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Arbitrary amplitude electron-acoustic solitons and double layers with Cairns–Tsallis-distributed hot electrons

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Abstract. In the present research paper, propagation attributes of nonlinear electron-acoustic (EA) waves have been investigated in an unmagnetised plasma system consisting of cool fluid electrons and hot electrons observing the hybrid Cairns–Tsallis distribution. Sagdeev pseudopotential method has been used to explore the occurrence of large-amplitude solitons and double layers, focussing on how their characteristics depend upon different parameters. The analysis is further extended to examine the dynamics of large- and small-amplitude double layers. It is revealed that the present plasma system supports the existence of negative potential solitons and double layers in certain region of parameter space. The numerical results show that the Cairns–Tsallis-distributed hot electrons may affect the spatial profiles of EA waves and double layers. The present investigation may be relevant to the observation from Viking satellite in the dayside auroral zone.

Keywords. Electron-acoustic waves; soliton; double layer; Cairns–Tsallis distribution; Sagdeev pseudopotential.

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1. Introduction

The disquisition of correlation, excitation and dissemination of nonlinear waves has become a matter of considerable importance in theoretical physics. Any deviation from equilibrium results in the formation of nonlinear structures such as solitons or solitary waves, double layers etc. Solitons come into existence from the counterbalance of phenomena like nonlinearity and dispersion and proceeds without change of characteristics such as shape and velocity. The propagation of electron-acoustic waves (EA waves) has gathered an appreciable amount of consideration owing to their relevance in many plasma-based laboratory explorations [1,2], space plasma inspections [3,4] and computational simulations [5–7]. Fried and Gould [8] had come up with the idea of EA mode while performing numerical solutions of linear Vlasov dispersion equations in an unmagnetised homogeneous plasma. The existence of EA waves in plasma was subsequently confirmed with two-electrons population cited as cold and hot with T_c and T_h as the corresponding temperatures. These are electrostatic modes [9] at elevated frequency in a plasma arrangement where cold electrons oscillate against a prevalent thermalised environment of inertialess hot electrons. In this case, the cool electrons provide inertia and restoring force comes from hot electron pressure [10]. The phase velocity of EA waves lies between thermal speeds of hot electron component and cold electrons and ions. The region of existence for EA waves is restricted by Landau damping as the phase velocity advances towards the thermal velocity of either electron components. After performing a parametric survey on these waves, it was found by Gary and Tokar [5] that for the propagation of EA wave, $T_h \gg T_c$. However, $n_h \gg n_c$ represents another state for the occurrence of this wave where n_h, n_c are the number densities of hot and cold electrons. EA solitary waves are frequently observed in auroral magnetosphere [11,12], Earth's bow-shock [3,4,6] and laboratory experiments [1,2] and also exist in strong currents associated with acceleration region. This high-frequency mode has been used to interpret broadband electrostatic noise (BEN), commonly observed by satellites in plasma sheath boundary layer [13,14]. To understand the various characteristics of BEN, Mace *et al* [15] reported the nonlinear propagation of EA waves. Analysis of double layers in plasma has generated a good deal of enthusiasm over a period of time in view of

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its congruity in cosmic applications [16–18]. A double layer (DL) comprises two oppositely charged parallel layers resulting in a strong electric field across the layer and can accelerate the electrons and ions in opposite directions thereby producing an electric current. Langmuir was the first to do a recognised work on DLs in the year 1929 and quoted it in his exposition named "The interaction of electron and positive ion space charges in cathode sheath" [19]. After 29 years, Schönhuber [20] conducted a thorough analysis of DLs in the laboratory. In the same year, the presence of DLs in connection with the aurora was proposed by Alfven [21]. In his work, he suggested that stimulation of the particles from the magnetosphere into the ionosphere lies in the crucial role of DLs [22,23]. Thenceforth, the studies had been started on DLs to analyse them numerically, theoretically and experimentally. At times, the DLs are also termed as shocks and kinks owing to changing values in their related criteria and electrostatic potential at the extremes. The name 'double layers' arises due to the presence of regions of positive and negative charges. The generation of such DLs is difficult as it necessitates the calibration of plasma parameters resulting in a more convoluted plasma configuration. This demands an ample leeway to follow the required constraints [24–26]. Extensive studies have been carried out by many researchers to investigate the properties of EA solitons and DLs in a variety of plasma systems owing to their relevance in cosmic plasmas and plasma thrusts [26–34].

Several observations demonstrate the non-equilibrium quality of plasma and inability of their particle distributions to be described by Maxwellian and for such considerations, non-Maxwellian distribution functions present good approximations. The confirmation of the presence of energetic electrons in various astrophysical plasma environments is made by a number of measurements and their distribution functions are revealed to be non-thermal. The non-thermal populations are found to exist in the vicinity of Moon [35] and Earth's foreshock [3]. The experimental confirmation of the presence of non-thermal electrons in Earth's bow-shock is made by Vela Satelite [36]. One of the common characteristics of auroral zone is the enhanced high-energy tails. Cairns *et al* [37] designed a distribution for its modelling that is named after him. Last decade has witnessed an immense focus on the non-extensive statistical approach or Tsallis statistics [38]. The long-range interactions or correlation in plasma are responsible for the non-extensive character. Recently, Tribeche *et al* [39] proposed a hybrid distribution known as Cairns–Tsallis distribution combining Cairn's non-thermal [37] and Tsallis's non-extensive [38] distribution. It offers enhanced parameter flexibility in modelling non-thermal plasmas. This two-parameter (α, q) distribution model is applicable to auroral region and magnetosphere of Earth. Amour *et al* [30] used this distribution to study EA solitary waves in plasma and observed the occurrence of rarefactive EA solitons. Further, Williams *et al* [40] re-examined the Cairns–Tsallis distribution and discussed the validity of its range. They showed that this distribution is applicable in (q, α) range $0 \leq \alpha < 0.25$ and $0.6 < q \leq 1$ where parameters α and q define number of non-thermal electrons and strength of non-extensivity. Bouzit *et al* [41] used Cairns–Tsallis distribution to study screening and sheath formation in plasma system. Dutta and Sahu [42] investigated the nonlinear features of solitons, DLs and supersolitons in a plasma consisting of warm ions and two-temperature non-thermal Tsallis-distributed electrons. Roshtampooran and Saviz [43] applied this hybrid model in collisionless unmagnetised two-component plasma system to study electromagnetic soliton. Bala *et al* [44] studied the small-amplitude nonlinear ion-acoustic dressed solitary structures in two-component plasma with electrons featuring hybrid q-non-extensive non-thermal distribution. Bansal *et al* [45] studied non-planer EA waves using Cairns–Tsallis distribution of hot electrons and obtained negative potential solitary structures.

Amour *et al* [30] considered cold fluid electrons, hot non-thermal q-distributed electrons and stationary ions to study the dynamics of rarefactive EA solitary waves. However, in the present case, the analysis of the effects of temperature is included which plays a crucial role. The study is further extended to the investigation of large- and small-amplitude DLs, which is missing in ref. [30]. The study of DLs becomes important from the fact that these are responsible for acceleration, deceleration or reflection of plasma particles. Sahu and Roychodhury [46] studied the propagation of EA solitons in relativistic plasma with non-thermal electrons. In their model, they considered relativistic ions and cold electrons and non-thermal hot electrons. They discussed relativistic and non-thermal effects on the existence domain of solitary waves. It is reported that the relativistic effect significantly restricts the region of existence for solitary waves. Verheest and Helberg [47] investigated the propagation of arbitrarily large EA solitons in a plasma composed of positive ions, adiabatic cool and isothermal hot electrons by including the effects of electron inertia. They assumed the Maxwell–Boltzmann distribution of all the species and presented a very detailed analysis of their study. Rufai [48] investigated the existence of ion-acoustic solitons and supersolitons in a magnetised auroral plasma consisting of cold ions, energetic non-thermal hot electrons and Boltzmann-distributed proton species. The studies reported in refs [30,46–48] may appear similar but the underlying beauty is that the analysis is different for each case. In the present paper, our study deals with the dynamics of arbitrary amplitude EA solitons and DLs in the plasma system comprising stationary ions, hot and cold electrons. The hot electrons are assumed to obey hybrid Cairns–Tsallis distribution proposed by Tribeche *et al* [39]. This distribution offers an enhanced parameter flexibility in modelling the non-thermal plasmas. The two-parameter representation of the distribution function might be useful for fitting to a wider range of observed plasmas. It may be mentioned that the hybrid model is suitably applicable as in the case of Cairn's non-thermal distribution. Due to its flexibility of two-parameter availability, this model is applicable to auroral region and magnetosphere of Earth. After a critical evaluation, Williams *et al* [40] have shown that Cairns–Tsallis distribution is applicable only for $0 \leq \alpha < 0.25$ and $0.6 < q < 1$ and α should satisfy the condition: $\alpha = (2q-1)/4$. The solitary waves exist in the sky as density waves in spiral galaxies, in giant Red Spot of Jupiter, Earth's magnetosphere, cometary tails, etc. and DLs are found in Earth's auroras, space plasmas etc. So to know about various fundamental underpinning processes, their extensive study is important. The paper is organised in the following fashion. In §2, expression for pseudopotential is derived by using the set of basic equations. In §3, nonlinear wave dynamics of EA waves and DL structures are studied using pseudopotential approach and the detailed discussion of numerical results is presented in §3.1, 3.2 and 3.3 respectively and §4 is devoted to conclusion.

2. Theoretical model and derivation of pseudopotential

The plasma with two-electron populations are known to exist frequently in space environment, where EA wave may play a significant role. In the present investigation, the nonlinear propagation of EA wave is studied in unmagnetised plasma composed of cold fluid electrons, hot electrons following hybrid Cairns–Tsallis distribution. The plasma fluid model consists of cold and hot components, referred to as subscripts c and h respectively. The presence of two non-drifting populations allows the existence of the EA waves itself. It may be mentioned that cold electron component does not mean $T_c = 0$ because in that case EA wave will not exist [49]. The propagation of the wave is assumed to be along the x-axis. The dynamics of the EA wave in two-electron component plasma can be described by a set of normalised fluid equations [34]

$$\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u_c}{\partial t} + u_c \frac{\partial u_c}{\partial x} + \frac{3\alpha(1+\alpha)^2}{\theta} n_c \frac{\partial n_c}{\partial x} - \alpha \frac{\partial \phi}{\partial x} = 0 \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\alpha} n_c + n_h - \left(1 + \frac{1}{\alpha}\right). \tag{3}$$

Here, number densities of cold and hot electrons, (n_c) and (n_h), are normalised by their equilibrium values n_{c0} and n_{h0}, ϕ represents the electrostatic wave potential normalised by $k_B T_h/e$, u_c is the fluid velocity of cold electrons normalised by $C_s = \sqrt{k_B T_h/\alpha m}$, $\alpha = n_{h0}/n_{c0}$, i.e. ratio of the number density of hot to cold electrons, m is the mass of electron, $\theta = T_h/T_c$, i.e. temperature ratio of hot to cold electrons, e is the electronic charge. The spatial variables are normalised by hot electron Debye length $\lambda_{Dh} = \sqrt{k_B T_h/(4\pi m_h e^2)}$ and temporal variables by inverse of cold electron plasma period $\omega_{pc}^{-1} = \sqrt{m/(4\pi m_h e^2)}$.

It is mentioned that in eq. (2), inertia of cold electron is included and cold population is assumed to respond adiabatically to electric field perturbation. The physical origin of the third term in eq. (2) is the pressure term where adiabatic cold electrons are considered and ratio of specific heats is taken as three and that $T_c \neq 0$ [34,50]. The conditions for the existence of EA mode are: (i) $T_h \gg T_c$ and (ii) θ cannot be zero (as cold electrons represent a significant fraction of plasma (more than 20%). The hot electrons are assumed to follow the following one-dimensional Cairns–Tsallis velocity distribution [39] as

$$f_e(v(x)) = C_{q,\beta}\left(1 + \beta \frac{v_x^4}{v_{te}^4}\right) \times \left(1 - (q-1)\frac{v_x^2}{2v_{te}^2}\right)^{1/(q-1)}. \tag{4}$$

Here $v_{te} = (T_e/m_e)^{1/2}$ is the thermal velocity of electrons, β stands for the number of non-thermal electrons in the distribution, q is a parameter representing the strength of non-extensivity, $C_{q,\beta}$ is the constant of normalisation given by

$$C_{q,\beta} = \frac{1}{\sqrt{2\pi} v_{te}} \left[\frac{\Gamma\left(\frac{1}{1-q}\right)(1-q)^{5/2}}{\left(\Gamma\left(\frac{1}{1-q}\right) - \frac{5}{2}\right)\left[3\beta\left(\frac{1}{q-1} + \frac{3}{2}\right)\left(\frac{1}{q-1} + \frac{5}{2}\right)(q-1)^2\right]} \right], \quad \text{for} \quad -1 < q \leq 1 \tag{5}$$

and

$$C_{q,\beta} = \frac{1}{\sqrt{2\pi}v_{te}}\left[\frac{\Gamma\left(\frac{1}{q-1}+\frac{3}{2}\right)(q-1)^{5/2}\left(\frac{1}{q-1}+\frac{5}{2}\right)\left(\frac{1}{q-1}+\frac{3}{2}\right)}{\left(\Gamma\left(\frac{1}{1-q}\right)+1\right)\left[3\beta+\left(\frac{1}{1-q}-\frac{3}{2}\right)\left(\frac{1}{1-q}-\frac{5}{2}\right)(1-q)^2\right]}\right], \quad \text{for} \quad q \geq 1 \quad (6)$$

For $q = 1$, Cairns distribution is recovered [37]. If $q > 1$, i.e. non-extensive character of non-thermal electrons increases, the distribution becomes less important. Therefore, the existence of higher energy states is less in this case and for $q < 1$, large number of high energy states exists [39]. Integrating eq. (4) over velocity space, we get the following expression for normalised number density:

$$n_e(\phi) = \int_{-\infty}^{+\infty} f_e(v_x)\,\mathrm{d}v_x \quad \text{for} \quad -1 < q < 1$$
$$n_e(\phi) = \int_{-v_{\max}}^{+v_{\max}} f_e(v_x)\,\mathrm{d}v_x \quad \text{for} \quad q > 1. \quad (7)$$

The normalised q-non-extensive non-thermal hot electron density profile is given by [39]

$$n_h(\phi) = (1+(q-1)\phi)^{\frac{q+1}{2(q-1)}}(1+A\phi+B\phi^2). \quad (8)$$

Here $A = -(16q\beta)/(3-14q+15q^2+12\beta)$, $B = 16q\beta(2q-1)/(3-14q+15q^2+12\beta)$ and the parameter q stands for the strength of non-extensivity of electrons. In the extensive limiting case ($q \to 1$) and $\beta = 0$, the above distribution reduces to the well-known Maxwell–Boltzmann velocity distribution. However, for $q \to 1$ and $\beta \neq 0$, the distribution (8) reduces to the well-known Cairns density given as

$$n_h(\phi) = \left(1 - \frac{4\beta}{1+3\beta}\phi + \frac{4\beta}{1+3\beta}\phi^2\right)\exp(\phi). \quad (9)$$

On the other hand, for $\beta = 0$, eq. (8) recovers q-non-extensive electron density as

$$n_h(\phi) = (1+(q-1)\phi)^{\frac{q+1}{2(q-1)}}. \quad (10)$$

In order to solve the nonlinear set of partial differential equations, we apply Sagdeev pseudopotential technique [51]. To study the time-independent EA solitary waves, all the dependent variables in eqs (1)–(3) are made to be functions of a single variable, i.e. $\xi = x - Mt$ as the only solution which depends on space and time. Here ξ is normalised by λ_D and M is the Mach number and is defined as the ratio of solitary wave speed and $C_s = \sqrt{k_B T_h/\alpha m}$. Using this transformation into eqs (1) and (2), we get

$$-M\frac{\partial n_c}{\partial \xi} + \frac{\partial n_c u_c}{\partial \xi} = 0 \quad (11)$$
$$-M\frac{\partial u_c}{\partial \xi} + u_c\frac{\partial u_c}{\partial \xi} + \frac{3\alpha(1+\alpha)^2}{\theta}n_c\frac{\partial n_c}{\partial \xi} - \alpha\frac{\partial \phi}{\partial \xi} = 0. \quad (12)$$

Integrating eqs (11) and (12), applying boundary conditions $n_c \to 1$, $u_c \to 0$ and $\xi \to \infty$ we get

$$n_c = \frac{M}{\left(M^2 - \frac{3\alpha(1+\alpha)^2}{\theta}\left(n_c^2 - 1\right) + 2\alpha\phi\right)^{1/2}}. \quad (13)$$

On further solving the above equation, a quadratic equation in n_c^2 is obtained from which two roots are obtained. After some standard mathematical procedure, we get the expression for n_c as

$$n_c = \frac{1}{2}\frac{1}{(1+\alpha)\sqrt{3\alpha/\theta}} \times[((M+(1+\alpha)\sqrt{3\alpha/\theta})^2+2\alpha\phi)^{1/2} +((M-(1+\alpha)\sqrt{3\alpha/\theta})^2+2\alpha\phi)^{1/2}]. \quad (14)$$

Using the transformation $\xi = x - Mt$ and value of n_c into eq. (3), integrating and multiplying both sides by $\mathrm{d}\phi/\mathrm{d}\xi$, we get

$$\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right)^2 + V(\phi) = 0, \quad (15)$$

where $V(\phi)$ stands for Sagdeev potential which reads as

$$V(\phi) = V_1(\phi) + V_2(\phi) + V_3(\phi), \quad (16)$$

where

$$V_1(\phi) = -\frac{2(1+(q-1)\phi)^{(3q-1)/(2(q-1))}}{(3q-1)} \times\left[(1+A\phi+B\phi^2) - \frac{2(A+2B\phi)(1+(q-1)\phi)}{(5q-3)} + \frac{8B(1+(q-1)\phi)^2}{(7q-5)(5q-3)}\right]$$

$$V_2(\phi) = \frac{2}{(5q-3)(3q-1)} \times \left[(5q-3) - 2A + \frac{8B}{(7q-5)}\right]$$

$$V_3(\phi) = \frac{1}{6\alpha^2(1+\alpha)\sqrt{3\alpha/\theta}} \times \Big[(M + (1+\alpha)\sqrt{3\alpha/\theta})^3 - (M - (1+\alpha)\sqrt{3\alpha/\theta})^3\Big] - \frac{1}{6\alpha^2(1+\alpha)\sqrt{3\alpha/\theta}} \times \Big[(2\alpha\phi + (M + (1+\alpha)\sqrt{3\alpha/\theta})^2)^{3/2} + (2\alpha\phi + (M - (1+\alpha)\sqrt{3\alpha/\theta})^2)^{3/2}\Big] + \left(1 + \frac{1}{\alpha}\right)\phi. \quad (17)$$

Further, eq. (15) represents an energy integral for a classical particle moving with velocity $\mathrm{d}\phi/\mathrm{d}\xi$ in a potential $V(\phi)$. It is considered as the motion of a particle whose pseudoposition is ϕ having pseudovelocity $\partial\phi/\partial\xi$ at pseudotime ξ in a pseudopotential well. This may be the reason behind Sagdeev potential being called as pseudopotential. It is pertinent to mention that the parameters chosen for the analysis are within the range of those values observed by the Viking satellite in the dayside auroral zone, where an electric field of amplitude $\approx 100\,\mathrm{mV/m}$ is observed [52,53]. Here we have chosen the parameters corresponding to dayside auroral zone, i.e. $T_c \approx 5$ eV, $T_h \approx 250$ eV, $n_{c0} \approx 0.5$ cm^{-3}, $n_{h0} \approx 2.5\ \mathrm{cm}^{-3}$ [52] thereby giving the electron density $\alpha = 5$, temperature ratio $\theta = 50$ for our calculations. These parameters correspond to $\lambda_{Dh} \approx 7340$ cm and normalised electrostatic wave potential amplitude as $\phi \approx 0.03$.

3. Results and discussion

3.1 *Existence of solitons*

A detailed discussion of large-amplitude theory of solitons has been presented by Buti [54] and Nishihara and Tajiri [55] in their respective research. It was mentioned that for both the solitons and DLs, the Sagdeev potential ϕ must be negative between two extreme points $\phi = 0$ and $\phi = \phi_m$. In order to have soliton solutions, the Sagdeev potential must satisfy the following conditions: (i) $V(\phi) = \mathrm{d}V(\phi)/\mathrm{d}\phi = 0$ at $\phi = 0$ and (ii) $\mathrm{d}^2V/\mathrm{d}\phi^2 < 0$ at $\phi = 0$ gives double roots. (iii) There exists a non-zero minimum or maximum value of ϕ, at which $V(\phi_m) \geq 0$ where ϕ_m represents the amplitude of the EA solitary wave. It means that a quasiparticle of zero total energy will be reflected at the position $\phi = \phi_m$. The nature of solitary structures depends upon the sign of ϕ_m, i.e. positive (negative) value of ϕ_m leads to a compressive (rarefactive) solitary wave. Further, $V(\phi) < 0$ when $0 < \phi < \phi_m$ for positive solitary waves and $\phi_m < \phi < 0$ for negative solitary waves. This means that $V(\phi)$ represents potential trough in which the quasiparticle can be trapped and experience oscillations. Applying the boundary conditions for the existence of localised structures we get Mach number as

$$M > \sqrt{\frac{2}{2A+q+1} + \frac{3\alpha(1+\alpha)^2}{\theta}} = M_1. \quad (18)$$

Here $M/M_1 = M_s$ and it may be noted that the domain of allowable Mach numbers (M) depends upon on various plasma parameters such as hot to cool electron density ratio (α), hot to cool temperature ratio (θ) and in particular, non-extensivity (q) and non-thermality (β) of the electrons. It may be reminded that the hybrid non-thermal non-extensive distribution is applicable in limited range of values of parameters (q, β), i.e. $0.6 < q \leq 1$ and $0 \leq \beta < 0.25$ [40]. It may be noted that in the limit $1/\theta \rightarrow 0$, our expression of Mach number reduces to that obtained by Amour *et al* [30]. It is clear from eq. (18) that the Mach number decreases (increases) with the temperature of hot electrons, i.e. $\theta = T_h/T_c(1/\theta)$. This becomes clear from figure 1a where we have shown a comparison of Mach number M_1 as a function of non-extensivity (q) with ref. [30]. Here solid line for $1/\theta = 0$ corresponds to ref. [30] and dotted line is for $1/\theta = 0.02$, i.e. $\theta = 50$ with $\alpha = 5$ and $\beta = 0.2$. It is obvious that Mach number M_1 increases with the introduction of temperature θ and decreases with entropic index q. Figure 1b portrays the corresponding comparison of Mach number M_1 with non-thermality (β). To investigate the effect of electron density ($\alpha = n_{h0}/n_{c0}$) on the potential structures, plot of Sagdeev potential $V(\phi)$ (eq. (16)) vs. ϕ is shown in figure 2a. Here $\theta = 50$, $\beta = 0.2381$, $q = 0.63$, $M_s = 1.028$ and $\alpha = 4.8, 5.0, 5.2$ correspond to dotted, solid and dashed lines respectively. It is observed that the amplitude of negative potential structures decreases with increase in electron density. For the parameters of figure 2a, figure 2b depicts the effect of temperature θ on solitons and it becomes clear that the amplitude decreases with θ. It is mentioned that the parameters $\alpha = 5$ and $\theta = 50$ correspond to the dayside auroral zone and the normalised potential amplitude (i.e. $\phi \approx 0.03$) is obtained for $\beta = 0.2381$ and $q = 0.63$ (see solid curves in figures 2a and 2b). It means that

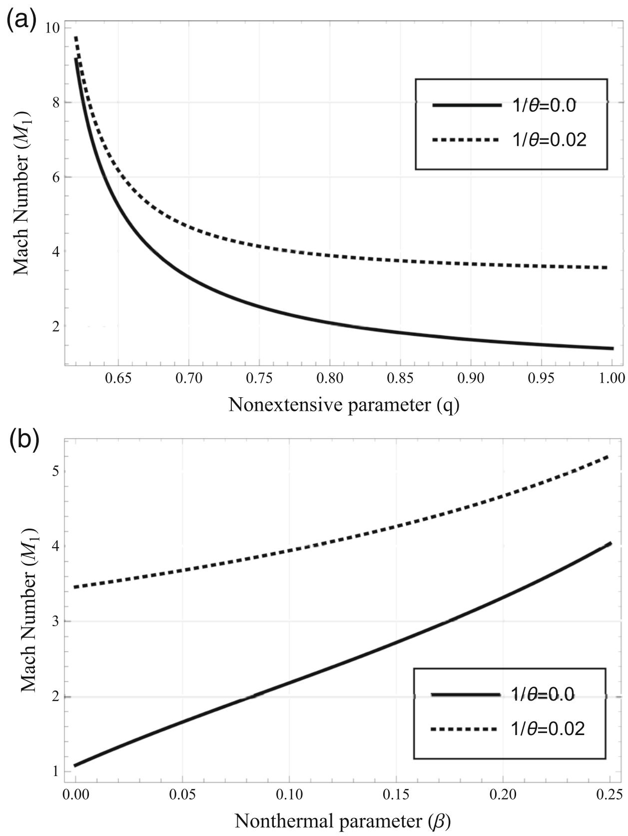

Figure 1. For $\alpha = 5$ and $\theta = 50$, plot showing the comparison of variation of Mach number M_1 of the present paper with with that of ref. [30] as a function of (**a**) non-extensivity q with $\beta = 0.2$ and (**b**) non-thermality β with $q = 0.7$.

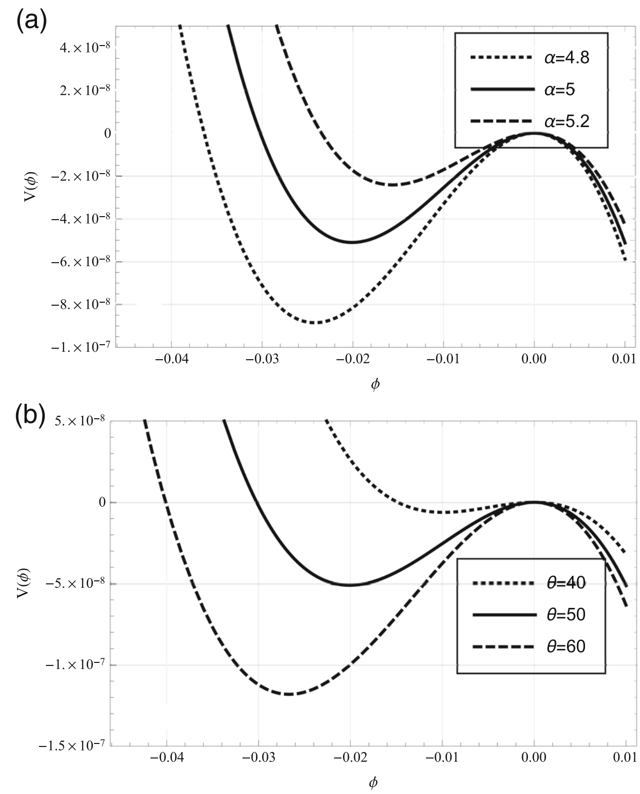

Figure 2. Plot of $V(\phi)$ vs. ϕ for (**a**) different values of α with $\theta = 50$ and (**b**) different values of θ with $\alpha = 5$. Other parameters are: $\beta = 0.2381$, $q = 0.63$, $M_s = 1.028$.

our results agree with the observations of Viking satellite in the dayside auroral zone where an electric field amplitude of $\approx$100 mV/m (i.e. $\phi \approx 0.03$) is observed [52,53]. It is worth mentioning that our plasma model supports the existence of rarefactive solitary structures for all ranges of non-thermal and non-extensive parameters. This is in good agreement with the findings of earlier investigations [30,46,55,56].

3.2 *Existence of large-amplitude double layers*

As discussed earlier, DLs are found to exist in a variety of space plasma environments such as solar wind, auroras and extragalactic jets and are responsible for reflection, acceleration or deceleration of plane polarities. For the formation of DL solution, in addition to the conditions of solitary waves, an additional condition that must be satisfied is $\mathrm{d}V(\phi)/\mathrm{d}\phi = 0$ at $\phi = \phi_m$. This condition shows the absence of reflection by the pseudoparticle at $\phi = \phi_m$ owing to the disappearance of pseudoforce and pseudovelocities. It proceeds towards another phase and generates asymmetrical DL with a net potential drop of ϕ_m. Here ϕ_m represents the amplitude of DL. The minimum value of Mach number above which DLs can be calculated is given by eq. (18). On increasing M above M_1, i.e. for $M > M_1$ or $M_s > 1$, EA solitons with increasing amplitude are formed until M_s reaches an upper limit beyond which no soliton would exist that may be due to the existence of DLs.

We have examined the Sagdeev potential $V(\phi)$ as a function of ϕ at three different Mach numbers $M_s = 2.975, 2.752, 3.643$ and is displayed in figure 3a. It is clear that for the set of parameters $\beta = 0.2$, $q = 0.69$, α =5, $\theta = 50$, EA DL exists at $M_s \approx 2.975$ (solid line) and for $M_s < 2.975$ (dotted line) solitons result. However, for $M_s > 2.975$ (dashed line), no soliton or DL exists. It is further reported that the amplitude of the DL is greater than that of the ordinary solitons. The corresponding phase portrait is depicted in figure 3b that shows the manifestation of allowed solutions. Here, closed curve (dotted lines) called homoclinic orbit refers to the solitary waves. Solid curve exhibits stable centre point and unstable saddle point, provides heteroclinic orbit that refer to kink solution of DL. By means of the large-amplitude Sagdeev pseudopotential approach and phase-portrait analysis, the plasma under consideration is found to support rarefactive large-amplitude DLs and solitary waves with certain features such as Mach number M_s, non-extensive parameter q, non-thermality

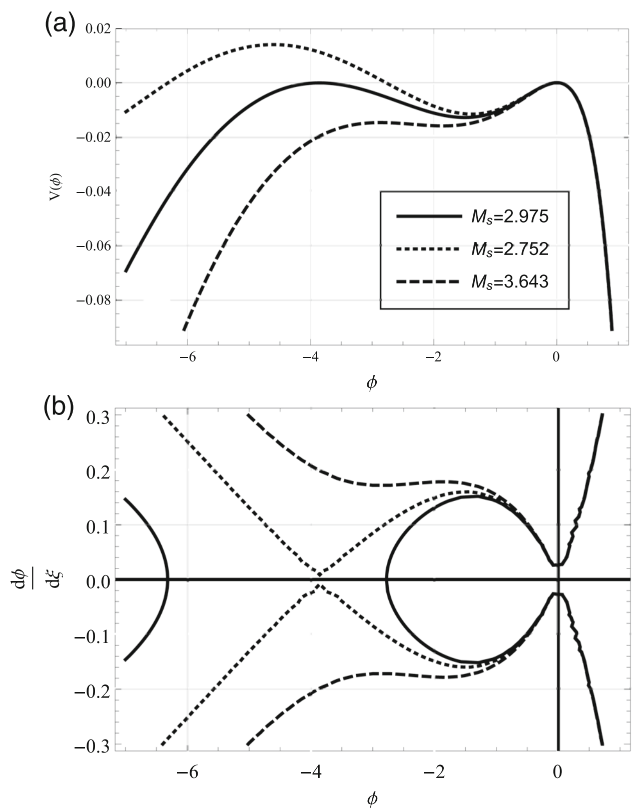

Figure 3. For large amplitude DLs, (**a**) plot of $V(\phi)$ vs. ϕ for different Mach numbers M_s with $\alpha = 5$, $\beta = 0.2$, $\theta = 50$ and $q = 0.69$ and (**b**) the corresponding phase portrait.

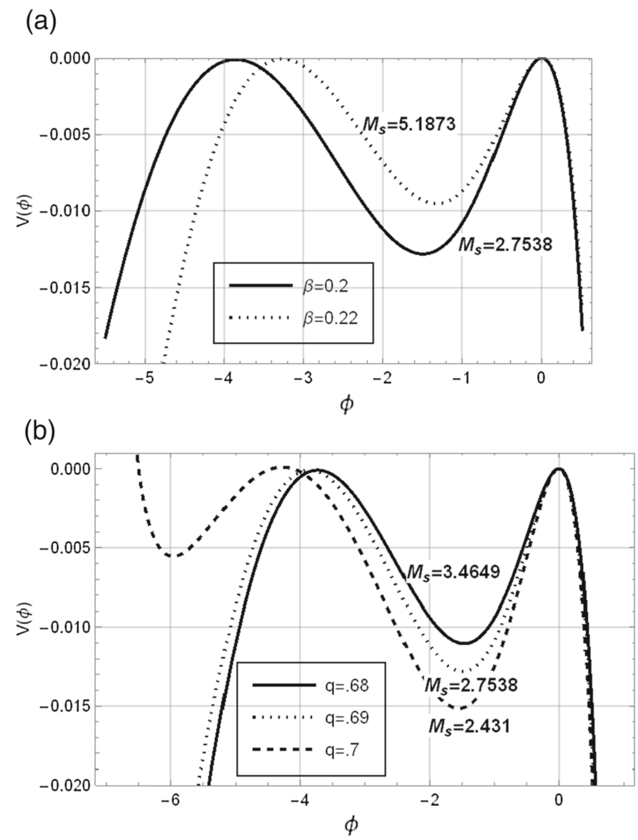

Figure 4. Sagdeev potential $V(\phi)$ vs. ϕ (**a**) for different values of β with $\theta = 50$ and $q = 0.69$ and (**b**) different values of non-extensive parameter q with $\alpha = 5$, $\beta = 0.2$.

β and number density α. To see the impact of non-thermality (β) on the nature of large-amplitude DLs, plot of $V(\phi)$ vs. ϕ is shown in figure 4a for $\beta = 0.2$, 0.22. Clearly, increase in β leads to decrease in amplitude and width of DL and decreases the steepness of electrostatic potential. However, the entropic index q has the opposite effect on the dynamics of DLs. From the plot of $V(\phi)$ vs. ϕ (figure 4b) for $q = 0.68, 0.69, 0.7$, it is clear that for a given non-thermality ($\beta = 0.2$ in this case), the amplitude of negative potential DLs increases slightly with q. Moreover, the width and depth of the potential also increase with q. It is further mentioned that the temperature of hot electrons and electron density do not affect the amplitude of large-amplitude EA DLs (not shown).

3.3 *Small-amplitude apporoximation*

The case of small amplitude should be considered to write down the explicit form of the DL solution. In the small-amplitude limit $\phi \ll 1$, eq. (16) can be written as

$$V(\phi) = A_1\phi^2 + A_2\phi^3 + A_3\phi^4. \tag{19}$$

Here A_1, A_2 and A_3 are given by

$$\begin{aligned}
A_1 &= -\frac{A(q+1)(7q-5)}{2(3q-1)(5q-3)} + \frac{8B(q-1)}{(3q-1)(5q-3)} \\
&\quad -\frac{(q+1)}{4} - \frac{1}{2(M^2-3\alpha(1+\alpha)^2/\theta)} \\
A_2 &= -\frac{A(q+1)}{6} + \frac{4B(q+3)}{3(5q-3)} + \frac{(q+1)(q-3)}{24} \\
&\quad -\frac{\alpha(3M^2+3\alpha(1+\alpha)^2/\theta}{6(M^2-3\alpha(1+\alpha)^2/\theta)^3} \\
A_3 &= \frac{A(q+1)(q-3)(9q+1)}{96(5q-3)} - \frac{B(q+1)}{8} \\
&\quad + \frac{(q+1)(q-3)(3q-5)}{192} + \frac{\alpha^2}{8} \\
&\quad \times \left(\frac{5M^4+30M^2\alpha(1+\alpha)^2/\theta+9\alpha^2(1+\alpha)^4/\theta^2}{(M^2-3\alpha(1+\alpha)^2/\theta)^5}\right).
\end{aligned} \tag{20}$$

From the boundary conditions (i) and (ii) specified in §3.1, we get $2\phi_m = -A_2/A_3$ for DLs and Sagdeev potential $V(\phi)$ is given by $V(\phi) = A_3\phi^2(\phi_m - \phi)^2$.

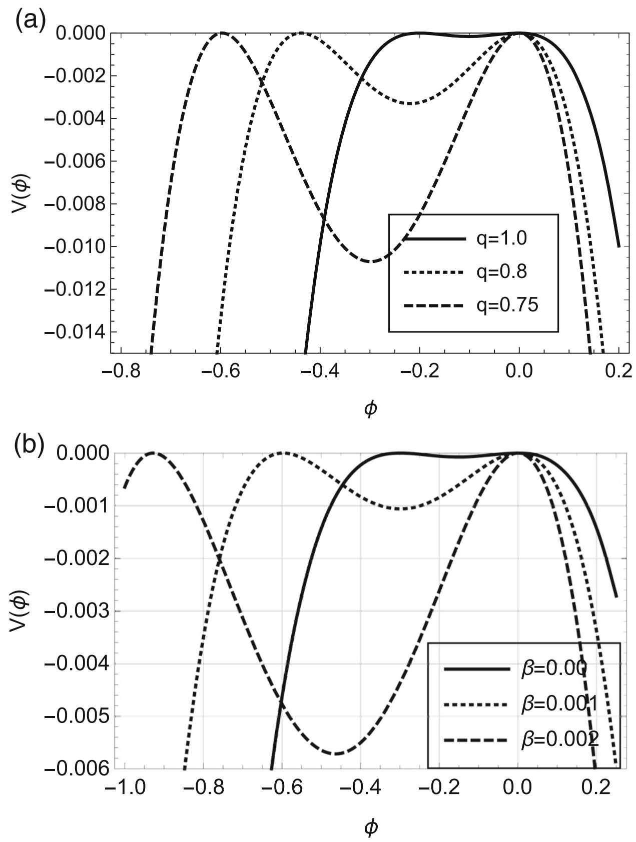

Figure 5. For the parameters $\alpha = 5$, $\theta = 50$ for small-amplitude DLs, plot of $V(\phi)$ vs. ϕ for (**a**) different values of q with $\beta = 0.1$ and $M = 2.5$ and (**b**) different values of β with $q = 0.7$ and $M = 2.1$.

Further, the DL solution can be obtained by

$$\phi = \frac{\phi_m}{2}\left[1 - \tanh\left(\frac{2\xi}{\Delta}\right)\right]. \tag{21}$$

Here $\Delta = \sqrt{-8/A_3}/|\phi_m|$ is the width of DL provided $A_3 < 0$. It may be mentioned here that the sign of A_2 determines the nature of DLs, i.e. for $A_2 > 0$, compressive (positive potential) DL exists whereas for $A_2 < 0$ rarefactive (negative potential) DL exists. We get the numerical computation of Mach number M_c corresponding to $A_2 = 0$ and find that $M > M_c$ ($M < M_c$) for compressive (rarefactive) DLs. Here M_c is the critical value of Mach number and is calculated from the numerical computation of the following relation:

$$3M_c^2 + \frac{3\alpha(1+\alpha)^2}{\theta} = \left(M_c^2 - \frac{3\alpha(1+\alpha)^2}{\theta}\right)^3 = \left[-\frac{A(q+1)}{\alpha} + \frac{8B(q+3)}{\alpha(5q-3)} + \frac{(q+1)(q-3)}{4\alpha}\right]. \tag{22}$$

Equation (22) is a cubic polynomial in M^2 and correspondingly we get six values of Mach numbers, determination of whose solutions is a laborious job. So we can numerically solve and choose only the real and positive value of the critical Mach number M_c for our computation. In order to discuss numerically the effect of non-extensivity on the properties of EA DLs, plot of Sagdeev potential $V(\phi)$ (given by eq. (19)) vs. ϕ is shown in figure 5a. It portrays negative potential EA DLs for three different values of non-extensive parameter q corresponding to $q = 1.0$ (solid line), $q = 0.8$ (dotted line) and $q = 0.75$ (dashed line) with $\alpha = 5$, $\theta = 50$, $\beta = 0.1$ and $M = 2.5$. The amplitude of the EA DL is affected if the electron density distribution deviates from the pure non-thermal (i.e. $q = 1, \beta \neq 0$) to the mixed one ($q \neq 1, \beta \neq 0$). From figure 5a, it is observed that the amplitude of negative potential EA DLs decreases with increasing q and is minimum for $q = 1$ (pure non-thermality). That is to say, any deviation from the non-extensivity leads to an increase in the amplitude of EA DLs. Further, it is also found that the depth of DL increases for an increased enhancement of non-extensive electrons, i.e. lowering the values of q. Also, for a given non-thermality, the negative potential DLs expand with decrease in q. It is also obvious that as non-extensive character increases, q decreases and the steepness of potential ϕ is increased. To take up the impact of non-thermal parameter (β) on DL dynamics, we have plotted $V(\phi)$ vs. ϕ at three different values of $\beta = 0.0, 0.001, 0.002$ (figure 5b) with $q = 0.7$, $M = 2.1$ with the other parameters of figure 5a. Here solid line stands for pure non-extensivity (i.e. $\beta = 0$). Again, it is remarked that even a small increase in non-thermality (β) leads to an increase in the amplitude of EA DLs and is minimum for $\beta = 0.0$, i.e. for pure non-extensivity. In this case, for a given non-extensivity, the depth of DL and steepness of electrostatic potential ϕ increase with increase in non-thermality β as obvious from the dotted and dashed lines.

The behaviour of the width of DL is displayed in figure 6a for three different values of non-thermal parameter β. From figure 6a, it is evident that the width of EA DL increases with increasing non-extensive parameter q, i.e. decreasing non-extensive character of electrons. Further, the width is found to decrease with non-thermality, i.e. increase in β. Hence, the nature of EA DLs depends sensitively on the entropic index q and non-thermal parameter β. To see the effect of electron density α and temperature ratio θ, plot of $|\phi_m|$ vs. α is shown in figure 6b for $\theta = 49$ (solid line), 50 (dotted line) and 51 (dashed line) with $q = 0.8$, $\beta = 0.1$ and $M = 2.6$. It is obvious that the absolute value of amplitude of small-amplitude EA DLs decreases with increase in temperature ratio with enhancement in the steepness

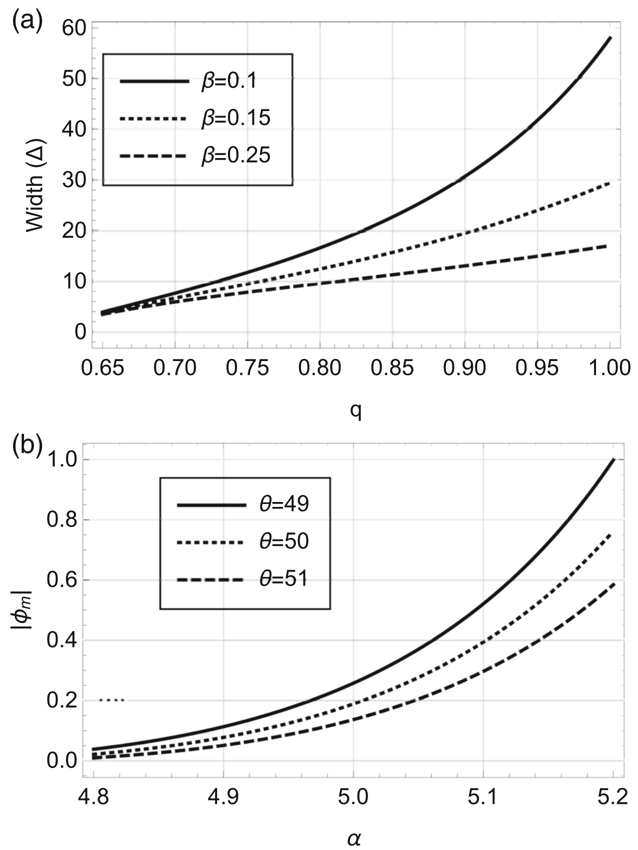

Figure 6. For $\alpha = 5$ and $\theta = 50$, variation of (**a**) width of DL as a function of q for different values of β with $M = 2.7$ and (**b**) amplitude $|\phi_m|$ vs. α for different values of θ with $M = 2.6$.

of potential whereas it is found to increase with α. It may be noted that for large-amplitude DLs, amplitude is independent of θ and α whereas these parameters have sufficient impact on the small-amplitude DLs.

4. Conclusion

To conclude, we have performed the nonlinear analysis of EA solitons and DLs in two-electron component plasma system consisting of Cairns–Tsallis-distributed hot electrons, adiabatic cool electrons and stationary ions. By employing a non-perturbative approach popularly known as Sagdeev pseudopotential method, large-amplitude EA solitary waves and DLs are investigated. The existence of solitary wave results within the range of allowable Mach number. It has been observed that Mach number M_1 decreases (increases) with non-extensivity (q) (non-thermality β). Introduction of temperature of hot electron leads to increase in the value of Mach number. Only negative potential solitary structures are observed whose amplitude decreases (increases) with the increase of number density (α) of hot to cool electrons (hot to cool electron temperature ratio θ). The study is further extended to examine the effects of various parameters on large- and small-amplitude DLs. The analysis of Sagdeev potential curves and phase portraits may allow only rarefactive DLs to exist in the model considered here. It is found that increase in non-thermal effects reduces the amplitude, width and steepness of large-amplitude DLs whereas the entropic index do the opposite. It is found that the width and amplitude of solitary waves and DLs depend upon the non-thermality and non-extensivity of the electrons. The parameters chosen are within the range of those values observed by the Viking satellite in the auroral zone of plasma. In the small-amplitude limit, amplitude of negative potential EA DLs decreases with decreasing non-thermal character (increasing q) and non-thermality and is minimum for pure non-thermality and non-extensivity. Further, increase in non-thermal and non-extensive character leads to increase in steepness of the potential and width of small-amplitude DLs. This indicated that the nature of EA DLs depends sensitively on the entropic index q and non-thermal parameter β. The absolute value of amplitude of DLs is further reported to increase (decrease) with α (θ). The present study may be beneficial to grasp striking attributes of nonlinear EA solitons and DLs both in laboratory and space plasma where two distinct groups of electrons are present.

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