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Enhancing traffic capacity of two-layer networks by link deletion

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Abstract. Two-layer or multilayer networks can more accurately reveal the transmission dynamics of the actual network and increase the traffic capacity. Therefore, we considered optimising the physical layer topology of the two-layer network, and proposed a link deletion strategy at the physical layer. The nodes with maximum weight (k_i*k_j) were deleted, where k_i and k_j are the degrees of nodes i and j, respectively, and the traffic load was redistributed from the central node to the non-central nodes. The simulation results showed that the strategy significantly increased the network traffic capacity. In two-layer network model, the logical layer network structure is unchanged and the shortest path routing strategy is used.

Keywords. Two-layer networks; traffic capacity; link deletion.

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1. Introduction

At the end of the last century, since Barabási and Albert discovered the scale-free property [1] and Watts and Strogatz found the small-world property [2], complex network theory has attracted the attention of many scientists from multiple fields. Scientists and researchers found that many real networks would display the characteristics of 'complex' [3–10], such as the Internet, the World Wide Web, aircraft route networks and phone networks. With the explosive growth of data, network congestion becomes more and more serious. Faced with the increasing demand for network performance [11–13], this problem is very important and needs to be solved urgently. It is urgent to reduce traffic congestion and increase traffic capacity of communication networks [14–19]. In order to improve network traffic capacity, three methods are proposed: designing routing strategy [20,21], changing network structure [22–24] and reallocating network resources [25,26].

Researchers have done a lot of work in improving traffic capacity. Yan *et al* [27] proposed a routing strategy called ER, where the path between nodes x and y is defined as the path in which the sum degree of nodes is a minimum. Wang *et al* [28] established two-layer network models for railway transportation system and proposed a new resource allocation method. Morris and Barthelemy [29] studied the process of data transmission over a two-layer coupled network and found that the shortest path of packets decreased with the increase of the degree of couplings. Zhang *et al* [30] proposed a new routing strategy to be applied in a two-layer network, and the simulation results showed that this routing strategy played an important role in improving the network traffic capacity. Liu *et al* [31] proposed a removing edge strategy by closing or cutting some links between some large-degree nodes. Another efficient link-removal strategy is proposed by Huang and Chow [32]. However, with the rapid development of society, the scale of real networks maintains high-speed growth. In order to maximise the network capacity, a rational adding link strategy is important. Huang and Chow [33] also showed that the traffic capacity can be enhanced by adding shortcut links to the existing networks. Jiang *et al* [34] proposed a scheme to incrementally enhance the network performance by adding a small fraction of edges or rewiring a fraction of edges for the network. Zhang *et al* [35] proposed a convenient method

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to enhance the transmission efficiency of scale-free networks dramatically by kicking out the edges linking to nodes with large betweenness. Zhuo *et al* [36] found that the physical layer played a greater role than the logical layer in the traffic capacity of the network, and it was proved that the physical layer had a higher tolerance under the premise of isomorphic physical topology. Ma *et al* [37] proposed a strategy to increase network traffic by adding some links according to the local centrality of nodes under the shortest path length.

In previous studies, most network models were assumed to be single-layer structures in the transmission dynamics. However, most real networks (for example, wired and wireless networks and peer-to-peer networks) have two-layer structures, and these layers are interactive and interdependent. With the vigorous development of communication networks, many application networks have been widely used in this infrastructure. In this article, we adopt the two-layer network model as the research object [38]. Physical layer and logical layer are the main components of the two-layer network, and many scholars have studied its structure. Jiang *et al* [34] and Zhang *et al* [35] have changed the logical layer of the two-layer network to increase the capacity by adding or deleting links. As the virtualisation process evolves, the physical layer can be reconstructed, and therefore, optimising the physical link structure is an efficient way to increase traffic capacity. Therefore, we propose a link deletion strategy on the physical layer.

This article is arranged as follows: in §2, the two-layer network and the traffic model are introduced. In §3, the link deletion strategy is described. In §4, the simulation results are showed and analysed. Finally, we summarize our results in §5.

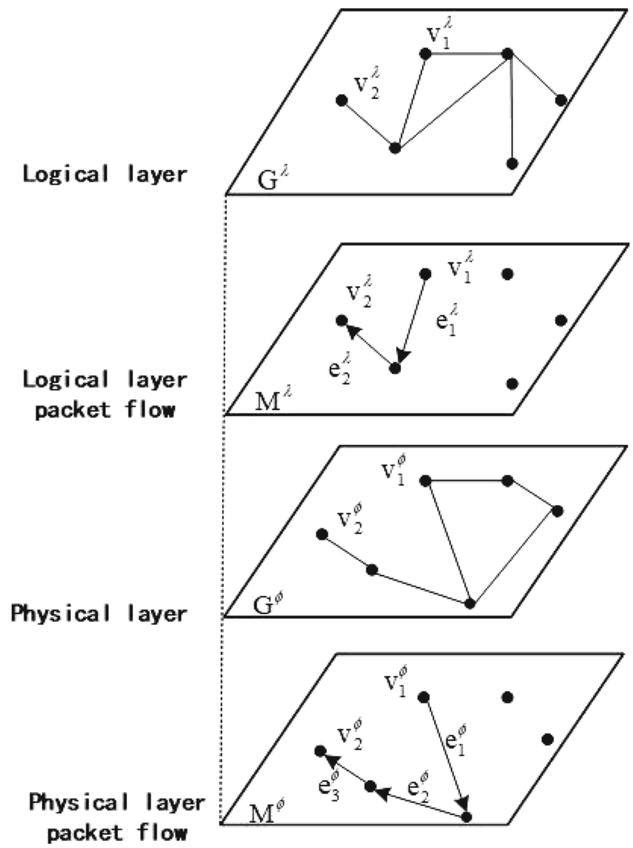

Figure 1. The transmission process of a packet under the two-layer network model. It shows a packet of the process of transmission from v_1^λ to v_2^λ. G^λ is the logical layer, G^ϕ is the physical layer, M^λ is the logical layer packet flow and M^ϕ is the physical layer packet flow.

2. The model

2.1 *The two-layer network model*

Multilayer network structure is a mature structure and is specially applied to actual network structure. In a two-layer network, the upper network is served by the lower network, while the lower network is served by the upper network. Similarly, the physical layer and logical layer of the two-layer network interact and are independent of each other. The logical layer is established on the physical layer to transmit packets according to specific routing strategies. The physical layer, as a 'black box', receives data packets from the logical layer for transmission and provides stable information exchange service for the logical layer. The physical layer can simply map the physical path, and the transmission process is not affected by the routing strategy selected by the logical layer. Logical edge $e^\lambda = (v_1^\lambda, v_2^\lambda)$ maps to the physical layer, generating the physical path, connected by physical nodes v_1^ϕ and v_2^ϕ, and corresponding to logical layer nodes v_1^λ and v_2^λ. The shortest path route strategy is used in this work. If there is more than one shortest path, we can randomly select one for transmission. Suppose that the node v_1^λ generates a packet and node v_2^λ is its destination node. The packet passes through e_1^λ and e_2^λ according to the shortest path on the physical layer G^ϕ (this corresponds to the logical layer G^λ). The shortest path from the source node to the destination node on the logical layer can be mapped to the physical layer. So we can represent the edges mapping e_1^λ to (e_1^ϕ, e_2^ϕ) and e_2^λ to e_3^ϕ. The logical layer packet travels along e_1^λ and e_2^λ to the destination node, but at the physical layer it goes through three nodes e_1^ϕ, e_2^ϕ, e_3^ϕ, and it reaches its destination in the physical layer G^ϕ. The detailed packet delivery process is shown in figure 1.

2.2 *Traffic model*

The traffic model is an important model to measure the network traffic performance in complex networks. Single-layer network model cannot fully explain the transmission process. So it is necessary to build a two-layer network model. The traffic model is described in detail as follows [39–41]:

- Each node has a special function and can act as either a router or a host. The length of the cache queue for each node is infinite, and the first input first output (FIFO) discipline is used on each queue after the packets arrive at the node. Source node and destination node are randomly selected in the network, and the processing capacity of each node is set to C. In this case, after the data packet is generated, it will be placed at the end of the node queue for processing.
- The function of the logical layer is routing. What it does is to find the shortest path between the source node and the destination node of the newly generated packet, along which the packet will be passed to the logical layer. For any set of source and destination nodes in the network, there is at least one shortest path between them. When a packet reaches its destination, it is deleted.
- Each layer of network has its own function, while the physical layer function is to transfer packets between nodes. In any period of time, given a physical path, the nodes of the physical layer pass C packets to the next node according to this path. When the packets reach the last node of the physical path, the logical layer will select the next physical path. This process is repeated until it is passed to the destination node.

In the previous study, the transmission performance of the whole network was measured by traffic capacity R_c , that is, the critical packet generation rate at which a continuous phase transition will occur from free-flow to congested phase. When $R < R_c$, the network is in free flow condition. Free-flow phase means that the number of generated packets are less than or equal to the removed packets and no congestion occurs. With the increase of the formation rate of packets, the number of packets in the network will increase, network transmission state will change, and when $R > R_c$, the network enters the congested phase, which means that the number of packets generated in the network exceeds the processing capacity of the network, and the network will become congested. That is, when $R = R_c$, the state will shift from free flow to congestion. The order function represents the packet transmission state and is expressed as [42]

$$H(R)= \lim_{t\to\infty} \frac{C}{R}\frac{\langle \Delta W\rangle}{\Delta \mathrm{t}}, \tag{1}$$

where $\langle \Delta W\rangle = W(t+\Delta t) - W(t)$, $\langle \Delta W\rangle$ represents the average value in the Δt time window and $W(t)$ represents the number of packets that can be generated on the network. When $H(R)$ is around zero, the inflow and outflow are balanced and no congestion occurs in the network. When $H(R)$ is a constant larger than zero, the number of packets in the network increase and the network is congested.

3. Link deletion strategy

In complex networks, degree is an important parameter. The degree of a node in a network is the number of connections it has with other nodes. Therefore, if the degree of a node is larger, the more important the node is in the network. The distribution function can well explain the distribution of node degree in the network, $P(k) \propto k^{-r}$. $P(k)$ represents the probability that the degree of the selected node is k when the node is randomly selected.

At first, we assume that the physical layer network structure is the same as the logical layer. Each physical link is mapped to one logical link. We consider deleting links for the physical layer. For convenience sake, the fraction of deleted links is denoted as

$$f = L'/L, \tag{2}$$

where L' represents the number of links to be deleted, while L represents the total number of links in the physical layer existing in the original network. When L' links are removed from the physical layer, the link deletion process is terminated. In the link deletion process, calculating the link L' with maximal value $(w_{ij} = k_i * k_j)$ which is computed at the beginning of link deletion process are removed in sequence. The most important thing is to keep the network connected while doing the experiment. In the process of deleting the link, if the physical layer link is deleted, the connectivity of the network will be affected. We shall not delete this link and continue to delete the next link. The specific process of this strategy algorithm is shown as follows:

A-1: Starting with $k = 0$, and G reflects the network topology of the physical layer.

A-2: Calculating the degree of nodes, and giving the weight $(w_{ij} = k_i * k_j)$ of each node based on the degree, where k_i and k_j clearly indicate the degree of nodes i and j. Arrange the weighted links w_{ij} to sort from high to low.

A-3: Finding the maximum weight from the queue and deleting the link. In this case, G' is the newly generated logical layer. Then setting $k = k + 1$, $G = G'$.

A-4: If $k < L'$, turn back to A-2, and if $k = L'$ and then G is the physical layer that we need.

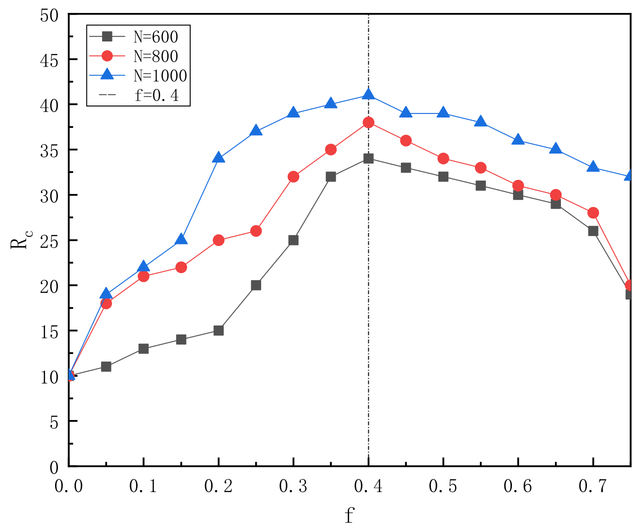

Figure 2. R_c vs. f under different network sizes N. When network size $N = 600$ and $L = 3000$, $R_c \approx 34$, when $N = 800$ and $L = 4000$, $R_c \approx 37$ and when $N = 1000$ and $L = 5000$, $R_c \approx 41$.

As the centre node is under more pressure of transmission congestion, the high weighted node will be under more transmission load. If the node with heavy weight is deleted, the transmission load will be transferred from the central node to the non-central node, thus easing the congestion of the network. Therefore, it is a feasible scheme to increase traffic capacity by deleting the link of the physical layer.

4. Simulation results

In the simulation, we adopt BA scale-free network model, setting the initial value $m = m_0 = 5$, the average degree of network model generation is $\langle k \rangle = 10$ (the average degree of all the network sizes mentioned below is 10). Without changing the logical layer structure and its routing strategy, modifying only the physical layer topology can enhance network traffic capacity. We use multiple parameters to verify the feasibility of the link deletion method for two-layer networks.

In figure 2, we first investigate the variations of traffic capacity R_c under different values of link deletion fraction f for different network sizes. When $f < 0.4$, the traffic capacity R_c increases and it decreases when f goes beyond 0.4 under all the three network sizes. We find that the optimal value is $f = 0.4$ under various BA network sizes. The traffic capacity can be enhanced several times. When $N = 800$, $R_c \approx 38$, the traffic capacity is about four times that of the previous network. It can be seen that the two-layer network model can improve the traffic capacity with our strategy.

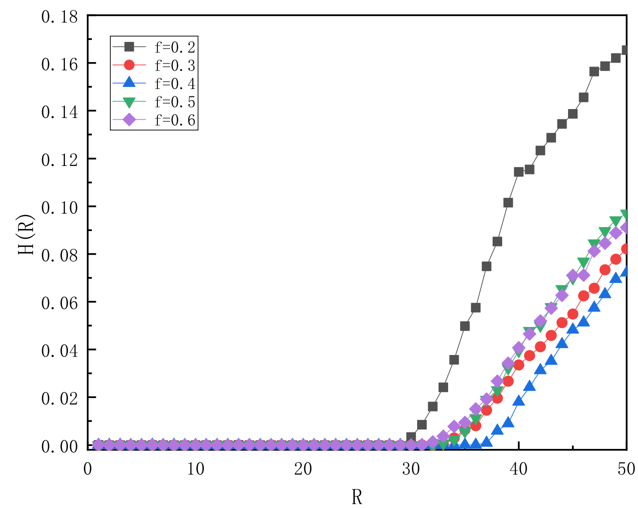

Figure 3. The order parameter $H(R)$ and packet generation rate R for different values of f. Network size $N = 800$, $L = 4000$.

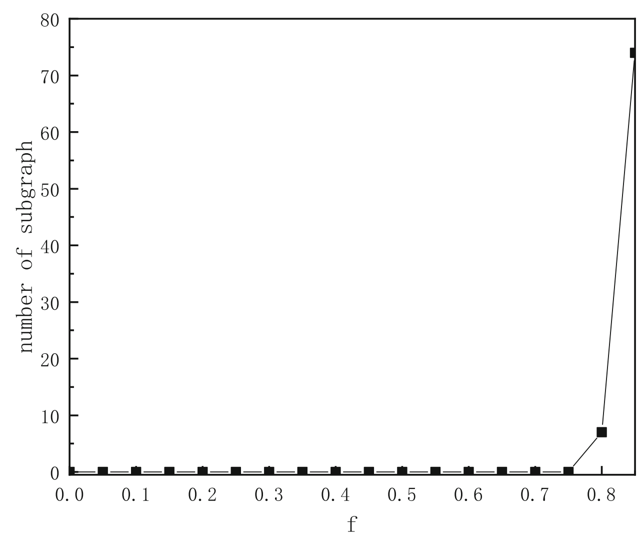

Figure 4. The connectivity of different network sizes related to the proportion of deleted links. Network size $N = 800$, $L = 4000$.

Then, take an example for network size $N = 800$, for different values of link deletion fraction f, and we study the relationship between order parameters $H(R)$ and packet generation rate R as shown in figure 3. When packet generation rate R is small, $H(R) \approx 0$, indicating that the network is free-flowing. When R becomes large, the sudden increase of $H(R)$ means that the network is becoming congested, and the packet generated exceeds the processing capacity of the network. In this view, this method effectively increases the traffic capacity. However, deleting a large number of links will affect the connection of physical layer network topology. Figure 4 shows the change of network connectivity as the

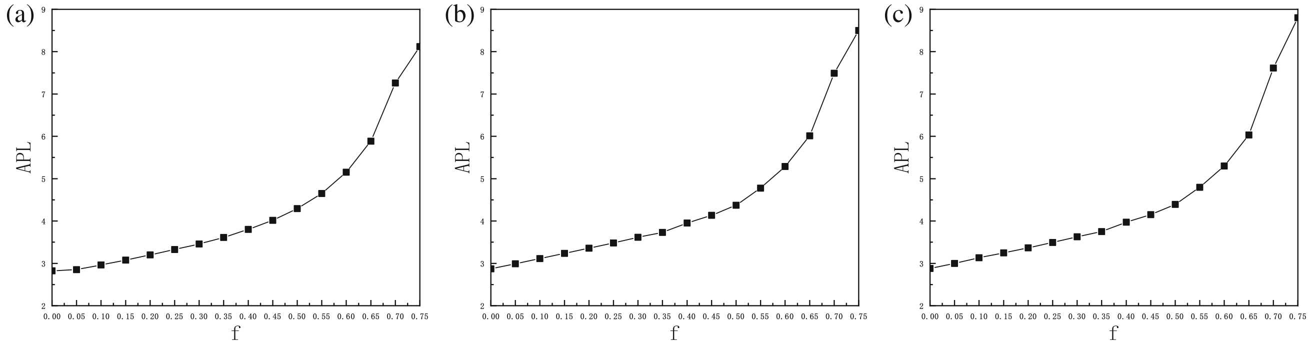

Figure 5. APL with different f values. (**a**) $N = 600$ and $L = 3000$, (**b**) $N = 800$ and $L = 4000$ and (**c**) $N = 1000$ and $L = 5000$. These results are the result of many experiments based on BA scale-free network.

proportion of deleted links increases when the network size is $N = 800$. Therefore, it can be seen that when the deleted link fraction exceeds $f = 0.75$, the network will be decomposed into multiple networks, and so the connectivity of the network cannot be guaranteed. At this point, the network model lost its research significance.

We study average path length (APL) with different f values adopting shortest routing strategy in figure 5. APL is defined as the average number of hops when a packet travels from the source node along the shortest path to the destination. The average distance between any two nodes can be expressed by APL in the complex network, which is defined as follows:

$$\mathrm{APL} = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}. \tag{3}$$

The average path length changes as the packet generation rate increases under the two-layer networks model. When $f = 0$, the network topology of physical layer and logical layer has not changed. At this time, the routing table is the same as the single layer network. As shown in figure 5, under different network sizes, APL increases with the increase of f. For example, when $f = 0.4$, $N = 800$, the APL is much larger than the original network. As the extension of the average path, traffic capacity is improved significantly. When network systems need high traffic capacity, the sacrifice is worth.

Generally, the proposed optimisation strategy always maximises the network traffic capacity and minimises the transmission time of packets as much as possible. Average transmission time is also an important parameter to measure network traffic dynamics. The average transmission time $\langle T \rangle$ represents the time required for transmission from the source node to the destination node, and includes the transmission time of the packet to travel to the destination node and the waiting time of the congestion node. The average transmission time is

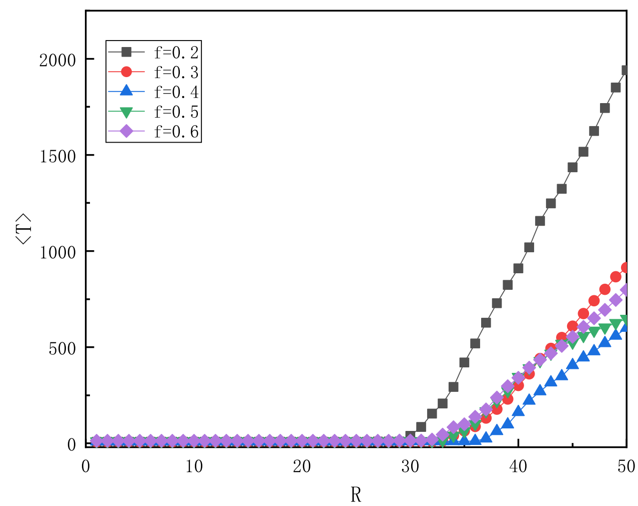

Figure 6. The average transmission time $\langle T \rangle$ with the change of relative packet generation rate under different values of f, and $N = 800$, $L = 4000$.

defined as

$$\langle T \rangle = \lim_{t \to \infty} \frac{1}{n} \sum_{i=1}^{n} T_i, \tag{4}$$

where n is the number of packets arriving in a given time and T_i represents the transmission time of packet i.

Average transmission time $\langle T \rangle$ is a measure of network transmission performance. Average transmission time $\langle T \rangle$ depends only on the transmission time when the network is free-flowing. Figure 6 shows the value of packet generation rate R under the shortest routing strategy and different f. When the packet generation rate R is small, the average transmission time $\langle T \rangle$ is almost zero, and with R increasing, the average transmission time increases sharply. It can be seen from the figure that there is an inflection point, which is roughly similar to the value of the critical packet generation rate R_c. APL is positively correlated with $\langle T \rangle$ in the free flow state.

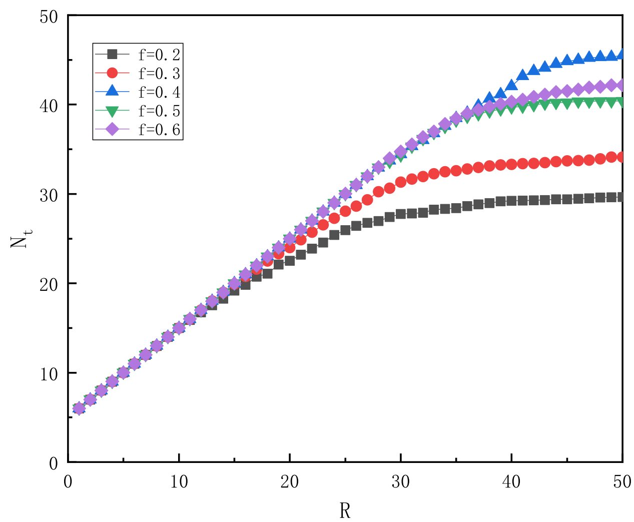

Figure 7. The relationship between network throughput N_t and packet generation rate R. Network size $N = 800$, $L = 4000$.

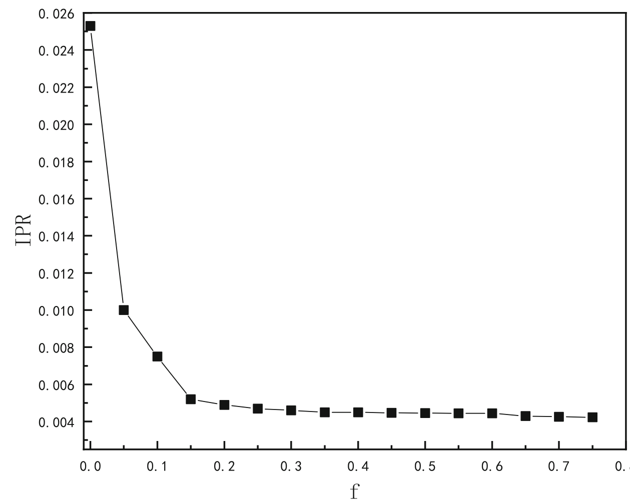

Figure 8. The evolution of physical layer network under link deletion.

Network throughput N_t is another important indicator for measuring network transmission performance. It is defined as the number of packets arriving in the network model. The average throughput is represented as

$$N_t = \lim_{t\to\infty} \frac{1}{\Delta t} \sum_{i=1}^{n} m_i\,(t)\,, \tag{5}$$

where t represents time, n is the quantity of all nodes and $m_i\,(t)$ is the number of packets arriving at the destination node in a specified time Δt.

In figure 7, we can see that the change of N_t can be divided into three states: the first state is the free flow state. When the packet generation rate R is very small, the network presents the free state. At this time, the average throughput of the network is positively correlated with the packet generation rate R. The second state is the congestion state. When the packet generation rate of the network is greater than the traffic capacity of the network, the network presents the congestion state. At this point, the packet generation rate is no longer proportional to the average throughput, but packets are also increasing slightly. The third state is the saturation state, where the average throughput no longer changes with the increase of packets. As can be seen from the figure, when $f = 0.4$, the average throughput N_t of the network shows the best trend. Given a routing strategy, transmission rate is large, when only the physical layer topology is changed and the simulation is carried out to verify the effectiveness of link deletion.

In addition, the network can be expressed as adjacency matrix $\mathbf{A}$, and the spectral characteristics of $\mathbf{A}$ are the eigenvalues $\boldsymbol{\lambda}$ and eigenvectors $\boldsymbol{f}$ of the analysis matrix. The expression formula is $\mathbf{A}\boldsymbol{f} = \boldsymbol{\lambda}\boldsymbol{f}$. The eigenvector corresponding to the maximum eigenvalue is called the principal eigenvector (PEV) of the network adjacency matrix [43]. The structure and dynamic characteristics of the underlying system are provided. We describe PEV in terms of inverse participation ratio (IPR), and it can be expressed as

$$\mathrm{IPR} = \sum_{i=1}^{N} f\,(x)_i^4, \tag{6}$$

where $(x)_i$ is the ith component of the eigenvector.

In figure 8, it is the evolution in IPR value. We note that the physical layer of the two-layer network model increases with the ratio of link deletion, IPR value decreases, and the evolution of the physical layer begins to provide a localized PEV, with the increase of the link deletion ratio f, the network structure and spectral characteristics change. For the whole link deletion process, the IPR value of the PEV reaches a stable state, and the positioning characteristics of the network model are prominent.

When $f = 0.4$, the degree distribution of the physical network also changes (see figure 9). Under different network scales, the degree distribution in the original network conforms to the scale-free feature. When the ratio of deleted links reaches the optimal value, the degree distribution becomes more uniform.

In addition, we analyse the clustering coefficient $\langle CC\rangle$. This coefficient is usually used to characterise the local structure of the network. The network after link deletion also presents a very low clustering coefficient in figure 10, which proves that the distribution of the network gradually becomes uniform.

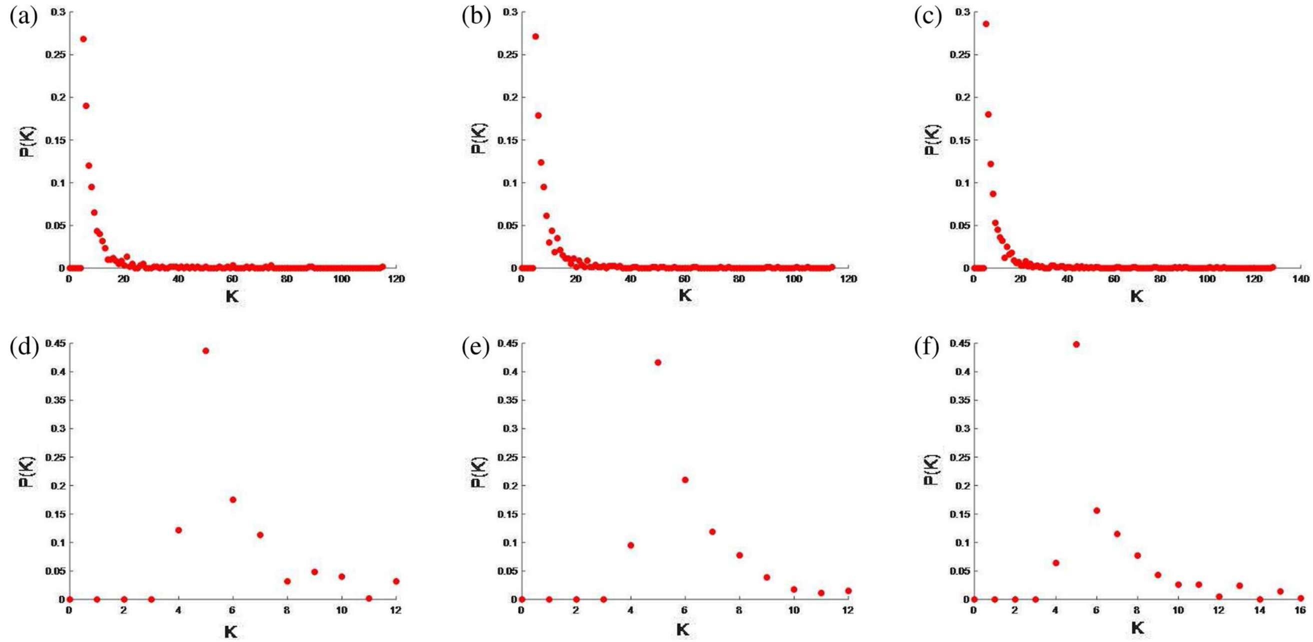

Figure 9. Degree distribution of the physical layer of (**a**) $N = 600$, (**b**) $N = 800$, (**c**) $N = 1000$. (**d**), (**e**) and (**f**) depict the degree distribution of link deletion.

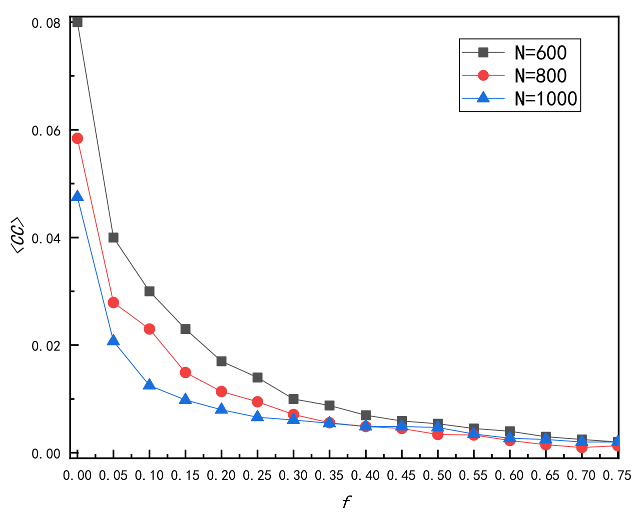

Figure 10. Physical layer clustering coefficient under different link deletion ratio f.

5. Conclusion

A single layer network cannot fully demonstrate all the key characteristics of a real network system. Fortunately, two-layer network model gives us a new perspective to understand the traffic dynamics on complex systems. Physical layer virtual link reconstruction can be easily realised. The main research object of this paper is the two-layer network, and in the two-layer network model, we adopt link deletion strategy to delete physical layer links. The simulation results show that the optimal parameter for deleting links is $f = 0.4$, traffic capacity has been maximised. Finally, we describe PEV in terms of inverse participation ratio IPR. The IPR value is reduced, the degree distribution is more uniform and has a very low clustering coefficient, the network structure is more uniform, and the traffic capacity is improved. Extensive simulation results prove the effectiveness of link deletion method on physical layer. The proposed link deletion strategy improves the traffic capacity, but it also has some disadvantages. It needs to sacrifice the average path length to achieve this strategy. The proposed strategy may affect the optimisation of certain actual traffic or communication networks.

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