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Empirical formula for beta-particle-induced bremsstrahlung yields

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Abstract. We have measured the β-particle-induced bremsstrahlung energy yield and photon yield in the energy range 0.1668–2.274 MeV using beta sources such as ^{35}S (0.1668), ^{99}Tc (0.293), ^{147}Pm (0.225), ^{90}Sr (0.5462), ^{204}Tl (0.76), ^{91}Y (1.5), ^{32}P (1.71) and ^{90}Y (2.274 MeV) in thick targets of atomic number range $13 < Z < 83$. We have used a NaI(Tl) detector to measure the bremsstrahlung radiations. Based on the experimental results, we have constructed a semiempirical formula for β-particle-induced bremsstrahlung energy yield and photon yield. This formula produces bremsstrahlung energy yield and photon yield in the energy range $0.1668\,\text{MeV} < E_{\text{max}} < 2.274\,\text{MeV}$ for thick targets within the atomic number range $13 < Z < 83$. The values produced by the present formula are compared with the experiments.

Keywords. Bremsstrahlung; X-ray; bremsstrahlung yield.

PACS Nos 34.80.−i; 78.70.−g; 33.20

1. Introduction

Bremsstrahlung is a continuous electromagnetic radiation emitted when an electron or a β-particle is deflected in the Coulomb field of the nucleus. Literature survey shows that studies were done earlier on the bremsstrahlung cross-sections using relativistic and Born approximations methods [1]. Manjunatha and Rudraswamy [2] proposed a numerical method to evaluate bremsstrahlung cross-section in compounds such as NaI, SiLi and GeLi using tabulated data given for elements. Haug [3] calculated bremsstrahlung cross-section with screening and Coulomb corrections at high energies using cross-sections of Somerfield–Maue functions with additional higher-order terms. Tessier and Kawrakow [4] reported numerical calculation of the electron–electron bremsstrahlung cross-section in the field of atomic electrons. Omar *et al* [5] evaluated bremsstrahlung cross-sections using Monte Carlo method and compared that with cross-sections obtained using different theories available in the literature. Manjunatha and Rudraswamy [6] estimated the theoretical data of bremsstrahlung radiation cross-section of the bone. Poškus [7] proposed a program for calculating spectra and angular distributions of bremsstrahlung at electron energies less than 3 MeV for exact screened calculations of atomic-field bremsstrahlung.

Singh *et al* [8] measured the bremsstrahlung spectrum generated in thick targets of oxides of lanthanides by ^{89}Sr β-particles in the photon energy region 1–100 keV. Sandrock *et al* [9] calculated radiative corrections to the average bremsstrahlung energy loss of high-energy muons using a modified Weizsäcker–Williams method. Singh [10] measured angular distributions of bremsstrahlung photons produced by 10–25 keV electrons incident on thick Ti and Cu targets using a Si-PIN photodiode detector. Jung [11] studied the electron-exchange and quantum shielding effects on the polarisation bremsstrahlung spectrum due to the electron-shielding sphere encounters in quantum plasmas. Prajapati *et al* [12] studied the anisotropy of bremsstrahlung photons emitted from 4.0 keV electrons by scattering by a free CH_4 molecule by measuring the cross-sections of bremsstrahlung photons. Kunashenko [13] developed the theory of coherent bremsstrahlung from neutrons in crystals in the framework of virtual photons.

Minter and Jenkins [14] proposed a program which calculates the double differential cross-section for bremsstrahlung production by electrons interacting with

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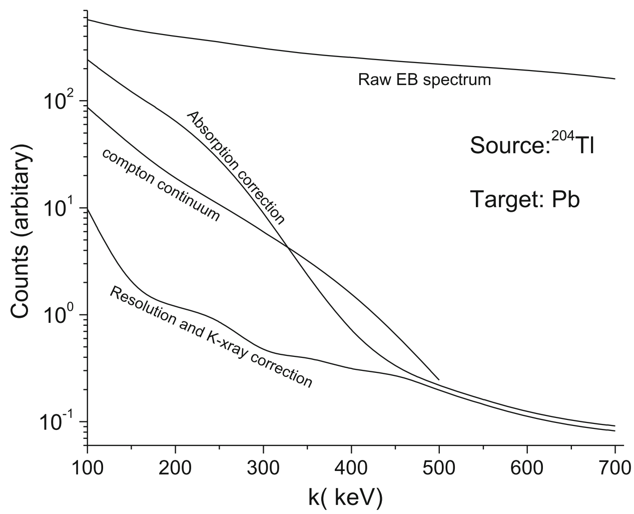

Figure 1. Raw bremsstrahlung spectra for the lead target excited by ^{204}Tl beta source along with different corrections such as absorption, Compton continuum, detector resolution and K.-X-ray correction.

a spinless point target. Al-Beteri and Raeside [15] calculated the electron bremsstrahlung cross-section using Monte Carlo transport simulation. Manjunatha [16,17] evaluated the β-particle-induced bremsstrahlung spectra and other dosimetric parameters in the bone. Mack and Mitter [18] calculated differential cross-section for bremsstrahlung emission in electron–electron collisions using computer programs for formula manipulations. Manjunatha [19] computed the β-particle-induced bremsstrahlung spectra produced by high-energy β-particle (>1MeV) in bones. Manjunatha and Rudraswamy [20] measured bremsstrahlung spectrum produced by the β-particles from ^{90}Sr to ^{90}Y, ^{147}Pm and ^{204}Tl in nuclear radiation detection compounds such as cesium iodide and sodium iodide.

In the present work, we have measured the β-particle-induced bremsstrahlung energy yield and photon yield in the energy range 0.1668–2.274 MeV using beta sources such as ^{35}S (0.1668), ^{99}Tc (0.293), ^{147}Pm (0.225), ^{90}Sr (0.5462), ^{204}Tl (0.76), ^{91}Y (1.5), ^{32}P (1.71), ^{90}Y (2.274) in thick targets within the atomic number range $13 < Z < 83$. Based on the experimental results, we have constructed the empirical formula for the β-particle-induced bremsstrahlung energy yield and photon yield.

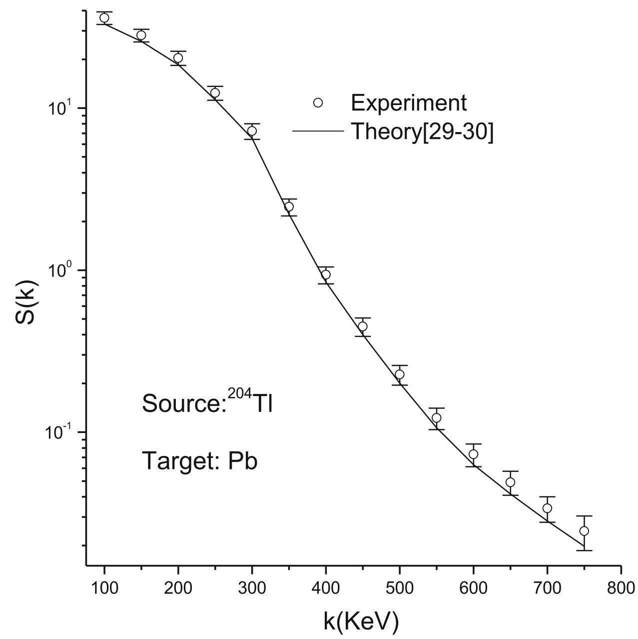

Figure 2. The bremsstrahlung spectrum produced in the lead target excited by the β-particles from ^{204}Tl along with theoretical prediction [29,30].

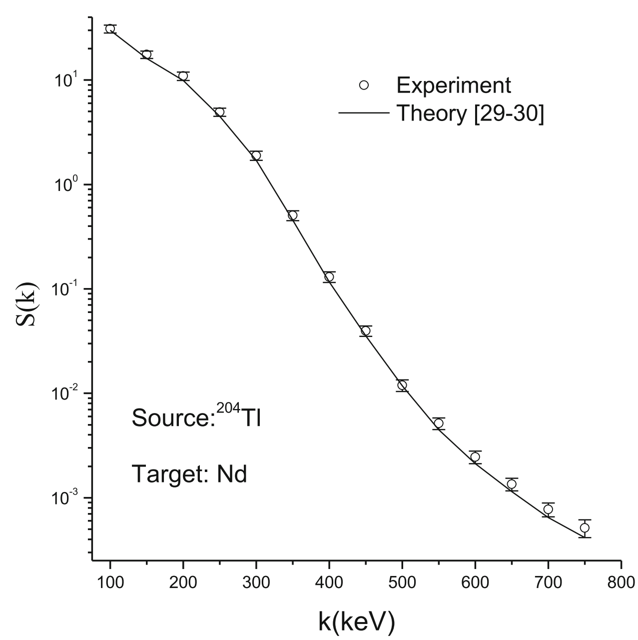

Figure 3. The spectral distributions of EB produced in neodymium target by the β-particles from ^{204}Tl along with theoretical predictions [29,30].

2. Present work

NaI(Tl) crystal detector mounted on a photomultiplier was coupled to a PC-based multichannel analyser. NaI(Tl) crystal was housed in a lead chamber and lead chamber was lined with aluminium. A detailed experimental procedure is given in [21–25]. In the present experiment, β-particles from ^{35}S (0.1668), ^{99}Tc (0.293), ^{147}Pm (0.225), ^{90}Sr (0.5462), ^{204}Tl (0.76), ^{91}Y (1.5), ^{32}P (1.71) and ^{90}Y (2.274) are used to produce bremsstrahlung in thick targets such as Al, V, Fe, Cu, Zn, Ca, Ba, Nd, Gd, Yb, W, Pb and Bi. Beta source was

Table 1. Bremsstrahlung energy yields (I) ($\times 10^{-3}$MeV/β-particle).

		^{35}S	^{99}Tc	^{147}Pm	^{90}Sr	^{204}Tl	^{91}Y	^{32}P	^{90}Y
Al	Expt.	0.362 ±0.03	0.636 ±0.06	0.684 ±0.07	2.910 ±0.11	4.040 ±0.36	7.170 ±0.71	8.170 ±0.77	12.300 ±1.15
	Theory	0.329	0.585	0.636	2.622	3.644	6.381	7.304	10.996
V	Expt.	0.832 ±0.07	1.460 ±0.14	1.570 ±0.15	6.680 ±0.26	9.290 ±0.83	16.500 ±1.63	18.800 ±1.77	28.400 ±2.65
	Theory	0.757	1.343	1.460	6.019	8.380	14.685	16.807	25.390
Fe	Expt.	0.832 ±0.07	1.460 ±0.14	1.570 ±0.15	6.680 ±0.26	9.290 ±0.83	16.500 ±1.63	18.800 ±1.77	28.400 ±2.65
	Theory	0.757	1.343	1.460	6.019	8.380	14.685	16.807	25.390
Cu	Expt.	0.949 ±0.08	1.670 ±0.16	1.790 ±0.17	7.610 ±0.30	10.600 ±0.95	18.800 ±1.86	21.400 ±2.02	32.300 ±3.01
	Theory	0.864	1.536	1.665	6.857	9.561	16.732	19.132	28.876
Zn	Expt.	0.988 ±0.09	1.740 ±0.17	1.870 ±0.18	7.930 ±0.31	11.000 ±0.98	19.500 ±1.93	22.300 ±2.10	33.700 ±3.14
	Theory	0.899	1.601	1.739	7.145	9.922	17.355	19.936	30.128
Ca	Expt.	0.607 ±0.05	1.070 ±0.10	1.150 ±0.11	4.870 ±0.19	6.780 ±0.60	12.000 ±1.19	13.700 ±1.29	20.700 ±1.93
	Theory	0.552	0.984	1.070	4.388	6.116	10.680	12.248	18.506
Ba	Expt.	2.090 ±0.18	3.670 ±0.35	3.950 ±0.38	16.800 ±0.66	23.300 ±2.08	41.300 ±4.09	47.100 ±4.44	71.200 ±6.64
	Theory	1.902	3.376	3.674	15.137	21.017	36.757	42.107	63.653
Nd	Expt.	2.270 ±0.20	3.990 ±0.38	4.290 ±0.41	18.200 ±0.71	25.300 ±2.26	44.900 ±4.45	51.200 ±4.83	77.400 ±7.22
	Theory	2.066	3.671	3.990	16.398	22.821	39.961	45.773	69.196
Gd	Expt.	2.450 ±0.21	4.310 ±0.41	4.630 ±0.45	19.700 ±0.77	27.400 ±2.44	48.500 ±4.80	55.300 ±5.22	83.600 ±7.80
	Theory	2.230	3.965	4.306	17.750	24.715	43.165	49.438	74.738
Yb	Expt.	2.730 ±0.24	4.800 ±0.46	5.160 ±0.50	21.900 ±0.85	30.500 ±2.72	54.000 ±5.35	61.600 ±5.81	93.100 ±8.68
	Theory	2.484	4.416	4.799	19.732	27.511	48.060	55.070	83.231
W	Expt.	2.920 ±0.25	5.130 ±0.49	5.510 ±0.53	23.400 ±0.91	32.600 ±2.91	57.800 ±5.72	65.800 ±6.21	99.500 ±9.28
	Theory	2.657	4.720	5.124	21.083	29.405	51.442	58.825	88.953
Pb	Expt.	3.300 ±0.29	5.800 ±0.56	6.240 ±0.60	26.500 ±1.03	36.900 ±3.29	65.300 ±6.46	74.500 ±7.03	113.000 ±10.54
	Theory	3.003	5.336	5.803	23.877	33.284	58.117	66.603	101.022
Bi	Expt.	3.350 ±0.29	5.890 ±0.57	6.330 ±0.61	26.900 ±1.05	37.400 ±3.34	66.300 ±6.56	75.600 ±7.13	114.000 ±10.63
	Theory	3.049	5.419	5.887	24.237	33.735	59.007	67.586	101.916

placed in a Perspex stand above the face of the detector. The target was placed between the detector and the source. The geometry was carefully adjusted so that the crystal was fully exposed to the bremsstrahlung emitted from the target. A Perspex sheet which is thick enough to stop all β-particles is placed on the top of the target compound and with source in position, the spectrum EB + IB + BG was taken. Here EB, IB and BG are external bremsstrahlung, internal bremsstrahlung and background respectively. The Perspex was then placed below the target compound and the spectrum IB + BG was recorded for the same time. The difference in the two spectra gives raw EB spectrum. The Liden–Starfelt procedure [26,27] was applied to unfold the measured

Table 2. Bremsstrahlung photon yields ($\times 10^{-3}$photons/β-particle).

		^{35}S	^{99}Tc	^{147}Pm	^{90}Sr	^{204}Tl	^{91}Y	^{32}P	^{90}Y
Al	Expt.	0.73	1.27	1.64	3.37	4.69	10.10	11.50	13.10
		±0.05	±0.07	±0.08	±0.23	±0.36	±0.83	±0.97	±1.21
	Theory	0.68	1.20	1.55	3.10	4.28	9.21	10.45	11.80
V	Expt.	1.79	3.14	4.04	8.30	11.60	24.90	28.40	32.20
		±0.11	±0.17	±0.20	±0.57	±0.90	±2.05	±2.40	±2.98
	Theory	1.66	2.95	3.81	7.64	10.58	22.72	25.82	29.01
Fe	Expt.	1.79	3.14	4.04	8.30	11.60	24.90	28.40	32.20
		±0.11	±0.17	±0.20	±0.57	±0.90	±2.05	±2.40	±2.98
	Theory	1.66	2.95	3.81	7.64	10.58	22.72	25.82	29.01
Cu	Expt.	2.06	3.62	4.66	9.57	13.30	28.70	32.70	37.10
		±0.13	±0.20	±0.24	±0.66	±1.03	±2.37	±2.77	±3.43
	Theory	1.92	3.41	4.39	8.81	12.13	26.18	29.72	33.43
Zn	Expt.	2.15	3.78	4.87	10.00	13.90	30.00	34.20	38.80
		±0.14	±0.21	±0.25	±0.69	±1.07	±2.47	±2.89	±3.59
	Theory	2.00	3.56	4.59	9.21	12.68	27.37	31.09	34.96
Ca	Expt.	1.27	2.23	2.87	5.90	8.21	17.70	20.20	22.90
		±0.08	±0.12	±0.15	±0.41	±0.63	±1.46	±1.71	±2.12
	Theory	1.18	2.10	2.71	5.43	7.49	16.15	18.36	20.63
Ba	Expt.	4.84	8.51	11.00	22.50	31.30	67.40	76.90	87.30
		±0.31	±0.47	±0.56	±1.55	±2.42	±5.55	±6.51	±8.07
	Theory	4.50	8.01	10.37	20.72	28.55	61.49	69.90	78.66
Nd	Expt.	5.30	9.31	12.00	24.60	34.30	73.80	84.10	95.50
		±0.34	±0.52	±0.61	±1.69	±2.65	±6.08	±7.12	±8.83
	Theory	4.93	8.76	11.32	22.66	31.29	67.33	76.45	86.05
Gd	Expt.	5.76	10.10	13.00	26.80	37.30	80.20	91.50	104.00
		±0.37	±0.56	±0.66	±1.84	±2.88	±6.61	±7.74	±9.62
	Theory	5.36	9.50	12.26	24.68	34.03	73.17	83.17	93.70
Yb	Expt.	6.47	11.40	14.60	30.10	41.90	90.10	103.00	117.00
		±0.41	±0.63	±0.74	±2.07	±3.23	±7.43	±8.72	±10.82
	Theory	6.02	10.73	13.77	27.72	38.23	82.20	93.63	105.42
W	Expt.	6.96	12.20	15.70	32.30	45.00	96.90	110.00	125.00
		±0.44	±0.68	±0.80	±2.22	±3.47	±7.99	±9.31	±11.56
	Theory	6.47	11.48	14.81	29.75	41.05	88.40	99.99	112.63
Pb	Expt.	7.95	14.00	18.00	37.00	51.40	111.00	126.00	143.00
		±0.51	±0.78	±0.91	±2.54	±3.97	±9.15	±10.66	±13.22
	Theory	7.39	13.17	16.97	34.08	46.89	101.27	114.53	128.84
Bi	Expt.	8.08	14.20	18.30	37.50	52.20	112.00	128.00	146.00
		±0.51	±0.79	±0.93	±2.58	±4.03	±9.23	±10.83	±13.50
	Theory	7.51	13.36	17.26	34.54	47.62	102.18	116.35	131.55

folded spectrum into the true photon spectrum $S(k)$ which gives the number of photons per m_0c^2 per β-particle. The raw bremsstrahlung spectra for the lead target excited by ^{204}Tl beta source along with different corrections such as absorption, Compton continuum, detector resolution and K.-X-ray correction are shown in figure 1. The true bremsstrahlung photon spectra $S(k)$ produced in lead and neodymium targets excited by the β-particles from ^{204}Tl along with theoretical prediction are shown in figures 2 and 3.

The main sources of error in the measurements are statistical error, error in determining intrinsic and geometric efficiencies, the energy resolution of the detector, EB absorption in the target compound, air, aluminium can, etc., and the beta source strength. In assessing these errors, we followed the methods

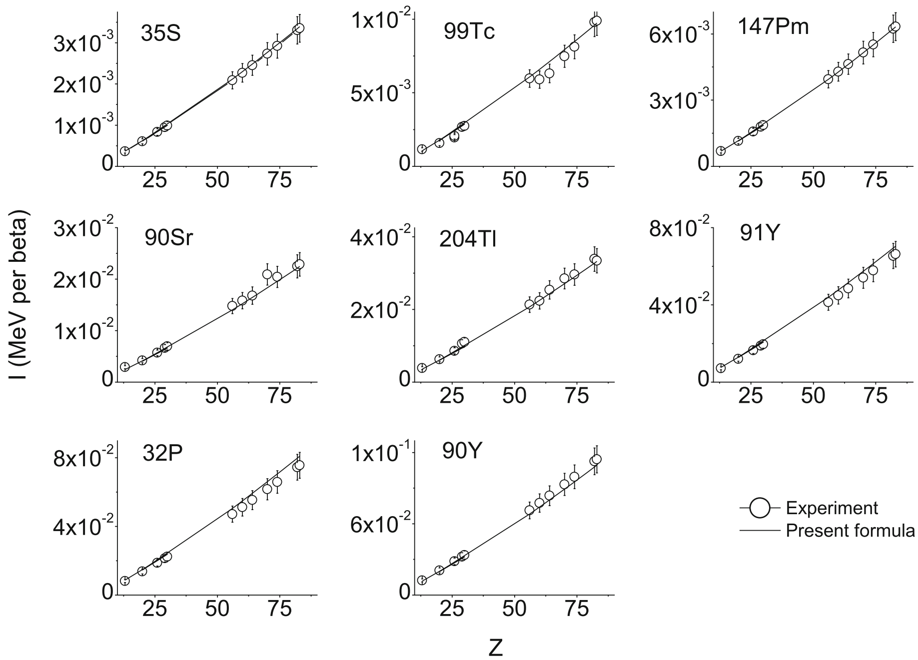

Figure 4. Comparison of the present formula for bremsstrahlung energy yield (I) with the experiment.

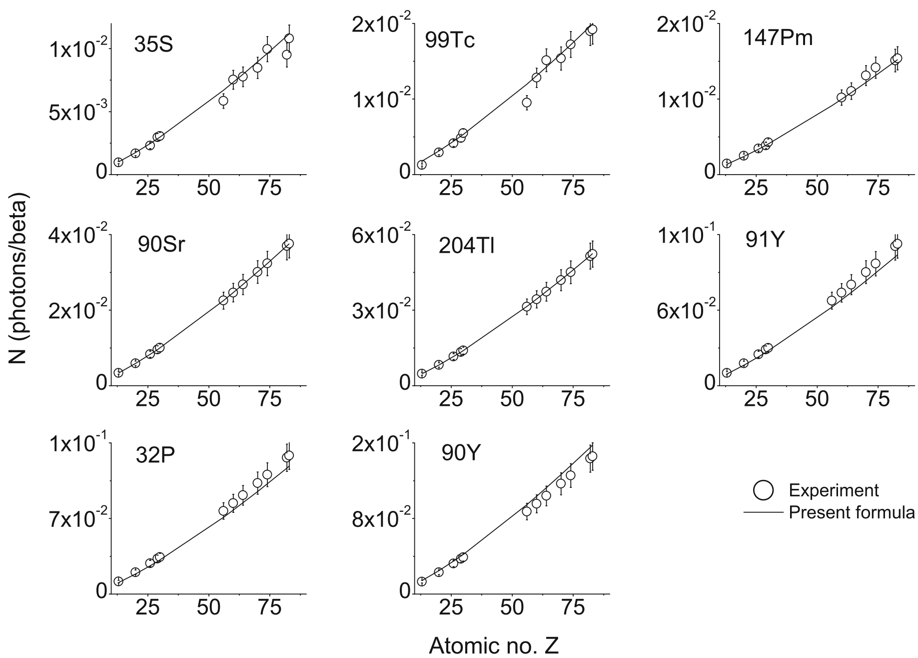

Figure 5. Comparison of the present formula for bremsstrahlung photon yield (N) with the experiment.

adopted by Liden *et al* [26] and Shivaramu [27]. The overall error was estimated to range from 4 to 7% from low to high energies. Bremsstrahlung photon yield (N) and energy yield (I) are evaluated from the measured spectrum $\{S(k)\}$ using the following expressions:

$$N = \int_{k_{\min}}^{k_{\max}} S(k)\mathrm{d}k, \tag{1}$$

$$I = \int_{k_{\min}}^{k_{\max}} kS(k)\mathrm{d}k, \tag{2}$$

where k is the photon energy, $k_{\min}$ and $k_{\max}$ are the minimum and maximum energy of the measured photon spectrum. We have evaluated the theoretical bremsstrahlung spectrum $S(k)$ using the procedure explained in [20,25] based on the Tseng–Pratt theory [29,30]. After computing the theoretical spectrum $S(k)$, we have calculated the theoretical bremsstrahlung yields using eqs (1) and (2).

3. Results and discussion

We have measured the β-particle-induced bremsstrahlung energy yield and photon yield in the energy range 0.1668–2.274 MeV for thick targets within the atomic number range $13 < Z < 83$. The measured bremsstrahlung yields are compared with theoretical prediction. The comparison of bremsstrahlung yields with experiments are shown in tables 1 and 2. Measured bremsstrahlung yields agree well with theoretical prediction. After measuring the bremsstrahlung yields, we have searched for the parametrisation. We have fitted appropriate functions for the bremsstrahlung energy yield (I) and photon yield (N). The constructed formulae for bremsstrahlung yields are as follows:

$$I = \left(2.50319761\times10^{-4}E_{\max} - 2.486389169\times10^{-5}\right) Z^{1.2} \quad (3)$$

$$N = \left(2.22711639\times10^{-4}E_{\max} - 1.379966839\times10^{-6}\right) Z^{1.3}. \quad (4)$$

In the above equation, bremsstrahlung energy yield (I) and photon yield (N) are in MeV/β-particle and photons/β-particle respectively. $E_{\max}$ is the maximum energy of the β particle in MeV. The values produced by the above formulae are compared with the experiments and this comparison is shown in figures 4 and 5. The constructed formula produces bremsstrahlung energy yield (I) and photon yield (N) with the simple inputs of atomic number (Z) and maximum energy of β-particles ($E_{\max}$). This formula can be extended to thick target compounds and mixtures by replacing Z by Z_{mod}, where Z_{mod} is the modified atomic number defined by Markowicz and VanGriken [28].

$$Z_{\text{mod}} = \sum_{i}^{l} \frac{W_i Z_i^2}{A_i} \Big/ \frac{W_i Z_i}{A_i}. \quad (5)$$

Here, W_i, Z_i and A_i are weight fraction, atomic number and atomic weight of the ith element in the compound/mixture.

4. Conclusion

We have measured the β-particle-induced bremsstrahlung energy yield and photon yield in the energy range 0.1668–2.274 MeV in thick targets within the atomic number range $13 < Z < 83$. Based on the experimental results, we have constructed a formula which produces bremsstrahlung energy yield (I) and photon yield (N) with the simple inputs of atomic number (Z) and maximum energy of β-particles ($E_{\max}$).

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