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Parametric amplification and dispersion characteristics of optical phonon mode in a semiconductor magnetoplasma

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Abstract. Using the classical hydrodynamic model of semiconductor plasmas, the parametric amplification and dispersion characteristics of optical phonon mode in a semiconductor magnetoplasma are investigated analytically. An expression for effective complex second-order optical susceptibility ($\chi_e^{(2)} = (\chi_e^{(2)})_r + i(\chi_e^{(2)})_i$) is obtained under off-resonant laser irradiation. The analysis deals with qualitative behaviour of threshold pump amplitude ($\xi_{0,\mathrm{th}}$) for the onset of parametric excitation, anomalous parametric dispersion (via $(\chi_e^{(2)})_r$) and parametric gain coefficient (g_{para} via $(\chi_e^{(2)})_i$) with respect to externally applied magnetostatic field (B_0) for different values of doping concentration (n_0). Numerical estimates are made for n-InSb–CO_2 laser system at 77 K. The analysis offers three achievable resonance conditions at which $\xi_{0,\mathrm{th}}$ reduces whereas g_{para} enhances by two orders of magnitude. The lowering in $\xi_{0,th}$ and enhancement in g_{para}, under proper selection of B_0 and n_0, confirms the chosen nonlinear medium as a potential candidate material for the fabrication of efficient optical parametric amplifiers. The negative and positive enhanced parametric dispersion may be of potential use in the study of squeezed state generation as well as in group velocity dispersion in semiconductor magnetoplasmas.

Keywords. Parametric amplification; parametric dispersion; optical phonon; semiconductor plasmas.

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1. Introduction

Parametric interactions (PIs) involving nonlinear mixing of three waves in a medium have proved their importance in the field of nonlinear optics. Optical parametric amplification (OPA), optical parametric dispersion (OPD), optical parametric oscillation (OPO), optical frequency conversion (OFC), optical phase conjugation (OPC), squeezed state generation (SSG), pulse compression etc., are some of the important processes whose origins lie in PIs in a nonlinear medium [1–8]. Among these, OPA process is receiving increasing attention of researchers due to its immense applications in science and technology [9–11]. Optical parametric amplifiers (devices based on OPA) have been recently used to detect weak signal which cannot be detected by detectors [12].

In OPA process, there is a nonlinear interaction among three distinct coherent waves viz. the pump wave, the idler wave and the signal wave and as a result of this interaction either or both the idler wave and signal wave undergo amplification depending upon the properties of nonlinear medium and geometry of the externally applied electric and/or magnetic fields. In this way, in OPA process, there are two distinct viewpoints: (i) properties of nonlinear medium and (ii) characteristics of wave propagation. These may be understood in terms of bunching of charge carriers present in the nonlinear medium under the influence of fields associated with generated waves and those applied externally. In this way any mechanism that affects the bunching of free charge carriers is expected to modify OPA process. In the presence of pump wave, the bunching of charge carriers induces density fluctuations and derives an electron-plasma wave (EPW) in the nonlinear medium. This EPW, in turn, couples nonlinearly with externally applied electric and/or magnetic fields and reinforces the original carrier density fluctuations at coherent coupled mode. In this way, OPA of coupled modes is the nonlinear interaction whereby energy from the pump field is transferred to the generated waves via resonant mechanism under the condition that pump field amplitude is sufficiently large to produce carrier density fluctuations in the nonlinear medium.

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While looking over continuous worldwide research activities on OPA, it has been found that an enhancement of parametric gain coefficient at low pump field in a variety of nonlinear media has been a significant issue to improve the efficiency and functionality of optical parametric amplifiers. Among the various nonlinear media, the doped III–V semiconductor crystals are appropriate hosts for the manufacture of optical parametric amplifiers [13,14]. In addition, in these crystals, there exist various optically excited coherent collective mode (for example, acoustical phonon mode, optical phonon mode, polaron mode, polariton mode etc.) and adopting the coupled mode theory a strong tunable electromagnetic Stokes mode may be achieved as a signal wave at the expense of the pump electromagnetic wave.

In high-density semiconductor plasmas, because the inter-fermion distances are much smaller than the de Broglie wavelength of plasma particles and because of the influence of Pauli exclusion rule, a lot of quantum effects may arise [15]. The variations in linear and nonlinear responses of quantum semiconductor plasmas from that of the classical ones have been reported in [16]. Most of these investigations are based on quantum hydrodynamic (QHD) model of semiconductor plasmas. This model is helpful in the analytical investigations of short-scale collective phenomena in high-density semiconductor plasmas [17]. By including quantum diffraction terms and statistical degeneracy pressure, QHD model becomes a generalisation of the usual fluid model. Using QHD model, the study of parametric amplification and dispersion characteristics of optical phonon (OP) mode in a quantum semiconductor plasma is the future plan of research of the present authors.

While looking over continuous worldwide research activities on OPA and the importance of semiconductor crystals (especially III–V semiconductors) in the fabrication of optical parametric devices, it has been found that up to now, OPA of various coherent excited modes (such as acoustical phonons, plasmons, magnons, polarons, polaritons, excitons etc.), in a variety of III–V semiconductor crystals have been studied by several research groups [18–26]. Available literature reveals that no analytical investigation has been made so far to study OPA in semiconductor plasmas (like n-InSb, n-GaAs, n-InAs, n-GaSb etc.) with OP mode acting as the idler wave. The investigation of the propagation characteristics of coherent OP modes is significant in the investigation of essential properties of crystals [27].

Keeping in view of the possible impact of PIs involving an OP mode, in the present paper, a theoretical formulation of OPA and OPD characteristics of OP mode is developed in a semiconductor magnetoplasma. For this study, the well-known classical hydrodynamic model for the one-component plasma is used. Coupled mode theory is used to analyse coupling among pump, OP and signal modes in the medium. The nonlinear mechanisms considered are: (i) nonlinear induced polarisation by the perturbed carrier density and (ii) nonlinear induced polarisation due to coupling between the pump wave and OP mode by virtue of molecular vibrations. Theoretical model has been developed under rotating wave approximation (RWA). The impacts of an externally applied magnetostatic field and doping concentration on threshold pump amplitude for the onset of OPA, parametric gain coefficient (well above the threshold pump field) and OPD characteristics are studied in detail.

2. Theoretical formulations

We consider the well-known classical hydrodynamic model (valid only in the limit $k_{\rm op} \cdot l \ll 1$; $k_{\rm op}$ is the OP wave number and l is the mean free path of charge carriers) of homogeneous semiconductor magnetoplasma [28]. In semiconductor plasmas, OPA and OPD of OP mode occurs as a result of the coupling between the OP mode and EPW in the presence of pump wave. In the multimode theory of PIs, the molecular vibrations generate electron density fluctuations in the nonlinear medium at molecular vibrational frequency which, in turn, couples nonlinearly with the pump field in the presence of externally applied magnetostatic field and derives an EPW at the sum and difference frequencies. Thus, under certain conditions, the EPW and OP mode derive each other in a semiconductor magnetoplasma at the expense of the pump wave.

We consider the parametric coupling among three waves, viz. an intense electromagnetic wave (pump) $\xi_0(x,t) = \xi_0 \exp[i(k_0 x - \Omega_0 t)]$, an induced OP mode (idler) $u(x,t) = u_0 \exp[i(k_{\rm op} x - \Omega_{\rm op} t)]$ and a scattered Stokes component of the pump wave (signal) $\xi_s(x,t) = \xi_s \exp[i(k_s x - \Omega_s t)]$. The phase matching conditions that these modes should satisfy are: $\hbar\vec{k}_0 = \hbar\vec{k}_s + \hbar\vec{k}_{\rm op}$ and $\hbar\Omega_0 = \hbar\Omega_s + \hbar\Omega_{\rm op}$. We consider the semiconductor plasma to be immersed in a transverse magnetostatic field $\vec{B}_0 = \hat{z}B_0$ (i.e. perpendicular to the direction of the pump wave). This type of field geometry is known as Voigt geometry [29].

Let the semiconductor plasma consists of N harmonic oscillators per unit volume, each oscillator being characterised by its molecular weight M, position x and normal vibrational coordinates $u(x,t)$. The equation of motion for a single oscillator (OP mode) is given by [30]

$$\frac{\partial^2 u}{\partial t^2} + \Gamma \frac{\partial u}{\partial t} + \Omega_t^2 u = \frac{F}{M}, \tag{1a}$$

where Γ is the damping constant which is taken to be equal to the phenomenological phonon collision frequency. Moreover, $\Gamma \approx 10^{-2}\Omega_t$, where Ω_t is the undamped molecular vibrational frequency which is taken to be equal to the transverse OP mode frequency. F stands for the driving force per unit volume experienced by the semiconductor magnetoplasma, which can be expressed as: $F = F^{(1)} + F^{(2)}$, where $F^{(1)} = q_s\xi$ and $F^{(2)} = 0.5\varepsilon\,\alpha_u\,\bar{\xi}^2(x,t)$ represent the forces arising due to Szigeti effective charge q_s and differential polarisability $\alpha_u = (\partial\,\alpha/\partial u)_0$ (say), respectively. The bar over ξ represents the averaging over a few optical cycles, as the molecules are unable to respond at OP mode frequencies. $\varepsilon = \varepsilon_0\varepsilon_\infty$; ε_0 and ε_∞ are the absolute and high frequency permittivities of the semiconductor magnetoplasma, respectively.

In the previously reported works, the origin of PIs has been taken into $F^{(2)}$; the contributions arising due to $F^{(1)}$ has normally been ignored. In the present analytical investigation, we included the forces arising due to Szigeti effective charge to study OPA and OPD characteristics of OP mode in a semiconductor magnetoplasma. After including the effects of Szigeti effective charge (substitution of F in eq. (1a)), the modified equation of motion for $u(x,t)$ of molecular vibrations in a semiconductor plasma becomes

$$\frac{\partial^2 u}{\partial t^2}+\Gamma\frac{\partial u}{\partial t}+\Omega_t^2 u=\frac{1}{M}\left[q_s\xi+\frac{1}{2}\varepsilon\alpha_u\bar{\xi}^2(x,t)\right]. \quad (1b)$$

The other basic equations in the formulation of $\chi_e^{(2)}$ are

$$\frac{\partial\vec{v}_0}{\partial t}+\nu\,\vec{v}_0=-\frac{e}{m}[\vec{\xi}_0+(\vec{v}_0\times\vec{B}_0)]=-\frac{e}{m}(\vec{\xi}_e) \quad (2)$$

$$\frac{\partial\vec{v}_1}{\partial t}+\nu\,\vec{v}_1+\left(\vec{v}_0\frac{\partial}{\partial x}\right)\vec{v}_1=-\frac{e}{m}[\vec{\xi}_1+(\vec{v}_1\times\vec{B}_0)] \quad (3)$$

$$\frac{\partial n_1}{\partial t}+n_0\frac{\partial v_1}{\partial x}+n_1\frac{\partial v_0}{\partial x}+v_0\frac{\partial n_1}{\partial x}=0 \quad (4)$$

$$\vec{P}_{mv}=\varepsilon N\alpha_u u^*\vec{\xi}_e \quad (5)$$

$$\frac{\partial\xi_{1x}}{\partial x}+\frac{1}{\varepsilon}\frac{\partial}{\partial x}(|\vec{P}_{mv}|)=-\frac{n_1 e}{\varepsilon}. \quad (6)$$

These equations have been used by many researchers [31,32] for theoretical modelling of Raman amplification in a semiconductor magneto-plasma. Here $\vec{v}_0$, $\vec{v}_1$, n_0, n_1 and ν represent the equilibrium oscillatory fluid velocity, perturbed oscillatory fluid velocity, equilibrium electron concentration, perturbed electron concentration and electron collision frequency, respectively. P_{mv} stands for nonlinear polarisation due to molecular vibrations of semiconductor plasma. $\vec{\xi}_1$ is the space charge electric field.

The molecular vibrations (at $\Omega_{\rm op}$) causes the modulation of dielectric constant of the semiconductor plasma medium which lead to an energy exchange between the electromagnetic fields separated in frequency by multiples of $\Omega_{\rm op}$ (i.e., $\Omega_0 \pm p\Omega_{\rm op}$, where $p = 1, 2, 3, \ldots$). The modes at sum frequencies (i.e. $\Omega_0 + p\Omega_{\rm op}$) are known as anti-Stokes modes, while modes at difference frequencies (i.e. $\Omega_0 - p\Omega_{\rm op}$) are known as Stokes modes. Here, only the first-order Stokes mode ($p = 1$) has been considered; the electron density fluctuations at off-resonant frequencies ($p \geq 2$) have been neglected [32].

Following Singh *et al* [20], we deduce the perturbed electron density ($n_{1\rm op}$) of the semiconductor magnetoplasma due to molecular vibrations from eqs (1)–(6) as

$$n_{1\rm op}=\frac{2M(\Omega_t^2-\Omega_{\rm op}^2+i\Gamma\Omega_{\rm op})-\varepsilon N(A)}{e\,\alpha_u(\xi_e)_x^*}(ik_{\rm op})u^*. \quad (7)$$

The electron density fluctuation associated with the molecular vibrations (at $\Omega_{\rm op}$) beats with the pump (at Ω_0) and produces fast components of electron density fluctuations. Following the procedure adopted by Singh *et al* [20], we obtain the Stokes mode of this component (at $\Omega_s = \Omega_0 - \Omega_{\rm op}$) as

$$n_{1s}=\frac{ie(k_0-k_{\rm op})(\xi_e)_x}{m(\Omega_{rs}^2-i\,\nu\,\Omega_s)}n_{1\rm op}^*. \quad (8)$$

In eq. (8), $\Omega_{rs}^2=\bar{\Omega}_r^2-\Omega_s^2$, where

$$\bar{\Omega}_r^2=\Omega_r^2\left(\frac{\nu^2+\Omega_{cx}^2}{\nu^2+\Omega_c^2}\right),$$

in which

$$\Omega_{cx,z}\left(=\frac{e}{m}B_{sx,z}\right)$$

are the components of cyclotron frequency $\Omega_c(=\sqrt{\Omega_{cx}^2+\Omega_{cz}^2})$ along the x- and z-axes,

$$\Omega_r^2=\frac{\Omega_p^2\Omega_l^2}{\Omega_t^2},\ \Omega_p=\left(\frac{n_0e^2}{m\varepsilon_0\varepsilon_L}\right)^{1/2}$$

(electron plasma frequency)

and

$$\frac{\Omega_l}{\Omega_t}=\left(\frac{\varepsilon_L}{\varepsilon_\infty}\right)^{1/2}.$$

Ω_l is the longitudinal OP mode frequency and is given by

$$\Omega_l=\frac{k_B\theta_D}{\hbar},$$

where k_B and θ_D are Boltzmann constant and Debye temperature of the semiconductor plasma, respectively. ε_L is the lattice dielectric constant.

From eq. (2), we obtain the oscillatory electron fluid velocity components in the presence of pump and externally applied magnetostatic fields as

$$v_{0x} = \frac{\bar{E}}{\nu - i\Omega_0}$$

and

$$v_{0y} = \frac{(e/m)[\Omega_{cz} + (\nu - i\Omega_0)]\xi_{0x}}{[\Omega_{cz}^2 + (\nu - i\Omega_0)^2]}. \tag{9}$$

The resonant Stokes component of the electron current density due to the finite nonlinear polarisation of semiconductor magnetoplasma can be expressed as [20]

$$J_{cd}(\Omega_s) = n_{1s}^* e v_{0x} = \frac{\varepsilon k_{\text{op}}(k_0 - k_{\text{op}})\,|\bar{\xi}_0|\,\xi_{1x}}{(\Omega_{rs}^2 + i\,\nu\,\Omega_s)(\nu - i\Omega_s)} \times \left[1 - \frac{\varepsilon N}{2M(\Omega_{r\text{op}}^2 + i\Gamma\Omega_{\text{op}})}\left(\frac{2q_s\,\alpha_u}{\varepsilon} - \alpha_u^2\,|(\xi_e)_x|^2\right)\right], \tag{10}$$

where $\Omega_{r\text{op}}^2 = \bar{\Omega}_r^2 - \Omega_{\text{op}}^2$.

Treating the nonlinear induced polarisation as the time integral of induced current density, we obtain

$$P_{cd}(\Omega_s) = \int J_{cd}(\Omega_s)\,\mathrm{d}t = \frac{\varepsilon_\infty e^2 k_{\text{op}}(k_0 - k_{\text{op}})\,|\bar{\xi}_0|\,\xi_{1x}}{m^2\Omega_0\Omega_s(\Omega_{rs}^2 + i\,\nu\,\Omega_s)} \times \left[1 - \frac{\varepsilon N}{2M(\Omega_{r\text{op}}^2 + i\Gamma\Omega_{\text{op}})}\left(\frac{2q_s\,\alpha_u}{\varepsilon} - \alpha_u^2\,|(\xi_e)_x|^2\right)\right] = \varepsilon_0\chi_{cd}^{(2)}\,|\bar{\xi}_0|\,\xi_{1x}. \tag{11}$$

Here, it is worth pointing out that in addition to the polarisation $P_{cd}(\Omega_s)$ due to the induced current density, the semiconductor magnetoplasma also possesses a polarisation $P_{mv}(\Omega_s)$ due to the interaction of the pump wave with the molecular vibrations generated within the medium. Using eqs (1) and (4), we obtain

$$P_{mv}(\Omega_s) = \frac{\varepsilon^2\Omega_0^2 N\,\alpha_u}{2M(\Omega_{r\text{op}}^2 + i\Gamma\Omega_{\text{op}})}\,|\xi_0|\,\xi_{1x} = \varepsilon_0\chi_{mv}^{(2)}\,|\xi_0|\,\xi_{1x}. \tag{12}$$

Thus the effective second-order optical susceptibility $\chi_e^{(2)}$ (at Ω_s) due to both nonlinear current density and molecular vibrations in a semiconductor magnetoplasma is given by

$$\chi_e^{(2)} = \chi_{mv}^{(2)} + \chi_{cd}^{(2)} = \frac{\varepsilon^2\Omega_0^2 N\,\alpha_u}{2\varepsilon_0 M(\Omega_{r\text{op}}^2 + i\Gamma\Omega_{\text{op}})} + \frac{\varepsilon_\infty e^2 k_{\text{op}}(k_0 - k_{\text{op}})}{\varepsilon_0 m^2\Omega_0\Omega_s(\Omega_{rs}^2 + i\,\nu\,\Omega_s)} \times \left[1 - \frac{\varepsilon N}{2M(\Omega_{r\text{op}}^2 + i\Gamma\Omega_{\text{op}})} \times \left(\frac{2q_s\,\alpha_u}{\varepsilon} - \alpha_u^2\,|(\xi_e)_x|^2\right)\right]. \tag{13}$$

From eq. (13), we observed that $\chi_e^{(2)}$ is a complex quantity. It can be put forward as: $\chi_e^{(2)} = (\chi_e^{(2)})_r + i(\chi_e^{(2)})_i$, where $(\chi_e^{(2)})_r$ and $(\chi_e^{(2)})_i$ stand for the real and imaginary parts of complex $\chi_e^{(2)}$. Rationalising eq. (15), we obtain

$$(\chi_e^{(2)})_r = \frac{\varepsilon^2\Omega_0^2 N\,\alpha_u\,\Omega_{r\text{op}}^2}{2\varepsilon_0 M(\Omega_{r\text{op}}^4 + \Gamma^2\Omega_{\text{op}}^2)} + \frac{\varepsilon_\infty e^2 k_{\text{op}}(k_0 - k_{\text{op}})\Omega_{rs}^2}{\varepsilon_0 m^2\Omega_0\Omega_s(\Omega_{rs}^4 + \nu^2\,\Omega_s^2)} \times \left[1 - \frac{\varepsilon N\Omega_{r\text{op}}^2}{2M(\Omega_{r\text{op}}^4 + \Gamma^2\Omega_{\text{op}}^2)} \times \left(\frac{2q_s\alpha_u}{\varepsilon} - \alpha_u^2|(\xi_e)_x|^2\right)\right] \tag{13a}$$

and

$$(\chi_e^{(2)})_i = \frac{\varepsilon^2\Gamma\Omega_0^2\Omega_{\text{op}}N\,\alpha_u}{2\varepsilon_0 M(\Omega_{r\text{op}}^4 + \Gamma^2\Omega_{\text{op}}^2)} + \frac{\varepsilon_\infty e^2 k_{\text{op}}(k_0 - k_{\text{op}})\,\nu\,\Omega_s}{\varepsilon_0 m^2\Omega_0\Omega_s(\Omega_{rs}^4 + \nu^2\,\Omega_s^2)} \times \left[1 - \frac{\varepsilon N\Gamma\Omega_{\text{op}}}{2M(\Omega_{r\text{op}}^4 + \Gamma^2\Omega_{\text{op}}^2)} \times \left(\frac{2q_s\,\alpha_u}{\varepsilon} - \alpha_u^2\,|(\xi_e)_x|^2\right)\right]. \tag{13b}$$

Here it is interesting to note that $(\chi_e^{(2)})_r$ is responsible for OPD while $(\chi_e^{(2)})_i$ gives rise to OPA as well as OPO.

It is well known that OPA occurs at pump field amplitudes well above a certain value, known as threshold pump amplitude. This threshold pump amplitude can be obtained by putting $(\chi_e^{(2)})_i = 0$. This condition yields

$$\xi_{0,\text{th}} = \frac{m}{ek_{\text{op}}}\frac{\Omega_{rs}\Omega_{r\text{op}}(\Omega_0^2 - \Omega_c^2)}{[(\Omega_0^2 - \Omega_{cx}^2) + \nu\,\Omega_{cz}]}. \tag{14}$$

The parametric gain coefficient (well above the threshold pump field) in a semiconductor magnetoplasma can be obtained by employing the relation [20]

$$g_{\text{para}} = \frac{\Omega_s}{\eta c}(\chi_e^{(2)})_i = \frac{\varepsilon^2\Gamma\Omega_0^2\Omega_{\text{op}}\Omega_s N\,\alpha_u}{2\varepsilon_0\eta c M(\Omega_{r\text{op}}^4 + \Gamma^2\Omega_{\text{op}}^2)} + \frac{\varepsilon_\infty e^2 k_{\text{op}}(k_0 - k_{\text{op}})\,\nu\,\Omega_s^2}{\varepsilon_0\eta c m^2\Omega_0\Omega_s(\Omega_{rs}^4 + \nu^2\,\Omega_s^2)}$$

$$\times\left[1-\frac{\varepsilon N\Gamma\Omega_{\rm op}}{2M(\Omega_{r\rm op}^4+\Gamma^2\Omega_{\rm op}^2)}\right.$$
$$\left.\times\left(\frac{2q_s\,\alpha_u}{\varepsilon}-\alpha_u^2\,|(\xi_e)_x|^2\right)\right]. \qquad (15)$$

Equations (16) and (17) reveal that both the threshold pump amplitude $\xi_{0,\rm th}$ for the onset of OPA and the parametric gain coefficient $g_{\rm para}$, are strongly influenced by material parameters and externally applied magnetostatic field B_0 (via parameter Ω_c and hence Ω_{rs}). The parametric gain of the signal as well as the idler waves can be possible only if $g_{\rm para}$ obtained from eq. (17) is positive for pump field $|\xi_0| > |\xi_{0,\rm th}|$.

3. Results and discussion

In order to establish the validity of the present theoretical formulation, we consider a III–V semiconductor magnetoplasma (viz. n-InSb) at 77 K irradiated by a pulsed 10.6 μm ($\Omega_0 = 1.78\times10^{14}$ s^{-1}) CO_2 laser. The physical parameters of the n-InSb crystal are given in refs [26–32].

Using the material parameters of n-InSb-CO_2 laser system, we determine the values of $(\chi_e^{(2)})_r$ and $(\chi_e^{(2)})_i$ for different pairs of B_0 and n_0. We find $(\chi_e^{(2)})_r = 3.40\times10^{-10}, 6.36\times10^{-8}, 4.79\times10^{-8}$ and 2.15×10^{-7} mV^{-1} when $(B_0, n_0) = (0$ T, 10^{23} m$^{-3})$, $(3$ T, 3.0×10^{23} m$^{-3})$, $(11$ T, 2.0×10^{23} m$^{-3})$ and $(14.2$ T, 1.5×10^{23} m$^{-3})$. Also, $(\chi_e^{(2)})_i = 3.28\times10^{-10}, 3.15\times10^{-8}, 1.64\times10^{-8}$ and 5.58×10^{-8} mV^{-1} when $(B_0, n_0) = (0$ T, 10^{23} m$^{-3})$, $(3$ T, 3.0×10^{23} m$^{-3})$, $(11$ T, 2.0×10^{23} m$^{-3})$ and $(14.2$ T, 1.5×10^{23} m$^{-3})$. The magnitudes of $(\chi_e^{(2)})_r$ and $(\chi_e^{(2)})_i$ agree well with other theoretically quoted values [14,33]. In the presence of magnetostatic field ($B_0 = 3, 11, 14.2$ T), both $(\chi_e^{(2)})_r$ and $(\chi_e^{(2)})_i$ are nearly 10^2 times higher than in the absence of magnetostatic field ($B_0 = 0$ T).

It is interesting to note from eq. (15) that for $\xi_0 < 7\times10^7$ V m^{-1},

$$\frac{2q_s\alpha_u}{\varepsilon} > \alpha_u^2\left|(\xi_e)_x\right|^2$$

and parametric amplification is due to finiteness of both q_s and α_u. However, for $\xi_0 > 7\times10^7$ Vm^{-1},

$$\frac{2q_s\,\alpha_u}{\varepsilon} < \alpha_u^2\,|(\xi_e)_x|^2$$

the contribution of q_s is wiped-off and parametric gain coefficient becomes dependent only on α_u. This result is in good agreement with the theory of stimulated Raman scattering of transverse OPs in weakly polar narrow band-gap semiconductor magnetoplasmas developed by Singh and Singh [32].

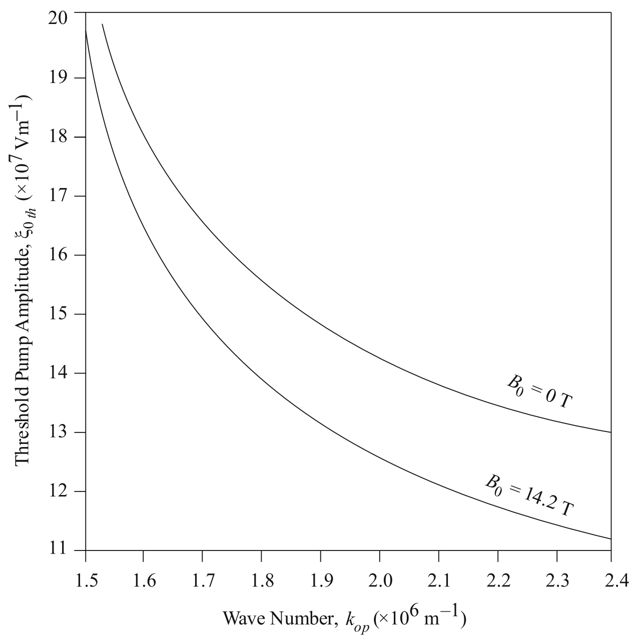

Figure 1. Variation of $\xi_{0,\rm th}$ with $k_{\rm op}$ for $B_0 = 0$ and 14.2 T at $n_0 = 1.5\times10^{23}$ m^{-3}.

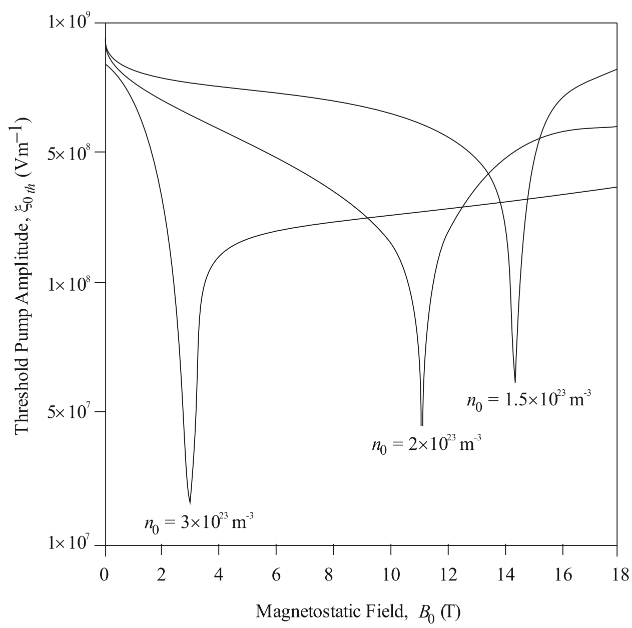

Figure 2. Variation of $\xi_{0,\rm th}$ with B_0 for $n_0 = 1.5\times10^{23}$, 2×10^{23} and 3×10^{23} m^{-3}.

The numerical estimations depicting the parametric amplification and dispersion characteristics of OPs (for n-InSb) are plotted in figures 1–6.

Figure 1 shows the variation of threshold pump amplitude $\xi_{0,\rm th}$ with wave number $k_{\rm op}$ for two different cases: (i) in the absence of magnetostatic field ($B_0 = 0$ T) and (ii) in the presence of magnetostatic field ($B_0 = 14.2$ T).

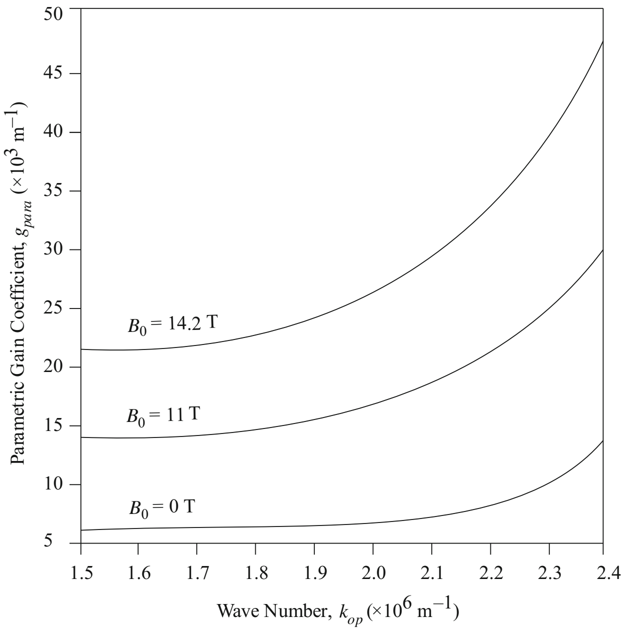

Figure 3. Variation of g_{para} with k_{op} for $B_0 = 0$, 11 and 14.2 T at $n_0 = 1.5 \times 10^{23}$ m^{-3} and $\xi_0 = 12.5 \times 10^8$ V m^{-1}.

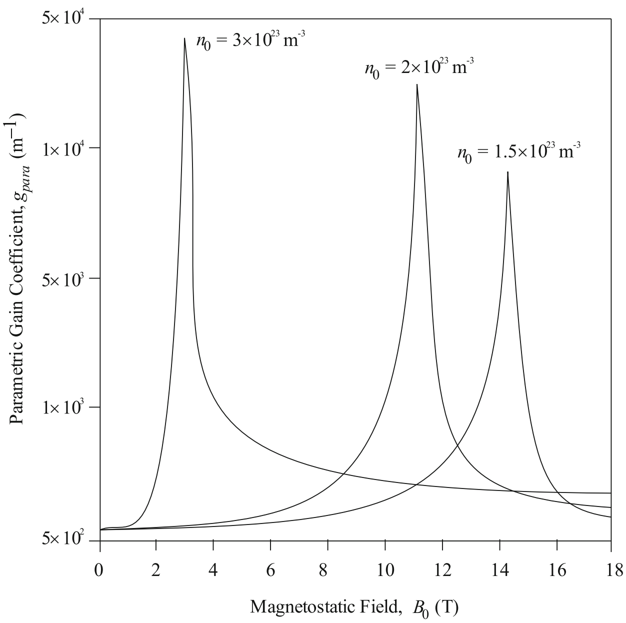

Figure 4. Variation of g_{para} with B_0 for $n_0 = 1.5 \times 10^{23}$, 2×10^{23} and 3×10^{23} m^{-3} at $\xi_0 = 12.5 \times 10^8$ V m^{-1}.

We observed that in both the cases, $\xi_{0,\text{th}}$ is comparatively larger for smaller values of k_{op}. With increasing k_{op}, $\xi_{0,\text{th}}$ decreases parabolically. This behaviour arises because $\xi_{0,\text{th}} \propto k_{\text{op}}^{-1}$, as suggested from eq. (14). A comparison between the two cases reveal that for the plotted regime of k_{op}, $\xi_{0,\text{th}}$ is smaller at $B_0 = 14.2$ T than at $B_0 = 0$ T. This is because around $B_0 = 14.2$ T, $\Omega_c^2 \sim \Omega_0^2$ and $(\Omega_0^2 - \Omega_c^2) \to 0$ [eq. (14)], thus lowering the value of $\xi_{0,\text{th}}$.

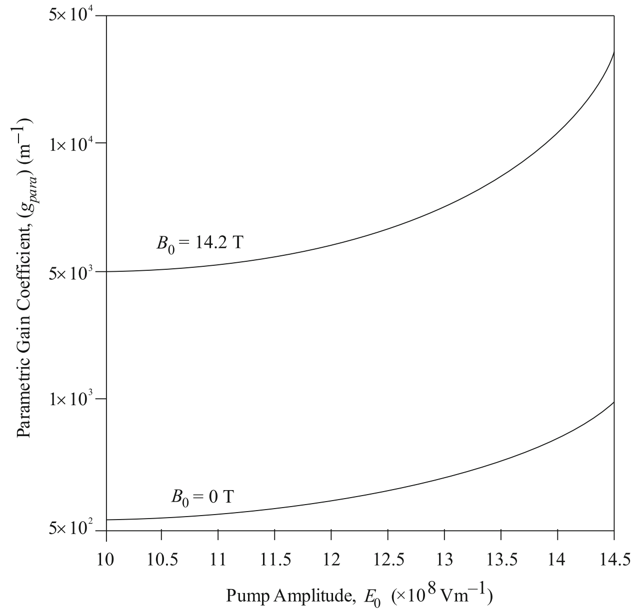

Figure 5. Variation of g_{para} with ξ_0 for $B_0 = 0$ and 14.2 T at $n_0 = 10^{22}$ m^{-3} and $\xi_0 = 12.5 \times 10^8$ V m^{-1}.

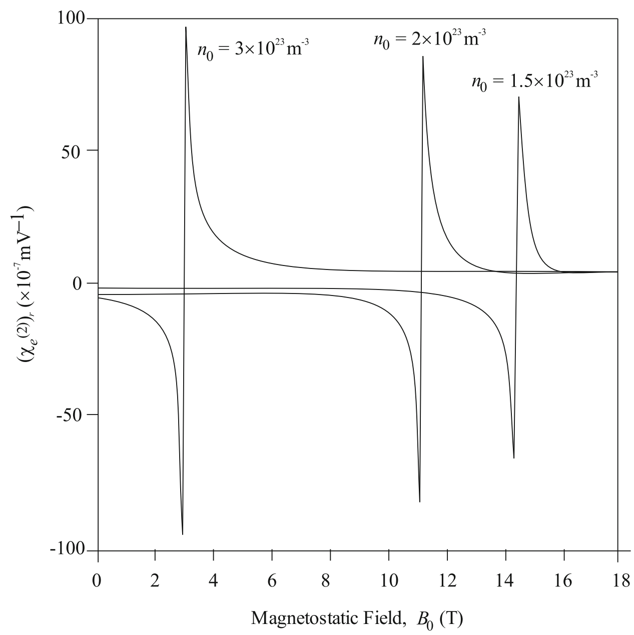

Figure 6. Variation of $(\chi_e^{(2)})_r$ with B_0 for $n_0 = 1.5 \times 10^{23}$, 2×10^{23} and 3×10^{23} m^{-3}.

Figure 2 shows the variation of threshold pump amplitude $\xi_{0,\text{th}}$ with magnetostatic field B_0 for three different values of doping concentration: (i) $n_0 = 1.5 \times 10^{23}$ m^{-3}, (ii) $n_0 = 2 \times 10^{23}$ m^{-3} and (iii) $n_0 = 3 \times 10^{23}$ m^{-3}. We observed that in all the three cases, $\xi_{0,\text{th}}$ is very large ($\sim 10^9$ V m^{-1}) at $B_0 = 0$ T. With increasing B_0, $\xi_{0,\text{th}}$ decreases sharply achieving a minimum value $(\xi_{0,\text{th}})_{\text{min}}$. Interestingly, with rising n_0, $(\xi_{0,\text{th}})_{\text{min}}$ decreases and shifts towards lower value of B_0. Using eq. (14), this behaviour may be explained as follows:

(i) When $n_0 = 1.5 \times 10^{23}\,\mathrm{m}^{-3}$, the minimum at $B_0 = 14.2$ T arises due to the resonance between electron cyclotron frequency and pump frequency ($\Omega_0^2 \sim \Omega_c^2$).

(ii) When $n_0 = 2\times10^{23}\,\mathrm{m}^{-3}$, the minimum at $B_0 = 11$ T arises due to the resonance between coupled cyclotron plasmon frequency and Stokes frequency ($\bar{\Omega}_r^2 \sim \Omega_s^2$) via parameter Ω_{rs}.

(iii) When $n_0 = 3 \times 10^{23}\,\mathrm{m}^{-3}$, the minimum at $B_0 = 3$ T arises due to the resonance between coupled cyclotron plasmon frequency and OP mode frequency ($\bar{\Omega}_r^2 \sim \Omega_{\mathrm{op}}^2$) via parameter $\Omega_{r\mathrm{op}}$.

We calculate the values of $(\xi_{0,\mathrm{th}})_{\min}$ for different pairs of B_0 and n_0. We find $(\xi_{0,\mathrm{th}})_{\min} = 8.0\times10^9$, 2.1×10^7, 4.8×10^7 and $6.0\times10^7\,\mathrm{V\,m}^{-1}$ when $(B_0, n_0) = (0$ T, $10^{23}\,\mathrm{m}^{-3})$, $(3$ T, $3.0\times10^{23}\,\mathrm{m}^{-3})$, $(11$ T, $2.0\times10^{23}\,\mathrm{m}^{-3})$ and $(14.2$ T, $1.5\times10^{23}\,\mathrm{m}^{-3})$. Thus, $\xi_{0,\mathrm{th}}$ may be lowered ($\sim10^2$ times) in III–V semiconductor magnetoplasmas around the resonance conditions, which may be achieved by proper selection of doping concentration and externally applied magnetostatic field. This is an interesting feature of the present analysis.

Figure 3 shows the variation of parametric gain coefficient g_{para} (at $\xi_0 = 12.5 \times 10^8\,\mathrm{V\,m}^{-1}$) with wave number k_{op} for the same cases considered in figure 1. We observed that in both the cases, g_{para} is comparatively smaller for smaller values of k_{op}. With increasing k_{op}, g_{para} increases quadratically. This behaviour arises because $g_{\mathrm{para}} \propto k_{\mathrm{op}}^2$ as suggested from eq. (15). A comparison between the two cases reveal that for the plotted regime of k_{op}, g_{para} is larger at $B_0 = 14.2$ T than at $B_0 = 0$ T because around $B_0 = 14.2$ T, $\Omega_c^2 \sim \Omega_0^2$ and $(\Omega_0^2 - \Omega_c^2) \to 0$ [eqs (15)], thus enhancing the value of g_{para}.

Figure 4 shows the variation of parametric gain coefficient g_{para} (at $\xi_0 = 12.5 \times 10^8$ V m^{-1}) with magnetostatic field B_0 for the three cases considered in figure 2. We observed that in all the three cases, g_{para} is very small ($\sim10^2\,\mathrm{m}^{-1}$) at $B_0 = 0$ T. With increasing B_0, g_{para} increases sharply achieving a maximum value $(g_{\mathrm{para}})_{\max}$. Interestingly, with rising n_0, $(g_{\mathrm{para}})_{\max}$ increases and shifts towards lower value of B_0. Using eq. (15), this behaviour may be explained in the same way as the behaviour of $\xi_{0,\mathrm{th}}$ with B_0 has been explained in figure 2. We calculate the values of $(g_{\mathrm{para}})_{\max}$ (for $\xi_0 = 12.5 \times 10^8\,\mathrm{V\,m}^{-1}$) for different pairs of B_0 and n_0. We find $(g_{\mathrm{para}})_{\max} = 5.0\times10^2, 4.8\times10^4, 2.5\times10^4$ and $8.5\times10^3\,\mathrm{m}^{-1}$ when $(B_0, n_0) = (0$ T, $10^{23}\,\mathrm{m}^{-3})$, $(3$ T, $3.0 \times 10^{23}\,\mathrm{m}^{-3})$, $(11$ T, $2.0 \times 10^{23}\,\mathrm{m}^{-3})$ and $(14.2$ T, $1.5 \times 10^{23}\,\mathrm{m}^{-3})$. Thus, g_{para} can be enhanced ($\sim10^2$ times) in III–V semiconductor magnetoplasmas around the resonance conditions. The magnetostatic field considered in the present numerical analysis ($0 \leq B_0 \leq 18$ T) is easily attainable in the laboratory. It should be worth pointing out that Generazio and Spector [34] have developed the theoretical formulation of free carrier absorption for n-InSb-CO_2 and n-InSb-CO laser systems at 77 K by placing the sample in an external magnetostatic field $B \leq 20$ T.

Figure 5 shows the variation of parametric gain coefficient g_{para} with pump amplitude $\xi_0(>\xi_{0,\mathrm{th}})$ for two different cases: (i) absence of magnetostatic field ($B_0 = 0$ T) and (ii) presence of magnetostatic field ($B_0 =14.2$ T). In both the cases g_{para} increases quadratically with respect to ξ_0. It may be recalled that pump field cannot be increased arbitrarily which may finally lead to the optical damage of the sample.

Being one of the principal objectives of the present analysis, the nature of dependence of the parametric dispersion arising due to $(\chi_e^{(2)})_r$ has been analysed in figure 6. Here, $(\chi_e^{(2)})_r$ is plotted with respect to magnetostatic field B_0 for three different values of doping concentration n_0 ($= 1.5 \times 10^{23}$, 2×10^{23} and 3×10^{23} m^{-3}). It is worth mentioning that there exists a distinct anomalous parametric dispersion regime that varies in magnitude with doping concentration. It can be observed that $(\chi_e^{(2)})_r$ can be both negative and positive under the anomalous regime. For $\Omega_0 < \Omega_c$, $\bar{\Omega}_r < \Omega_s$ and $\bar{\Omega}_r < \Omega_{\mathrm{op}}$, $(\chi_e^{(2)})_r$ is a negative quantity at $n_0 = 1.5 \times 10^{23}$, 2×10^{23} and 3×10^{23} m^{-3}, respectively. A slight increase in tuning between Ω_0 and Ω_c, $\bar{\Omega}_r$ and Ω_s and between $\bar{\Omega}_r$ and Ω_{op} beyond the above conditions causes a sharp rise in $(\chi_e^{(2)})_r$; making it vanish when $\Omega_0 \sim \Omega_c$, $\bar{\Omega}_r \sim \Omega_s$ and $\bar{\Omega}_r \sim \Omega_{\mathrm{op}}$ at respective values of doping concentration. After a particular resonance condition, $(\chi_e^{(2)})_r$ increases very sharply. By further increasing the value of B_0, $(\chi_e^{(2)})_r$ decreases very rapidly and saturate at larger values of magnetostatic field. This figure reveals that a proper selection of doping concentration and externally applied magnetostatic field enable one to achieve either negative or positive enhanced parametric dispersion. This result can be appropriately exploited in the generation of squeezed states. It can also be envisaged that a practical demonstration of the aforementioned parametric dispersion may lead to the possibility of observation of group velocity dispersion in semiconductor magnetoplasmas.

4. Conclusions

On the basis of the above discussion, the following conclusions may be drawn:

1. CHD model of semiconductor plasma has been successfully applied to study OPA and OPD

characteristics of the OP mode in semiconductor magnetoplasma medium under off-resonant laser irradiation.

2. Resonance between (a) electron cyclotron frequency and pump frequency, (b) coupled cyclotron plasmon frequency and Stokes frequency and (c) coupled cyclotron plasmon frequency and OP mode frequency reduces the threshold pump amplitude for the onset of parametric process and enhances the parametric gain coefficient of OP mode by about 10^2 times in III–V semiconductor magnetoplasmas.
3. The parametric gain coefficient can be enhanced by increasing the wave number magnitude and simultaneous application of externally applied static magnetostatic field. Moreover, higher pump field yields higher parametric gain coefficient.
4. A significant enhancement in the parametric dispersion (both negative and positive) can be achieved by the proper selection of doping concentration and externally applied magnetostatic field. This can be of potential use in the study of the generation of squeezed states as well as in group velocity dispersion in semiconductor magnetoplasmas.

The present analytical investigation presents a model most appropriate for the finite solid-state plasma and it may be used as a guide to an experimentalist who wants to fabricate efficient parametric amplifiers based on pump-optical phonon interaction in semiconductor magnetoplasmas.

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