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Perfect fluid and heat flow in $f(R, T)$ theory

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Abstract. Here we have studied locally rotationally symmetric Bianchi-V Universe in the presence of modified theory for gravitation [$f(R, T)$ theory] and for that, we considered a perfect fluid with heat conduction as the energy source. We used the law of variation for the deceleration parameter (DP) to solve field equations, as it gives a constant value of DP and is related to the average scale factor metric. Also, we have discussed the physical and geometrical properties of the model in detail.

Keywords. Bianchi Universe; $f(R, T)$ theory; perfect fluid; heat flow; deceleration parameter.

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1. Introduction

Nowadays modern cosmology attracts much attention of the researchers because of its ability to explain the late-time acceleration of the Universe. This is the main reason why the modern cosmology is the fastest-growing field in the study of the Universe. Modern cosmology achieved a new path because of the idea of accelerated expansion of the Universe. This idea was observed by type-Ia supernovae experiments, suggesting that the Universe is undergoing an accelerated expansion [1–6]. Some researchers are making notable efforts to observe the Universe filled with dark energy, in Einstein's theory. From observational data, they conclude that the Universe is dominated by negative pressure, dubbed as dark energy. To study the nature of accelerated expansion of the Universe, we have several choices of the theoretical models of the dark energy, namely the quintessence scalar field models [7,8], the phantom field [9–11], K-essence [12,13], tachyon field [14,15], quintom [16,17] and Chaplygin gas [18,19].

Einstein's theory of gravitation has made a very good impact on constructing a cosmological model and explaining the origin and evolution of the Universe. But Einstein's theory could not explain the late-time acceleration, which is one of the important problems in modern cosmology. Many attempts have been made to modify the gravity theory to explain the present accelerated phase. Many researchers from the cosmology field developed a number of alternative theories for the general theory of relativity because of its lacuna and they verified these theories with a brief explanation for the late-time accelerated expansion of the Universe. We are interested and motivated by one of the alternative theories of gravitation which is well known as $f(R, T)$ theory of gravity, where R is the Ricci scalar and T is a trace of the stress-energy tensor which was proposed by Harko *et al* [20] and obtained the gravitational field equations in the metric formalism, as well as the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor. There are different forms of a function for $f(R, T)$ theory which have been discussed by the researchers of this particular field. Godani and Samanta [21] have proposed a different type of function for $f(R, T)$ gravity in the form of $f(R, T) = R + \xi T^{1/2}$, where ξ, R and T are constant, scalar curvature and trace of stress–energy tensor respectively. They have studied Friedmann–Robertson–Walker (FRW) model and analysed energy conditions. Also, they used 57 redshift data for the estimation of the age of the Universe. Samanta and Dhal [22] studied $f(R, T)$ theory for the higher dimensional cosmological model by taking the same form of a function for $f(R, T)$, as we have chosen in this paper. They have discussed Hubble parameter, luminosity distance and distance modulus with redshift.

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Recently, Aktaş and Aygün [23] have discussed magnetised strange quark matter solutions in $f(R, T)$ gravity with a cosmological constant and they found that $f(R, T)$ theory can explain the late-time acceleration of the Universe. Samanta and Myrzakulov [24] studied bulk viscous fluid in $f(R, T)$ theory. Very recently, Pawar *et al* [25] have discussed the Bianchi-V model in the presence of $f(R, T)$ gravity using modified holographic Ricci dark energy and they found negative value of the deceleration parameter (DP) which indicates that the Universe is in the accelerated expansion phase and they observed that the Universe is isotropic throughout the evolution. Similarly, Pawar *et al* [26] and Sharif [27] have analysed $f(R, T)$ theory with different energy sources and in different cosmological models. Samanta [28] has investigated $f(R, T)$ gravity for the Bianchi type-V Universe filled with wet dark fluid. In [28] proper distance, look-back time and some other astrophysical phenomena with redshift are discussed and it is found that volume is zero at the initial time but the pressure and density are infinite for $t = 0$. We have good agreement with this work.

Also, there is an interesting function of $f(R, T)$ theory of gravity in the form of $f(R, T) = R + \lambda R^2 + 2\beta \ln(T)$, where λ and β are constants and other notations have the same meaning as defined already. Elizalde *et al* [29] found that in the neighbourhood of bouncing point (initial time) all the energy conditions are satisfied and hence they concluded that the null energy condition in general relativity within the framework of spatially at four-dimensional FLRW model can be avoidable and it is not necessary. Samanta [30] studied $f(R, T)$ theory by taking Kantowski–Sachs metric along with perfect fluid. Agrawal and Pawar [31] discussed $f(R, T)$ theory and analysed a plane-symmetric model with quark and strange quark matter and they found that the model does not approach isotropy. Agrawal and Pawar [32] have discussed the Bianchi type-V Universe model with magnetised domain walls in $f(R, T)$ theory of gravity and they concluded that the Universe is expanding endlessly under the influence of dark energy. Katore *et al* [33] have discussed domain walls in $f(R, T)$ theory. Samanta *et al* [34] have done a comparative study in the framework of $f(R)$, $f(R, T)$ theory and general relativity for wormhole structure with exponential shape function. They found that the exponential shape function is a very good choice to explain the existence of wormhole solutions filled with very less amount of exotic matter near the throat of a wormhole.

The $f(R, T)$ theory has attracted a lot of attention of the astrophysicists in recent times and hence discussion is going on by many researchers in this modified theory because of its ability to explain mysterious things in cosmology and astrophysics (for more details, one can refer [35–41]). The most curious mystery of the Universe is Big-Bang singularity and hence it is very obvious that researchers are interested to study the behaviour of the Universe near the Big-Bang singularity. The distribution of matter is essentially inhomogeneous and anisotropic. In the early stage of the evolution of the Universe, matter is not expected to reach thermal equilibrium, and this is the reason that in the Universe there would be heat flow. Shri Ram *et al* [42] have studied perfect fluid with heat flow as an energy source for anisotropic Bianchi type-V Universe in Saez Ballester theory, and they have presented power law solution as well as exponential-type solution. For that they have used variation law of Hubble parameter. Singh *et al* [43] have found the exact solution in Einstein theory of the Universe considering perfect fluid with heat flow. So many researchers have studied perfect fluid as an energy source. Pawar *et al* [44] have discussed the role of constant DP by considering perfect fluid and dark energy. In our work, we also have constant DP because of the variation law for the Hubble parameter, which is suitable to explain the present day Universe. Presently, Bianchi Universes are playing important roles in observational cosmology, as the WMAP data [45,46] seem to require an addition to the standard cosmological model with a positive cosmological constant that bears a likeness to the Bianchi morphology [47–50]. Hence, Bianchi Universe has taken more attention from researchers and here we used LRS- Bianchi type-V cosmological model.

From the above work, we got a motivation to study the behaviour of the Universe by considering LRS Bianchi-V space–time, filled with perfect fluid with heat conduction in $f(R, T)$ theory. The physical and dynamical behaviour of the Universe is also observed. The paper is organised as follows: Section 2 discusses the $f(R, T)$ gravity. In §3, we have studied the metric (Bianchi type-V) and field equations for $f(R, T)$ gravity. In §4, we have discussed variation law for the Hubble parameter. Section 5 is devoted to the solutions of field equations. In §6, we have discussed the physical and dynamical parameters. Finally, in §7, we have concluded our work.

2. $f(R, T)$ Theory and field equation

Hilbert–Einstein variational principle on which field equation of $f(R, T)$ theory is formed, is given by

$$S = \frac{1}{2\kappa}\int f(R,T)\sqrt{-g}\mathrm{d}^4x + \int L_m\sqrt{-g}\mathrm{d}^4x. \quad (1)$$

The gravitational field equations for $f(R,T)$ gravity are

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j)f_R(R,T) = \kappa T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij}, \quad (2)$$

where

$$\theta_{ij} = g^{\alpha\beta}\frac{\partial T_{\alpha\beta}}{\partial g^{ij}}, \quad f_R = \frac{\partial f(R,T)}{\partial R}, \quad f_T = \frac{\partial f(R,T)}{\partial T}$$

and ∇_j is the co-variant derivative. We choose

$$\kappa = \frac{8\pi G}{c^4},$$

where G is the Newtonian gravitational constant and c is the speed of light in vacuum. T_{ij} is the standard matter energy–momentum tensor derived from the Lagrangian L_m.

Form of the energy–momentum tensor given by [51] for a perfect fluid with heat flow is

$$T_{ij} = (\rho + p)u_iu_j + p\,g_{ij} + h_i\,u_j + h_j\,u_i, \quad (3)$$

where ρ is the energy density, p is the thermodynamic pressure, u_i is the four-velocity of the fluid, h_i is the heat flow vector satisfying

$$h^i\,u_i = 0, \quad h^i\,u_i > 0. \quad (4)$$

Let us consider that $u^i = \delta^i_0$. Then, the field equation and eq. (4) give that the heat flow is in the x-direction only, and therefore we have

$$h_i = (0, h_1(t), 0, 0). \quad (5)$$

In the present work, we have taken the particular functional as $f(R,T) = R + 2f(T)$. Otherwise functional can be taken in different ways corresponding to viable models. Here $f(T)$ is a function of the trace of the energy–momentum tensor. By using this functional, field equation can be rewritten as

$$R_{ij} - \frac{1}{2}Rg_{ij} = \kappa T_{ij} + 2f_T T_{ij} + [f(T) + 2pf_T]g_{ij}, \quad (6)$$

where f_T is a partial derivative of f with respect to T.

Assuming $f(T) = \lambda T$, λ being constant, we have chosen a system for $\kappa = 1$.

Here our intention is to observe the law of variation for the mean Hubble parameter, which gives a constant value of DP in a perfect fluid with heat flow by considering the LRS Bianchi type-V model. This law of variation gives a new path to solve field equations of cosmological models and this is somewhat general and appropriate to describe the present-day Universe.

3. Metric and field equations

We consider locally rotationally symmetric (LRS) Bianchi type-V space–time described by the line element

$$\mathrm{d}s^2 = A^2\mathrm{d}x^2 + B^2\mathrm{e}^{2x}(\mathrm{d}y^2 + \mathrm{d}z^2) - \mathrm{d}t^2. \quad (7)$$

Here, A and B are functions of cosmic time t only.

Now using a co-moving coordinate system, the field equation (6) with the help of eqs (3) and (4) for the metric equation (7), can be explicitly written as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -p, \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -p, \quad (9)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} = \rho - 2h_1, \quad (10)$$

$$2\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = h_1. \quad (11)$$

Here dot means derivative with respect to t.

Dynamical parameters for Bianchi Type-V are defined as follows:

The average scale factor

$$a(t) = (AB^2)^{1/3}. \quad (12)$$

The spatial volume

$$V = a^3(t) = AB^2. \quad (13)$$

The directional Hubble parameters

$$H_x = \frac{\dot{A}}{A}, \quad H_y = H_z = \frac{\dot{B}}{B}. \quad (14)$$

The average Hubble parameter

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right). \quad (15)$$

The dynamical scalar expansion θ and shear scalar σ^2 are

$$\theta = 3H \quad (16)$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2. \quad (17)$$

The average anisotropic parameter

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \quad (18)$$

Here H_i represents the directional Hubble parameters $(i = 1, 2, 3)$.

The deceleration parameter (DP) is

$$q = -1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right) \quad \text{or} \quad q = -\left[\frac{a\ddot{a}}{\dot{a}^2}\right]. \quad (19)$$

Now, eqs (8)–(11) can be written in terms of $H,\ q,\ \sigma^2$ as

$$\rho = 3H^2 - \sigma^2 - \frac{3}{A^2} \tag{20}$$

$$p = H^2(2q-1) - \sigma^2 + \frac{1}{A^2}. \tag{21}$$

4. Variation law of Hubble's parameter

We have to find a solution of eqs (8)–(11), which contain heat conduction. Here we consider the relation between H and a

$$H = la^{-n} = l(AB^2)^{-n/3}, \tag{22}$$

where $l > 0$ and $n \geq 0$ are constants.

This relation was proposed by Berman [52], and Berman and Gomide [53] have studied some relations for finding the solutions of field equations in FRW models. Now from eqs (15) and (22), we get

$$\dot{a} = la^{-n+1} \tag{23}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1}. \tag{24}$$

From eqs (19)–(21) we obtain

$$q = n - 1. \tag{25}$$

Now, using eqs (22) and (25), the solution of eq. (19) gives the law of variation of the average scale factor of the form,

$$a = (nlt)^{1/n}, \quad \text{for } \ n \neq 0. \tag{26}$$

5. Solution of the field equations

The field equations (8)–(11) reduce to a system of four non-linear equations in five unknowns parameters, A, B, p, ρ, h_1. Hence, to find the determinate solution of the system, we used the law of variation.

Now, from eqs (8) and (9), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} = 0. \tag{27}$$

This, on integration gives,

$$\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = \frac{c_1}{AB^2}, \tag{28}$$

where c_1 is the constant of integration.

Using eq. (12) in (28) and integrating again, we get

$$\frac{B}{A} = c_2 \ \exp\left(\int \frac{c_1}{a^3}\mathrm{d}t\right). \tag{29}$$

The metric functions A and B in terms of average scale factor $a(t)$ are given by

$$A(t) = c_2^{-2/3} \ a \ \exp\left(-\frac{2c_1}{3}\int a^{-3}\mathrm{d}t\right) \tag{30}$$

$$B(t) = c_2^{1/3} \ a \ \exp\left(\frac{c_1}{3}\int a^{-3}\mathrm{d}t\right). \tag{31}$$

Now using eq. (26) in eqs (30) and (31), we get

$$A(t) = c_2^{-2/3} \ (nlt)^{1/n} \ \exp\left(-\frac{2c_1}{3l(n-3)}(nlt)^{(n-3)/n}\right) \tag{32}$$

$$B(t) = c_2^{1/3} \ (nlt)^{1/n} \ \exp\left(\frac{c_1}{3l(n-3)}(nlt)^{(n-3)/n}\right), \tag{33}$$

where $n \neq 3$.

6. Dynamical parameters and their physical discussion

Dynamical parameters are quite significant in the discussion of the physical properties of the cosmological model and to develop a cosmological theory in $f(R,T)$ theory of gravity. We compute the following cosmological parameters for the model given by eq. (22).

The spatial volume of the metric is

$$V = a^3(t) = (nlt)^{3/n}. \tag{34}$$

The average Hubble parameter

$$H = (nt)^{-1}. \tag{35}$$

The dynamical scalar expansion θ and shear scalar σ^2 are

$$\theta = 3(nt)^{-1} \tag{36}$$

$$\sigma^2 = {c_1}^2(nlt)^{-6/n}. \tag{37}$$

The average anisotropic parameter

$$\Delta = \frac{2{c_1}^2}{l^2}(nlt)^{(2n-1)/n}. \tag{38}$$

Now using the above equations in eqs (20) and (21), we get energy density and pressure as follows:

$$\rho = 3(nt)^{-2} - {c_1}^2(nlt)^{-6/n} - 3{c_2}^{4/3} \ (nlt)^{-2/n} \times \exp\left[\frac{4c_1}{3l(n-3)}(nlt)^{(n-3)/n}\right] \tag{39}$$

$$p = (2n-3)(nt)^{-2} - {c_1}^2(nlt)^{-6/n} + {c_2}^{4/3}(nlt)^{-2/n} \times \exp\left[\frac{4c_1}{3l(n-3)}(nlt)^{(n-3)/n}\right]. \tag{40}$$

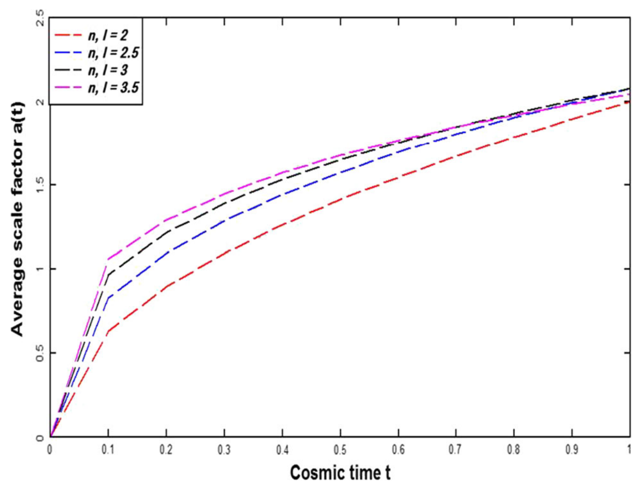

Figure 1. Variation of average scale factor $a(t)$ with cosmic time t with varying constants $n, l = 2,\ 2.5,\ 3,\ 3.5$.

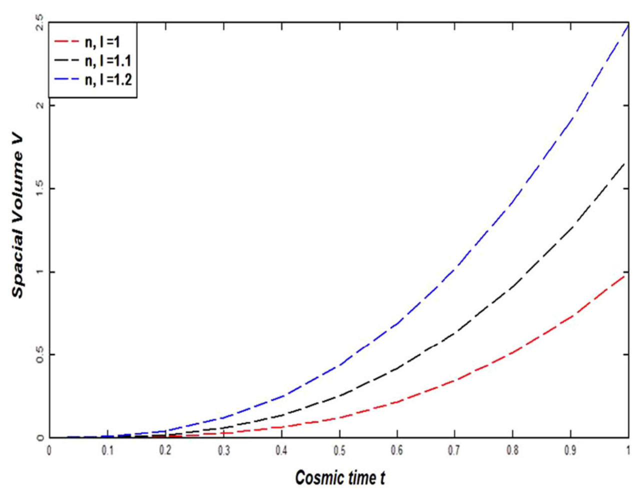

Figure 2. Variation of spatial volume V with cosmic time t with varying constants $n,\ l =\ 1,\ 1.1,\ 1.2$.

In eq. (11) using eqs (32) and (33) the solution of heat conduction can be obtained as

$$h_1 = 2c_1(nlt)^{-3/n}. \tag{41}$$

7. Conclusion

We have studied the LRS Bianchi type-V cosmological model in the $f(R, T)$ theory of gravity and for that we considered a perfect fluid with heat conduction. So we found solutions in the presence of heat conduction only and we have not discussed the solutions without heat conduction because results are obvious and already discussed by many researchers. In order to obtain the solutions of field equations, we used the law of variation for the Hubble parameter. Here we found some

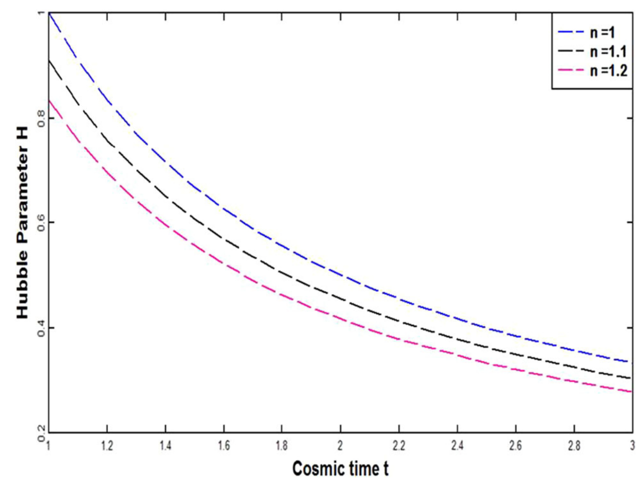

Figure 3. Variation of Hubble parameter H with cosmic time t with varying constants $n,\ l =\ 1,\ 1.1,\ 1.2$.

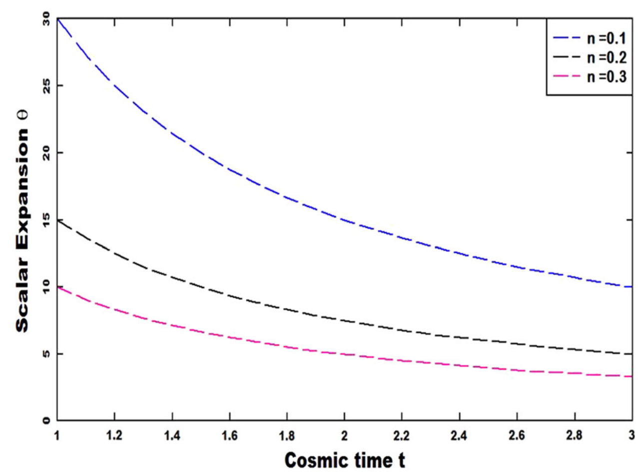

Figure 4. Variation of scalar expansion θ with cosmic time t with varying constants $n,\ l =\ 0.1,\ 0.2,\ 0.3$.

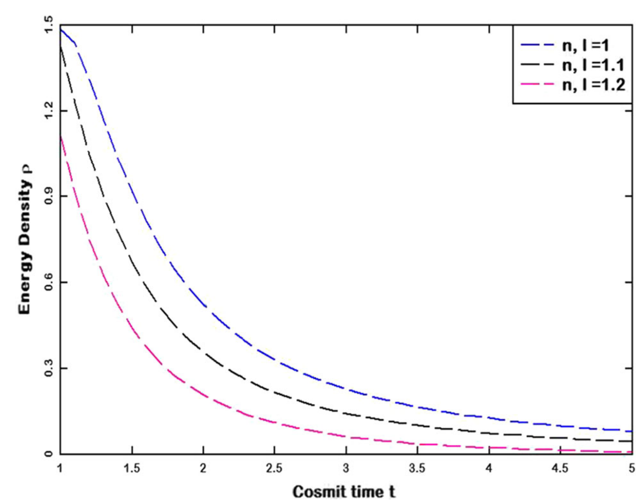

Figure 5. Variation of energy density ρ with cosmic time t with varying constants $n,\ l =\ 1,\ 1.1,\ 1.2$ and for $c_1 = c_2 =\ 1$.

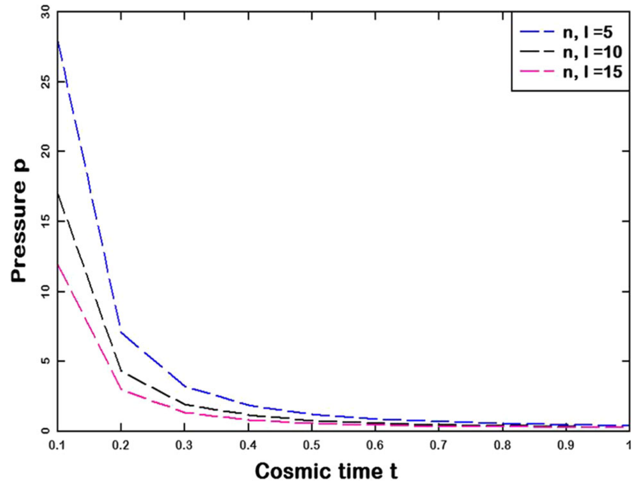

Figure 6. Variation of pressure p with cosmic time t with varying constants $n,\ l = 5,\ 10,\ 15$ and for $c_1 = c_2 = 1$.

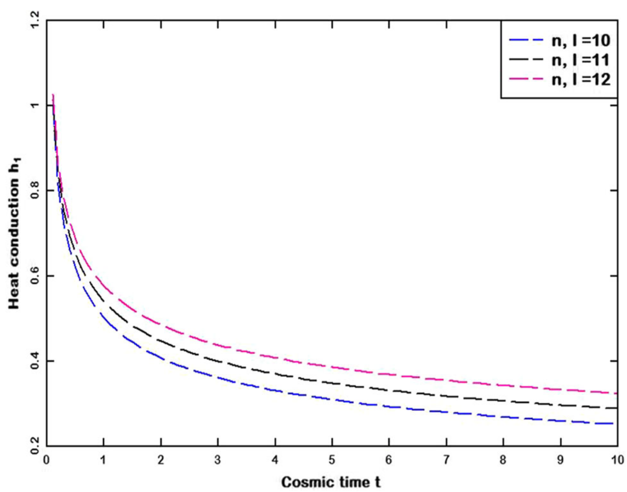

Figure 7. Variation of heat conduction h_1 with cosmic time t with varying constants $n,\ l = 10,\ 11,\ 12$ and for $c_1 = 1$.

of the dynamical parameters, metric potentials, internal pressure, density and mainly the heat flow for which our work is devoted. We observed that at an initial time, metric potentials $A(t)$ and $B(t)$ are zero. Also, it tends to zero as well as infinity depending on the value of n, i.e. for $n < 3$ and $n > 3$ respectively.

Graphically, we can observe that the average scale factor or cosmic scale factor increases as time (t) increases (figure 1). Spatial volume is zero when $t = 0$ and it increases as time increases. It means expansion of the Universe starts with finite volume and it is expanding as t increases (figure 2). The average Hubble parameter (H), dynamical scalar expansion (θ) and shear scalar (σ) are functions of time t and have a singularity at $t = 0$ and it tends to zero for large t. Also, H decreases as t increases (figure 3). The dynamical scalar expansion (θ) also decreases as t increases (figure 4) but the positive value of Hubble parameter and expansion scalar throughout the evolution show that the Universe is expanding gradually. Here anisotropy parameter (Δ) is zero at an initial time only, i.e. the model approaches isotropy at $t = 0$. As time increases, the Universe approaches anisotropy throughout the evolution for $n > 3$. This result is matched with the result of Agrawal and Pawar [32]. For large values of t, the model approaches isotropy.

$$t \to \infty,\ \left(\frac{\sigma}{\theta}\right)^2 \to 0 \quad \text{for } n < 3.$$

Also, energy density and pressure are monotonically decreasing functions of time t (figures 5 and 6) but infinite at the initial epoch. Heat flow is a decreasing function of time t (figure 7). It is infinite at initial epoch but it will be vanished for large t. From the behaviour of all physical quantities, we have observed that, the volume of the Universe is zero at the initial time but anisotropic parameter is infinite for $n < 3$. Expansion scalar and heat flow are infinite at this stage. This shows that our Universe begins with zero volume and large heat flow having expansion rate is infinite. Here we agreed with the result of [42]. Also the negative value of the DP for $n < 1$ shows that the Universe is in the accelerating phase, and our result is in good agreement with [21,25,28]. At the initial time, we have infinite energy density, infinite internal pressure and as we discussed earlier, infinite heat flow. This means that our Universe has an initial singularity. We matched our result with a researcher from cosmology [43].

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