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Bipartite entanglement in Auger ionisation of N_2

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Abstract. Quantum entanglement and its paradoxical properties are of paramount importance in quantum information theory. In recent years, there has been an increasing interest in the studies of high-dimensional quantum states and their impact on quantum communication as it can encode and process more data. Photonic entanglement is usually an evanescent property as it is destroyed easily by its interaction with an external environment. Electronic qubits are stable and can store information for a long time. However, qudit systems are more efficient, stable and allow noise robustness than qubit system. In this article, we investigate bipartite entanglement between doubly ionised molecular qudit and electronic qubit in the Auger emission process for N_2 molecule following the absorption of a single photon without observing spin-orbit interaction (SOI). In the absence of SOI, Russell–Saunders coupling (L-S coupling) is applicable. The entanglement properties are estimated on the basis of negativity of partial transpose of the density matrix for Auger ionisation. We find that the entanglement depends on the spins of the singly ionised excited states and doubly ionised states of the molecules as well as on the directions of spin quantisation and of ejection of Auger electrons. A significant effect on the variation of negativity due to the linear dichroism (LD) has also been observed.

Keywords. Quantum entanglement; qudit; qubit; Auger ionisation; negativity; density matrix; linear dichroism.

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1. Introduction

Quantum entanglement [1] is the key resource of quantum information technology (QIT) [2–4]. The production and characterisation of entangled states [5] of two or more particles are drawing great interest recently. In QIT qubit is the basic unit of quantum computing [6,7]. Qubits, using properties of quantum entanglement and superposition, have enormous computing power [8–10]. There is currently a great interest in using high-dimensional (dimension d>2) quantum states known as qudit for various communication and computational tasks [11,12]. High-dimensional quantum states provide an efficient and robust means for encoding information, where each qudit can encode a maximum of $\log_2 d$ bits of information [13,14].

Investigations on entanglement of photonic qudits have already been done by several groups [13,15–17]. However, photons are not suitable for storing information for long time as they are incredibly unstable [18]. Theories have been developed for studying entanglement of electronic qubit and ionic qudit produced by single [19] and double photoionisation [20,21] of atoms. Bipartite as well as tripartite entanglement have been studied in qubit system for atoms in Auger decay process [22,23]. Here we present a theoretical analysis of high-dimensional quantum entanglement between electronic qubit and doubly ionised molecular qudit in the Auger emission process for N_2 molecule. For quantifying the degree of entanglement we use negativity [24,25] of partial transpose of the density matrix (DM) for Auger decay. Negativity is an additive and operational measure of entanglement and it can also be generalised to multipartite and higher-dimensional entanglement [24]. It has been observed that the character of entanglement depends on the directions of ejection and spin quantisation of emitted Auger electrons. The degree of entanglement also varies with linear dichroic effect.

In §2 we calculate the density operator (DO) and density matrix (DM) for qudit–qubit system in Auger

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ionisation of molecule. This DM is used in §3 to study the quantum entanglement between qudit and qubit systems in N_2 molecule. However, the theoretical formulations developed in the preceding section can be applied to any other molecular system. In §4 we have discussed about how the entanglement properties depend on the directions of ejection and of spin quantisation of emitted Auger electrons. The effect of linear dichroism (LD) on the entanglement has also been shown. Section 5 contains the conclusion part of this work.

2. Density operator and density matrix for the qudit–qubit system in Auger ionisation of molecule

Let us consider the following process:

$$h\nu_r(|\,l_r|=1,m_r)+M\,|0\rangle \rightarrow M^{+*}\,|1\rangle+e_p \tag{1}$$

followed by

$$M\,|1\rangle^{+*} \rightarrow M\,|2\rangle^{++}+e_a\left(\vec{k}_a;\mu_a\hat{u}_a\right) \tag{2}$$

caused by the absorption of a photon of frequency ν_r in a molecule M. m_r represents the state of polarisation of incident photon. M^{+*} in (1) is the photoexcited state of the molecule formed after the emission of photoelectron e_p. Emission of the Auger electron e_a results in the formation of residual doubly charged molecular ion M^{++} in (2). Here $(\vec{L}_0,\vec{L}_1,\vec{L}_2)$ are the total orbital angular momenta and $(\vec{S}_0,\vec{S}_1,\vec{S}_2)$ are the spin angular momenta of M, M^{+*} and M^{++} respectively. We use the symbols $M_{L_0}, M_{S_0}, M_{L_1}, M_{S_1}, M_{L_2}$ and M_{S_2} to represent the respective projections of $\vec{L}_0,\vec{S}_0,\vec{L}_1,\vec{S}_1,\vec{L}_2$ and $\vec{S}_2$ along the polar axis of the space frame. The electronic states of M, M^{+*} and M^{++} in L-S coupling are given by [26] as $|0\rangle \equiv \left|L_0,S_0,M_{L_0},M_{S_0}\right\rangle$, $\left|1\right\rangle = \left|L_1,S_1,M_{L_1},M_{S_1}\right\rangle$ and $|2\rangle \equiv \left|L_2,S_2,M_{L_2},M_{S_2}\right\rangle$ respectively. Here the unit vector $\hat{u}_a(\theta,\phi)$ is the direction of spin quantisation, $\mu_a(=\pm 1/2)$ is the projection of the spin angular momentum along the spin quantisation and the vector $\vec{k}_a(k_a,\hat{k}_a(\theta_a,\phi_a))$ is the direction of propagation of e_a (figure 1).

The density matrix for Auger decay process can be written as

$$\begin{aligned}&\left\langle L_2S_2M_{S_2};\mu_a\hat{u}_a\vec{k}_a|\rho|L_2S_2M'_{S_2};\mu'_a\hat{u}_a\vec{k}_a\right\rangle\\&=\frac{K_a}{(2L_0+1)(2S_0+1)}\\&\times\sum_{M_{L_0}M_{S_0}M_{L_1}M_{S_1}M_{L_2}}\left\langle L_1S_1M_{S_1}|F_p|L_0S_0M_{S_0};1m_r\right\rangle\\&\times\left\langle L_2S_2M_{S_2};\mu_a\hat{u}_a\vec{k}_a|F_a|L_1S_1M_{S_1}\right\rangle\\&\times\left\langle L_0S_0M_{S_0};1m_r|F_p^*|L_1S_1M_{S_1}\right\rangle\\&\times\left\langle L_1S_1M_{S_1}|F_a^*|L_2S_2M'_{S_2};\mu'_a\hat{u}_a\vec{k}_a\right\rangle.\end{aligned} \tag{3}$$

Here ρ is the density operator, F_p is the photoabsorption operator and F_a is the Auger transition operator [27]. The value of the constant K_a depends on the energy of the emitted electrons in Auger decay process. If the orbital angular momentum of the Auger electron e_a is $\vec{l}_a$ with its spin angular momenta $s_a=(\frac{1}{2})_a$, then we have

$$\vec{L}_1=\vec{L}_2+\vec{l}_a \tag{4}$$

and

$$\vec{S}_1=\vec{S}_2+\vec{s}_a\left[=\left(\frac{1}{2}\right)_a\right]. \tag{5}$$

Here $\left\langle L_2S_2M_{S_2};\mu_a\hat{u}_a\vec{k}_a|F_a|L_1S_1M_{S_1}\right\rangle$ is the Auger decay amplitude and $\left\langle L_1S_1M_{S_1}|F_p|L_0S_0M_{S_0};1m_r\right\rangle$ is the photoabsorption amplitude. For evaluating (3) we substitute in it the values of Auger decay amplitude, photoabsorption amplitude, their Hermitian conjugates as well as (4) and (5). The result can be simplified by analytically evaluating as many sums as possible present therein by applying Racah algebra. It needs, for example, the use of (a) the addition theorems from [28] for rotational and spherical harmonics, (b) eq. (6.2.5) of [28] for converting a single sum of the product of three 3-j symbols into a product of one 3-j and one 6-j symbols, (c) identity (2) given on p. 453 in ref. [29] for converting a single sum of the product of two 3-j symbols into a product of two 3-j and one 6-j symbols summed over two variables, (d) eq. (14.42) from [30] which transforms a quadruple sum of the product of four 3-j symbols into a double sum containing two 3-j and one 6-j symbols, (e) eq. (3.7.9) from [28] for changing a phase factor into a 3-j symbol, (f) orthogonality (3.7.7) from [28] of 3-j symbols.

The simplified DM for process (2) in the absence of SOI can be written in the following form:

$$\begin{aligned}&\left\langle L_2S_2M_{S_2};\mu_a\hat{u}_a\vec{k}_a|\rho|L_2S_2M'_{S_2};\mu'_a\hat{u}_a\vec{k}_a\right\rangle\\&=\frac{\mathrm{d}\sigma(m_r)}{\mathrm{d}\hat{k}_a}\sigma(S_1;S_2;\hat{u}_a)_{\mu_aM_{S_2};\mu'_aM'_{S_2}}.\end{aligned} \tag{6}$$

The first term $(\frac{\mathrm{d}\sigma(m_r)}{\mathrm{d}\hat{k}_a})$ on the right-hand side of (6) is the angular part of density matrix (APDM) of spin-unresolved Auger electron emitted following photoabsorpsion in electric dipole approximation in a free, gaseous molecule. This expression can be written in the following form:

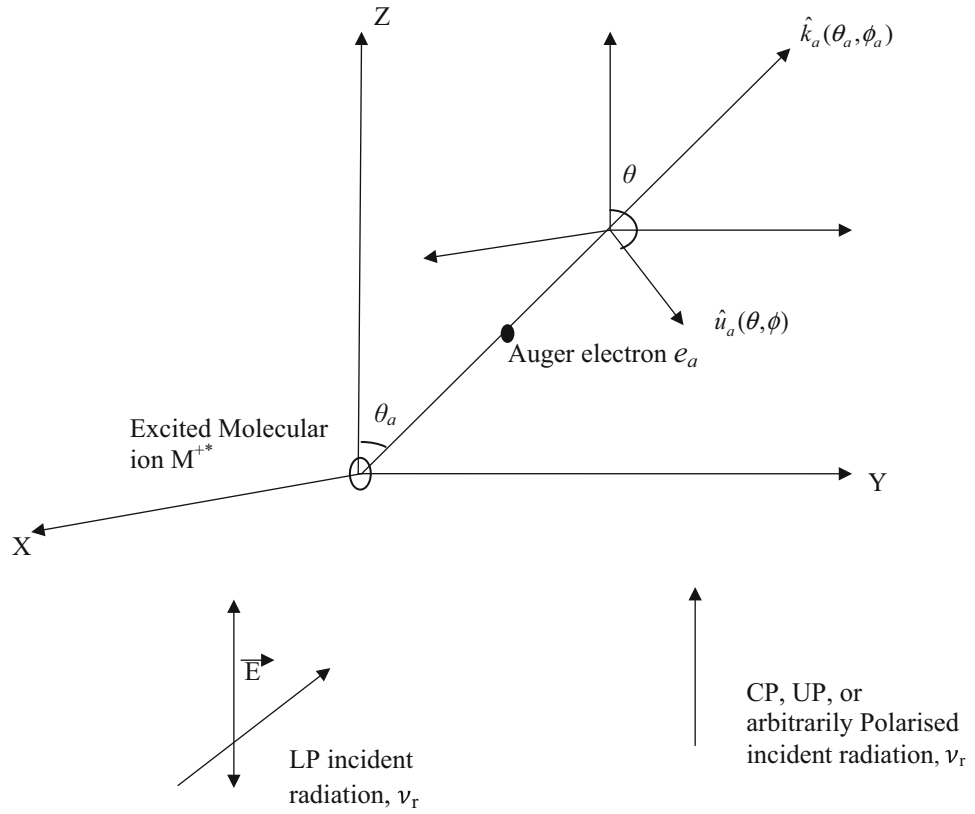

Figure 1. Emission of Auger electron (e_a) from the excited molecular ion (M^{+*}) following photoabsorption.

$$\frac{\mathrm{d}\sigma(m_r)}{\mathrm{d}\hat{k}_a} = [1 - 2m_r\alpha_a\cos\theta_a + \frac{1}{2}(2-3m_r^2)\beta_a P_2(\cos\theta_a)], \tag{7}$$

where

$$\alpha_a = \frac{\pi}{\hbar}\sqrt{\frac{3}{2}} \sum_{\substack{l_a l'_a m_a m'_a \lambda_r \\ \lambda'_r L M L' M'}} (-i)^{l_a - l'_a}(-1)^{l_a+m_a-l'_a-m'_a+L} \times \mathrm{e}^{i(\sigma l_a - \sigma l'_a)} \begin{pmatrix} l_a & l'_a & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_a & 1 & L \\ m_a & \lambda_r & M \end{pmatrix} \begin{pmatrix} l'_a & 1 & L' \\ m'_a & \lambda'_r & M' \end{pmatrix} \times \begin{Bmatrix} 1 & 1 & 1 \\ l_a & l'_a & L \end{Bmatrix} \langle L_1 | F_p | L_0 1\lambda_r \rangle \langle (L_2 l_a m_a) L | F_a | L_1 \rangle \times \langle L_1 | F_p | L_0 1\lambda'_r \rangle^* \langle (L_2 l'_a m'_a) L' | F_a | L_1 \rangle^* \tag{8}$$

$$\beta_a = \frac{2\pi}{\hbar}\sqrt{\frac{10}{3}} \sum_{\substack{l_a l'_a m_a m'_a \lambda_r \\ \lambda'_r L M L' M'}} (-i)^{l_a - l'_a}(-1)^{l_a+m_a-l'_a-m'_a+L} \times \mathrm{e}^{i(\sigma l_a - \sigma l'_a)} \begin{pmatrix} l_a & l'_a & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_a & 1 & L \\ m_a & \lambda_r & M \end{pmatrix} \begin{pmatrix} l'_a & 1 & L' \\ m'_a & \lambda'_r & M' \end{pmatrix} \times \begin{Bmatrix} 1 & 1 & 2 \\ l_a & l'_a & L \end{Bmatrix} \langle L_1 | F_p | L_0 1\lambda_r \rangle \langle (L_2 l_a m_a) L | F_a | L_1 \rangle \times \langle L_1 | F_p | L_0 1\lambda'_r \rangle^* \langle (L_2 l'_a m'_a) L' | F_a | L_1 \rangle^* \tag{9}$$

α_a and β_a are the Auger decay parameters containing Auger decay matrix elements.

The dynamical terms $\langle L_1 | F_p | L_0 1\lambda_r \rangle$ and $\langle (L_2 l_a m_a) L | F_a | L_1 \rangle$ are defined as reduced photoabsorption and Auger amplitudes respectively. Thus, we can see from eqs (7)–(9) that the term $\frac{\mathrm{d}\sigma(m_r)}{\mathrm{d}\hat{k}a}$ is related to the angular correlations between e_a and M^{++} ion. It is a positive quantity and acts as a multiplicative factor in the DM in (3) for Auger process (2). The second term (i.e., $\sigma(S_1; S_2; \hat{u}_a)_{M_{S_2}\mu_a; M'_{S_2}\mu'_a}$) on the right-hand side of DM is the (6×6)-dimensional spin-correlation density matrix (SCDM). It completely determines the entanglement properties among electronic qubit (e_a) and ionic qudit (M^{++}). Then SCDM can be written as

$$\sigma(S_1; S_2; \hat{u}_a)_{\mu_a M_{S_2}; \mu'_a M'_{S_2}} = (-1)^{\mu'_a + S_1 + M_{S_2}} \times \sum_{Smn} (2S+1) \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S \\ \mu_a & -\mu'_a & m \end{pmatrix} \begin{pmatrix} S_2 & S_2 & S \\ M_{S_2} & -M'_{S_2} & -n \end{pmatrix} \times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S \\ S_2 & S_2 & S_1 \end{Bmatrix} D^S_{mn}(\omega), \tag{10}$$

where D is the rotational harmonic [28] with $\omega_a(\theta, \phi, 0)$, the Euler angle which rotates the polar axis of the

space frame into the spin quantisation direction $\hat{u}_a$ (figure 1). In order to study the entanglement properties for Auger ionisation in (2), we need to calculate partial transpose (PT) [6,7] of the SCDM (10) with respect to Auger electron (e_a) and M^{++} ion. The PT can be written as

$$\begin{aligned}\sigma^T(S_1;S_2;\hat{u}_a)_{\mu_a M_{S_2};\mu'_a M'_{S_2}} &= \sigma(S_1;S_2;\hat{u}_a)_{\mu_a M'_{S_2};\mu'_a M_{S_2}} \\ &= \sigma(S_1;S_2;\hat{u}_a)_{\mu'_a M_{S_2};\mu_a M'_{S_2}}. \end{aligned} \tag{11}$$

As a measure of the degree of entanglement, we consider the negativity [24,25] which can be defined as

$$N = \max(0, -\lambda_{\text{neg}}). \tag{12}$$

where λ_{neg} is the sum of the negative eigenvalues of the partial transpose, $\sigma^T(S_1;S_2;\hat{u}_a)_{\mu_a M_{S_2};\mu'_a M'_{S_2}}$.

3. Spin-correlation density matrix (SCDM) between qudit and qubit system in N_2 molecule

In this section, we calculate the SCDM obtained in (10) for Auger decay process of N_2 molecule. The electronic configuration [31] of the ground state of N_2 can be written as

$$\left(\sigma_g 2s\right)^2 (\sigma_u 2s)^2 (\pi_u 2p)^2 \left(\sigma_g 2p\right)^2, {}^1\Sigma_g^+. \tag{13}$$

As N_2 has only closed orbital in the ground state (13), one of the possible states of N_2^{++}molecular ion is given by the state of the following 2-hole configuration [31]

$$\left(\sigma_g 2s\right)^{-1} (\sigma_u 2p)^{-1}, {}^3\Sigma_u^+. \tag{14}$$

According to (13) and (14) we have the values for S_1 and S_2 are 1/2 and 1 respectively. In order to calculate the SCDM of qudit–qubit system, according to the process followed by ref. [32], we have considered only the real part of the outgoing wave function and the density matrix in this case. We now obtain the following SCDM for (10) and its PT are respectively.

$\sigma(S_1;S_2;\hat{u}_a)_{\mu_a M_{S_2};\mu'_a M'_{S_2}} =$

$\mu_a M_{S_2}$ \ $\mu'_a M'_{S_2}$	$\frac{1}{2}1$	$\frac{1}{2}0$	$\frac{1}{2}-1$	$-\frac{1}{2}1$	$-\frac{1}{2}0$	$-\frac{1}{2}-1$
$\frac{1}{2}1$	$\frac{1}{6}(1-\cos\theta)$	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	0	$\frac{\sin\theta}{6}$	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$	0
$\frac{1}{2}0$	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}$	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$	0	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$
$\frac{1}{2}-1$	0	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}(1+\cos\theta)$	0	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$	$-\frac{\sin\theta}{6}$
$-\frac{1}{2}1$	$\frac{\sin\theta}{6}$	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$	0	$\frac{(1+\cos\theta)}{6}$	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	0
$-\frac{1}{2}0$	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$	0	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}$	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$
$-\frac{1}{2}-1$	0	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$	$-\frac{\sin\theta}{6}$	0	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}(1-\cos\theta)$

Table 1. SCDM of qudit–qubit system.

$\sigma^T(S_1;S_2;\hat{u}_a)_{\mu_a M_{S_2};\mu'_a M'_{S_2}} =$

$\mu_a M_{S_2}$ / $\mu'_a M'_{S_2}$	$\frac{1}{2}1$	$\frac{1}{2}0$	$\frac{1}{2}-1$	$-\frac{1}{2}1$	$-\frac{1}{2}0$	$-\frac{1}{2}-1$
$\frac{1}{2}1$	$\frac{1}{6}(1-\cos\theta)$	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	0	$\frac{\sin\theta}{6}$	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$	0
$\frac{1}{2}0$	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}$	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$	0	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$
$\frac{1}{2}-1$	0	$-\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}(1+\cos\theta)$	0	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$	$-\frac{\sin\theta}{6}$
$-\frac{1}{2}1$	$\frac{\sin\theta}{6}$	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$	0	$\frac{(1+\cos\theta)}{6}$	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	0
$-\frac{1}{2}0$	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$	0	$\frac{(1-\cos\theta)}{6\sqrt{2}}\cos\phi$	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}$	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$
$-\frac{1}{2}-1$	0	$-\frac{(1+\cos\theta)}{6\sqrt{2}}\cos\phi$	$-\frac{\sin\theta}{6}$	0	$\frac{\sin\theta\cos\phi}{6\sqrt{2}}$	$\frac{1}{6}(1-\cos\theta)$

Table 2. PT of SCDM of qudit–qubit system.

From table 1 we have calculated the eigenvalues of SCDM which are

$$\left(0,0,\frac{1}{12}\left(3-\sqrt{5+4\cos 2\phi}\right),\frac{1}{12}\left(3-\sqrt{5+4\cos 2\phi}\right),\frac{1}{12}\left(3+\sqrt{5+4\cos 2\phi}\right),\frac{1}{12}\left(3+\sqrt{5+4\cos 2\phi}\right)\right). \tag{15}$$

From (15) we find that the density matrix of table 1 has more than one non-zero eigenvalues. So all the states for S_2 =1 form mixed states [33].

The eigenvalues of PT from table 2 are

$$\left(\frac{1}{3},\frac{1}{3},\frac{1}{12}\left(1-\sqrt{5+4\cos 2\phi}\right),\frac{1}{12}\left(1-\sqrt{5+4\cos 2\phi}\right),\frac{1}{12}\left(1+\sqrt{5+4\cos 2\phi}\right),\frac{1}{12}\left(1+\sqrt{5+4\cos 2\phi}\right)\right). \tag{16}$$

For quantifying entanglement, we have calculated negativity using table 2, according to the definition given in (12), which yields

$$N=\frac{1}{6}\left(-1+\sqrt{5+4\cos 2\phi}\right). \tag{17}$$

The variation of negativity (N) with respect to direction (ϕ) of spin quantisation for $S_2 = 1$ is given in figure 2.

Figure 2 shows that the system is entangled for all values of ϕ.

4. Bipartite entanglement with respect to APDM and LD in Auger ionisation of N_2 molecule

In order to study bipartite entanglement with respect to APDM ($\mathrm{d}\sigma(m_r)/\mathrm{d}\hat{k}_a$), we first calculate the values of

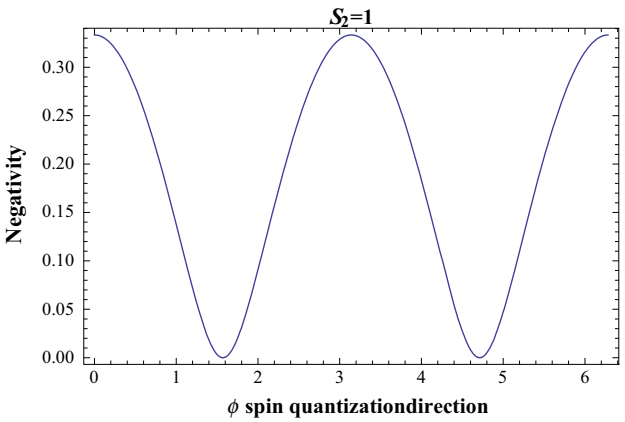

Figure 2. Variation of SCDM negativity with spin quantisation direction (ϕ).

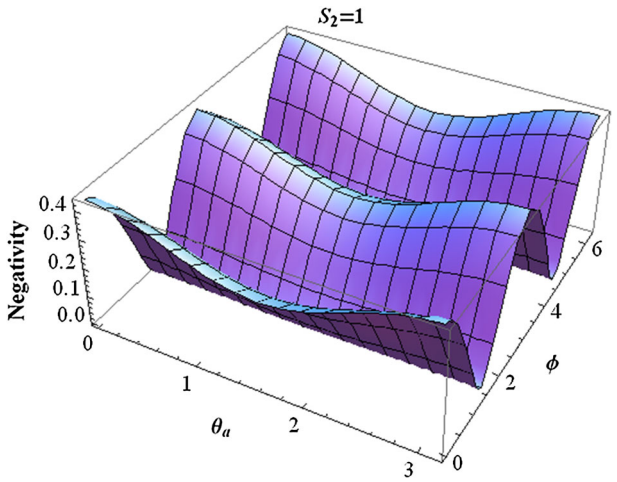

Figure 3. Variation of APDM-affected negativity with respect to θ_a and ϕ.

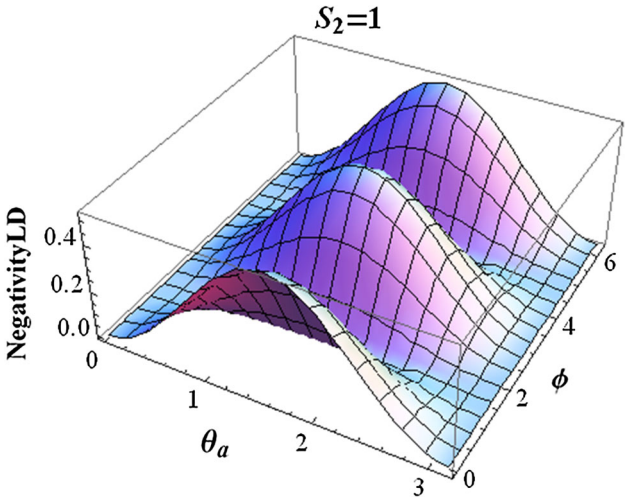

Figure 4. Variation of LD-affected negativity with respect to θ_a and ϕ.

α_a and β_a in (8) and (9) respectively. The Auger and photoabsorption amplitudes for linear molecules have the following properties [23]:

$$\langle (L_2 l_a - m_a) L | \, F_a \, | L_1 \rangle = \langle (L_2 l_a m_a) L | \, F_a \, | L_1 \rangle \quad (18)$$

and

$$\langle L_1 | \, F_p \, | L_0 1 \lambda_r \rangle = \langle L_1 | \, F_p \, | L_0 1 - \lambda_r \rangle \quad (19)$$

because the nuclear fields are cylindrically symmetric in such molecules. Using (8), (9), (18) and (19) we obtain

$$\alpha_a = 0 \quad (20)$$

and

$$\frac{\mathrm{d}\sigma(m_r = 0)}{\mathrm{d}\hat{k}_a} = 1 + 0.145\left(3\cos^2\theta_a - 1\right). \quad (21)$$

In order to show the effect of APDM on the entanglement of N_2 molecule, we plot negativity of DM (6) with respect to the directions of ejection as well as of spin quantisation of Auger electrons in figure 3.

From figure 3 we see that depending on the values of θ_a and ϕ, the Auger decay states for $S_2 = 1$ in N_2 molecule are entangled and magnitude of negativity is also effected by APDM.

Next we study how the entanglement properties are affected by LD of qudit–qubit system in N_2 molecule. LD from (7) is given by

$$\frac{\mathrm{d}\sigma^{\mathrm{LD}}}{\mathrm{d}\hat{k}_a} = \frac{\mathrm{d}\sigma(m_r = 0;\, \hat{p}(\frac{\pi}{2}, 0))}{\mathrm{d}\hat{k}_a} - \frac{\mathrm{d}\sigma(m_r = 0;\, \hat{p}(\frac{\pi}{2}, \frac{\pi}{2}))}{\mathrm{d}\hat{k}_a} = \sqrt{\frac{15}{2}}\beta_a \sin^2\theta_a \cos\phi_a. \quad (22)$$

Here $\hat{p}(\theta_p, \phi_p)$ is the direction of the polar axis of photon frame with respect to space frame. The variation of LD-affected negativity with respect to the directions of ejection as well as of spin quantisation of Auger electrons is shown in figure 4.

Thus, the variation of LD-affected negativity has a significantly different nature compared to that of APDM-affected negativity.

5. Conclusion

Nowadays one of the most ambitious technological goals is to develop quantum computer which is based on quantum superposition and entanglement. In this article we have tried to show that an Auger electron (qubit) and a doubly ionised N_2 molecule (qudit) can produce high-dimensional entanglement in Auger emission process. Entanglement between qudit system and qubit system is much stronger and lasts longer and it can be used more easily for quantum computation and information. The bipartite system is entangled for $S_2 = 1$ state of doubly ionised N_2 and there is a variation of entangled properties with respect to the directions of spin quantisation and of the ejection of Auger electrons. We have also seen that linear dichroism (LD) has a significant effect on the entangled properties of the system.

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