Pramana – J. Phys. (2020) 94:121
https://doi.org/10.1007/s12043-020-01994-x

Equation of state of PNJL model under the influence of thermal mass and magnetic field

ANJU DAHIYA and S SOMORENDRO SINGH*

Department of Physics and Astrophysics, University of Delhi, New Delhi, Delhi 110 007, India
*Corresponding author. E-mail: sssingh@physics.du.ac.in

MS received 2 March 2020; revised 29 May 2020; accepted 8 June 2020

Abstract. We present analytical results of the equation of state (EOS) described by a model of thermal quark mass and magnetic potential term introduced in Polyakov–Nambu–Jona–Lasinio (PNJL) model for two-flavour quarks. Under the influence of thermal quark mass and magnetic field term in the potential, the calculated results of EOS using the model are enhanced in a good pattern up to the temperature $T = 2.2T_c$ MeV and can follow result similar to the unmagnetised field when the temperature is increased beyond $T = 2.2T_c$ MeV. The result shows that the thermodynamic behaviour agrees well with the standard properties of quantum chromodynamics (QCD) thermodynamics and enhance the result up to $2.2T_c$ from the earlier predicted results and show the same behaviour beyond $2.2T_c$ MeV.

Keywords. Quantum chromodynamics; quark-gluon plasma.

PACS Nos 25.75.Ld; 12.38.Mh

1. Introduction

Quantum chromodynamics (QCD) describes the theory of strong interactions at non-zero density and at very high temperature. Two of the striking features of QCD are the spontaneous symmetry breaking of chiral symmetry and deconfinement. Both have non-perturbative origin. The interactions among quarks get weaker with distance, and this feature is known as asymptotic freedom [1]. Moreover, the interaction becomes stronger as the particle separation increases, which is found in the case of hadrons. A quantitative understanding of this mechanism, called confinement, is hard, even though we know the underlying theory. Due to the non-Abelian nature of QCD, an additional gluon self-interaction term is created by colour interaction of gluons. It is very difficult to solve the QCD equations on a purely mathematical ground [2]. We can use the perturbation theory at short distance due to the asymptotic property of QCD, which is to solve QCD in the strong coupling regime, relevant to nuclear physics. It can be done through numerical calculations of QCD on a discrete four-dimensional space–time lattice quantum chromodynamics (LQCD) [3]. So, deconfinement in a gauge theory is really due to spontaneous symmetry breaking of global centre symmetry. The Polyakov loop constitutes an order parameter for the centre symmetry [4]. So, basically the definition of deconfinement is nothing but a phase transition from colourless bound states to colour unbound states, i.e., from bound hadrons to unbound quarks and gluons in QCD or from glueballs to unbound gluons in pure gauge theory [5] where the issue of symmetry breaking to the restoration of chiral symmetry breaking happened. Therefore, due to the release of degrees of freedom, we expect a sharp transition from a confined hadronic phase to a deconfined phase of non-interacting colour quarks and gluons. Thus, the presence of dynamical quarks in QCD explicitly breaks the centre symmetry. Nevertheless, the Polyakov loop remains small up to a certain temperature and then increases rapidly in a very narrow temperature interval, which coincides with the rapid increase of the energy density, indicating a sudden change in the number of degrees of freedom from bound to unbound colour matter. In addition to these ideas, if we consider the entangled Polyakov–Nambu–Jona–Lasinio (PNJL) model with a Polyakov loop scale parameter that depends on the magnetic field, it is possible to obtain an earlier rise of the Polyakov loop with the increase of the magnetic field. The entanglement brings inverse magnetic catalysis in the lattice QCD calculations [6]. The NJL-type models and LQCD calculations indicate the signal for enhance-

Published online: 13 August 2020

ment of the condensate in the presence of the magnetic field though the temperature is not involved. These effects lead to an increase in the transition temperature for chiral symmetry restoration as a function of magnetic field B. Likewise, some critical behaviour present in the gauge theory seems to persist even in the presence of dynamical quarks [7]. Therefore, the prediction of the matter as deconfined state known as quark-gluon plasma (QGP) is confirmed at non-zero density and at very high temperature in the evolution of the early Universe. As the matter exists, it subsequently cools down and these free quarks condensate and form a confined matter of hadrons. So the nature of the Universe has become a complicated phenomenon during the process of phase transition. Therefore, this phase transition phenomenon is studied by the theorists and experimentalists since long and they try to identify the position of critical point in the QCD phase structure. So the deconfined state has been of great interest for the last two/three decades [8]. During these decades, many high-energy laboratories are also set up around the globe and these experiments focus on finding how the early Universe started and they try to recreate the system again. In addition to these experiments, there are a number of phenomenological methods which try to solve these complicated phenomena. In a similar fashion, we also try to solve the problem by considering a phenomenological model or thermal mass model which replaces the mass created by symmetry breaking due to the presence of magnetic field in the Lagrangian density of the PNJL model. This is because of the fact that Universe has become a highly correlated system of temperature-dependent matter. Besides this temperature effect, the matter is mainly composed of highly charged particles. Due to these two features of temperature and charge particle, we cannot neglect the temperature effect on quark mass and the magnetic field effect created by these particles in the study of QGP evolution. So the introduction of the thermal mass model and applied magnetic field is very much essential in the calculation of QCD phase structure. However, we find from many researchers the introduction of magnetic field in the PNJL model and its value is considered to be around 10^{12} GeV^2. So, in this paper, we focus on finding the equation of state using thermal quark mass and magnetised field incorporating in the PNJL model. Moreover, this introduction of thermal mass and magnetic field produces enhancement in the results of phase structure.

The paper is organised as follows: In §2, we briefly try to construct the Lagrangian density of the system incorporating the electromagnetic field in the potential so that the Lagrangian density is obtained through the external magnetised field in the PNJL model. In §3, we present all the thermodynamic properties to determine the EOS of the system symbolising the structure of QGP. In §4, we give our analytical solutions for this kind of phenomenological potential model. At last we give the conclusion with the details of information about QGP structure under the effect of thermal mass and magnetising potential model.

2. Lagrangian density of magnetised PNJL model

In this section we briefly describe the formalism of the PNJL model incorporating thermal mass manually replacing the mass created through symmetry breaking and applied magnetic field in the potential [9]. In order to study the PNJL model using this simple model, we first introduce the NJL model which is described by the following Lagrangian density [10]:

$$\mathcal{L} = \bar{\psi}(\imath\gamma_\mu D^\mu - m)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\tau\psi)^2], \quad (1)$$

where ψ represents the quark field and $D^\mu = \partial^\mu - \imath A^\mu$. G is the coupling strength of chirally symmetric four-fermion interactions. Again we incorporate the Polyakov loop potential and applied magnetic field to the Lagrangian density of NJL model. The Polyakov–Nambu–Jona–Lasinio (PNJL) model attempts to describe in a simple way the two characteristic phenomena of QCD, namely deconfinement and chiral symmetry breaking. To enhance the characteristic features of the PNJL model, we introduce the quasimodel of quark and incorporate the external magnetic field in the PNJL potential model. In the presence of thermal quark mass and magnetic field, the early characteristic feature of the PNJL model is re-examined. Now the Lagrangian density in the presence of thermal quark mass and applied magnetic field is defined as [11,12]

$$\begin{aligned}\mathcal{L} =& \bar{\psi}(\imath\gamma_\mu D^\mu - m)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\tau\psi)^2] \\ & - U(\phi^*, \phi, T) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},\end{aligned} \quad (2)$$

where

$$\begin{aligned}U(\phi^*, \phi, T) =& -\frac{1}{2}b_2(T)\phi^*\phi \\ & + b_4(T)\ln[1 - 6\phi^*\phi + 4(\phi^{*3} + \phi^3) \\ & - 3(\phi^*\phi)^2]\end{aligned} \quad (3)$$

in which

$$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}, \quad b_4(T) = b_4\frac{T_0^3}{T^3}, \quad (4)$$

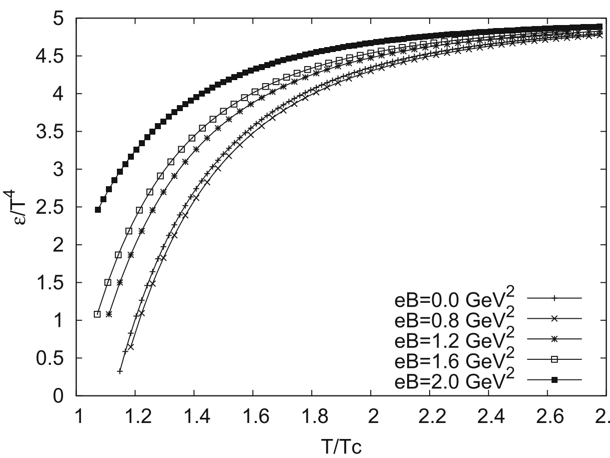

Figure 1. Energy density as a function of temperature when $eB = (0,\ 0.8,\ 1.2,\ 1.6,\ 2.0)$ GeV^2.

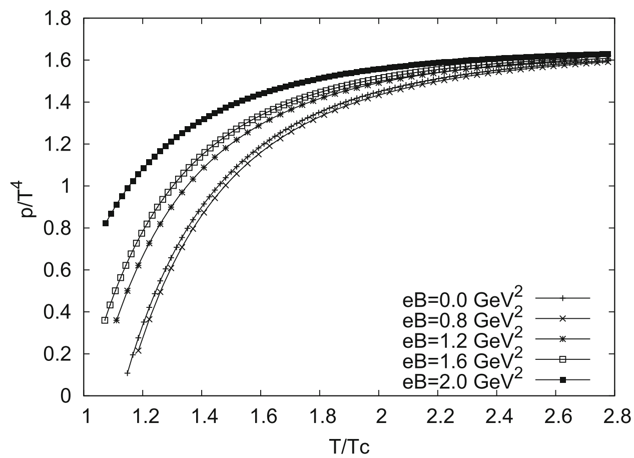

Figure 2. Pressure as a function of temperature when $eB = (0,\ 0.8,\ 1.2,\ 1.6,\ 2.0)$ GeV^2.

where the coefficient parameters have the same value as considered earlier by Ratti's PNJL model without magnetic field and these are $a_0 = 3.51$, $a_1 = -2.41$, $a_2 = 15.22$, $b_4 = -1.75$. The effective potential $U(\phi, \phi^*, T)$ in the Lagrangian density governs the dynamics of Polyakov loop of field $\phi = Tr_c(L)/3$ and its conjugate $\phi^\dagger = Tr_c(L^\dagger)/3$ [13]. The potential $U(\phi, \phi^*, T)$ obeys the following general features that satisfy the $Z(3)$ centre symmetry like the pure gauge QCD Lagrangian. In addition, it accords with the lattice prediction for the behaviour of the Polyakov loop of field ϕ and the potential U having an absolute minimum at $\phi = 0$ at small temperature, while above the critical temperature, the minimum value of $\phi = 0$ would be shifted to a finite value of ϕ. When temperature T tends to infinity, then the field ϕ goes to unity. So we have chosen the simplest possible forms for $U(\phi, \phi^*)$, namely polynomial in ϕ, ϕ^*. So, a precision fitting the parameter (a_i, b_i) is performed to check the result produced by the lattice with the incorporation of thermal mass and magnetic field. The results of the combined fit are shown in figures 1–4 in which the critical temperature is fixed as 170 MeV in order to agree with the other result produced by [14]. Now the thermodynamical potential for the two-flavour quark in the presence of thermal quark mass and magnetic field is written as [15]

$$\Omega(\phi, \phi^*; T, \mu) = U(\phi, \phi^*, T) + \frac{m(T) - m_0}{4G_1}$$
$$- 2N_c N_f \int_\pi d^3 p E_p \frac{2}{\pi^3} - 2N_f eBT$$
$$\times \int d^3 p \frac{2}{\pi^3} [Tr \ln(1 + L e^{(E_p - \mu)/T})$$
$$+ Tr \ln(1 + L^\dagger e^{-(E_p - \mu)/T})]$$

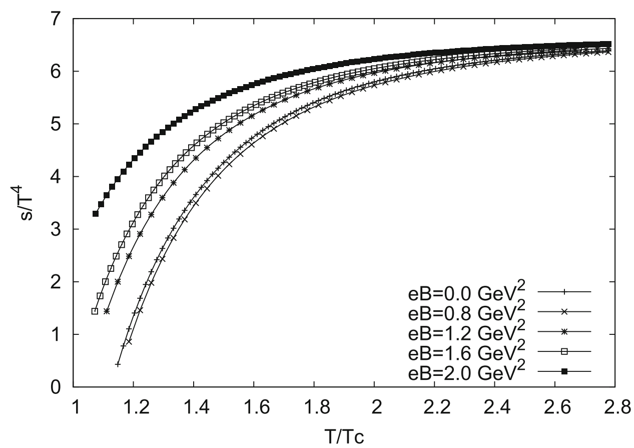

Figure 3. Entropy as a function of temperature when $eB = (0,\ 0.8,\ 1.2,\ 1.6,\ 2.0)$ GeV^2.

$$- \frac{3(qB)^2}{2\pi^2} \left[\zeta'(-1, x_f) - \frac{1}{2}(x_f^2) \right.$$
$$\left. - x_f \ln(x_f) + \frac{x_f^2}{4} \right] \quad (5)$$

in which

$$x_f = \frac{M_f^2}{2q_f B}, \quad \zeta'(-1, x_f) = \left. \frac{d\zeta}{dz} \right|_{z=-1},$$

where $\zeta(z, x_f)$ is the Reimann–Hurwitz zeta function. In the above formula $E_p = \sqrt{(p^2 + m^2(T) + 2neB)}$ is the Hartree single quasiparticle energy calculated with the thermal quark mass [16]. The thermal mass is defined as

$$m^2(T) = 16\pi \gamma_{q,g} \alpha_s(E_p) T^2, \quad (6)$$

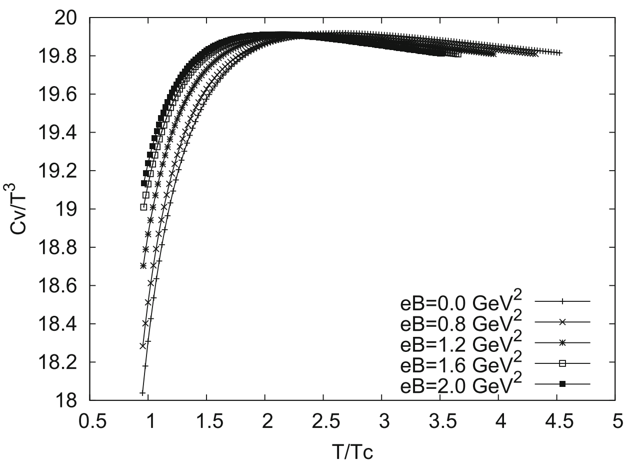

Figure 4. Specific heat as a function of temperature when $eB = (0,\ 0.8,\ 1.2,\ 1.6,\ 2.0)$ GeV^2.

where

$$\gamma_{g,q} = \frac{\sqrt{2(\gamma_g^2 + \gamma_q^2)}}{\gamma_g \gamma_q}$$

with $\gamma_g = 6\gamma q$ and $\gamma_q = \frac{1}{6}$.

These values are taken from the earlier calculation of QGP evolution phenomena of thermal quark mass. Depending on the results of QGP droplet formation, the parameter is fixed with the stable QGP droplets. $\alpha_s(E_p)$ is the running coupling strength which is a decreasing function of the magnetic field and it is also defined in terms of the energy cut-off value and QCD parameter Λ as

$$\alpha_s(E_p) = 4\pi/\left[(33 - 2n_f)\ln\left(1 + \frac{E_p^2}{\Lambda^2}\right)\right].$$

E_p is the energy cut off to truncate the divergence of the system and Λ is the QCD parameter taken as 150 MeV. In the present work, we use mean-field limit which implies $\phi = \phi^*$ in which we take $\mu = 0$. Considering the external magnetic field effect we get the QCD phase diagram structure by calculating the bulk thermodynamic properties of the system. The introduction of thermal mass and magnetic field enhances the chiral condensate due to the opening of the gap between the Landau levels, increasing the low-energy contributions to the chiral condensate [17]. However, it leads to the suppression of the condensate due to strong screening effect of the gluon interactions in the region of the low momenta relevant for the chiral symmetry breaking mechanism [18]. The suppression of quark condensate is also known as inverse magnetic catalysis (IMC) and this study has been done by many [19] in the increasing function of the Polyakov loop. So, many successful mechanisms are explained in this regard for the exploration of QCD phase diagram [20]. These mechanisms are used for introducing the initial temperature T_0 in the Polyakov loop potential [21,22]. Moreover, the PNJL model couples with G_s connecting through the running coupling constants [23,24]. Due to this, it can decrease the effect of magnetic field allowing to contribute the effect of α_s. So we also focus on studies to determine the properties of QCD considering the Polyakov loop through the magnetic field and thermal mass in which the coupling constant is involved.

3. Thermodynamic parameters of the magnetised PNJL model

We briefly discuss the bulk thermodynamic properties like energy density, pressure, entropy, specific heat, susceptibility etc., which can be derived from the Gibbs potential incorporated by the thermal mass and magnetic field in the PNJL model. Theoretically, we take the derivative of Gibb's potential with respect to temperature for finding the energy density. So it is defined as the standard definition of energy density.

$$\epsilon(T,\mu)|_{\mu=0} = -T^2 \frac{\partial(\frac{\Omega(T,\mu)}{T})}{\partial(T)}\bigg|_{\mu=0}. \quad (7)$$

From this, we get the energy density at zero chemical potential and it is equivalent to $\epsilon = 3P(T,0)$. However, the pressure is related to the function of temperature T and chemical potential μ [25]. It is therefore given by the following relation:

$$P(T,\mu)|_{\mu=0} = -\Omega(T,\mu)|_{\mu=0}. \quad (8)$$

It implies that the pressure of the system is the negative of thermodynamic potential at finite temperature and chemical potential. So, we can obtain the pressure in terms of temperature and chemical potential μ fixing as 0. We then obtain the entropy of the system as it is important to determine the equilibrating state and disordered condensate states of the system. Moreover, some researchers reported about determining the order of phase transition on the basis of first and second derivatives of the Gibbs potential energy. Its value at zero chemical potential in terms of energy density is equal to

$$s(T,\mu=0) = \frac{4}{3}\frac{\epsilon}{T}.$$

However, the entropy contribution is mainly dependent on temperature. So we define the entropy in the following expression as a function of temperature:

$$s(T,\mu)|_{\mu=0} = -\frac{4}{3} T \frac{\partial(\frac{\Omega(T,\mu)}{T})}{\partial(T)}\bigg|_{\mu=0}. \quad (9)$$

We further calculate specific heat of the system, which is defined as the rate of change of energy density with temperature at constant volume.

$$C_v(T,\mu)|_{\mu=0} = \left.\frac{\partial\epsilon(T,\mu)}{\partial T}\right|_{\mu=0}. \quad (10)$$

The specific heat increases with increasing temperature and reaches a peak at $2.0T_c$. It decreases slightly for a short range of temperature. We look forward quark susceptibility χ_q at zero quark chemical potential and determine the effect of thermal quark mass and magnetic field. To calculate susceptibility, we use the following relation:

$$\chi_q(T,\mu)|_{\mu=0} = -\left.\frac{\partial^2\Omega(T,\mu)}{\partial\mu^2}\right|_{\mu=0}. \quad (11)$$

The calculation shows that χ_q is the response of the thermodynamic potential $\Omega(T,\mu)$ at zero chemical potential. In addition to susceptibility, we can obtain Taylor expansion coefficient of the Gibbs potential $\Omega(T,\mu)$ around a fixed temperature and at zero chemical potential. The Taylor expansion coefficients at temperature T give the value of second-order coefficient C_2 and fourth-order Taylor coefficients C_4. To get these coefficients, we perform Taylor expansion of the Gibbs potential incorporated by the thermal mass and magnetised PNJL model. Now we have

$$C_n(T,\mu)|_{\mu=0} = \frac{1}{n!}\left.\frac{\partial^n(p(T,\mu)/(T^4))}{\partial(\mu/T)^n}\right|_{\mu=0}. \quad (12)$$

From the above definition we get the corresponding second- and fourth-order coefficients. We represent the fitted values of Taylor's coefficients for a few values of temperature to show the dependence of these coefficients on the temperature and number of terms in the expansion of polynomial about zero chemical potential. The calculation up to the fourth order is kept as limitation of numerical accuracy and time factor. So we have to choose a maximum term of the polynomial up to the fourth order of zero chemical potential. Then we look for the comparison of these two coefficients with the earlier results. These are demonstrated in figures 6 and 7 in comparison to the results produced by other researchers as a function of temperature. Thus, we obtain all the thermodynamic properties of strongly interacting matter incorporated by the thermal mass and magnetised PNJL model.

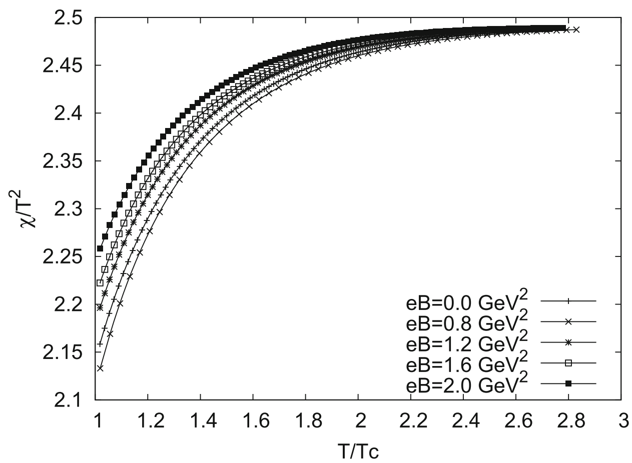

Figure 5. The susceptibility as a function of temperature when $eB = (0,\ 0.8,\ 1.2,\ 1.6,\ 2.0)\ \mathrm{GeV}^2$.

4. Results

In order to discuss the QGP phase structure, we look at the bulk thermodynamic parameters at various temperatures for the thermal mass which is considered manually to represent the mass formed by symmetry breaking due to the presence of magnetic field applied in the PNJL model.

In figures 1–5, we plot energy density, pressure, entropy, specific heat and susceptibility with respect to temperature at zero chemical potential ($\mu = 0$) with the thermal mass and magnetised PNJL model. In addition to these figures, we extend to draw more figures, different from the figures of thermodynamic parameters in order to describe the phase structure. From the figures, we can see that the values of energy density at critical temperature is different for different values of eB. As eB increases, ϵ/T^4 is found to increase. It means that the contribution is mainly dominated by free colour quarks–antiquarks and gluons. When the temperature is raised beyond the critical temperature, there is an exponential rise in the energy density of the system and it becomes constant at temperature around $2.2T_c$. During this change of energy density, the matter completely consists of colour-free quarks, antiquarks and gluons. On the other hand, when the temperature is below the critical temperature $T = 170$ MeV the energy density is much lower and the behaviour seems to change a lot. This is not shown here and the model shows the formation of an ambiguous matter of probable mixed phase for a short temperature range.

Moreover, if we keep on looking for the influence of the thermal mass and magnetic field, then the energy density is improved from the result without the magnetic field in the Lagrangian density of the PNJL model. The increase is very little in terms of magnitude when we consider the magnetic field strength as small and it suddenly enhances when the field strength reaches eB =1.2 GeV^2. Besides this, there is another effect on energy density due to the variation of temperature. At temperature below $2.2T_c$ the energy density is much larger than the result without magnetic field. So the model of thermal mass and magnetic field has larger impact between the critical temperature and temperature below $2.2T_c$.

Depending on the temperature and the temperature above $2.2T_c$, the behaviour of the system is similar to that of the other researchers' works. It means that the effect of thermal mass and magnetic field does not show much change in the energy density with increasing temperature beyond $2.2T_c$. It indicates that there is no minimum effect of the thermal mass and magnetic field beyond $2.2T_c$ whereas at temperature below $2.2T_c$, when there is an enhancement in energy density, the quarks and antiquarks are affected by the magnetic potential and thermal mass. It implies that in this temperature zone, the symmetry breaking in the presence of magnetic field put forward the creation of mass effectively and enhances the energy density of the system. This is one contribution of thermal mass in the study of energy density to explain the QCD phase structure through the thermal mass and magnetic field approach to the PNJL model.

Similarly, the behaviour of the pressure is the same with the energy density up to the temperature around $2.2T_c$ and obviously, pressure is equivalent to the energy density as $P = \frac{\epsilon}{3}$ in terms of magnitude. The behaviour of pressure is the same with the energy density and only the difference of magnitude of energy density is found for every increased value of eB. It implies that pressure increases up to the temperature $2.2T_c$ after incorporating the thermal mass and magnetic field in the PNJL model. It then reaches a constant value which matches with other work at high temperature beyond $2.2T_c$. This shows that the effect of thermal mass and magnetic field is visible during the intermediate zone of temperature T_c–$2.2T_c$. The visible output in the pressure is correspondingly due to enhancement in the interaction of charge particles boosted by the magnetic field and effective thermal mass of these charged particles.

In figure 3, we plot entropy as a function of temperature and the result shows enhancement in the entropy compared to the result obtained by other researchers' work. The enhancement in the result is correspondingly obtained as found in the case of energy density and pressure. Accordingly, the entropy is enhanced because of enhancement in energy density and pressure. The result is almost similar when the temperature is increased beyond $2.2T_c$ in which there is less effective influence of thermal mass and magnetic field as already stated in energy density and pressure. This shows that the thermal mass and magnetised potential has influence up to $2.2T_c$ and beyond this value the effect is negligible. Moreover, entropy measurement indicates the phase transition of QGP as reported in many literatures that first and second order can be obtained by the first and second derivatives of Gibbs' potential energy. However, it also describes the equilibrating state and disordered condensates of the system.

Again in figure 4 we show the variation of specific heat with temperature. The result exactly follows the standard QCD pattern up to $T = 2.2T_c$. Beyond this temperature, the system begins to suppress because of the thermodynamic change with the increasing magnetic field. It may be due to the condensation of quark–antiquarks and increasing critical temperature due to the magnetic effect.

Then we calculate susceptibility for these incorporated thermal mass and magnetised PNJL model. The susceptibility result of the model calculation is plotted in figure 5. The figure follows exactly in similar direction and the result is improved slightly for small magnetic field from the result of unmagnetised fields. Subsequently, for the increase of magnetic field eB, say from 0 GeV^2 to 2.0 GeV^2, the susceptibility is correspondingly increased uniformly up to $2.2T_c$. Beyond this temperature, the effects of magnetic field and thermal mass seem to be minimised and the value of susceptibility is the same as the values from the works of earlier researchers and our earlier value [26]. It shows that at higher temperature, say beyond $2.2T_c$, there is no change in the susceptibility for the entire range of magnetic field and it is almost the same with the earliest susceptibility works. The pattern is the same for all the thermodynamic parameters with this model of thermal mass and magnetic field. The enhancement is found for the change of eB, say from 0.0 to 2.0 GeV^2, with the increase of temperature up to $T = 2.2T_c$ under the effect of thermal mass and magnetic field. It means that the effect of thermal mass and magnetic field is saturated after reaching the temperature $2.2T_c$ and no increment is found in the susceptibility beyond this temperature. Obviously, the improvement in the intermediate temperature, say (T_c–$2.2T_c$), is due to the effect of thermal mass and magnetic field present in the PNJL model. It implies that the model shows more respective linear function with the temperature up to $T = 2.2T_c$ with the introduction of thermal mass and magnetic field. The improvement in susceptibility indeed shows the picturisation of good enhancement in the phase structure with the thermal mass and magnetised PNJL model. Beyond $2.2T_c$, the magnetic field in the PNJL model of the corrected Gibbs potential results in magnetic catalysis called inverse magnetic catalysis (IMC) obtained

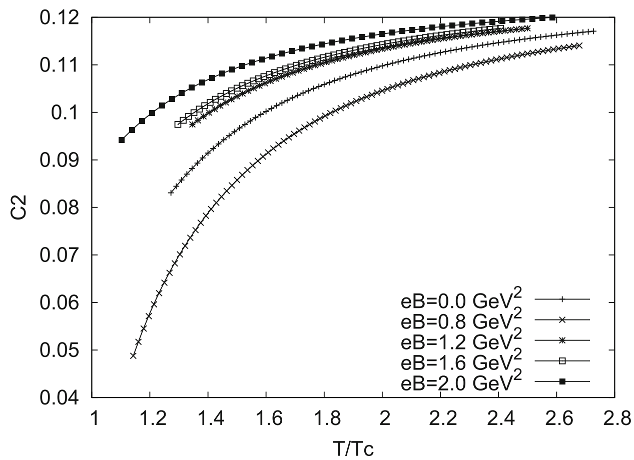

Figure 6. The Taylor coefficient of second-order function of temperature when $eB = (0,\ 0.8,\ 1.2,\ 1.6,\ 2.0)$ GeV^2.

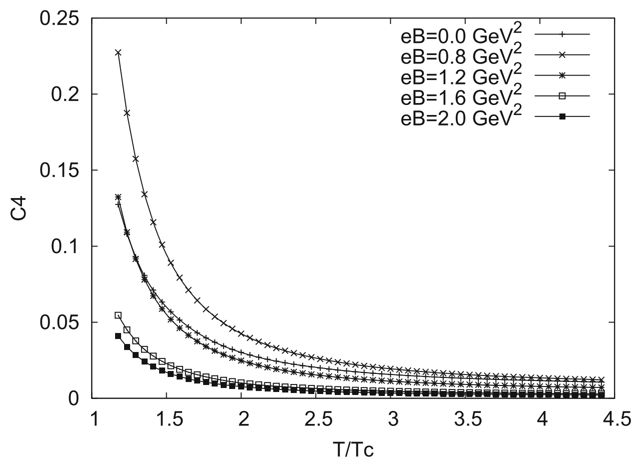

Figure 7. The Taylor coefficient of order four as a function of temperature when $eB = (0,\ 0.8,\ 1.2,\ 1.6,\ 2.0)$ GeV^2.

due to the suppression of quark condensation in the system [27,28].

In figures 6 and 7 we plot the first- and second-order coefficients of Taylor expansion. The calculation of these coefficients enable to control the continuum and volume extrapolation as the terms are expanded to a large number. Moreover, these coefficients are not affected by the polarity of any thermodynamic parameter as these are normally defined by the expectation value. So the calculated results are almost the same as the predicted results of the PNJL model concerning QCD thermodynamics at zero chemical potential.

Figure 6 shows the second-order coefficient for various values of the magnetic field with the change of temperature. The coefficient keeps on increasing with variation of temperature for lower values of magnetic field eB up to 1.2 GeV^2. Then by increasing the magnetic

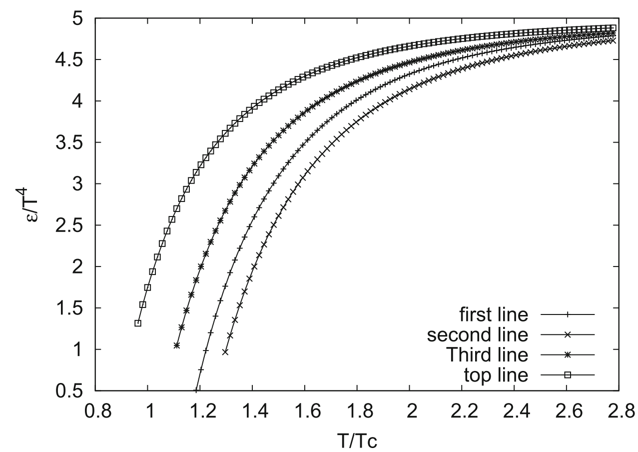

Figure 8. Energy density vs. temperature. Fourth line: $eB = 0.8$ GeV^2, $M \neq 0$; third line: $eB = 0.0$ GeV^2, $M \neq 0$, second line: $eB = 0.8$, $M = 0$ and first line: $eB = 0$, $M = 0$.

field, the coefficient starts increasing reaching up to the value where the closest magnitudes of magnetic field $eB = 1.6$ GeV^2 are almost similar and slightly more in the case of $eB = 2.0$ GeV^2. These increments are obtained with the variation of temperature up to $2.2T_c$, and become almost constant with the increase of temperature beyond $2.2T_c$. At this point, the effects of thermal mass and magnetic field disappear in the Taylor coefficients. Moreover, in calculating the fourth coefficient the variation of Taylor coefficient is the same with the earlier calculations. The results are compared with the corresponding result of fourth coefficient produced by Ratti's model, considered without the effect of thermal mass and magnetised potential. It indicates that our sample model with the consideration of thermal mass and magnetic field has shown improvement in all the thermodynamic parameters and therefore the model is well fitting to describe the QCD phase structure.

In figure 8, the characteristic feature of energy density at one particular magnetic field and finite mass is plotted with the incorporation of the thermal mass and magnetic field. It shows large effect with the inclusion of the magnetic field and thermal mass whereas, if we consider their contributions separately, then the contribution is dominated by the thermal mass over the magnetic field. It can be observed from the second and third lines of the figure. We also plot when both thermal mass and magnetic field are zero. Adding either of these two parameters shows enhancement in the energy density with the increase of temperature. This shows that the model which includes the thermal mass and magnetic field in the potential exhibits overall results. Figure 9 shows the variation of the QCD field with the magnetic field strength at different finite temperatures.

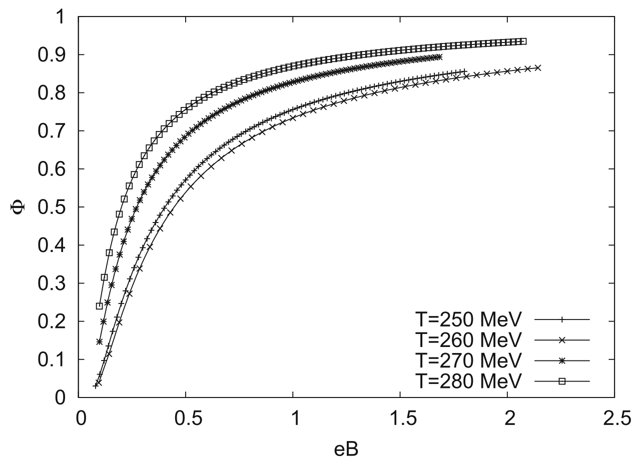

Figure 9. ϕ as a function of eB when $T = (250, 260, 270, 280)$ MeV.

The figure is absolutely in the direction of QCD result. Only the field intensity is dependent on the temperature. As the temperature is raised, the intensity of the field is increased from the predicted lattice result [29]. Thus, thoroughly studying the thermodynamical properties of the PNJL model helps us to understand the PNJL model quite accurately. So the PNJL model predicts the critical temperature as $T_c = 170$ MeV for phase transition and the temperature seems to be increased as the magnetic field strength increases.

5. Conclusion

The NJL model has been successful in describing the results of two/three flavour of QCD thermodynamic properties. The result is in agreement with the results obtained from other calculations when $m_q = 0$ and $eB = 0$ are considered. Now the Polyakov loop which is an extended NJL model can also reproduce almost the same two/three flavour thermodynamic results, viz. energy density, pressure and other properties. The coupling of quarks to the Polyakov loop produces a statistical suppression in thermodynamical properties of the one- and two-quark flavour model. We successfully manage to produce the result by introducing thermal mass and magnetic field in the PNJL model. The behaviour of quark condensates under the influence of an external magnetic field is studied within the PNJL model [30]. The chiral and deconfinement transition temperature increases in the presence of an external magnetic field, although the deconfinement transition temperature suffers a much weaker effect. It behaves differently with the magnetic field and also shows enhancement when considered with the thermal mass and applied magnetic field. The deconfinement temperature suffers just a small increase compared to the huge increase of the chiral transition temperature within the PNJL model. We then investigate its phase structure at finite temperature T and zero chemical potential. In the presence of magnetic field in the PNJL model, quarks condensate more and magnetic catalysis may occur in the system [31,32]. While evaluating and plotting the bulk thermodynamic properties at finite temperature and zero chemical potential, the result exactly follows similar pattern of other works at high temperature, say above $2.2T_c$. It implies that the effect of thermal mass and magnetic field is minimum at high temperature. It is due to the fact that high energy phenomena can dominate the effect of magnetic field. On other hand, the effect of thermal mass and magnetic field is clearly visible at low temperature and the result of our model is enhanced from the result produced without magnetic field and thermal mass. It indicates that the role of thermal mass and magnetic field dominates at the intermediate temperature zone, say T_c–$2.2T_c$ where free quarks, antiquarks are slightly coupled by the magnetic field. So all the bulk thermodynamic parameters can be reproduced as a qualitative measurement with the change of temperature. These qualitative results are already predicted by many models of demagnetisation. The result is reproduced and enhanced by using the thermal mass and magnetic field in the Lagrangian of Polyakov loop in the range of temperature (T_c–$2.2T_c$). So we consider SU(2) environment to have significant contribution of the magnetic field [33]. It concludes that the PNJL model gives QGP around the temperature $T = (160–170)$ MeV with the enhancement of thermodynamic parameters in the intermediate range of temperatures (T_c–$2.2T_c$). However, the inclusion of thermal mass and magnetic field boosts the transition temperature slightly from $T_c = 170$ MeV and it can be around $T_c = (170–200)$ MeV. Yet we assumed the transition temperature as a standard value, say $T = 170$ MeV which is used in most of the calculations of QCD phase structure and using this transition temperature, the calculated results with thermal mass and magnetised field show improved behaviour in all the thermodynamic parameters in comparison with the other models of non-thermal mass and non-magnetised field [34].

Within the PNJL model, the temperature of the deconfinement transition is almost insensitive to the magnetic field when compared with the chiral transition temperature. The transition temperature increases slightly with the magnetic field and no evidence for a disentanglement of both phase transitions was found, at least for magnetic fields up to 2.0 GeV^2 [35,36]. Similar arguments in this direction have been initiated by QCD lattice that the magnetic field would be dependent on initial temperature so that IMC produced by gluon sector can be recovered in order to reproduce the correct behaviour of transition temperature and QCD phase structure at large magnetic field as well as temperature [37]. Nevertheless, the same mechanism also can give rise to a first-order phase transition at a quite low magnetic field. So the effective model of QCD has accounted a lot in the features of both chiral symmetry breaking and deconfinement by using the effective potential obtained in pure gauge sector [38]. So the incorporation of thermal mass and magnetic field in the PNJL model can describe significantly the formation and the structure of the QCD phase diagram.

Acknowledgements

The authors thank R Ramanathan for his suggestions in fulfilling the manuscript's writing and for his useful discussions in preparing the manuscript. The author, Anju thanks the University and CSIR, Delhi for providing financial support in terms of Junior Research Fellowship for this work.

References

[1] Y Nambu and G J Lasinio, *Phys. Rev. D* **122**, 345 (1996)
[2] P N Meisinger and M C Ogilvie, *Phys. Lett. B* **379**, 163 (1996)
[3] S Lawley, W Bentz and M C Ogilvie, *J. Phys. G* **32**, 667 (2004)
[4] H Hansen, W M Alberico, A Beraudo, A Molinari, M Nardi and C Ratti, *Phys. Rev. D* **75**, 065004 (2007)
[5] K Fukushima, *Phys. Rev. D* **77**, 114028 (2008)
[6] M Ferreira, P Costa and C Providencia, *Phys. Rev. D* **89**, 036006 (2014)
[7] G Aarts, *Pramana – J. Phys.* **84**, 787 (2015)
[8] P Costa, C A Sousa, M C Ruivo and Y L Kalinovsky, *Phys. Lett. B* **647**, 431 (2007)
[9] S Roesser, C Ratti and W Weise, *Phys. Rev. D* **75**, 034007 (2007)
[10] K Fukushima, *Phys. Lett. B* **591**, 277 (2004)
[11] C Ratti, M A Thaler and W Weise, *Phys. Rev. D* **73**, 014019 (2006)
[12] P Cheng *et* al, *Phys. Rev. D* **94**, 123014 (2016)
[13] F Karsch, *AIP Conf. Proc.* **842**, 20 (2006)
[14] W Fu, Z Zhang and Y Liu, *Phys. Rev. D* **77**, 014006 (2008)
[15] A I Chowdhury, A Bandyopadhyay, P K Roy and S Sarkar, *Phys. Rev. D* **99**, 094028 (2019)
[16] S S Singh and Y Kumar, *Can. J. Phys.* **92**, 31 (2014)
[17] S P Klevansky, *Rev. Mod. Phys.* **64**, 649 (1992)
[18] P Wang, A W Thomas and A G Williams, *Phys. Rev. C* **75**, 045202 (2007)
[19] A M Polyakov, *Phys. Lett. B* **72**, 477 (1978)
[20] A Gocksch and M Ogilvie, *Phys. Rev. D* **31**, 877 (1985)
[21] M Fukugita and A Ukawa, *Phys. Rev. Lett. B* **57**, 503 (1986)
[22] S K Ghosh, T K Mukherjee, M G Mustafa and R Ray, *Phys. Rev. D* **73**, 114007 (2006)
[23] R V Gavai, S Gupta and S Mukherjee, *Pramana – J. Phys.* **71**, 487 (2008)
[24] C Ratti, S Roessner and W Weise, *Phys. Lett. B* **649**, 57 (2007)
[25] G S Bali, F Bruckmann, G Endródi, Z Fodor, S D Katz and A Schäfer, *Phys. Rev. D* **86**, 071502 (2012)
[26] S S Singh and G Saxena, *Pramana – J. Phys.* **88**: 85 (2017)
[27] P R Subramanian, H Stocker and W Greiner, *Phys. Lett. B* **173**, 468 (1986).
[28] M G Alford, K Rajagopal and F Wilczek, *Nucl. Phys. B* **537**, 443 (1999)
[29] Y Hatta and T Ikeda, *Phys. Rev. D* **67**, 014028 (2003)
[30] M Ferreira, P Costa, D P Menezes, C Providencia and N N Scoccola, *Phys. Rev. D* **89**, 016002 (2014)
[31] D H Rischke, D T Son and M A Stephanov, *Phys. Rev. Lett. B* **87**, 062001 (2001)
[32] D Blaschke, S Fredriksson, H Grigorian, A M Oztas and F Sandin, *Phys. Rev. D* **72**, 065020 (2005)
[33] N D Ippolito, G Nardulli and M Ruggieri, *J. High Energy Phys.* **0704**, 036 (2007)
[34] Y Aoki, Z Fodor, S D Katz and K K Szabo, *J. High Energy Phys.* **0601**, 089 (2006)
[35] K Fukushima, *Phys. Rev. D* **69**, 097502 (2009)
[36] D Ebert, Y L Kalinovski, L Munchow and M K Vilkov, *Int. J. Mod. Phys.* **8**, 1295 (1993)
[37] Z F Cui, Y L Du and H S Zong, *Int. J. Mod. Phys.* **29**, 1460232 (2014)
[38] C R Allton *et al*, *Phys. Rev. D* **71**, 054508 (2005)