



# Effect of point/line heat source and Hall current on free convective flow between two vertical walls

NAVEEN DWIVEDI<sup>✉</sup>\* and ASHOK KUMAR SINGH

Department of Mathematics, Institute of Science, Banaras Hindu University, Varanasi 221 005, India

\*Corresponding author. E-mail: naveen.dwivedi5@bhu.ac.in

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**Abstract.** The influence of a point heat source and Hall current on the laminar hydromagnetic free convective flow of an incompressible and electrically conducting viscous liquid between two vertical walls has been studied. A wavelet function is utilised to mathematically formulate the point or line heat source. The incidental equations on the flow have been processed subject to the Boussinesq approximation. A unified analytical solution of basic equations like thermal energy and momentum has been derived by employing Laplace transform technique. The impacts of the pertinent physical parameters, such as Hall parameter, magnetic field and point heat source, on the velocity field are explained graphically. The valuable result from the investigation is that an increase in the length of the point heat source leads to the enhancement of the velocity profiles. Moreover, it is noticeable that an enhancement of Hall current has a direct connection with the primary factor of the volumetric flow rate and skin friction.

**Keywords.** Magnetohydrodynamic flow; Laplace transformation; Hall current; point/line heat source.

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## 1. Introduction

Magnetohydrodynamics designates the frontier area combining electrodynamics and classical fluid mechanics. It deals with the flow of electrically conducting liquids which are subject to a magnetic field. Today, magnetohydrodynamics has developed into a broader field of fundamental as well as applied research in physical science and engineering. It has several uses in engineering and science, for example, solar energy collectors, nuclear reactors, astrophysics, aerospace, electromagnetics, geomechanics, electrical heaters, plasma confinement, oceanography, geophysics, magnetic drug targeting, etc. In natural convection, the liquid motion is produced by buoyancy effects because of the density differences caused by the differences in liquid temperature or because fluid–solid interfaces are at dissimilar temperatures or because of the changes in densities of distinct fluids adjoining to each other. MHD natural convective flow of an incompressible viscous liquid in the vertical walls has been the subject of numerous previous researches due to its potential relevance in several industrial processes.

First of all, Osterl and Young [1] studied the influence of applied magnetic field and viscous dissipation on the

free convective flow of a liquid in two heated vertical walls. Poots [2] investigated laminar hydromagnetic free convective flow between two parallel plane surfaces and in a circular tube. The Couette flow of an electrically conducting and incompressible viscous liquid with low magnetic Reynolds number in the presence of a transverse magnetic field was studied by Katagiri [3]. Aung *et al* [4] studied the free convective flow between vertical flat plates due to asymmetric heating. They found out the solution for cases of both constant temperatures and constant heat fluxes of the wall. An extensive review of the works on the hydromagnetic natural convection can be seen in [5]. Soundalgekar and Haldavnekar [6] analysed the hydromagnetic fully-developed free convective flow between two vertical plates. Wirtz and Stutzman [7] studied the free convective flow in vertical plates due to symmetric heating. Ghosh and Bhattacharjee [8] investigated the influences of forced as well as free convection on a viscous, electrically conducting and incompressible liquid flowing in a rotating parallel channel with completely conducting walls. Umavathi and Malashetty [9] have done a comprehensive analysis of the hydromagnetic forced and free convection between a vertical channel with the Joulean dissipations by taking asymmetric and symmetric boundary

heating. In recent years, much attention has been given to the study of hydromagnetic effects on the mixed and natural convective flows between vertical channels [10–14].

The existence of constant heat source/sink mostly has a central role in the discussion of MHD free convective flow. Fabrication and quality of the eventual product depends on these motivations as the circulation of temperature inside the fluid depends on the heat source/sink procedure. More relevant studies bringing out the influence of heat source/sink are in [15–17].

Important engineering applications of the hydromagnetic free convective flow under the influence of Hall current can be seen in MHD pumps and power generators, refrigeration coils, Hall accelerators, in-flight MHD, electric transformers, the solar cycle, the design of magnetic stars, cool combustors, electronic system cooling, thermal energy storage, oil extraction etc. Also, the Hall effects play a valuable role in the dynamics of the liquid and the magnetic field of numerous astrophysical objects, like dense molecular clouds, fluctuation in accretion disks or the construction of white dwarfs.

Sutton and Sherman [18] studied an electrically conducting liquid flow in the channel corresponding to the distribution of fully developed limiting temperature. Eraslan [19] considered the Hall effects on temperature distributions in hydromagnetic channel flow. In order to examine the Hall effects with convective transport in hydromagnetic flow, several investigators studied this pattern. For instance, Javeri [20] considered the influence of ion slip, Hall effects and viscosity on the channel flow. Further, Javeri [21] studied the combined impact of ion slip, Hall effects, Joule heating and viscous dissipation on hydromagnetic heat transfer, between the walls. Considering the importance of the Stokes fluid flow problems, Singh [22–24] examined the MHD free convective flow of an incompressible, electrically conducting and viscous liquid in a system by studying several phases of the problem. Further, Singh [25] studied the influence of Hall current on the natural convective flow of an incompressible, viscous electrically conducting liquid along an accelerated vertical porous plate, under the influence of a strong magnetic field. Gosh [26] investigated the effect of magnetic field and Hall current on hydromagnetic Couette flow through a rotating channel. Further studies related to the influence of Hall current are presented in refs [27,28]. The impact of Hall current on free convective boundary layer flow of hydromagnetic incompressible and viscous liquid from a permeable vertical plate with a transverse magnetic field have been studied by Saha *et al* [29]. Other interesting studies regarding the Hall effects can be seen in [30–35]. Singh *et al* [36] studied the influence of wall conductance and Hall current on the hydromagnetic

free and forced convective flow between rotating parallel plates in a porous medium. However, Krishna and Chamkha [37,38] investigated the hydromagnetic squeezing flow and rotating flow between parallel disks [37] and on the vertical plate [38] respectively under the influence of Hall current. Dwivedi and Singh [39,40] studied the influence of Hall current on MHD free convective flow in cylindrical geometries with the heat source/sink. Recently, Krishna and his colleagues [41–46] have examined the influence of Hall current and ion slip on the flow of different fluids in various geometries.

The objective of this study is to investigate the impact of line/point heat source and Hall current on the hydromagnetic free convective flow of an incompressible viscous and electrically conducting liquid between two vertical walls. To the authors' knowledge, the physical characteristic of developing free convection flow due to the point heat source between two vertical walls subjected to Hall effects is studied for the first time. The present study is an extension of the idea of Dwivedi *et al* [47] by incorporating the Hall effects. The novelty of the current study is that we are examining the effect of Hall current and point source on free convection in vertical walls with the magnetic field. It is expected that this research will help in considering flow formation minimisation and enhancement due to the line and point heat source. The solutions of basic equations (momentum and thermal energy) are obtained using the Laplace method. The effects of several controlling variables on the liquid flow are presented using tables and graphs. Moreover, the result acquired through the exact solution can provide a criterion for different numerical computations.

## 2. Mathematical formulation

Consider the hydromagnetic incompressible, viscous liquid flow between two infinite vertical walls with the point heat source. Let  $d$  be the distance between the two walls. The flow schematic of the channel with the coordinates is displayed in figure 1. The  $x$ -axis is taken in the axial direction along the flow, the  $y$ -axis is taken perpendicular to the walls and the  $z$ -axis is normal to the  $xy$ -plane. A uniform magnetic field of strength  $B_0$  is applied in the  $y$  direction. Let  $T_c$  be the temperature of both walls. As the walls are infinitely long adjacent to the  $z$  and  $x$  directions, all resultant parameters will be dependent on  $y$  only. Here, we suppose that the magnetic Reynolds number is very small in order to neglect the induced magnetic field. In addition, we have the constant point heat source to generate heat at rate  $Q_0$  in the liquid. The free convective flow takes place between the plates because of the point heat source.

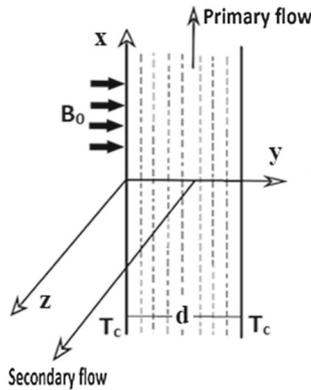


Figure 1. Schematic of the problem.

We take a wavelet function  $F(y, a, b)$  for the mathematical formulation of the line/point heat source which is defined by Dwivedi *et al* [47] as

$$F(y, a, b) = H(y - a) - H(y - b) = \begin{cases} 0, & y < a \\ d, & a \leq y < b \\ 0, & y \geq b \end{cases}, \quad (1)$$

where  $a$  and  $b$  are arbitrary constants.

We assume the relation between  $a$  and  $b$  as  $0 < a < b < d$ . Here, the function will act as a point heat source if the differences between  $a$  and  $b$  are too small. Otherwise the function will be a line source. Also, the unit step function has the following property:

$$H(y * d) = \frac{1}{|d|} H(y).$$

Under these inspections, the momentum equation, generalised Ohm’s law and thermal energy equation with the line/point heat source subjected to the Boussinesq’s approximations are as follows (Dwivedi *et al* [47], Singh [25]):

$$v \frac{d^2 u}{dy^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(u+mw) + g\beta(T-T_c) = 0, \quad (2)$$

$$v \frac{d^2 w}{dy^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mu-w) = 0, \quad (3)$$

$$\frac{\kappa}{\rho C_p} \frac{d^2 T}{dy^2} + \frac{Q_0}{\rho C_p} F(y, a, b) = 0, \quad (4)$$

where  $B_0$  is the constant magnetic field,  $C_p$  is the specific heat of the fluid at constant pressure,  $g$  is the acceleration due to gravity,  $m$  is the Hall current parameter ( $\omega_e \tau_e$ ),  $T$  is the fluid temperature,  $T_c$  is the temperature of the walls,  $u, w$  are the velocities of the

liquid in the  $x$  and  $z$  directions,  $\beta$  is the coefficient of thermal expansion and  $\rho$  is the fluid density.

The boundary conditions for the present model are expressed as follows (Dwivedi *et al* [47]):

$$u = 0, w = 0, T = T_c \quad \text{at } y = 0 \quad (5)$$

$$u = 0, w = 0, T = T_c \quad \text{at } y = d. \quad (6)$$

Dimensionless form of the basic equations is obtained by using dimensionless variables such as:

$$u^* = \frac{u}{u_0}, \quad w^* = \frac{w}{u_0}, \quad y^* = \frac{y}{d}, \quad u_0 = \frac{g\beta d^2}{\nu} T_c, \\ T^* = \frac{T - T_c}{T_c}, \quad a^* = \frac{a}{d}, \quad b^* = \frac{b}{d}, \quad (7)$$

where  $d$  is the distance between the walls,  $u_0$  is the characteristic velocity of the liquid and  $u^*, w^*$  are the dimensionless velocities of the fluid in the  $x$  and  $z$  directions.

By incorporating eq. (7), eqs (2)–(4) will be in the following dimensionless form:

$$\frac{d^2 u^*}{dy^{*2}} - \frac{Ha^2}{1+m^2}(u^* + mw^*) + T^* = 0, \quad (8)$$

$$\frac{d^2 w^*}{dy^{*2}} + \frac{Ha^2}{1+m^2}(mu^* - w^*) = 0, \quad (9)$$

$$\frac{d^2 T^*}{dy^{*2}} + SF(y^*, a^*, b^*) = 0, \quad (10)$$

where  $Ha = B_0 d \sqrt{\sigma/\mu}$  is the Hartmann number ( $\sigma$  is the electrical conductivity,  $\mu$  is the coefficient of viscosity),  $S = Q_0 d/\kappa T_c$  ( $\kappa$  is the thermal conductivity) is the heat source parameter,  $T^*$  is the dimensionless temperature of the liquid and

$$F(y^*, a^*, b^*) = H(y^* - a^*) - H(y^* - b^*) = \begin{cases} 0, & y^* < a^* \\ 1, & a^* \leq y^* < b^* \\ 0, & y^* \geq b^* \end{cases}$$

with  $0 < a^* < b^* < 1$ .

The dimensionless form of the boundary conditions (5) and (6) are as follows:

$$u^* = 0, \quad w^* = 0, \quad T^* = 0 \quad \text{at } y^* = 0, \quad (11)$$

$$u^* = 0, \quad w^* = 0, \quad T^* = 0 \quad \text{at } y^* = 1. \quad (12)$$

Combining eqs (8) and (9) we have

$$\frac{d^2 V^*}{dy^{*2}} - \delta^2 V^* + T^* = 0, \quad (13)$$

where  $V^* = u^* + iw^*$  and  $\delta = \alpha + i\beta$  as  $\alpha, \beta = Ha \left\{ \frac{(\sqrt{m^2+1}\pm 1)}{2(m^2+1)} \right\}^{1/2}$ .

In order to derive the solution of eqs (10) and (13) with the boundary conditions (11) and (12), we use the Laplace transform approach which is defined as

$$\bar{g}(s) = \int_0^\infty g(y)e^{-sy} dy,$$

where the Laplace transformation of  $g(y)$  is  $\bar{g}(s)$ .

The solutions of ODEs given by (10) and (13) subject to the concerned boundary conditions are achieved by utilising the Laplace transformation, which is derived as follows:

$$\bar{T}^*(s) = T^{*'}(0)/s^2 - S \left[ \frac{\exp(-a^*s)}{-\exp(-b^*s)} \right] /s^3 \tag{14}$$

$$\bar{V}^*(s) = V^{*'}(0)/s^2 - \delta^2 - \bar{T}^*(s)/s^2 - \delta^2. \tag{15}$$

Taking the inverse Laplace transforms of both sides of eqs (14) and (15), we get the required velocity and temperature expressions having the form

$$V^* = \frac{y^* T^{*'}(0)}{\delta^2} - S \sinh(\delta y^*) \left[ \frac{\cosh(\delta(a^* - 1))}{-\cosh(\delta(b^* - 1))} \right] / \delta^4 \sinh \delta + S(\psi(y^*, b^*) - \psi(y^*, a^*)) / 2\delta^4, \tag{16}$$

$$T^* = y^* T^{*'}(0) - S[\xi(y^*, a^*) - \xi(y^*, b^*)] / 2, \tag{17}$$

where

$$\begin{aligned} T^{*'}(0) &= S(a^{*2} - b^{*2} - 2a^* + 2b^*)/2, \\ \xi(y^*, a^*) &= H(y^* - a^*) (y^{*2} - 2a^*y^* + a^{*2}) \\ \xi(y^*, b^*) &= H(y^* - b^*) (y^{*2} - 2b^*y^* + b^{*2}) \\ \psi(y^*, a^*) &= H(y^* - a^*) \left[ \frac{2 + \delta^2(a^* - y^*)^2}{-2 \cosh(\delta(a^* - y^*))} \right] \\ \psi(y^*, b^*) &= H(y^* - b^*) \left[ \frac{2 + \delta^2(b^* - y^*)^2}{-2 \cosh(\delta(b^* - y^*))} \right]. \end{aligned}$$

The coefficients of skin friction at the surface of the walls are considered as

$$\begin{aligned} \tau_0 &= (\tau_x + i\tau_z)_{y^*=0} = \left( \frac{dV^*}{dy^*} \right)_{y^*=0} \\ &= T^{*'}(0)/\delta^2 - S \left[ \frac{\cosh(\delta(a^* - 1))}{-\cosh(\delta(b^* - 1))} \right] / \delta^3 \sinh(\delta) \tag{18} \end{aligned}$$

$$\begin{aligned} \tau_1 &= (\tau_x + i\tau_z)_{y^*=1} = \left( \frac{dV^*}{dy^*} \right)_{y^*=1} \\ &= S \cosh(\delta) \left[ \frac{\cosh(\delta(b^* - 1))}{-\cosh(\delta(a^* - 1))} \right] / \delta^3 \sinh(\delta) \end{aligned}$$

$$+ S \left[ \frac{\delta(a^* - b^*) + \sinh(\delta(1 - a^*))}{\sinh(\delta(1 - b^*))} \right] / \delta^3 + T^{*'}(0)/\delta^2. \tag{19}$$

Using eq. (16), the volumetric flow rate for the present flow is given by

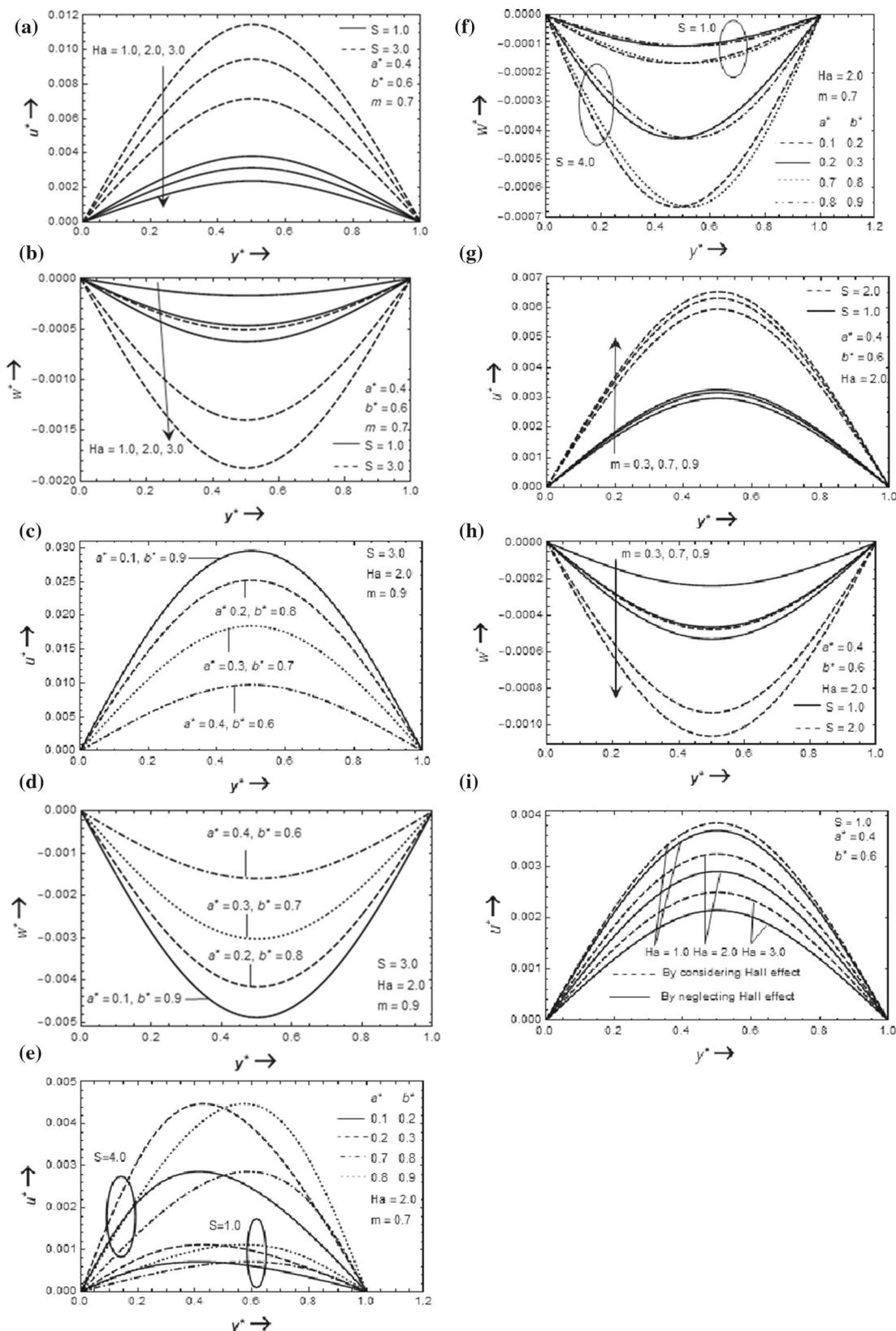
$$\begin{aligned} Q &= (Q_x + iQ_z) = \int_0^1 V^* dy^* \\ &= T^{*'}(0)/2\delta^2 - S(\cosh(\delta) - 1) \left[ \frac{\cosh(\delta(a^* - 1))}{-\cosh(\delta(b^* - 1))} \right] / \delta^5 \sinh(\delta) \\ &+ S \left[ \frac{3\delta^2(b^{*2} - a^{*2} - b^* + a^*)}{-\delta^2(b^{*3} - a^{*3}) - 6(b^* - a^*)} \right] / 6\delta^4 \\ &- S[\sinh(\delta(1 - b^*)) - \sinh(\delta(1 - a^*))] / \delta^5. \tag{20} \end{aligned}$$

### 3. Results and discussion

The influences of several parameters on the hydromagnetic free convection of an incompressible, electrically conducting liquid in vertical walls with the point heat source and Hall current is analytically investigated. The coupled equations are solved analytically subject to the related boundary conditions. The influence of several controlling parameters on the liquid velocity, volume flow rate and skin friction in primary and secondary directions of flows are explained using tables and graphs. These parametric effects, namely the Hartmann number (Ha), point/line heat source, heat source ( $S$ ) and Hall parameter ( $m$ ), are demonstrated graphically in figures 2a–2i. The values of the volume flow rate and skin friction are given in tabular form.

The expression for temperature given in eq. (10) with the boundary conditions (11) and (12) is similar to the one obtained by Dwivedi *et al* [47]. The influence of the line/point heat source parameter and heat source parameter ( $S$ ) on the liquid temperature and heat transfer rate are examined in their work. Consequently, these results will not be re-examined in the present analysis. The main point of this investigation is to examine the characteristics of the controlling parameters on the configuration of flow.

Figures 2a and 2b demonstrate the effect of constant heat source and magnetic field on the velocity field by considering the point heat source in the middle region ( $a^* = 0.4, b^* = 0.6$ ). These figures reveal that Ha has a reducing effect on the primary and secondary velocities of the liquid, indicating that the magnetic field inhibits the impact on the velocity of the liquid in the primary



**Figure 2.** (a) Impact of Ha on primary velocity, (b) impact of Ha on secondary velocity, (c) impact of line heat source on primary velocity, (d) impact of line heat source on secondary velocity, (e) impact of point heat source on primary velocity, (f) impact of point heat source on secondary velocity, (g) impact of Hall current on primary velocity, (h) impact of Hall current on secondary velocity and (i) impact of Hall parameter ( $m$ ) and Hartmann number ( $Ha$ ) on primary velocity profiles.

**Table 1.** Effects of Ha and  $m$  on skin friction and mass flow rate in the primary and secondary directions.

$S = 1.0$				Primary component			Secondary component		
$a^*$	$b^*$	$m$	Ha	$(\tau_x)_{y^*=0}$	$(\tau_x)_{y^*=1}$	$Q_x$	$(\tau_z)_{y^*=0}$	$(\tau_z)_{y^*=1}$	$Q_z$
0.1	0.5	0.7	1	0.02061	-0.01615	0.00370	-0.00084	0.00078	-0.00016
			3	0.01338	-0.00950	0.00229	-0.00319	0.00282	-0.00060
		1.5	1	0.02128	-0.01678	0.00383	-0.00088	0.00083	-0.00017
			3	0.01599	-0.01180	0.00279	-0.00473	0.00431	-0.00091
0.2	0.4	0.7	1	0.01096	-0.00835	0.00194	-0.00044	0.00041	-0.00008
			3	0.00714	-0.00487	0.00120	-0.00169	0.00147	-0.00032
		1.5	1	0.01132	-0.00867	0.00201	-0.00046	0.00043	-0.00009
			3	0.00852	-0.00607	0.00146	-0.00251	0.00225	-0.00048
0.2	0.3	0.7	1	0.00512	-0.00361	0.00086	-0.00019	0.00018	-0.00003
			3	0.00340	-0.00208	0.00053	-0.00076	0.00064	-0.00014
		1.5	1	0.00528	-0.00375	0.00089	-0.00020	0.00019	-0.00004
			3	0.00403	-0.00260	0.00065	-0.00113	0.00098	-0.00021

and secondary directions of flow. This is because the electromagnetic force has the tendency of resisting the liquid motion. These consequences quantitatively agree with the expectations, because the magnetic field utilises the retarding force on free convective flow. Also, the influence of heat source ( $S = 1.0, 3.0$ ) enhances the primary component of the velocity field and reverses the phenomenon for the secondary velocity component. This incident agrees with the physical realities because the heat source generates internal heat in the region. As required, the velocity profiles of the liquid in the axial direction ( $u^*$ ) are higher than in the transverse direction ( $w^*$ ). The shape of the velocity fields in the primary, as well as secondary directions of flow, is parabolic in the upward and downward directions respectively.

The influence of line heat source on the primary and secondary velocity fields in the presence of Hall current, magnetic field and heat source are shown in figures 2c and 2d. Here, we can conclude that as the length of the line heat source reduces, velocity in the primary direction decreases and the reverse occurs for the flow in the secondary direction. From these figures, we can deduce that the velocity field in the primary direction reduces and secondary direction enhances very sharply as the line source is changed to the point source. These outcomes are expected physically because the line heat source generates more internal energy than the point heat source.

Figures 2e and 2f present the impact of point heat source position on both components of the velocity fields with different values of the heat source ( $S = 1.0, 4.0$ ). From these figures, we can state that the strength of the heat source has its usual impact as in figures 2a and 2b. Furthermore, the velocity components are maximum when the point heat source is located in the middle of the walls. From this outcome, we can see

that with the help of point heat source the flow formation in primary and secondary directions may be controlled. Consequently, we can deduce that the flow between of the fluid in both primary and secondary directions in the vertical plates depends on the fluctuation of the constants ( $a^*$  and  $b^*$ ). These outcomes are in exact agreement with the results of Dwivedi *et al* [47] without Hall effects.

Figures 2g and 2h depict the impact of Hall current on the liquid velocity in both directions of flow. It is revealed from figures 2g and 2h that the influence of Hall current enhances the component of the primary velocity field. The liquid’s electrical conductivity reduces with increment in Hall currents which eventually diminishes the magnetic force and the reverse will happen for the secondary velocity field. Therefore, we conclude that Hall current supports the flow onward the primary flow direction while the opposite effect is noticed along the secondary flow direction.

In figure 2i, we have shown the impact of Hall current by comparing the action of velocity fields with and without taking Hall current into consideration. It is noticed that a considerable reduction in velocity takes place when Ha increases for both cases.

Table 1 demonstrates the influence of line/point heat source, Hall current and magnetic field, on both components of mass flux and skin friction at the wall’s surface. This table clearly demonstrates that due to the enhancement in Hall current, the primary component of the skin friction enhances at the wall  $y^* = 0$  but diminishes at the wall  $y^* = 1$  but the reverse happens for the secondary factor of the skin friction. The influence of point heat source (i.e. point heat source changes into line heat source), on both factors of the skin friction, is just the same as that of the Hall current. Moreover, the impact of Ha on both factors of the skin friction is

to diminish at the wall  $y^* = 0$  but enhance at the wall  $y^* = 1$ . The increment in Hall current enhances the mass flow rate in the primary direction and reduces it in the secondary direction, and also, have the same action when the gap between  $a^*$  and  $b^*$  enhances (the point heat source transforms into the line heat source). Furthermore, as expected the influence of  $Ha$  reduces the primary and secondary factors of the mass flow rate.

#### 4. Conclusions

The combined influence of Hall current and line/point heat source on the hydrodynamic free convective flow of an incompressible, viscous and electrically conducting liquid between two vertical walls are studied. The important outcomes are as follows:

- It is noticed that the resultant velocity reduces with growth in Hartmann number.
- An enhancement in the heat source and Hall parameters enhances the velocity factor for the primary flow but an opposite effect is seen for the secondary flow.
- The fluid velocity with the line heat source is higher than that of the point heat source for the primary flow but the reverse happens for the secondary flow.
- For the primary flow, the value of skin friction at the right wall reduces and enhances at the left wall with the Hall parameter but the opposite happens for the secondary flow.
- The impact of Hall current enhances the mass flow rate in the primary direction and reduces the mass flow rate in secondary direction.
- The primary component of the mass flow rate is more in the presence of the line heat source than in the presence of the point heat source but opposite happens for the secondary component of the mass flow rate.

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