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Bulk viscous accelerating Universe in $f(R, T)$ theory of gravity

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Abstract. In this paper, we propose that the late-time acceleration of the Universe is due to bulk viscous fluid and trace of energy–momentum tensor T in $f(R, T)$ theory of gravity. We assume that $f(R, T) = f(R) + 2f(T)$ with $f(R) = R$ and $f(T) = \lambda T$ where λ is a constant, R and T are the Ricci scalar and trace of energy–momentum tensor. First, we obtain an exact solution of the bulk viscous Universe in $f(R, T)$ gravity, then we use observational Hubble data (OHD), the baryon acoustic oscillation (BAO) distance ratio data as well as SN Ia data to constrain the parameters of the derived bulk viscous Universe. Our estimations show that in the model under consideration $H_0 = 69.089$ km/Mpc/s which is in good agreement with recent astrophysical observations. We ascertain the present age of the derived Universe as well as the signature flipping behaviour of deceleration parameter. Some physical properties of the derived model are also discussed.

Keywords. Bulk viscosity; $f(R, T)$ gravity; accelerating Universe; deceleration parameter.

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1. Introduction

The late-time accelerated expansion of the Universe is one of the most challenging problem of theoretical physics [1,2]. To describe this late-time acceleration, a large number of dark energy cosmological models have been investigated but the existence of dark energy have posed a fundamental theoretical challenge to gravitation theories [3–15]. The possibility of explaining the observed late-time accelerated expansion of the Universe without considering dark energy components is plausible with a more general action in which the famous Einstein–Hilbert action is replaced by an arbitrary function of Ricci scalar R. These cosmological models are simply named as $f(R)$ modified theories of gravity. Some useful applications of $f(R)$ theories of gravity in describing the late-time acceleration of the observed Universe is given in ref. [16]. In 2010, Sotiriou and Faraoni [17] have presented a review on $f(R)$ generalised gravity models. Later on, Nojiri and Odintsov [18] have described the unified cosmic history of the Universe in $f(R)$ theory of gravity. Bertolami *et al* [19] have investigated a generalised form of $f(R)$ modified theories of gravity by taking into account an explicit coupling between Ricci scalar R and matter Lagrangian density L_m. In 2011, Harko *et al* [20] have proposed $f(R, T)$ theory of gravity in which the gravitational Lagrangian is defined by an arbitrary function of R and trace T of the energy–momentum tensor. This means that in $f(R, T)$ gravity, the gravitational part of acceleration depends on trace T which is coupled with geometry through R. Ming and Liang [21] have investigated quintom model in $f(R, T)$ theory of gravitation by taking into account the periodic variation of deceleration parameter. Capozziello and Francaviglia [22] have reviewed extended theories of gravity and pointed out its astrophysical applications. The main objective of this study was to search an alternative approach to solve the puzzles associated with dark components. Capozziello *et al* [23] have discussed more profoundly the energy conditions in modified theory of gravity. Biswas *et al* [24] have studied strange stars in Krori–Barua space–time in the framework of $f(R, T)$ gravity while Paul *et al* [25] have studied the cooling of neutron star including axion emission by nucleon–nucleon axion bremsstrahlung. Recently, Rahaman *et al* [26] have studied the existence of compact spherical systems representing anisotropic

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matter distributions within the scenario of $f(R,T)$ theory of gravity. In refs [27–33] the usefulness of the $f(R,T)$ theory of gravity is studied in different physical contexts.

Moreover, an accelerated expansion of the Universe is explained with the negative pressure of the cosmic fluid – a plausible scenario is provided by the viscous fluid because its pressure is affected by a bulk viscosity term. Thus, the inclusion of bulk viscosity contributes negative pressure to the cosmic fluid, making effective total pressure more negative and hence stimulating repulsive gravity. This repulsive gravity overcomes the attractive gravity and pushes the Universe in accelerated expansion. Carlevaro and Montani [34] have derived the stability of early Universe due to the presence of bulk viscous fluid. In fact, the bulk viscous terms in relativistic fluid was introduced by Eckart [35] to investigate the exact process of evolution of early Universe. Later on, it has been proved that bulk viscosity appears as dissipative phenomenon which stimulates the rate of expansion of the present Universe [36]. Some more useful applications of bulk viscosity in the evolution of early Universe are given in refs [37–40]. Mark and Harko [41] have investigated the effect of bulk viscosity in Brans–Dicke theory of gravitation. Cataldo *et al* [42] have obtained bulk viscous cosmological solution that exhibits big rip singularity at the boundary between $p = 0$ and $-\rho$. In previous studies [43,44], we have investigated bulk viscous embedded homogeneous but anisotropic cosmological model in $f(R,T)$ theory of gravity by taking into account the hybrid law for scale factor [45]. In this paper, we confine ourselves to investigate a model of accelerating Universe without including dark energy components or cosmological constant (either it is variable or constant) in energy budget of the Universe so that the derived model does not suffer with the cosmological constant problems on the theoretical grounds. That is why we consider bulk viscous cosmic fluid in the framework of $f(R,T) = f(R) + 2f(T)$ gravity. Recently, Odintsov *et al* [46] have investigated some cosmological scenarios of bulk viscous dark energy fluid and tested it with the latest astronomical data.

The paper is organised as follows. In §2, we construct the model and its basic formalism. Section 3 deals with the method to find observational constraints on model parameter and fitting of derived model with data. The physical properties of the derived model has been discussed in §4. Finally in §5, we have summarised our findings.

2. The model and basic formalism

The isotropic and homogeneous space–time is given by

$$ds^2 = dt^2 - a^2(dx^2 + dy^2 + dz^2), \tag{1}$$

where a denotes the scale factor of metric (1).

The geometrically modified action in $f(R,T) = f(R) + 2f(T)$ theory of gravitation reads as

$$S = \frac{1}{16\pi}\int [f(R) + 2f(T)]\sqrt{-g}\,d^4x + \int L_m\sqrt{-g}\,d^4x, \tag{2}$$

where $f(R)$ and $f(T)$ are arbitrary functions of Ricci scalar R and trace T of the stress–energy tensor of the matter T_{ij}.

Following Harko *et al* [20], we choose $f(R) = R$ and $f(T) = \lambda T$, where λ is a constant. Thus, the corresponding field equation reads as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2\frac{\partial f}{\partial T}T_{ij} + \left[f(T) + 2p_v\frac{\partial f}{\partial T}\right]g_{ij}. \tag{3}$$

Here p_v is the pressure of the bulk viscous fluid and it is defined as $p_v = p - 3\zeta H$, where p, ζ and H are pressure of the perfect fluid, coefficient of bulk viscosity and Hubble's function respectively. At present, the Universe is filled with pressure-less matter (dust) and therefore, we obtain $p_v = -3\zeta H$.

For field equation (3), the line element (1) yields the following field equations:

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -(8\pi + 3\lambda)p_v + \lambda\rho \tag{4}$$

$$\frac{\dot{a}^2}{a^2} = (8\pi + 3\lambda)\rho - \lambda p_v. \tag{5}$$

We assume that $\mu = 3\lambda/8\pi$.

Hence, field equations (4) and (5) are rewritten as

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = 8\pi\left((1+\mu)3\zeta H + \frac{\mu\rho}{3}\right) \tag{6}$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi((1+\mu)\rho + \mu\zeta H). \tag{7}$$

Equation (7) leads to

$$1 = \frac{8\pi\rho}{3H^2} + \frac{8\pi\mu(\rho + \zeta H)}{3H^2} \Rightarrow 8\pi\mu\zeta = 3H_0((1 - (1+\mu)\Omega_m), \tag{8}$$

where

$$\Omega_m = \frac{8\pi\rho}{3H^2} \quad \text{and} \quad H = \frac{\dot{a}}{a}.$$

Solving eqs (6) and (7), we obtain

$$(2\mu + 3)\left(H - \frac{8}{3}\pi\zeta(4\mu + 3)\right) - 2(\mu + 1)(z + 1)H' = 0, \quad (9)$$

where H' denotes the first derivative of H with respect to z.

Thus, the expression for Hubble's function of the derived model is computed as

$$H(z) = H_0\left[\alpha + (1 - \alpha)(z + 1)^{\frac{2\mu+3}{2(\mu+1)}}\right], \quad (10)$$

where

$$\alpha = \frac{4\mu + 3}{\mu}[(1 - (1 + \mu)\Omega_m]$$

and H_0 denotes the present value of Hubble's constant.

3. Observational confrontation

In this section, we describe the observational data and the statistical methodology used to constrain the model parameters of the derived model.

- **Observational Hubble Data (OHD)**: We adopt 46 $H(z)$ data points over the redshift range of $0 \leq z \leq 2.36$ obtained from cosmic chronometric technique. All these 46 $H(z)$ data points are compiled in table I of ref. [47].
- **Baryon acoustic oscillations (BAO)**: We use 10 baryon acoustic oscillations data extracted from the 6dFGS [48], SDSS-MGS [49], BOSS [50] and WiggleZ [51] surveys.
- **SN Ia data**: We use SN Ia observational data in the redshift range $0.01 < z < 1.4$, which include 276 SNIa ($0.03 < z < 0.65$) obtained by the Pan-STARRS1 MDS and SNIa distance estimates from SDSS, SNLS and low-z HST samples.

For statistical analysis, we have used χ^2 test for parameters with the likelihood given by $\varphi \propto e^{-\chi^2/2}$. Therefore, the χ^2 function for OHD is written as

$$\chi^2_{\text{OHD}} = \sum_{i=1}^{46}\left[\frac{H(z_i, s) - H_{\text{obs}}(z_i)}{\sigma_i}\right]^2, \quad (11)$$

where s and σ_i respectively denote the parameter vector and standard error in experimental values of Hubble's function H.

Similarly, the joint χ^2 reads as

$$\chi^2_{\text{joint}} = \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}}. \quad (12)$$

The numerical result of the statistical analysis of the derived model with recent observational data is listed in table 1. Figure 1 shows one-dimensional marginalised posterior distributions for H_0 and α of the derived model with OHD, BAO and OHD + BAO. We obtain the best-fit value of H_0 as 69.089 km/Mpc/s by bounding the derived model with joint OHD and BAO data. This value of H_0 is very close to that of the corresponding value of H_0, obtained by PLANK Collaboration [52].

Table 1. Numerical result of the statistical analysis of the derived model.

Parameters	OHD	BAO	OHD + BAO
H_0 (km/Mpc/s)	66.080	70.734	69.089
α	0.634	0.792	0.650

The luminosity distance D_L is obtained as

$$D_L = \frac{(1+z)}{H_0}\int_0^z \frac{dz}{\left(\alpha + (1-\alpha)(z+1)^{\frac{2\mu+3}{2(\mu+1)}}\right)}. \quad (13)$$

The distance modulus μ and apparent distance m_b of the derived model read as

$$\mu = 25 + 5\log_{10}\left(\frac{(1+z)}{H_0}\int_0^z \frac{dz}{\left(\alpha + (1-\alpha)(z+1)^{\frac{2\mu+3}{2(\mu+1)}}\right)}\right) \quad (14)$$

$$m_b = 16.08 + 5\log_{10}\left(\frac{1+z}{0.026}\int_0^z \frac{dz}{\left(\alpha + (1-\alpha)(z+1)^{\frac{2\mu+3}{2(\mu+1)}}\right)}\right). \quad (15)$$

Figure 2 shows the best-fit curve of distance modulus for the derived model with SN Ia observational data. From figure 2, we observe that the derived model is in good agreement with SN Ia observations. In figure 3, we have plotted Hubble rate with respect to red-shift error bars and shown the consistency of the derived model with OHD points.

4. Physical properties of the model

4.1 *Age of the Universe*

The age of the Universe is computed as

$$dt = -\frac{dz}{(1+z)H} \Rightarrow \int_t^{t_0} dt = -\int_z^0 \frac{1}{(1+z)H}dz. \quad (16)$$

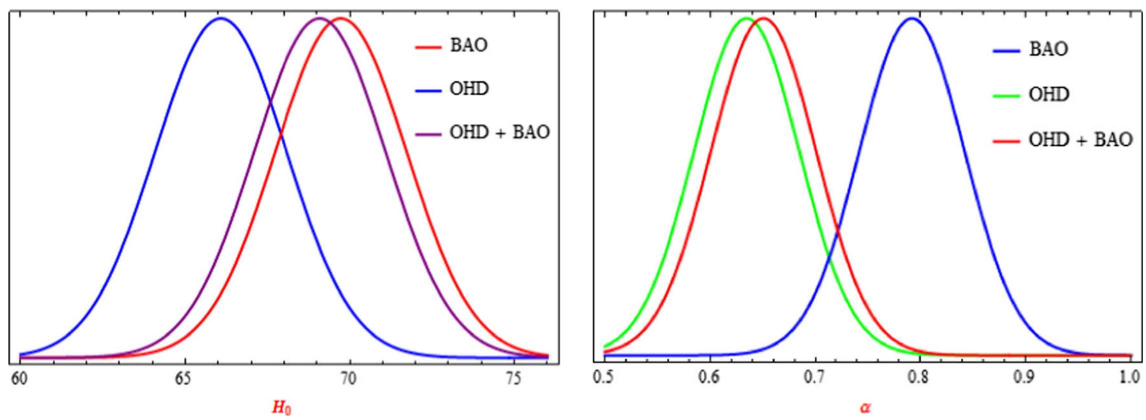

Figure 1. One-dimensional marginalised posterior distributions for Hubble's constant H_0 (left panel) and model parameter α (right panel) of the derived model with OHD, BAO and OHD + BAO.

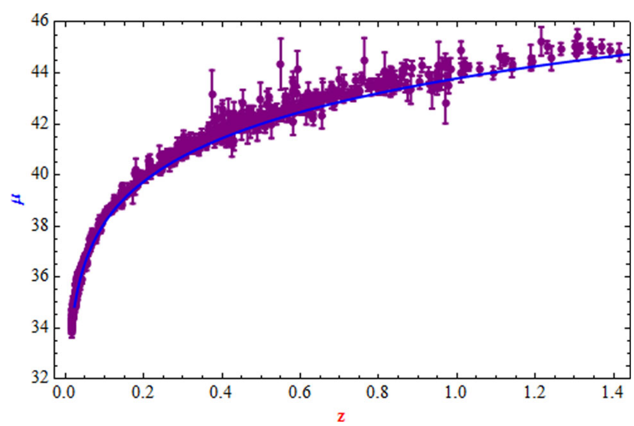

Figure 2. The best-fit curve of the distance modulus for the derived model with SN Ia observational data for $H_0 = 69.089$ km/Mpc/s and $\alpha = 0.6508$. The points with error bar (purple colour) denotes the observed values of distance modulus μ and solid line (blue colour) denotes the corresponding theoretical values of μ.

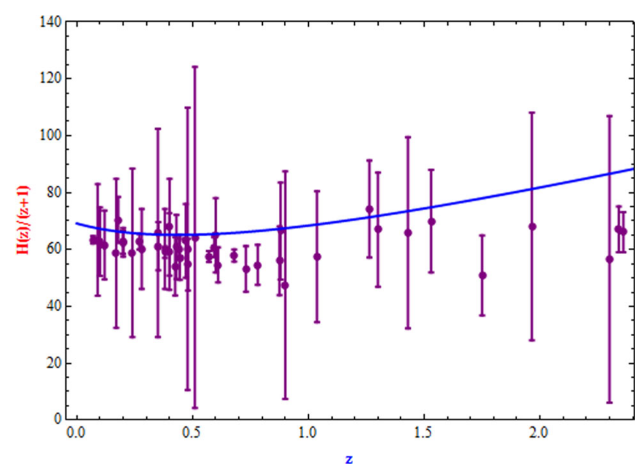

Figure 3. The Hubble rate vs. red-shift error bar plot with OHD points. The solid blue line represents our derived model.

Using eq. (10) in eq. (16), we have

$$t_0 - t = \int_0^x \frac{\mathrm{d}z}{H_0(1+z)\left(\alpha + (1-\alpha)(z+1)^{\frac{2\mu+3}{2(\mu+1)}}\right)}. \tag{17}$$

Here, t_0 denotes the present age of the Universe.

From eq. (17), it is convenient to estimate the present age of the Universe by considering $t \to 0$ as $z \to \infty$. Therefore,

$$t_0 = \lim_{x\to\infty} \int_0^x \frac{\mathrm{d}z}{H_0(1+z)\left(\alpha + (1-\alpha)(z+1)^{\frac{2\mu+3}{2(\mu+1)}}\right)}. \tag{18}$$

The numerical integration of eq. (18) leads to

$$H_0 t_0 = 0.839036. \tag{19}$$

Therefore, the present age of the derived Universe is obtained as $t_0 = 12.4483$ Gyrs. The plot of $H_0(t_0 - t)$ vs. red-shift z is shown in figure 4. From figure 4, we observe that at the present time, i.e. for $z = 0$, $H_0(t_0 - t)$ becomes null and $t = t_0$ which verify the statement that at $z = 0$, $t = t_0$.

4.2 *Deceleration parameter*

The expression of deceleration parameter q is obtained as

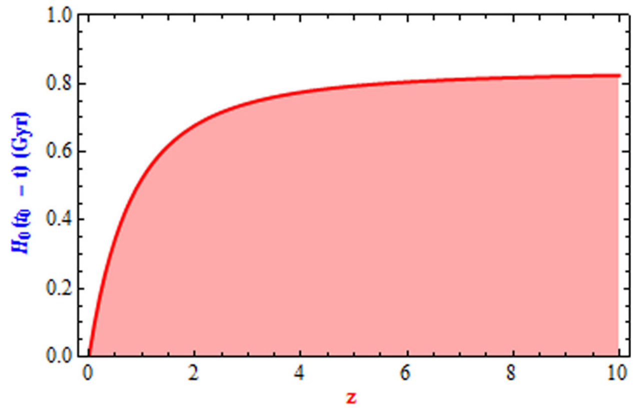

Figure 4. The variation of $H_0(t - t_0)$ vs. z.

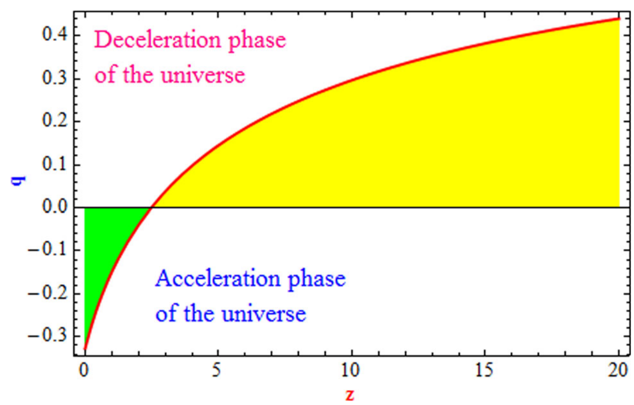

Figure 5. The variation of q vs. z.

$$q = \frac{1}{2(\mu+1)} - \frac{\alpha(2\mu+3)}{2(\mu+1)\left(\alpha + (1-\alpha)(z+1)^{\frac{2\mu+3}{2(\mu+1)}}\right)}. \tag{20}$$

The behaviour of deceleration parameter q vs. z is shown in figure 5. It is evident that the past Universe was in decelerating mode while the current Universe is in accelerating mode. Therefore, the derived model represents the model of transitioning Universe with transition red-shift $z = 2.47$. The present value of deceleration parameter is obtained as $q_0 = -0.327$. It is important to note that the derived bulk viscous Universe in $f(R, T)$ gravity describes the late-time acceleration of the Universe without including dark energy components or cosmological constant Λ. This late-time acceleration of the Universe is due to bulk viscous fluid and trace of energy–momentum tensor. Thus, the model under consideration does not suffer with cosmic coincidence and fine tuning problems.

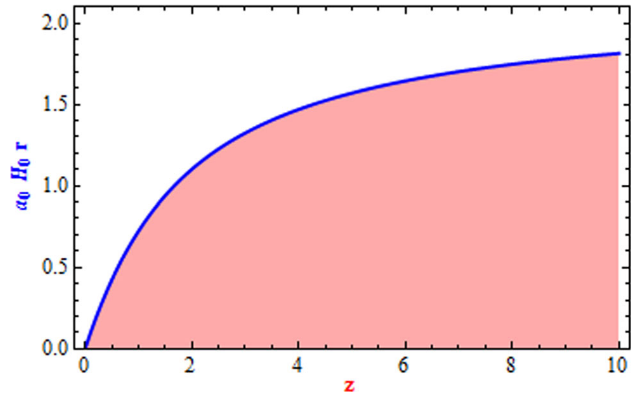

Figure 6. The plot of $a_0 H_0 r$ vs. z.

4.3 *Particle horizon*

Following Bentabol *et al* [53], the particle horizon measures the total distance the light signal could have travelled from the first event occurred in the past till today of the observable Universe and hence it is represented by proper distance measured by light signal coming from $t = 0$ to t_0. Thus, the particle horizon for the derived model is obtained as

$$R_p = \lim_{t_p \to 0} a_0 \int_{t_p}^{t_0} \frac{\mathrm{d}t}{a(t)} = \lim_{z\to\infty} \int_0^z \frac{\mathrm{d}z}{H(z)}, \tag{21}$$

where t_p denotes time in the past at which the light signal was emitted from the source.

Equations (10) and (21) lead to

$$R_p = \lim_{z\to\infty} \int_0^z \frac{\mathrm{d}z}{H_0\left[\alpha + (1-\alpha)(z+1)^{\frac{2\mu+3}{2(\mu+1)}}\right]}. \tag{22}$$

Integrating eq. (22), we obtain

$$R_p = \frac{2.1459}{H_0}. \tag{23}$$

The variation of $a_0 H_0 r$ vs. z is shown in figure 6. Here r measures the proper distance. From figure 6, it is evident that at $z = 0$, $r = 0$ and at $z \to \infty$, $r \to \infty$. This means that we are at infinite distance from the first event occurred in the past.

4.4 *Jerk parameter*

The jerk parameter (j) is given by

$$j = 1 - (1+z)\frac{H'}{H} + \frac{1}{2}(1+z)^2\frac{[H'']^2}{[H]^2}. \tag{24}$$

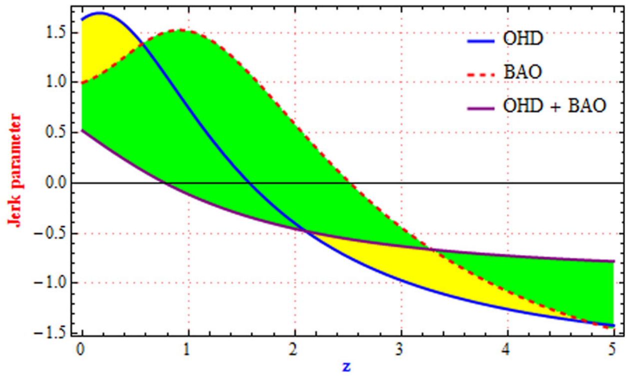

Figure 7. The evolution of jerk parameter j vs. z.

Using eq. (10) in eq. (24), the expression of jerk parameter of the derived model reads as

$$j = 1 + \frac{(\alpha-1)^2(2\mu+3)^2(z+1)^{\frac{1}{\mu+1}}}{32(\mu+1)^4\left(\alpha-\alpha(z+1)^{\frac{2\mu+3}{2\mu+2}}+(z+1)^{\frac{2\mu+3}{2\mu+2}}\right)^2} - \frac{(1-\alpha)(2\mu+3)(z+1)^{\frac{2\mu+3}{2\mu+2}}}{2(\mu+1)\left(\alpha+(1-\alpha)(z+1)^{\frac{2\mu+3}{2\mu+2}}\right)}. \quad (25)$$

The evolution of the jerk parameter j vs. z is shown in figure 7. We observe that the jerk parameter was evolving with negative sign in the past due to the decelerated expansion of the Universe and in the present time it becomes positive. This signature flipping of jerk parameter leads the criteria that $j > 0$ when $q < 0$. In the literature, for ΛCDM model, the present value of jerk parameter is found to be $j_0 = 1$ [54,55].

5. Concluding remarks

In this paper, we have investigated the bulk viscous accelerating Universe in $f(R, T)$ theory of gravity. It is observed that appearance of the late-time acceleration of the Universe is due to bulk viscous fluid and trace of energy–momentum tensor. This late-time accelerated expansion of the Universe is one of the important features of $f(R, T)$ theory of gravity. The derived model shows a signature flipping from early decelerating phase to current accelerating phase at transition red-shift $z_t = 2.47$. In the model under consideration, we estimate the present value of H_0 as 69.089 km/Mpc/s by bounding it with joint OHD and BAO observational data sets. This value of H_0 matches well with the recent astrophysical observations [52]. The best-fit curves of the derived model for distance modulus and Hubble's rate with SN Ia and OHD data sets are shown in figures 2 and 3 respectively. The present age of the Universe is obtained as $t_0 = 12.4483$ Gyrs. As a final comment, we note that the derived model describes late-time acceleration of the Universe without including dark energy components or cosmological constant in the energy budget of the Universe.

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