectric permittivity are required. Using dielectric permittivity elements (19)–(27) and electric field components (39) and (42), the average force in units of volume V can be acquired.

We proceed now to the derivation of the wave equation. This can be done by using the EM field components presented above. A corresponding minimal set of coupled equations can be obtained based on the field components $E_z^{(I)}$ and $B_z^{(I)}$ as

$$\frac{\chi\psi_{12}^{(I)}}{\beta^{(I)}}\nabla_\perp^2 E_z^{(I)} - P_{\text{op}}\frac{\partial E_z^{(I)}}{\partial\eta} - U_{\text{op}}\frac{\partial E_z^{(I)}}{\partial\xi} - Y E_z^{(I)} = -\frac{\omega^2}{c^2}B_z^{(I)} - \frac{1}{\beta^{(I)}}(\psi_{11}^{(I)} + \psi_{22}^{(I)})\nabla_\perp^2 B_z^{(I)} + M_{\text{op}}\frac{\partial B_z^{(I)}}{\partial\xi} + N_{\text{op}}\frac{\partial B_z^{(I)}}{\partial\eta} \quad (43)$$

$$S_{\text{op}}\frac{\partial B_z^{(I)}}{\partial\xi} + W_{\text{op}}\frac{\partial B_z^{(I)}}{\partial\eta} - \chi\psi_{12}^{(I)}h\nabla_\perp^2 B_z^{(I)} = -Q_{\text{op}}\frac{\partial E_z^{(I)}}{\partial\xi} + (\beta^{(I)} + \chi^2(\psi_{11}^{(I)} + \psi_{22}^{(I)}))h\nabla_\perp^2 E_z^{(I)} - R_{\text{op}}\frac{\partial E_z^{(I)}}{\partial\eta} - T E_z^{(I)} \quad (44)$$

from which the Laplace operator is defined as

$$\nabla_\perp^2 = \frac{1}{h^2}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right).$$

For deriving these equations we have introduced the operational parameters

$$P_{\text{op}} = \frac{c^2}{h^2\omega^2}\left[\frac{\partial}{\partial\xi}\left(\frac{\chi\psi_{11}^{(I)}}{\beta^{(I)}}\right) + \frac{\partial}{\partial\eta}\left(\frac{\chi\psi_{12}^{(I)}}{\beta^{(I)}}\right) - \frac{i\omega h}{\beta^{(I)}c}\Gamma_1^{(I)} + \frac{\chi}{\beta^{(I)}}\left(2\psi_{12}^{(I)}\frac{\partial}{\partial\eta} + \psi_{11}^{(I)}\frac{\partial}{\partial\xi}\right)\right] \quad (45)$$

$$M_{\text{op}} = -\frac{c^2}{h^2\omega^2}\left[\frac{\partial}{\partial\xi}\left(\frac{\psi_{11}^{(I)}}{\beta^{(I)}}\right) + \frac{\partial}{\partial\eta}\left(\frac{\psi_{12}^{(I)}}{\beta^{(I)}}\right) + \frac{\psi_{12}^{(I)}}{\beta^{(I)}}\frac{\partial}{\partial\eta} - \frac{\psi_{22}^{(I)}}{\beta^{(I)}}\frac{\partial}{\partial\xi}\right] \quad (46)$$

$$N_{\text{op}} = -\frac{c^2}{h^2\omega^2} \times \left[\frac{\partial}{\partial\xi}\left(\frac{\psi_{12}^{(I)}}{\beta^{(I)}}\right) + \frac{\partial}{\partial\eta}\left(\frac{\psi_{22}^{(I)}}{\beta^{(I)}}\right) + \frac{\psi_{12}^{(I)}}{\beta^{(I)}}\frac{\partial}{\partial\xi} - \frac{\psi_{11}^{(I)}}{\beta^{(I)}}\frac{\partial}{\partial\eta}\right] \quad (47)$$

$$U_{\text{op}} = \frac{c^2}{h^2\omega^2} \times \left[-\frac{\partial}{\partial\xi}\left(\frac{\chi\psi_{12}^{(I)}}{\beta^{(I)}}\right) - \frac{\partial}{\partial\eta}\left(\frac{\chi\psi_{22}^{(I)}}{\beta^{(I)}}\right) + \frac{i\omega h}{\beta^{(I)}c}\Gamma_2^{(I)} - \frac{\chi\psi_{22}^{(I)}}{\beta^{(I)}}\frac{\partial}{\partial\eta}\right] \quad (48)$$

$$Q_{\text{op}} = -\frac{i\omega}{c}\chi\Gamma_3^{(I)} + \frac{\beta^{(I)}}{h} \times \left[\frac{\partial}{\partial\eta}\left(\frac{\chi^2\psi_{12}^{(I)}}{\beta^{(I)}}\right) - \frac{\partial}{\partial\xi}\left(1 + \frac{\chi^2\psi_{22}^{(I)}}{\beta^{(I)}}\right) - \frac{i\omega h}{c\beta}\chi\Gamma_1^{(I)} + \frac{1}{h}\left(\chi^2\varphi_{12}^{(I)} \times \frac{\partial}{\partial\eta} + (\beta^{(I)} + \chi^2\psi_{11}^{(I)})\frac{\partial}{\partial\xi}\right)\right] \quad (49)$$

$$R_{\text{op}} = -\frac{i\omega}{c}\chi\Gamma_4^{(I)} + \frac{\beta^{(I)}}{h}\left[\frac{\partial}{\partial\xi}\left(\frac{\chi^2\psi_{12}^{(I)}}{\beta^{(I)}}\right) - \frac{\partial}{\partial\eta}\left(1 + \frac{\chi^2\psi_{11}^{(I)}}{\beta^{(I)}}\right) - \frac{i\omega h}{c\beta^{(I)}}\chi\Gamma_2^{(I)} + \frac{1}{h}\left(\chi^2\varphi_{12}^{(I)}\frac{\partial}{\partial\xi} + (\beta^{(I)} + \chi^2\psi_{22}^{(I)})\frac{\partial}{\partial\eta}\right)\right] \quad (50)$$

$$W_{\text{op}} = \frac{-i\omega}{c}\Gamma_3^{(I)} + \frac{\beta^{(I)}}{h}\left(\frac{\partial}{\partial\eta}\left(\frac{\chi\psi_{12}^{(I)}}{\beta^{(I)}}\right) - \frac{\partial}{\partial\xi}\left(\frac{\chi\psi_{22}^{(I)}}{\beta^{(I)}}\right)\right) - \frac{\chi}{h}\left(\psi_{22}^{(I)}\frac{\partial}{\partial\xi} - 2\psi_{12}^{(I)}\frac{\partial}{\partial\eta}\right) \quad (51)$$

$$S_{\text{op}} = \frac{i\omega}{c}\Gamma_4^{(I)} + \frac{\beta^{(I)}}{h}\left(\frac{\partial}{\partial\eta}\left(\frac{\chi\psi_{11}^{(I)}}{\beta^{(I)}}\right) - \frac{\partial}{\partial\xi}\left(\frac{\chi\psi_{12}^{(I)}}{\beta^{(I)}}\right)\right) + \frac{\chi\psi_{11}^{(I)}}{h}\frac{\partial}{\partial\eta}, \quad (52)$$

where they act on all the functions located in their right-hand side. In addition, in eqs (43) and (44) we have defined the non-operator parameters

$$Y = \frac{ic}{\omega h^2}\left(\frac{\partial}{\partial\xi}\left(\frac{h\Gamma_2^{(I)}}{\beta^{(I)}}\right) - \frac{\partial}{\partial\eta}\left(\frac{h\Gamma_1^{(I)}}{\beta^{(I)}}\right)\right) \quad (53)$$

$$T = \frac{\omega^2 h}{c^2}\left(\psi_{31}^{(I)}\Gamma_1^{(I)} + \psi_{32}^{(I)}\Gamma_2^{(I)} + \frac{i\omega h}{c}\psi_{33}^{(I)}\right) - \frac{i\omega\beta^{(I)}}{ch}\left(\frac{\partial}{\partial\xi}\left(\frac{h\chi\Gamma_1^{(I)}}{\beta^{(I)}}\right) + \frac{\partial}{\partial\eta}\left(\frac{h\chi\Gamma_2^{(I)}}{\beta^{(I)}}\right)\right). \quad (54)$$

Therefore, it can be concluded that RMF causes a system of coupled equations and hybrid wave propagation in an inhomogeneous drift plasma column of elliptic cross-section.

5. Summary and conclusion

In this work, we considered a long column with elliptical cross-section containing multilayer cold collisionless magnetised inhomogeneous drift plasmas. An external magnetic field rotating with uniform angular velocity and amplitude was applied to it. Using two-fluid equations, the generalised field equations for each layer were presented. All the boundaries of the layers were assumed to be confocal elliptic. The dielectric tensor for each layer was obtained. It was shown that the dielectric tensor has two principle parts, the non-operational Hermitian tensor and the pure operator tensor. It was shown that the second part will be omitted when the plasma is considered without initial drift velocity irrespective of whether plasma is homogeneous or inhomogeneous. The transverse electric and magnetic fields were obtained in terms of the longitudinal electric and magnetic fields. The generalised coupled equations of longitudinal electric and magnetic fields were presented. It was shown that in these systems the waves are strongly hybrid. The power absorption related to the anti-Hermitian part of the dielectric tensor was estimated. It is believed that this approach can be useful in helping to investigate the wave propagation, wave absorption, ponderomotive force and particle acceleration in the general cases.

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