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Hadron energy estimation from atmospheric neutrino events

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Abstract. The iron calorimeter (ICAL) at India-based neutrino observatory (INO) is designed to mainly observe the muons produced in the charged current interactions of atmospheric muon neutrinos and antineutrinos. The track of the muon is reconstructed using the hits they produce in the detector. From this track, the charge, the energy and the direction of the muon are estimated, which are used to do oscillation physics analysis. In a large fraction of events, a number of hadrons are also produced in addition to the muons. The charged hadrons also leave hits in the detector which can be utilised to estimate the hadron energy. In this work, we generate atmospheric neutrino events using two different neutrino event generators: NUANCE and GENIE. The generated events are passed through the Geant4 simulator of ICAL. In each case, we study the relation between hadron hits, defined to be the difference between the total number of hits and the muon track hits, and the hadron energy. We find that a non-negligible number of baryons are produced in atmospheric neutrino interactions. For $E_{\text{had}} < 5$ GeV, almost all the hadron energy is carried by these baryons. Finally, we formulate a procedure by which the hadron energy can be estimated from the number of hadron hits.

Keywords. Atmospheric neutrinos; energy of hadrons; energy of baryons.

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1. Introduction

The iron calorimeter (ICAL) at the India-based neutrino observatory (INO) is a proposed giant magnetised neutrino detector. Its primary goal is to study the interactions of the atmospheric muon neutrinos and antineutrinos. It aims to determine neutrino mass hierarchy and also add to the precision of neutrino oscillation parameters [1].

ICAL has a mass of 50 ktons and it consists of three modules, each of which has dimensions 16 m × 16 m × 14.4 m. Each module contains 151 layers of 5.6 cm thick iron plates interspersed with resistive plate chambers (RPCs). The iron plates act as targets to atmospheric neutrinos. The charged current (CC) interactions of ν_μ and $\bar{\nu}_\mu$ produce muons. Because of the 1.3 T magnetic field in the iron plates, ICAL can determine the sign of the muon charge and distinguish between the interactions of ν_μ and $\bar{\nu}_\mu$. Approximately 30,000 RPCs are used in the detector. Each RPC has a surface area of 1.84 m × 1.84 m and carries a potential difference of 10 kV across its electrodes. Copper strips, of width 1.96 cm, are laid in parallel on the top (bottom) surface of the RPC in X (Y) direction. The vertical direction is taken to be the Z-direction. A charged particle, passing through the RPC, creates an avalanche. The signal due to this avalanche is read by the X and Y copper strips. These readings, together with the RPC layer number, give (X, Y, Z) coordinates of a 'hit'.

The muons produced in the detector, being minimum ionising particles, pass through a number of iron layers and RPCs. Based on the hits produced by the muon in different RPCs, it is possible to reconstruct the track of the muon. The hits which make up the track are defined as 'track hits'. From the reconstructed track, the charge, the energy and the direction of the muon are estimated with good precision. In the first set of physics studies, the physics capabilities of ICAL were estimated using the kinematical information of only the muons [2–4].

In addition to a muon, an atmospheric neutrino interaction also produces a set of hadrons (mesons and baryons). The energy of these hadrons can be estimated based on the hits produced by the charged hadrons in the detector. However, we need to develop a different technique for this estimation. Most of the time, the hits due to the hadrons cannot be reconstructed into tracks because (a) the energy of a typical hadron is much smaller than the energy of the muon and (b) the hadrons can be absorbed by the detector nuclei. Thus, the inclusion of the hadron energy in the kinematic reconstruction of an event poses a great challenge. The first estimate of the hadron energy of atmospheric neutrino events in ICAL was done in ref. [5].

Various efforts were made to estimate the hadron energy in ICAL [6,7]. However, in these efforts, a charged pion of known energy is injected into Geant4 simulator and the corresponding hit pattern was studied. This process was repeated for pions of different energies and in different directions with respect to the zenith. Through these simulations, a correlation between the pion energy and the number of hits was established and the resolution in pion energy was estimated. It was assumed that these correlations and the resolutions will hold for all hadrons produced in atmospheric neutrino interactions. The physics capabilities of ICAL are re-calculated using three kinematical variables: muon energy, muon direction and the hadron energy, estimated from the pion simulations mentioned above. With this '3D analysis', it was shown that the physics capabilities of ICAL detector are enhanced [8,9].

In this work, we performed a systematic study of particle production in atmospheric neutrino events. We found that a significant number of baryons are produced in a large fraction of these events. For hadron energy less than 5 GeV, these baryons carry almost all the hadron energy. Therefore, we believe that the hit pattern produced by an isolated, single charged pion does not represent the hit pattern produced by the hadrons in an atmospheric neutrino event properly. We establish a correlation between the hits produced by the baryons and the baryon energy in atmospheric neutrino events and show that this correlation is very different from the correlation found for pions in ref. [7]. We then obtain a relation between the number of hits produced by all the hadrons in an atmospheric neutrino event and the hadron energy. As mentioned before, such a study was done before in ref. [5]. However, we include two features which were not present in ref. [5], which will be discussed later.

To obtain a realistic correlation between the hadron energy in atmospheric neutrino events and the number of hits in ICAL, we used the following procedure: We generated 100 years of un-oscillated atmospheric neutrino events in ICAL through two different neutrino event generators: NUANCE [10] and GENIE [11]. We performed a full Geant4 simulation of all the generated ν_μ-CC and $\bar{\nu}_\mu$-CC events. We selected those events for which at least one muon track is reconstructed. We obtained 489124 (452632) events from NUANCE (GENIE) simulation. The difference in the two numbers is mainly due to the different cross-sections used in the two event generators. For these events, we isolated the hits produced by the hadrons by eliminating the muon track hits from the total number of hits. We used this hadron hit bank information to estimate the hadron energy and the energy resolution.

In the analysis below, we use only the total number of hits in an event but not the detailed information of the coordinates of each hit. This helps us in avoiding problems related to 'ghost hits' [6]. Ghost hits arise in situations where two or more charged hadrons pass through different regions of the same RPC. If one hadron passes through the point (X_1, Y_1) and the other through (X_2, Y_2), then we get signals from the two 'X' strips at X_1 and X_2 and from the two 'Y' strips Y_1 and Y_2. Such a signal could have arisen due to the particles passing through the points (X_1, Y_2) and (X_2, Y_1). Such points are referred to as 'ghost hit' points. In our analysis, we avoid this problem by taking the number of hits in an RPC to be the larger of the two numbers: number of X strips with a signal and number of Y strips with a signal.

2. Baryons in atmospheric neutrino events

The analysis presented in this section is also based on 100 years of atmospheric neutrino events generated by NUANCE. In the last section, we shall do a systematic comparison of the results obtained using NUANCE with those obtained by using GENIE. As mentioned in the Introduction, we believe that doing a simple simulation of isolated charged pions does not reflect the true picture of hadron production in atmospheric neutrino events. This is so because a fair number of baryons are produced in these events and they carry a non-negligible fraction of the hadron energy. This is illustrated in figure 1. In this figure, the two variables E_{had} and E_{baryons} are obtained from NUANCE by the following definitions:

$$\begin{aligned} E_{\mathrm{had}} &= E_\nu - E_\mu, \\ E_{\mathrm{baryon}} &= E_\nu - E_\mu - E_{\mathrm{meson}}, \end{aligned} \tag{1}$$

where E_ν is the energy of the neutrino, E_μ is the energy of the muon and E_{meson} is the sum of the energies of all the mesons. From the first two panels of this figure, we see that baryons carry almost all the hadron energy for a vast majority of events when $0 \le E_{\mathrm{had}} \le 5$ GeV. For larger values of hadron energy, the energy fraction carried by the baryons becomes smaller until it becomes negligibly small for $E_{\mathrm{had}} > 20$ GeV.

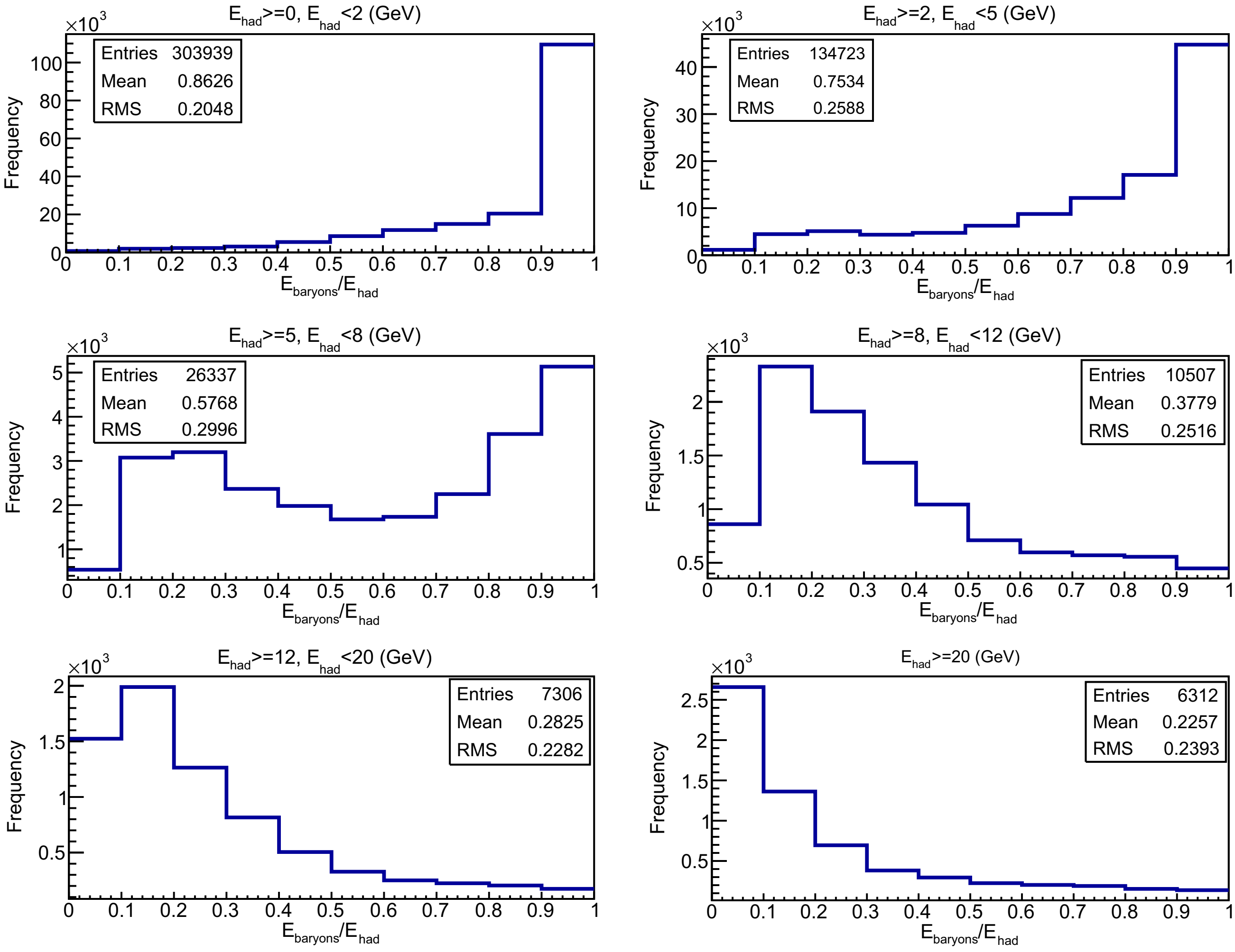

Figure 1. $E_{\text{baryons}}/E_{\text{had}}$ vs. frequency using NUANCE simulation.

Therefore, in this section, we study the correlation between the energy carried by the baryons and the hits produced by them in ICAL.

2.1 *Baryon hits*

In order to do this analysis, we need to know (a) the energy carried by all the baryons (E_{baryon}) and (b) the hits produced by these baryons (‘baryon hits’) in each event. It is straightforward to obtain E_{baryon} from NUANCE. To calculate baryon hits, we used the following method: We took all the ν_μ CC events generated by NUANCE and looked at the particle content of all the mesons in these events. We found that essentially all these mesons are pions and kaons and the heavier mesons form less than 1% of all the mesons. So, we performed a Geant4 simulation of these events after turning off the muons, the pions and the kaons. It is expected that the resulting number of hits is essentially due to the baryons produced in these events.

After obtaining E_{baryon} and baryon hits for each event, we define a set of ranges of baryon hits. For each range we plot the histogram of frequency vs. E_{baryon} and fit it with Vavilov distribution [12]. We tested a number of different sets of ranges until we found an optimal set of ranges for which the fitted distributions matched the histograms very well. From the fits done for this optimal set of ranges, we determined the average energy of the baryons $E_{\text{baryon-mean}}$ and the associated resolution σ_{Eb} for each range. A sample of these fits is shown in figure 2. The values of $E_{\text{baryon-mean}}$ and σ_{Eb} for each baryon hit range are listed in table 1. For each distribution, these quantities are defined by

$$E_{\text{baryon-mean}} = (\gamma - 1 - \ln p0 - p1)p3 + p2,$$
$$\sigma_{\text{Eb}} = \sqrt{\frac{(2-p1)}{2p0}(p3)^2}, \tag{2}$$

where γ is the Euler’s constant and pl ($l = 0, 1, 2, 3$) are the parameters of the Vavilov fit. Figure 3 shows

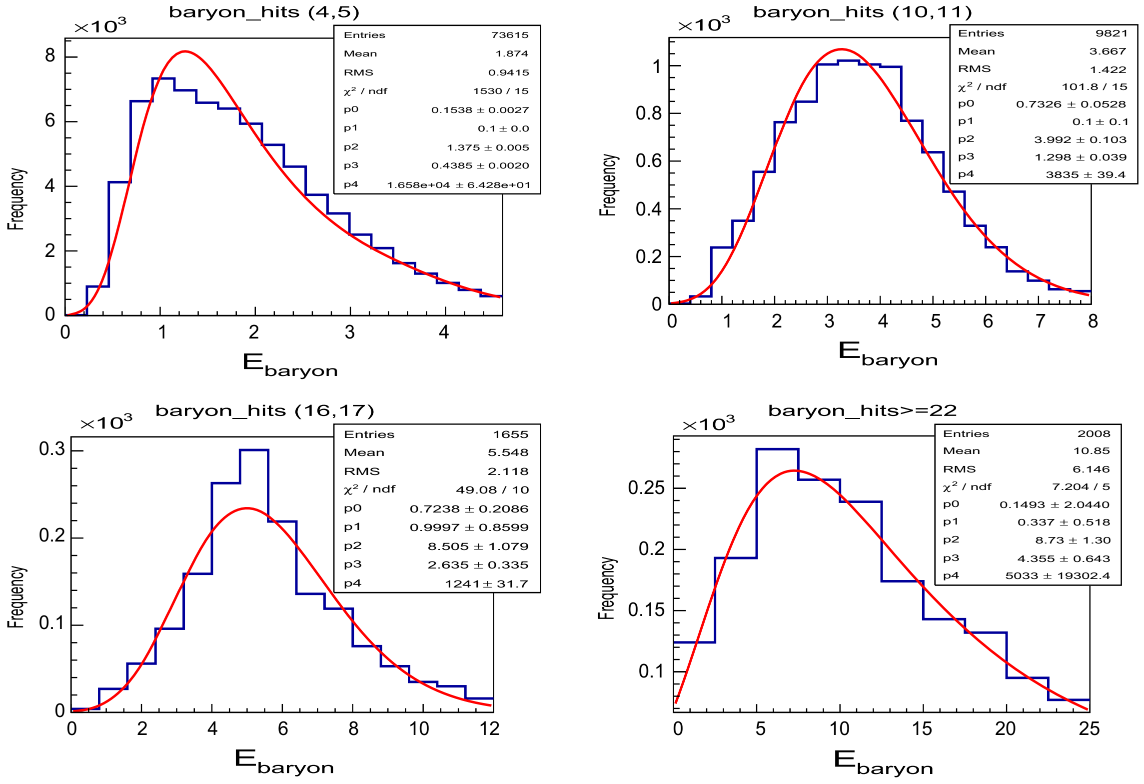

Figure 2. E_{baryon} distribution for hit ranges (4, 5), (10, 11), (16, 17) and (≥22). The smooth curve is the Vavilov fit.

Table 1. Number of baryon hits, $E_{\text{baryon-mean}}$ and σ_{Eb}.

Baryon hits	$E_{\text{baryon-mean}}$	σ_{Eb}
2–3	1.340	0.845
4–5	1.967	1.090
6–7	2.569	1.237
8–9	3.179	1.369
10–11	3.717	1.477
12–13	4.299	1.607
14–15	4.837	1.774
16–17	5.609	2.190
18–21	6.603	2.855
≥22	13.561	10.079

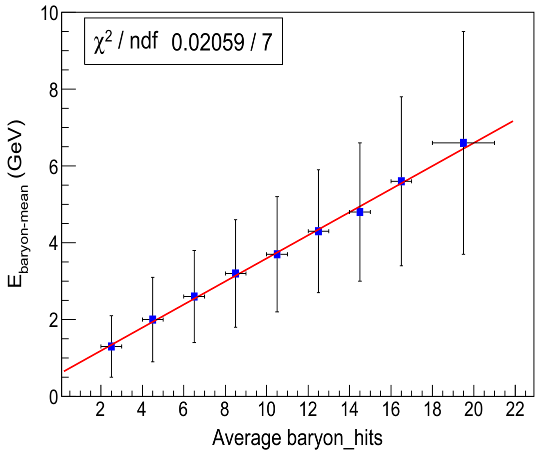

Figure 3. $E_{\text{baryon-mean}}$ vs. average baryon hits.

the relation between the average number of baryon hits in a bin vs. the corresponding $E_{\text{baryon-mean}}$. The X-axis error bars represent the bin width and the Y-axis error bars represent σ_{Eb} in that baryon hits bin. This relation is well described by the linear fit

$$E_{\text{baryon-mean}} = 0.29\ (\text{baryon hits}) + 0.70. \tag{3}$$

We compare this relation with the relation obtained from the simulation of isolated pions. This latter simulation was done in ref. [7]. The left panel of figure 2 of this reference gives the plots of mean no. of hits vs. E_{pion} for various thickness values of the iron plates of ICAL. The Geant4 simulation used in our analysis set the iron plate thickness to be 5.6 cm. We took the data for this thickness from figure 2 of ref. [7] and replotted as E_{pion} vs. mean no. of hits in figure 4.

We see that this data also show a linear relationship between these two variables and a linear fit gives the relation

$$E_{\text{pion}} = 0.64\ (\text{mean no. of hits}) - 1.57. \tag{4}$$

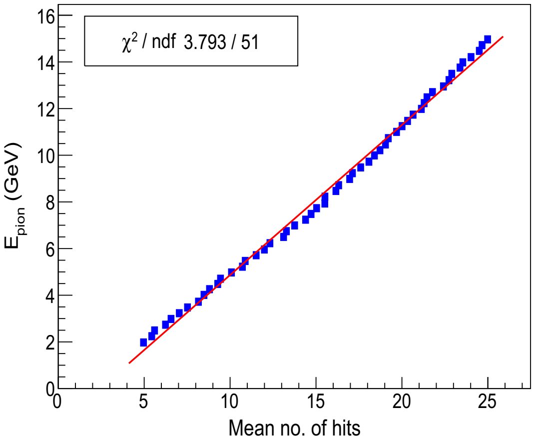

Figure 4. E_{pion} vs. mean no. of hits for 5.60 cm thick iron plates of ICAL.

Even though there is a linear relation between the no. of hits and the hadron energy for both baryons and pions, as seen from eqs (3) and (4), the slope of the pion fit is close to the slope of the baryon fit but the intercepts in the two cases are very different. Therefore, we argue that a proper estimation of hadron energy in atmospheric neutrino events requires doing a full Geant4 simulation of these events and establishing a relation between the number of hits produced by the hadrons and the hadron energy [5].

3. Hadron hit analysis

In this section, we analyse the hits generated by the hadrons produced in the ν_μ CC events of atmospheric neutrinos. We establish a correlation between these hadron hits and the energy of the hadrons. We shall also calculate the energy resolution for each given hadron energy.

3.1 *Hadron hit bank*

The Geant4 simulation of atmospheric neutrino event gives the full information on hits due to all the charged particles. This information, in particular, contains the (X, Y, Z) coordinates. The full set of this data is called the 'hit bank'. The ICAL reconstruction code looks to form a track with a subset of the total number of hits. If a track is not constructed, the event is not processed any further. In our analysis, we do not consider those events for which no track is reconstructed. If a track is constructed, it is identified with a muon and its energy and direction are calculated. If more than one track is constructed, the longest track is identified with a muon and its energy and direction are taken to be the muon kinematic variables. Once the muon track is identified, the hits which make up such a track are removed from the hit bank information. The remaining hits form 'hadron hit bank'. In the analysis below, the events without hadron hits are discarded. The hadron hit bank is likely to contain ghost hits. The problem of ghost hits is avoided using the procedure described in the Introduction. The resultant hits, after the implementation of this procedure, are defined as 'hadron hits'. The difference between the neutrino energy and the muon energy for a given event, obtained from NUANCE, is defined as 'E_{had}' of that event.

The hit distributions of this hit bank are given in figure 5. From the hit distribution, we see that ~22% of the events have less than five hits and ~55% have less than 10 hits. In the following analysis, we shall establish a correlation between the number of hits and the hadron energy. We first divided the event sample into different bins, each with its own range in the number of hits. For each bin, we plotted the histogram of frequency

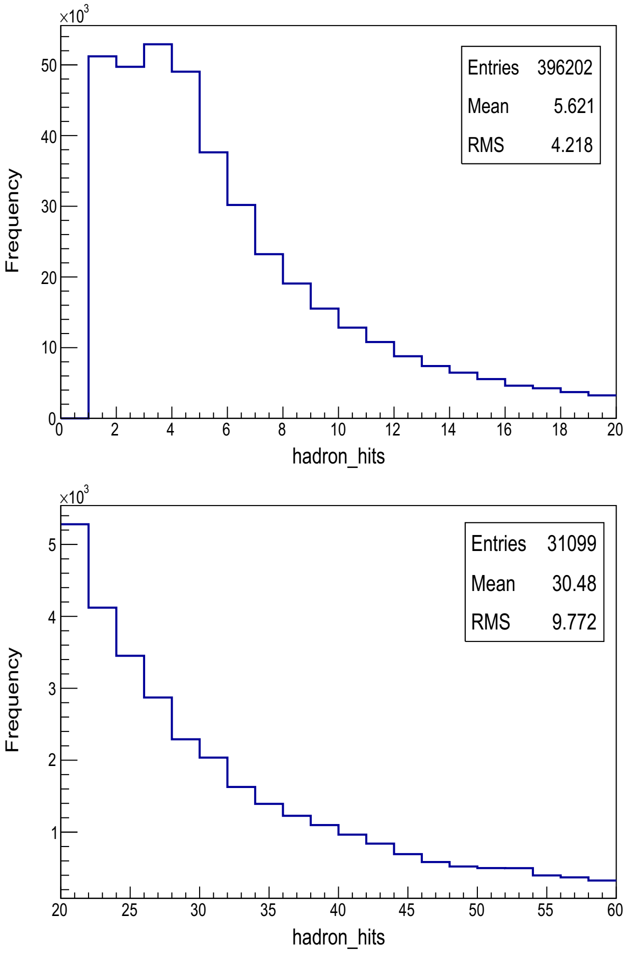

Figure 5. The top (bottom) panel shows the distribution for the number of hits < 20 (≥ 20).

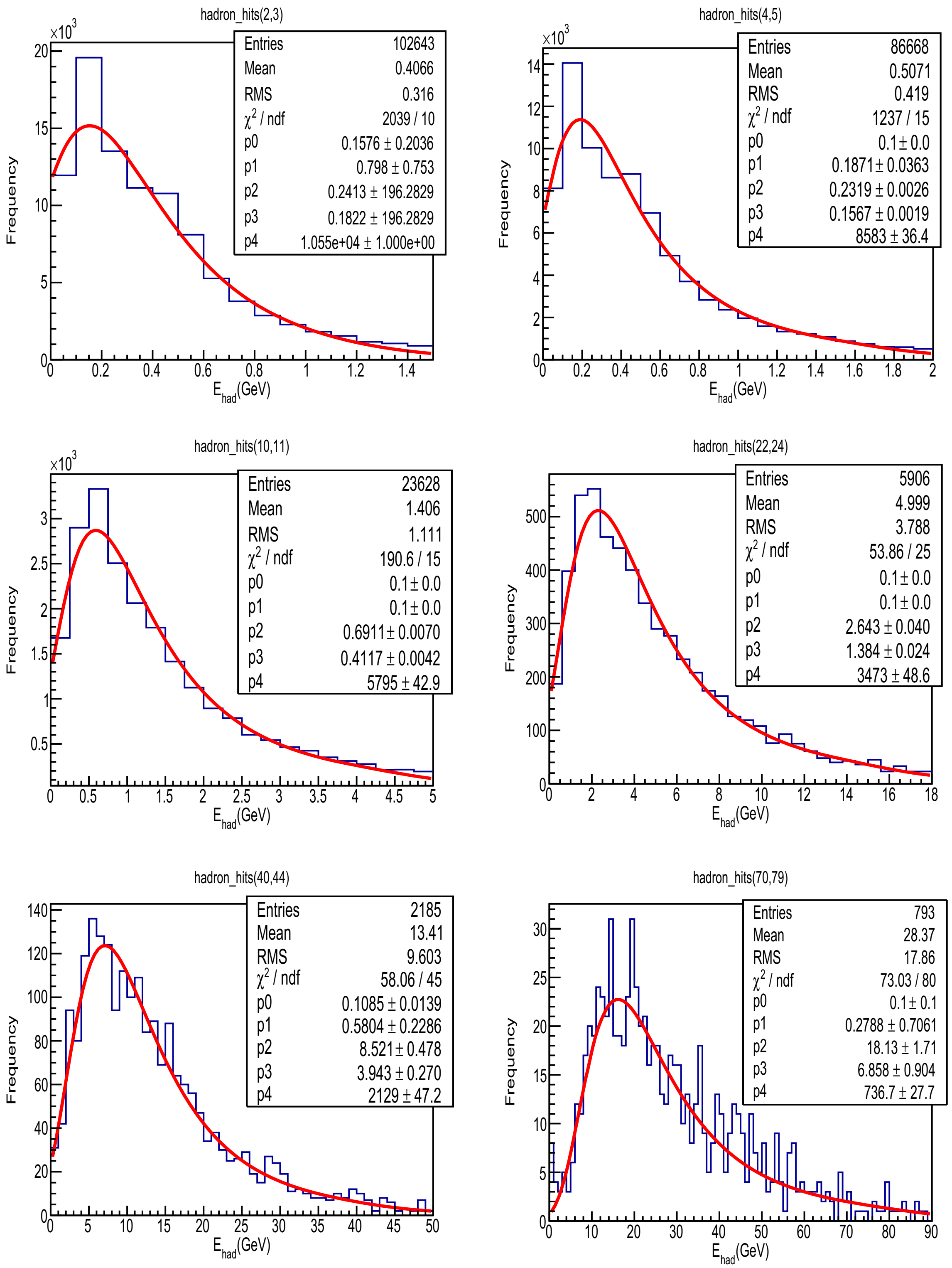

Figure 6. E_{had} distribution for hit ranges (2, 3), (4,5), (10,11), (22,24), (40,44) and (70,79). The smooth curve is the Vavilov fit.

vs. E_{had} and fit it to a Vavilov distribution. Various hit ranges were tried and the procedure was repeated till we obtained an optimal set of hit ranges for which the Vavilov distribution provided a good fit for each of the frequency vs. E_{had} histograms. A sample of these histograms, along with the fitted Vavilov distributions, are shown in figure 6. For the bin with (2,3) hits, the Vavilov distribution was not a good fit. Therefore, this

Table 2. The ranges of hadron hits and the corresponding fit values of Vavilov distributions.

Hadron hits	$E_{\text{had-mean}}$	σ_{Eh}
2–3	0.356	0.356
4–5	0.497	0.472
6–7	0.939	1.111
8–9	1.220	1.250
10–11	1.424	1.269
12–13	1.892	1.687
14-15	2.315	1.976
16–17	2.850	2.440
18–19	3.639	3.094
20–21	4.130	3.367
22–24	5.106	4.266
25–29	6.572	5.411
30–34	8.391	6.718
35–39	11.397	9.069
40–44	13.320	10.083
45–49	15.311	10.891
50–54	16.694	11.586
55–59	20.203	14.196
60–69	24.510	16.834
70–79	29.105	20.119
80–99	32.444	21.135
≥100	44.4066	24.679

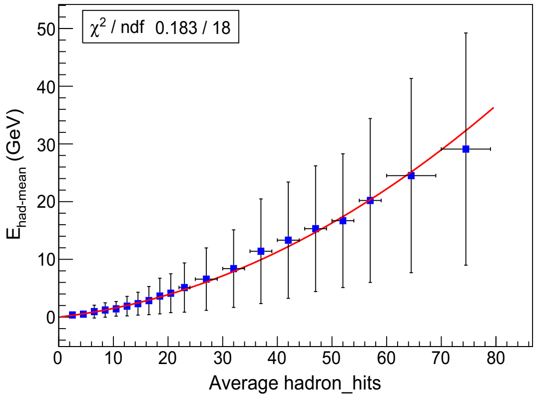

Figure 7. $E_{\text{had-mean}}$ vs. average hadron hits.

bin was not used in further analysis. Moreover, such low number of hits may also occur due to just noise. For these reasons, we drop this bin. The hadron hit ranges used and the corresponding Vavilov fit values are shown in table 2.

The values of $E_{\text{had-mean}}$ and σ_{Eh} are calculated for each distribution from the corresponding Vavilov fit parameters pl ($l = 0, 1, 2, 3$), using equations similar to those in eq. (2). We plotted the average number of hadron hits in a bin vs. $E_{\text{had-mean}}$ in figure 7. The X-axis error bars represent the bin width and the Y-axis error bars represent σ_{Eh} in that hadron hits bin. When this plot was fitted with a linear function, the estimate of hadron energy was too low for hadron hits > 40. Therefore, we performed a fit with a quadratic function and obtained

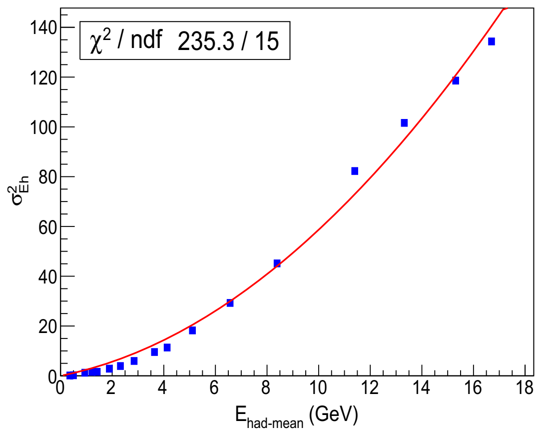

Figure 8. σ^2_{Eh} vs. $E_{\text{had-mean}}$.

$$E_{\text{had-mean}} = 0.11x + 0.004x^2, \tag{5}$$

where x is the number of hadron hits. We also plotted σ^2_{Eh} vs. $E_{\text{had-mean}}$ in figure 8. This energy resolution is parametrised as $\sigma(E)/E = \sqrt{a^2/E + b^2}$ [7]. A fit to the plot gives the values

$$\left(\frac{\sigma_{\text{Eh}}}{E_{\text{had-mean}}}\right) = \sqrt{\frac{2.06 \pm 0.44}{E_{\text{had-mean}}} + 0.38 \pm 0.03}, \tag{6}$$

leading to $a = 1.44 \pm 0.15$ and $b = 0.62 \pm 0.02$.

We believe that $E_{\text{had-mean}}$ and σ_{Eh} obtained in this section form the correct representation of hadron energy and its resolution in atmospheric neutrino events. In a previous work, Devi *et al* [5] also have used the hadron hit information from the Geant4 simulation of NUANCE-generated atmospheric neutrino events. There are a number of differences in the procedure they used and in the procedure used in this work.

- Their data set consists of 1000 years of atmospheric neutrinoevents, whereas our set consists of 100 years of data.
- They obtained hadron hit bank information by doing the Geant4 simulation of an event with the muon turned off at the input level. In our case, we performed the full Geant4 simulation of all the charged particles in the event and subtracted the hits which went into the track reconstruction. This is the procedure which will be utilised in the case of actual data.
- The avalanche produced in an RPC by one charged particle can, quite often, produce hits in two adjacent strips. Thus, the number of hadron hits in an RPC is likely to be larger than the number of charged particles passing through it. This feature is built into

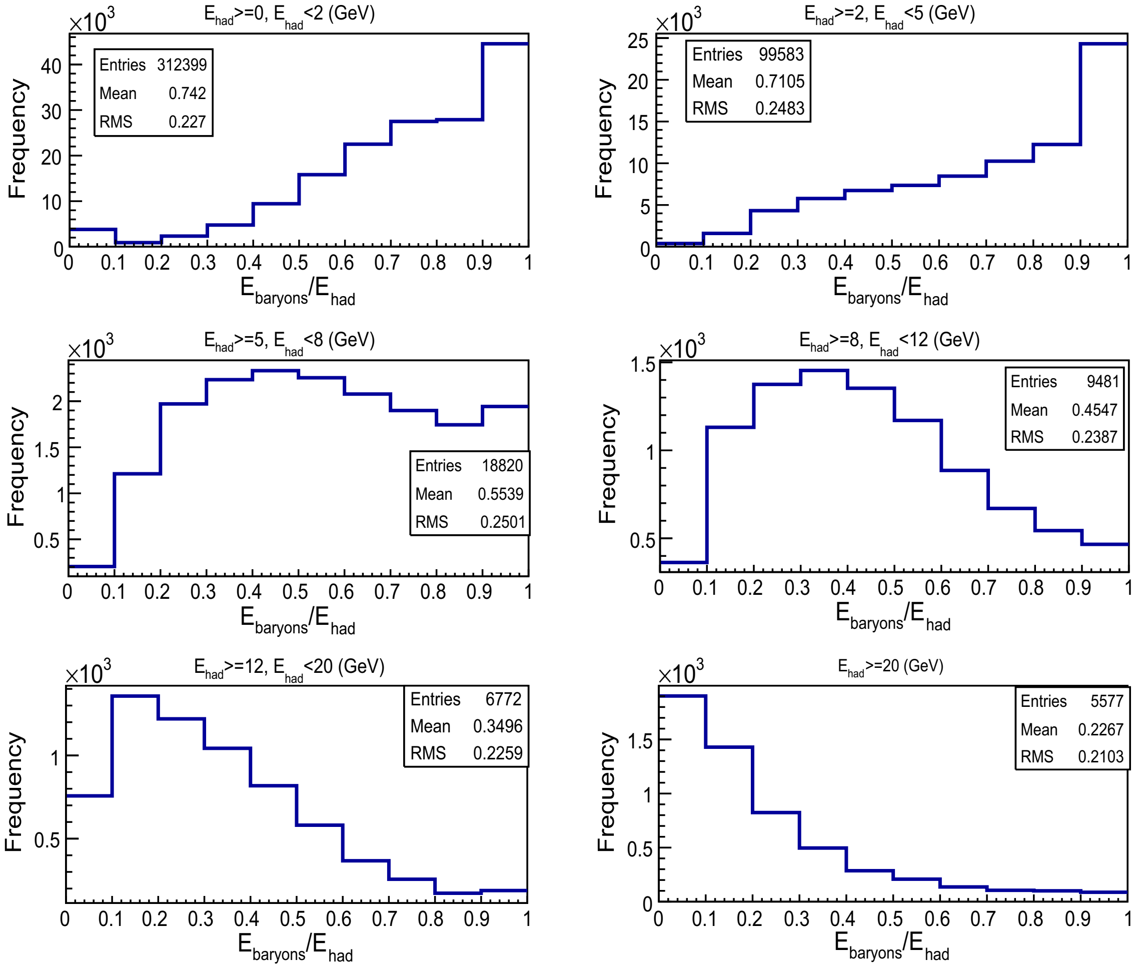

Figure 9. $E_{\text{baryons}}/E_{\text{had}}$ vs. frequency from GENIE simulation.

Geant4 through the option ‘multiplicity’. Devi *et al* [5] kept this option ‘off’ and hence obtained a smaller number of hits for a given hadron energy. In our case, we kept the multiplicity option ‘on’ and obtained about 30 to 40% larger number of hits for the same hadron energy. This is a more realistic simulation of the detector.

Because of points 2 and 3, the procedure we used is modelled more closely to what happens in the detector.

4. Comparison of GENIE results with NUANCE results

NUANCE event generator was developed almost 20 years ago [10]. It included all the neutrino–nucleon/nucleus scattering data available then and the hadronisation models used to describe that data. However, there have been no updates of NUANCE in the past few years. On the other hand, many experiments collected neutrino scattering data in the energy range 0.1−3.0 GeV, which is the most relevant for neutrino oscillations. All these data and the corresponding hadronisation models developed to describe this data, are included in the generator GENIE. It is continuously updated and it aims to become a canonical neutrino event generator with wide applicability [11].

Hence, we repeated our analysis using the GENIE Monte Carlo event generator. In this section, we present the GENIE results and compare them with NUANCE results.

In analysing the atmospheric neutrino events by GENIE, we used exactly the same procedure that we used in analysing the events by NUANCE. We see the same general features in both sets of events. For example, we find that baryons carry most of the energy for $E_{\text{had}} < 5$ GeV, in the case of GENIE also. This is illustrated in the top two panels of figure 9. We established the relation between the number of baryon hits and $E_{\text{baryon-mean}}$, which is shown in the left panel of figure 10. The corresponding figure for NUANCE-generated events is shown in the right panel of the same figure

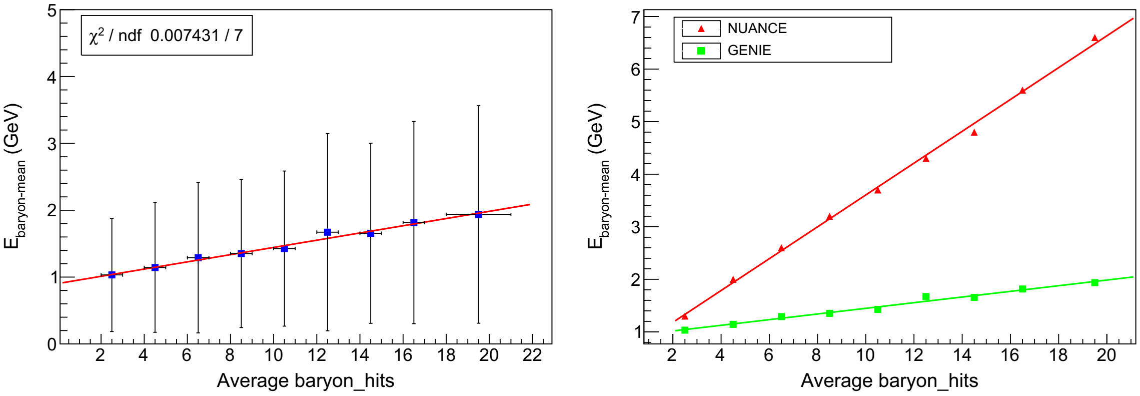

Figure 10. $E_{\text{baryon-mean}}$ vs. average baryon hits from GENIE (left) and comparison with NUANCE (right).

Table 3. Baryon hits and $E_{\text{baryon-mean}}$ table from GENIE.

Baryon hits	$E_{\text{baryon-mean}}$ GENIE	σ_{Eb} GENIE	$E_{\text{baryon-mean}}$ NUANCE	σ_{Eb} NUANCE
2–3	1.03	0.85	1.3	0.8
4–5	1.14	0.97	2.0	1.1
6–7	1.29	1.12	2.6	1.2
8–9	1.35	1.11	3.2	1.4
10–11	1.43	1.16	3.7	1.5
12–13	1.67	1.47	4.3	1.6
14–15	1.65	1.35	4.8	1.8
16–17	1.82	1.51	5.6	2.2
18–21	1.94	1.63	6.6	2.9
≥22	2.62	2.27	13.6	10.1

The corresponding numbers from NUANCE are also listed for comparison.

for comparison. Even though we find a linear relationship in both cases, $E_{\text{baryon-mean}}$ for GENIE is less than half of that for NUANCE. We also see this feature in table 3 where the fitted values of $E_{\text{baryon-mean}}$ and σ_{Eb} for GENIE and NUANCE are listed as a function of 'baryon_hits'. It seems as if NUANCE generates more higher energy baryons compared to GENIE. The fit equation for $E_{\text{baryon-mean}}$ vs. baryon hits for GENIE data is given in eq. (7).

$$E_{\text{baryon-mean}} = 0.05\ (\text{baryonhits}) + 0.93. \tag{7}$$

Turning our attention to all the hadrons produced in atmospheric neutrino events, the variation of $E_{\text{had-mean}}$ and σ_{Eh} with respect to hadron hits does look similar qualitatively but there are important quantitative differences. These are illustrated in figures 11 and 12 and in table 4. The values of $E_{\text{had-mean}}$ for GENIE are systematically higher (by about 50%) than the NUANCE value, though the values of σ_{Eh} are close. We do note, however, that the difference between the $E_{\text{had-mean}}$ values of the two generators is smaller than σ_{Eh} of either generator. That is, the results of the two generators are compatible with each other. The fit equations between $E_{\text{had-mean}}$ vs. hadron hits and σ_{Eh} vs. hadron hits are given in eq. (8).

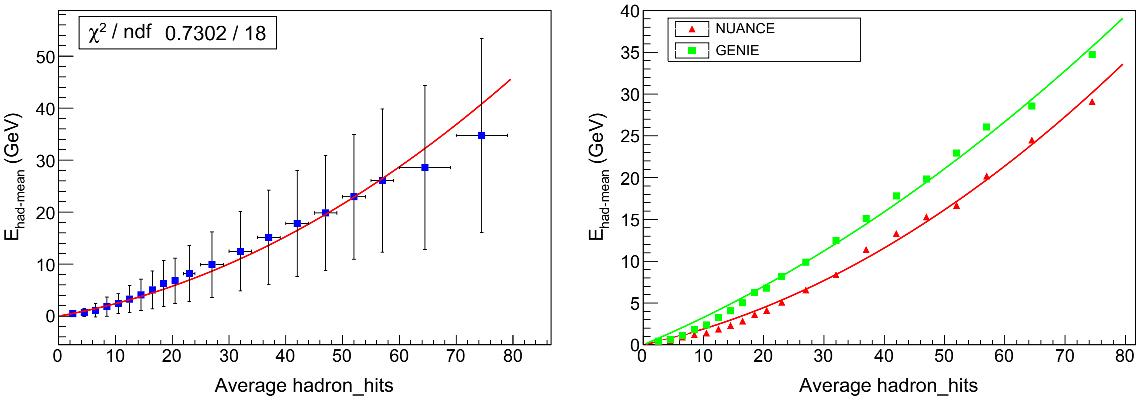

Figure 11. $E_{\text{had-mean}}$ vs. average hadron hits from GENIE (left) and comparison with NUANCE (right).

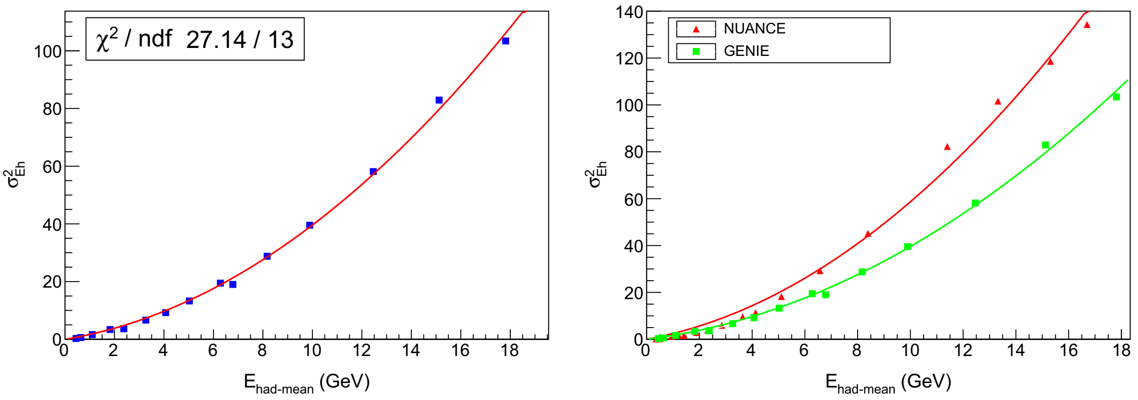

Figure 12. σ^2_{Eh} vs. $E_{\text{had-mean}}$ from GENIE (left) and comparison with NUANCE (right).

Table 4. Hadron hits and $E_{\text{had-mean}}$ table from GENIE.

Hadron hits	$E_{\text{had-mean}}$ GENIE	σ_{Eh} GENIE	$E_{\text{had-mean}}$ NUANCE	σ_{Eh} NUANCE
2–3	0.4	0.5	0.4	0.4
4–5	0.6	0.8	0.5	0.5
6–7	1.1	1.3	0.9	1.1
8–9	1.8	1.8	1.2	1.3
10–11	2.4	1.9	1.4	1.3
12–13	3.3	2.6	1.9	1.7
14–15	4.1	3.1	2.3	2.0
16–17	5.0	3.6	2.9	2.4
18–19	6.3	4.4	3.7	3.1
20–21	6.8	4.4	4.1	3.4
22–24	8.2	5.4	5.1	4.3
25–29	9.9	6.3	6.6	5.4
30–34	12.5	7.6	8.4	6.8
35-39	15.1	9.1	11.4	9.1
40–44	17.8	10.2	13.3	10.1
45–49	19.8	11.0	15.3	10.9
50–54	22.9	12.0	16.7	11.6
55–59	26.1	13.8	20.2	14.2
60–69	28.6	15.8	24.5	16.9
70–79	34.7	18.7	29.1	20.2
80–99	37.4	19.2	32.4	21.1
≥100	49.2	23.7	44.4	24.7

The corresponding numbers from NUANCE are also listed for comparison.

$$E_{\text{had-mean}} = 0.19x + 0.005x^2,$$

$$\left(\frac{\sigma_{\mathrm{Eh}}}{E_{\text{had-mean}}}\right) = \sqrt{\frac{1.40 \pm 0.14}{E_{\text{had-mean}}} + 0.26 \pm 0.01}, \qquad (8)$$

where x represents the number of hadron hits, leading to $a = 1.18 \pm 0.06$ and $b = 0.51 \pm 0.01$. From figure 12, we note that the fitted values of σ^2_{Eh} for GENIE are better described by the fit expression $\sigma(E)/E = \sqrt{a^2/E + b^2}$ than those of NUANCE.

5. Conclusion

In this work, we attempted to obtain an estimate of the energy of hadrons produced in a charged current interaction of an atmospheric muon neutrino/antineutrino in ICAL at INO. This was done by performing a full Geant4 simulation of atmospheric neutrino events generated by the neutrino event generators NUANCE and GENIE. We have used the un-oscillated data simulated for a period of 100 years. The events generated by both the generators show the following features:

- For $E_{\text{had}} < 5$ GeV, almost all the hadron energy is carried by the baryons.
- The relation between the number of hits and the energy of hadrons is very different for the two cases when the hadrons are mesons and when the hadrons are baryons.
- When the events are classified into bins with different number of hadron hits, the resulting spectra are reasonably well described by Vavilov distributions.
- There is a good correlation between the number of hits and the mean value of E_{had} of the Vavilov distributions.
- For each Vavilov distribution, the width (σ) is calculated. A relation between σ and the mean energy ($E_{\text{had-mean}} = E$), of the form $\sigma(E)/E = \sqrt{(a^2/E + b^2)}$, is established. Values of σ and E from GENIE fit the above form much better than those from NUANCE.

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