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An experimental study of recoil capillary waves and break up of vertically flowing down water jets

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Abstract. Break up of water jets under gravity is a ubiquitous phenomenon. The role of surface tension on the instability of uniform water jets was recognised long ago by Plateau and Rayleigh. According to the Plateau–Rayleigh theory, external (or internal) perturbation waves create necks and bulges all along the uniform jet length. The perturbation waves of wavelengths larger than a certain value keep growing with time and ultimately cause the continuous jet to break up into individual drops. The effect of external perturbation waves was investigated experimentally, in most cases under gravity, and found to confirm the essentials of the theory. Recently, the idea of recoil capillary waves as a possible internal source of perturbations was emphasised. According to this idea, immediately after the break up of the jet, the tip of the remaining continuous jet (after a drop is detached) recoils. Its effect travels upstream as a recoil capillary wave which gets reflected at the mouth of the jet-issuing nozzle. The reflected capillary wave travels downstream along the jet with its Doppler-shifted wavelength as a reinforcing perturbation wave and, as a result, affecting the break up length of the jet. We set up and perform an experiment to verify the existence of these tip contraction recoil capillary waves. The results of our experiment support the existence of these recoil capillary waves. However, the effect of these capillary waves on the jet break up length is found to be small.

Keywords. Water jet; under gravity; capillary waves; jet break up length.

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1. Introduction

A continuous water jet falling gently downward through a tap and its subsequent break up into discontinuous drops is a common phenomenon which can be witnessed everyday. There have been many attempts theoretically as well as experimentally to understand the physical mechanism behind the breaking up of continuous jets into drops instead of continuing to keep reducing its cross-sectional area forever. However, the complete understanding of this macroscopic phenomenon has eluded so far and its study continues to be of current interest [1–5].

The scientific study of this phenomenon has a long history. Savart (1833) experimentally observed the instability of liquid jets that clearly indicated that the liquid column (jet) undergoes changes before breaking up into disjoint droplets. Savart's experimental observations suggested that the liquid column develops necks and bulges in the form of waves, as one moves down the jet length, before the jet breaks up. This problem of instability of uniform inviscid liquid jets was taken up by Plateau and then by Rayleigh (1879). They developed a theory which came to be known as Plateau–Rayleigh theory. Rayleigh [6,7] showed that perturbations of wavelengths larger than π times the jet diameter d_1 grow rapidly with time which ultimately make the uniform column unstable against the formation of droplets. The theory is based on surface energy considerations of inviscid liquids. Chandrasekhar [8] later extended the theory to viscous liquids. Many experimental investigations have earlier been conducted to examine the validity of Plateau–Rayleigh theory (see for example [9–11]).

The Plateau–Rayleigh theory describes how the perturbations of wavelengths (λ) larger than d_1 keep growing with time whereas those with lower wavelengths decay with time. It is the fastest growing perturbation ($\lambda \approx 4.508 \times d_1$) that ultimately breaks the jet. However, the theory is silent about the origin of the perturbations. The experiment of Goedde and Yuen, for example,

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applied external perturbations to study the length of the liquid jet (measured from the root of the jet) before it breaks up [10]. However, even if no external perturbations are applied, one obtains a finite deterministic length of the water jet. Obviously, the perturbations must have their origin at the root of the jet (at the mouth of the issuing nozzle) or all through the length of the jet due to the presence of noise in the laboratory. Naturally, in the latter case, the perturbations will have amplitudes of random sizes. As a consequence, the length of the jet should have random values. However, on the average, the length turns out to be a deterministically fixed number depending on the conditions of the experiment.

In some recent works, Umemura and co-workers [12,13] emphasised the idea that the perturbations get generated and sustained self-consistently. This is based on the observed fact that soon after the jet breaks up (at one of the points on the lowermost neck), the tip of the remaining column contracts to make its shape round once again to minimise its surface energy. The tip contraction (recoil) gives rise to an upstream propagating capillary wave which upon reflection at the mouth of the nozzle moves downstream with Doppler-modified wavelengths. Some of these waves with the right wavelength (as discussed by Rayleigh) cause the liquid column to break producing another contraction of the tip of the column and the process repeats once again. In the present work, we set up and conduct an experiment to verify the existence of these recoil capillary waves. We undertake this task by damping the recoil capillary waves by bringing the jet, immediately after break up, in contact with a stationary water surface.

2. Experimental set-up and experiment

One of the main components of the experimental set-up (figure 1) consists of a transparent rectangular beaker with a level outlet on one of its vertical sides. The beaker is placed vertically below a jet-issuing (vertically aligned) nozzle so that the vertical water jet falls directly on the water kept in the beaker. The water level in the beaker is maintained fixed by letting the excess water flow out through the level outlet of the beaker. The beaker is placed on a horizontal platform fitted to the vertical stand of a travelling microscope (vernier scale least count = 0.001 cm) so that the beaker can be smoothly moved vertically and its position measured. A laser-pointer-and-detector arrangement [14,15] is also fitted to the platform so that the horizontal laser beam is incident normally on the vertical surface of the beaker and passes through the path of the water jet and then through the opposite surface of the beaker before it is collected by the detector (figure 1). The laser-detector arrangement can also be moved vertically (with respect to the same horizontal platform) and fixed as desired. A digital counter mounted with a clock is connected to the detector to count the number of drops. Note that the 'jet-length' is defined (and measured) as the vertical distance between the mouth of the nozzle and the position of the horizontal laser beam through the jet.

Our experimental set-up is similar in essentials to that of Goedde and Yuen (compare figure 1 of ref. [10]). Also, the laser-detector arrangement is similar to what was used in ref. [16]. The only new and important addition is the intervening water containing beaker (figure 1)

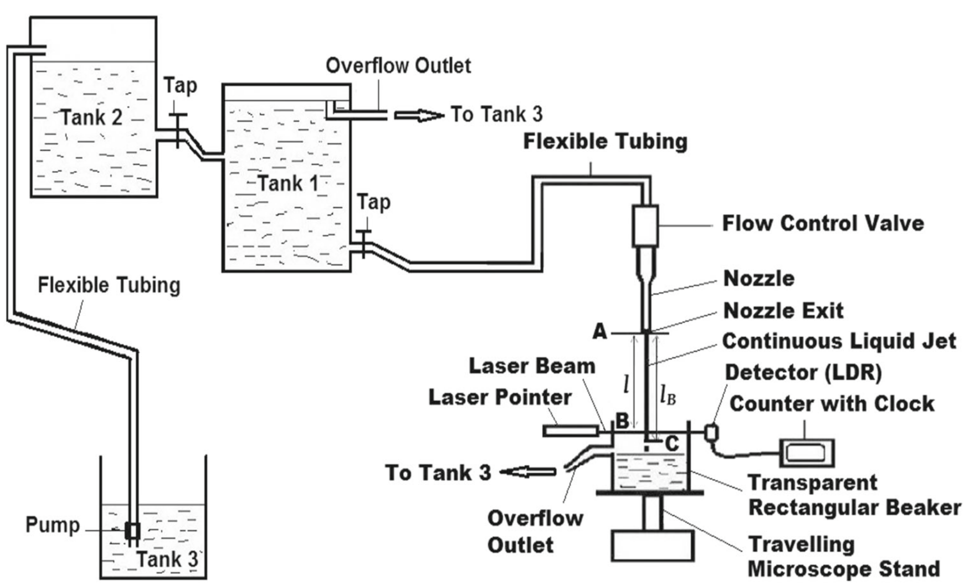

Figure 1. The diagramatic sketch of the experimental set-up. Note that the overflow outlet of the beaker is actually not on the surface through which the laser beam passes but on the transverse side and not like what is shown in the diagram.

as in the case of the one employed by Hancock and Bush [17]. The ambient condition of temperature and humidity is controlled and allowed to settle down at desired values before the experiment is conducted and the condition is kept fixed throughout the duration of experiment. Moreover, we have made an independent experimental check to make sure that the nozzle remains stationary (vibration free) during the experiment. Also, the pressure of the water level at the position of the nozzle is kept fixed with the help of a water-tank arrangement (figure 1).

The water-tank arrangement in the experimental set-up (figure 1) consists of three water tanks, the first one (Tank 1) is placed at a height lower than the second tank (Tank 2) which supplies water at a constant flow rate to Tank 1. In order to maintain a constant level of water (i.e., to keep the pressure same throughout the experiment) in Tank 1, a level outlet tube is fixed with the tank. Both these tanks are fitted with one tap each. The tap of Tank 2 is connected to the inlet of Tank 1 by a rubber pipe. The vertically aligned glass nozzle is connected to Tank 1 by a rubber pipe through a flow control valve, which permits us to control the flow rate of water. The excess water in Tank 1 flows out through the level outlet to the lower reservoir tank (Tank 3) (Tank 3 is kept conveniently far away to reduce noise). The water from Tank 3 is gently pumped up to Tank 2 by using a water pump. All the tanks are filled with freshly produced distilled water.

The distilled water is jetted vertically downward through the glass nozzle (of length larger than about ten times its internal diameter ($d = 2a_0$)) into the water in the beaker through the stagnant ambient air atmosphere. As long as the jet remains continuous, the detector remains quiescent. However, a break up of the jet into a drop is detected as a count if the laser beam happens to pass through unobstructed by a discontinuity in the jet. The jet length at the position of break up of the jet gives the break-up length ($l = l_{\mathrm{B}}$). Initially, the laser beam is made to face the continuous jet by moving the platform up and then the platform is gradually lowered so that the jet length increases. At every position of the laser-detector arrangement the counts are recorded for two minutes each for several times and their average calculated to obtain the average number of drop-counts per second. The platform is slowly and gradually lowered by small steps and at each position the experiment is repeated to obtain the average count rate. Naturally, the count rate begins from zero, reaches a threshold at which the rate just begins to show non-zero value and then gradually keeps increasing as the platform is slowly lowered. The jet length at the very threshold point is termed here as the first break-up length ($l = l_{\mathrm{FB}}$) of the jet. The process is continued till the rate reaches a maximum (saturation) value. Throughout the above process, the water flow rate [18] is kept fixed. The same process is then repeated for several values of flow rates. Note that after each change of the flow rate, the flow and the jet are allowed to become steady before the measurement process is begun afresh.

We perform two distinct kinds (sets) of experiments. In one we let the horizontal laser beam pass through the beaker just about 0.2 cm above the water surface on the beaker (Set 1), as shown in the inset of figure 2. In the other, the beam is kept at a height of about 1.5 cm above the water surface (Set 2) (inset of figure 3). How these two sets of experiments serve our purpose is explained below.

Consider a liquid column (jet) whose lower-most portion is about to get detached (as a drop). Let its length be l_1. Now, consider the instant at which the lower-most portion (part of the neck and the globule of the pendant) just gets detached. Let the length of the residual jet be l_2. Obviously, $l_2 < l_1$. Let the water surface in the beaker be close at hand so that as the drop gets detached it coalesces with the water reservoir. The residual jet tip will have just a little distance to travel before it comes in contact with the water surface. If this distance is less than, say, 0.2 cm, the tip will not have enough time to recoil (as it touches the water surface) but the time is enough to allow the laser beam to pass through to give a count in the detector counter. The absence of tip recoil, however, means the recoil capillary wave propagating upstream along the jet length is effectively damped. This is what happens in the first set of experiment at threshold and for this case, the first break-up length $l_{\mathrm{FB}} \approx l_2$. For the same jet length in the second set of experiments, however, with the laser beam being at a much larger distance from the water surface below, the jet tip finds enough time to recoil before coming in contact with the water surface. Therefore, the first set of experiments has the chance to detect break up in the situation when the recoil capillary wave gets damped whereas in the other set, for the same condition, the recoil capillary wave propagates upstream undamped. Therefore, if the recoil capillary waves exist and have any influence on the first break-up lengths, l_{FB}, in the two cases must have different values. Our experiment exactly seeks to verify the same.

The experiment is performed in an enclosure where the air current is minimised and, as stated earlier, the temperature and humidity are kept fixed for all sets of measurements. Thus, the enclosure inside the lab is made sure to be stagnant. The entire set-up, except Tank 3, is placed on a firm concrete slab platform. The pump motor vibration is very feeble and is almost ineffective as Tank 3 is kept a little far away. Thus, the external disturbances, such as sound, etc., on the jet are minimised as far as possible.

3. Experimental results

Our measurements mainly consist of counting the number of drops into which the jet breaks up in a given time interval. Of course, it depends on the internal diameter of the nozzle and the flow rate. The jet break up does not take place at a fixed distance from the nozzle but the break-up point spreads over a length and hence the relevance of break-up length distribution. Figures 2 and 3 show the typical measurements of drop-count rates at various jet lengths (l) and also the corresponding break-up length distributions. Rest of the figures give the detailed summary of these measurements for different experimental parameters.

Figure 4 shows the average number of counts (drops) per second, for a water flow rate of 35.0 cc/min and d = 0.78 mm ($\sqrt{We}$ =2.837, in terms of the Weber number $We = \rho_{\mathrm{w}} u^2 d/\sigma$, where u is the jet issuing speed, σ is the surface tension and ρ_{w} is the density of water), as a function of l. Note that, as stated earlier, the jet length is the vertical distance of the laser beam from the lower tip of the nozzle. Each point on the graph corresponds to an average of three sets of counts for a duration of 2 min each. The counts begin from zero (indicating continuous jet) to a saturation value (indicating the finality of the break-up process). From these data, we calculate the distribution, ρ, of break-up lengths (figures 2, 3 and 5). The probability density ρ of the break-up lengths is roughly calculated as the derivative of the count rate with respect to l. For better visibility (in the same graphs), we have chosen the scale of ρ such that $\int \rho \, \mathrm{d}l = 100$. Therefore, naturally, the normalised distribution $\rho_n = \rho/100$. The distribution provides information about the mean value of break-up lengths ($\overline{l_{\mathrm{B}}}$). The mean values $\overline{l_{\mathrm{B}}}$ are plotted in figure 6 for the same d =0.78 mm but for different flow rates ($\sqrt{We}$) for both sets of experiments (with and without damping). The two sets of mean values lie in a single band. The approximate equality of the two mean values is understandable: once the water level in the beaker is

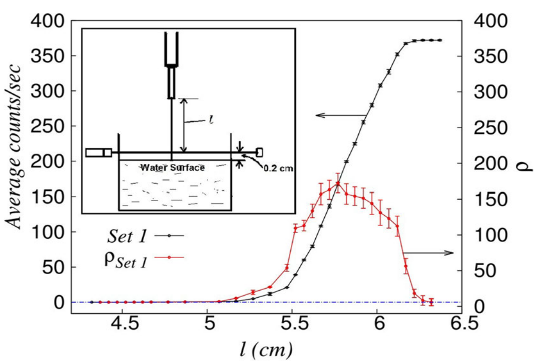

Figure 2. Average count rate (s^{-1}) as a function of l (only for Set 1) at the flow rate of 35.0 cc/min and d = 0.78 mm. It also shows the corresponding ρ.

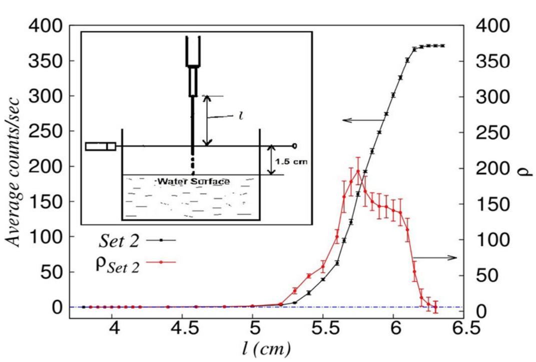

Figure 3. Average count rate (s^{-1}) as a function of l (only for Set 2) at the flow rate of 35.0 cc/min and d = 0.78 mm. It also shows the corresponding ρ.

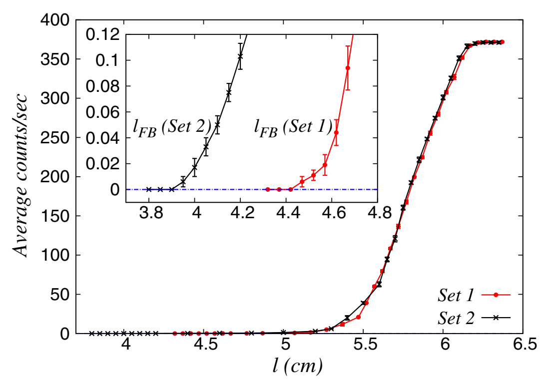

Figure 4. Average count rate (s^{-1}) as a function of l at the flow rate of 35.0 cc/min and d = 0.78 mm for the two sets of experiments. The inset shows a magnified graph to show l_{FB} in the two sets of experiments.

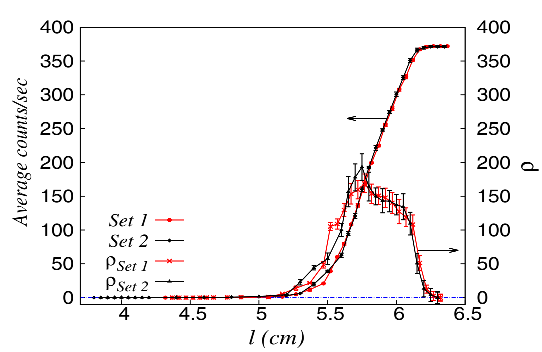

Figure 5. Average count rate (s^{-1}) and the corresponding ρ as a function of l at the flow rate of 35.0 cc/min and d = 0.78 mm for the two sets of experiments.

well below the break-up point the drop counts in the two sets are identical.

The inset of figure 4 gives a magnified diagram of the curves of figure 4 in the lower range of l. From this graph, one can obtain l_{FB} in the two cases (with and without damping) and are plotted in figure 6 as a function of $\sqrt{We}$. The figure clearly shows that l_{FB} (Set 1) is consistently larger than l_{FB} (Set 2) for all values of $\sqrt{We}$ measured. Though the amount of increase of break-up length of the jet as a result of damping is not very large, the figure shows the effect of damping of the recoil capillary waves on l_{FB} consistently for all values of We. As explained earlier, the difference in the first break-up lengths in the two sets of experiments for the same experimental parameters shows the role played by recoil capillary waves, hence confirming their existence.

The mean breakup lengths $\overline{l_{\mathrm{B}}}$ plotted in figure 6, though not exactly identical to the earlier reported results (for example, p. 104 of ref. [3]), the qualitative features are very similar, showing various regimes of jet breakup. However, information on l_{FB} is entirely new. It also shows that l_{FB} peak at a smaller Reynolds number (Re $\approx$ 1415.65) and square root of Weber number ($\sqrt{We}$ $\approx$ 3.679) than in the case of $\overline{l_{\mathrm{B}}}$ (Re $\approx$ 1506.98, $\sqrt{We}$ $\approx$ 4.01). Our measurements are done at the temperature $(25\pm.5)$°C and at relative humidity (80 ± 5)%. However, in order to calculate $\mathrm{Re} = \rho_{\mathrm{w}} u d/\mu$ and $We = \rho_{\mathrm{w}} u^2 d/\sigma$, we have used the tabulated values of surface tension of water $\sigma = 71.99 \times 10^{-3}\,\mathrm{N\,m^{-1}}$ and coefficient of dynamic viscosity $\mu = 8.9 \times 10^{-4}$ kg $\mathrm{m^{-1}\,s^{-1}}$. The issuing jet speed u is calculated (assuming plug flow) as the ratio of the flow rate and the inner area of the nozzle exit and the density of water $\rho_{\mathrm{w}} = 997.05$ kg $\mathrm{m^{-3}}$.

The experiment was repeated for other nozzles with diameters, d = 0.70 mm and 1.19 mm, and the corresponding results are plotted in figures 7 and 8, respectively. For all the nozzles, the results are consistently similar. We also note that, at small flow rates (before the $\overline{l_{\mathrm{B}}}$ curves bend), $\overline{l_{\mathrm{B}}}$ shows nearly linear dependence on $\sqrt{We}$ for all nozzle diameters. We have chosen the range of nozzle of internal diameters (d) between 0.70 mm and 1.19 mm because this is the range we could handle without having to face the difficulty of very small and very large break-up lengths. We repeat that for all the nozzles and we arrive at the same conclusion about the existence of recoil capillary waves.

4. Discussion and conclusion

We have performed a very simple experiment to measure the break-up lengths of a water jet under gravity. The experiment is based on a simple idea of damping

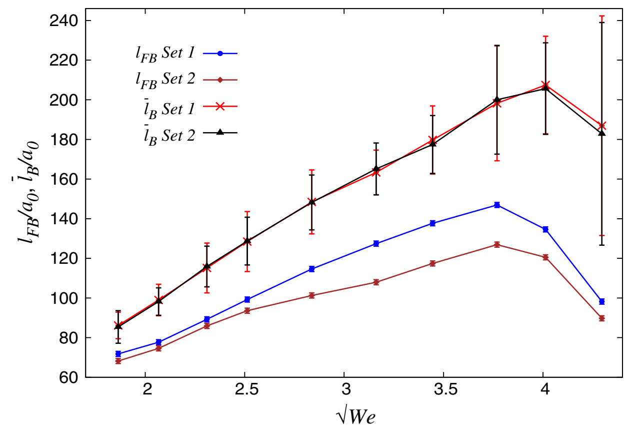

Figure 6. Plots of scaled lengths l_{FB} (the lower set of two curves) and $\overline{l_{\mathrm{B}}}$ (the upper set of two curves) as a function of $\sqrt{We}$ for $d = 0.78$ mm. The vertical bars on $\overline{l_{\mathrm{B}}}$ curves represent the full-width at half-maximum of the break-up length distribution $\rho(l_{\mathrm{B}})$. The experimental error bars are small and of the size of the experimental points on the graph.

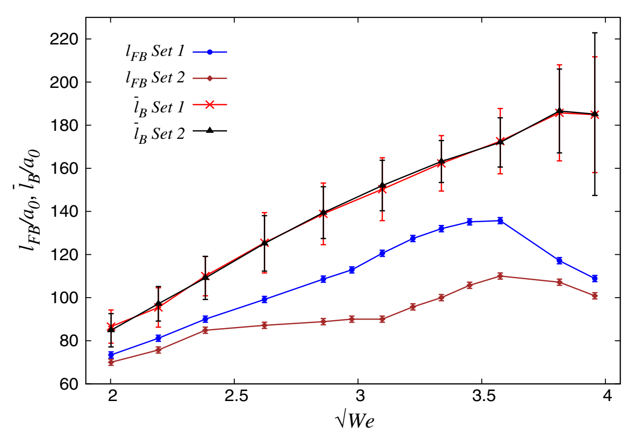

Figure 7. Similar plots as in figure 6 but for $d = 0.70$ mm.

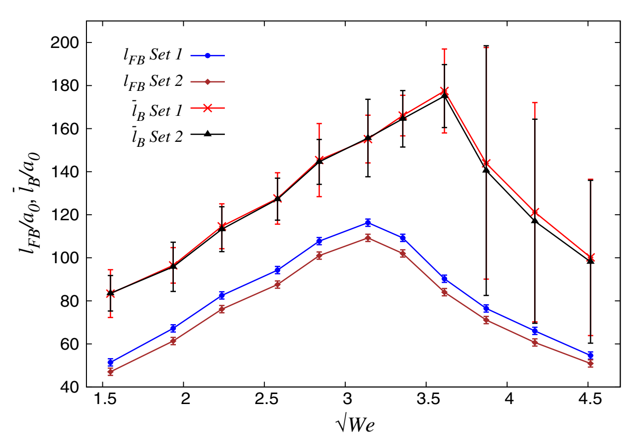

Figure 8. Similar plots as in figure 6 but for $d = 1.19$ mm.

recoil capillary waves by means of an almost immediate water contact of the jet tip. Note that these capillary waves are not due to the impact of the water jet on the water surface [17]. The effect of impact of the water jet on the water surface is being investigated at present.

Note that the vertical position of the laser beam is obtained by measuring the position of the horizontal platform on which the laser-detector arrangement is fixed with the help of rigid stands. The position of the laser beam with respect to the platform (and hence with respect to the water level in the beaker) is changed by raising or lowering the stands with the help of screws. There is always a chance of making a mistake of fixing the beam position leading to error in the measurement of break-up length and hence altering the main conclusion. (However, adequate care is taken to avoid such a possibility.) Yet, the relative values of l_{FB} have the same nature in the case of the two sets. The experiment for the whole range of l is performed in one go without altering, in between, the positions of the laser-detector arrangement stands. The consistency of the relative values of l_{FB} (*vis-à-vis* $\overline{l_{\mathrm{B}}}$) shows that it is not due to any possible error in fixing the laser beam position.

The observation of a difference in the first break-up lengths l_{FB} in the two sets of experiments provided a sure but somewhat indirect evidence of the existence of the recoil capillary waves. A high-speed photograph of the jet in the two sets of experiments could, perhaps, have given a more direct evidence of the recoil capillary waves with the laser beam positioned at the location of the first jet break up. However, unavailability of such a camera prevented us from making such an attempt.

The not so large value of the lengthening of l_{FB} due to the damping of the recoil capillary wave shows that the effect of this capillary wave is marginal. That is, even if the recoil waves are stopped completely, the process of break up of the jet cannot be postponed for long. This leaves open the question of the origin(s) of perturbation waves. In the case of viscous liquids, e.g. silicone oil, causes of perturbation at the nozzle and all along the jet are discussed in detail by Dizes and Villermaux [5]. However, as stated earlier, the effect of perturbation waves is well established [10].

The experimental error bars on $\overline{l_{\mathrm{B}}}$ in both cases are small. The presented vertical bars (figures 6–8) on the experimental points of mean $\overline{l_{\mathrm{B}}}$ are not the experimental error bars but they indicate the corresponding full-width at half-maximum of the normalised distribution ρ_n. The vertical bars suddenly become large at and after the peak of the mean $\overline{l_{\mathrm{B}}}$ curves because l_{B} are more scattered at large range of l values.

The results of the experiment may be considered to give a 'proof' of the existence of recoil capillary waves and their effect on the jet break-up length. The qualitative features shown are unmistakable.

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