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M-lump and lump–kink solutions of (2 + 1)-dimensional Caudrey–Dodd-Gibbon–Kotera–Sawada equation

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Abstract. In this paper, M-lump and interaction between lumps and kink solitons of the (2 + 1)-dimensional Caudrey–Dodd–Gibbon–Kotera–Sawada equation are studied based on Hirota bilinear form. M-lump solutions are derived by taking a 'long wave' limit of the N-soliton solutions, and the lump–kink solutions are presented consequently. In addition, evolutions of solutions are shown by choosing certain parameters.

Keywords. Caudrey–Dodd–Gibbon–Kotera–Sawada equation; lump solution; lump–kink solution; Hirota bilinear form.

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1. Introduction

Lump waves can well explain the rogue wave dynamics which often appear in some disciplines of mechanical waves, such as optics and acoustics. Therefore, lump wave has attracted much attention since it was discovered. Manakov *et al* [1] first gave lump wave solutions to the Kadomtsev–Petviashvili equation a rational form. From their work, it can be seen that the lump solutions have good behaviours. Lump waves are reduced to zero in each direction, and they do not interact with each other. Subsequently, different methods for solving 1-lump solutions of high-dimensional nonlinear partial differential equations were also established, such as Hirota bilinear method [2–8], Darboux transformation method [9], Bäklund transformation method [10,11], inverse scattering method [12,13], etc.

In a more general case, Satsuma and Ablowitz [14] studied multiple collisions of lump waves. They obtained M-lump solutions to the nonlinear evolution equations by taking a 'long wave' limit of the corresponding N-soliton solutions. After that, many results following their work have been obtained, and the significant physical meanings of M-lump solutions are shown in [15,16]. Based on these studies, the researchers also investigated the interaction between lump waves and solitons in nonlinear systems, and constructed a series of interactive solutions such as lump–kink solutions [17–19] and lump–strip solutions [20,21]. Lump solutions to some nonlinear partial differential equations retain their shapes, amplitudes, velocities and some other physical properties after the interaction with some solitons. In other words, the interaction is elastic. However, for some other nonlinear equations, the interaction between lump waves and solitons may not be elastic under certain conditions. Our subsequent results can also prove it.

Here we investigate a $(2+1)$-dimensional integrable nonlinear evolution equation called $(2+1)$-dimensional Caudrey–Dodd–Gibbon–Kotera–Sawada (CDGKS) equation as follows:

$$3v_t + (v_{xxxx} + 15vv_{xx} + 15v^3)_x - 5u_{yy} - 5(v_{xxy} + 3vv_y + 3v_x u_y) = 0, \tag{1.1}$$

where $u = u(x, y, t)$ and $v = u_x = v(x, y, t)$ are two real functions with variables x, y and t. The equation can be written in the following form:

$$3u_{xt} - 5u_{yy} - 5u_{xxxy} + u_{xxxxxx} - 15u_x u_{xy} - 15u_y u_{xx} + 15u_x u_{xxxx} + 15u_{xx}u_{xxx} + 45u_x^2 u_{xx} = 0. \tag{1.2}$$

The linear term of (1.2) is

$$3u_{xt} - 5u_{yy} - 5u_{xxxy} + u_{xxxxxx} = 0. \tag{1.3}$$

Caudrey–Dodd–Gibbon [22] and Kotera–Sawada [23] studied a class of higher-order KdV equations which had

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N-soliton solutions but did not satisfy infinity of polynomial conservation laws. CDGKS equation was generalised by this class of equations. Later, Konopelchenko–Dubrovsky [24] gave the $(2+1)$-dimensional analogue of the CDGKS equation by considering the commutativity condition about two explicitly given operators. Afterwards, much of the significant work about the $(2+1)$-dimensional CDGKS equation have been done [25–28]. In this paper, the M-lump solutions and the corresponding lump–kink solutions of eq. (1.1) will be constructed.

The outline of this paper is as follows: In §2, the M-lump solutions of eq. (1.1) are obtained through the N-soliton solutions which can be obtained by applying Hirota bilinear method. Some figures are shown to illustrate the dynamical properties of these M-lump solutions which describe the collision of multiple lumps. Then in §3, the interaction phenomenon of lumps and kink solitons is studied. Finally, we conclude our work in §4.

2. M-lump solutions

In this section, the N-soliton solutions of the CDGKS equation are derived using Hirota bilinear method, and then the corresponding M-lump solutions can be obtained. The properties of this solutions are analysed and shown by some graphs.

2.1 *N-soliton solutions*

Consider the following transformation:

$$u = R(\ln f)_x, \tag{2.1}$$

where R is a constant, and $f = f(x, y, t)$ is a real positive function with variables x, y and t. By using this transformation to eq. (1.2), the N-soliton solutions of eq. (1.1) can be written as

$$f = f_N = \sum_{\alpha=0,1} \exp\left\{\sum_{i<j}^{N} \alpha_i\alpha_j\beta_{ij} + \sum_{i=1}^{N}\alpha_i\theta_i\right\}. \tag{2.2}$$

This expression tells us to sum all terms $\exp\{\sum_{i<j}^{N}\alpha_i\alpha_j \beta_{ij} + \sum_{i=1}^{N}\alpha_i\theta_i\}$ for every possible combinations of $\alpha = (\alpha_1, \ldots, \alpha_N)$, where every α_i is 0 or 1. θ_i in the expression is defined by

$$\theta_i = l_i x + m_i y - c_i t, \quad i = 1, 2, \ldots, N, \tag{2.3}$$

where l_i, m_i and c_i are constants. Substitute $u^* = \mathrm{e}^{\theta_i}$ into the linear terms (1.3) of the equation to find the relation between l_i, m_i and c_i,

$$c_i = \frac{1}{3}l_i^5 - \frac{5}{3}l_i^2 m_i - \frac{5}{3}\frac{m_i^2}{l_i}.$$

Then θ_i can be written as

$$\theta_i = l_i x + m_i y - \left(\frac{1}{3}l_i^5 - \frac{5}{3}l_i^2 m_i - \frac{5}{3}\frac{m_i^2}{l_i}\right)t. \tag{2.4}$$

We consider the following 1-soliton solution to calculate R:

$$f_1 = 1 + \mathrm{e}^{\theta_1} \tag{2.5}$$

and

$$u_1 = R(\ln f_1)_x. \tag{2.6}$$

Substituting (2.6) into (1.2), we obtain

$$R = 2. \tag{2.7}$$

The Hirota bilinear form of (1.2) can be obtained by applying the transformation (2.1)

$$(3D_x D_t - 5D_y^2 - 5D_x^3 D_y + D_x^6)(f \cdot f) = 0, \tag{2.8}$$

where D_x, D_y and D_t are Hirota bilinear operators, which are defined by

$$\begin{aligned} &D_x^m D_y^n D_t^l (f_1 \cdot f_2) \\ &\quad = \frac{\partial^m}{\partial x^m}\frac{\partial^n}{\partial y^n}\frac{\partial^l}{\partial t^l} f_1(x+\varepsilon_1, y+\varepsilon_2, t+\varepsilon_3) \\ &\qquad \times f_2(x-\varepsilon_1, y-\varepsilon_2, t-\varepsilon_3)\big|_{\varepsilon_1=\varepsilon_2=\varepsilon_3=0}. \end{aligned} \tag{2.9}$$

Formula (2.8) can also be written as

$$\begin{aligned} &3(f_{xt}f - f_x f_t) - 5(f_{yy}f - f_y^2) + (f_{xxxxxx}f \\ &\quad -6f_x f_{xxxxx} + 15 f_{xx}f_{xxxx} - 10 f_{xxx}^2) \\ &\quad -5(f_{xxxy}f - f_{xxx}f_y + 3f_{xy}f_{xx} \\ &\quad -3f_x f_{xxy}) = 0. \end{aligned} \tag{2.10}$$

In order to determine β_{ij} in (2.2), we consider the 2-soliton solutions

$$f_2 = 1 + \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2} + \mathrm{e}^{\theta_1+\theta_2+\beta_{12}} \tag{2.11}$$

and

$$u_2 = 2(\ln f_2)_x. \tag{2.12}$$

Substituting (2.12) into (1.2), and solving the equation for the phase shift $\mathrm{e}^{\beta_{12}}$, we can obtain

$$\begin{aligned} \mathrm{e}^{\beta_{12}} = {} & [l_1^2 l_2^2 (l_1 - l_2)^2 (l_1^2 - l_1 l_2 + l_2^2) - l_1 l_2 (l_1 - l_2) \\ & \times (l_2 m_1 (2l_1 - l_2) - l_1 m_2 (2l_2 - l_1)) \\ & + (l_1 m_2 - l_2 m_1)^2]/[l_1^2 l_2^2 (l_1 + l_2)^2 (l_1^2 \\ & + l_1 l_2 + l_2^2) - l_1 l_2 (l_1 + l_2) \\ & \times (l_2 m_1 (2l_1 + l_2) + l_1 m_2 (2l_2 + l_1)) \\ & + (l_1 m_2 - l_2 m_1)^2], \end{aligned} \tag{2.13}$$

and this can be generalised to

$$\begin{aligned}e^{\beta_{ij}} = {} & [l_i^2 l_j^2 (l_i - l_j)^2 (l_i^2 - l_i l_j + l_j^2) - l_i l_j (l_i - l_j) \\ & \times (l_j m_i (2l_i - l_j) - l_i m_j (2l_j - l_i)) \\ & + (l_i m_j - l_j m_i)^2]/[l_i^2 l_j^2 (l_i + l_j)^2 (l_i^2 + l_i l_j + l_j^2) \\ & - l_i l_j (l_i + l_j)(l_j m_i (2l_i + l_j) \\ & + l_i m_j (2l_j + l_i)) + (l_i m_j - l_j m_i)^2] \end{aligned} \tag{2.14}$$

for $1 \le i < j \le N$. Then all N-soliton solutions of the equation can be written explicitly. For example, we give the expressions of f_1 and f_2 as (2.5) and (2.11) respectively. And f_3 can also be written as follows:

$$\begin{aligned} f_3 = {} & 1 + e^{\theta_1} + e^{\theta_2} + e^{\theta_3} + e^{\beta_{12}} e^{\theta_1+\theta_2} + e^{\beta_{23}} e^{\theta_2+\theta_3} \\ & + e^{\beta_{13}} e^{\theta_1+\theta_3} + e^{\beta_{12}+\beta_{23}+\beta_{13}} e^{\theta_1+\theta_2+\theta_3}, \end{aligned} \tag{2.15}$$

where θ_i and β_{ij} are defined by (2.4) and (2.14) respectively. For $N > 3$, the N-soliton solutions can be obtained by taking a similar method. The corresponding u_i can be obtained using transformation (2.1).

2.2 *M-lump solutions*

In order to obtain the M-lump solutions of (1.1), we consider the following f_N^*:

$$f_N^* = \sum_{\alpha=0,1} (-1)^{\sum_{i=1}^N \alpha_i} \exp\left\{ \sum_{i<j}^N \alpha_i \alpha_j \beta_{ij} + \sum_{i=1}^N \alpha_i \theta_i \right\}. \tag{2.16}$$

Setting

$$m_i = l_i p_i \tag{2.17}$$

we have

$$\theta_i = l_i \left(x + p_i y - \left(\frac{1}{3} l_i^4 - \frac{5}{3} l_i^2 p_i - \frac{5}{3} p_i^2 \right) t \right) \tag{2.18}$$

and

$$\begin{aligned} e^{\beta_{ij}} = {} & [(l_i - l_j)^2 (l_i^2 - l_i l_j + l_j^2) - (l_i - l_j) \\ & \times (2l_i p_i - 2l_j p_j + l_i p_j - l_j p_i) + (p_i - p_j)^2] \\ & /[(l_i + l_j)^2 (l_i^2 + l_i l_j + l_j^2) - (l_i + l_j) \\ & \times (2l_i p_i + 2l_j p_j + l_i p_j + l_j p_i) \\ & + (p_i - p_j)^2]. \end{aligned} \tag{2.19}$$

By taking every l_i approaching 0, and supposing that every l_i has the same asymptotic order, we have

$$\begin{aligned} f_N^* = {} & \sum_{\alpha=0,1} \prod_{i=1}^N (-1)^{\alpha_i} (1 + \alpha_i l_i \xi_i) \prod_{i<j}^N (1 + \alpha_i \alpha_j l_i l_j B_{ij}) \\ & + O(l^{N+1}), \end{aligned} \tag{2.20}$$

where

$$\xi_i = x + p_i y + \frac{5}{3} p_i^2 t \tag{2.21}$$

and

$$B_{ij} = \frac{6(p_i + p_j)}{(p_i - p_j)^2}. \tag{2.22}$$

Omitting the term $O(l^{N+1})$ in (2.20), we consider

$$F_N = \sum_{\alpha=0,1} \prod_{i=1}^N (-1)^{\alpha_i} (1 + \alpha_i l_i \xi_i) \prod_{i<j}^N (1 + \alpha_i \alpha_j l_i l_j B_{ij}). \tag{2.23}$$

By expanding the expression of F_N, we have

$$F_N = \prod_{i=1}^N l_i \left\{ \sum_{m=0}^{[N/2]} \left[\frac{1}{m! 2^m} \sum_{i_1,\ldots,i_{2m}} \left(\prod_{j=1}^m B_{i_{2j-1} i_{2j}} \prod_{\substack{i \neq i_k \\ k=1,\ldots,2m}} \xi_i \right) \right] + O(l) \right\}, \tag{2.24}$$

where $[N/2]$ is the largest integer not greater than $N/2$. Let $i_1, \ldots, i_{2m}$ be $2m$ different integers in 1 to N,

$$\prod_{i \neq i_k k=1,\ldots,2m} \xi_i = \frac{\prod_{i=1}^N \xi_i}{\prod_{k=1}^{2m} \xi_{i_k}}$$

and $\sum_{i_1,\ldots,i_{2m}} (\prod_{j=1}^m B_{i_{2j-1} i_{2j}} \prod_{\substack{i \neq i_k \\ k=1,\ldots,2m}} \xi_i)$ is the sum of all $\prod_{j=1}^m B_{i_{2j-1} i_{2j}} \prod_{\substack{i \neq i_k \\ k=1,\ldots,2m}} \xi_i$ for every possible combination of $(i_1, \ldots, i_{2m})$.

By the property of transformation (2.1), if f is a solution of (2.10), then f multiplied by a constant is also a solution of (2.10). Therefore, we can omit the constant factor $\prod_{i=1}^N l_i$ of F_N. When every l_i approaches 0, we can also omit the term $O(l)$. If N is even, and let $M = N/2$, we can set $p_{i+1} = \overline{p_i}$ $(i = 1, 3, \ldots, N-1)$, that is, p_{i+1} is the complex conjugate of p_i. Then we get a class of non-singular rational solutions of (2.10) with the form expressed by

$$F_M^* = \sum_{m=0}^{M}\left[\frac{1}{m!2^m}\sum_{i_1,\ldots,i_{2m}}\left(\prod_{j=1}^{m} B_{i_{2j-1}i_{2j}} \prod_{\substack{i\neq i_k \\ k=1,\ldots,2m}} \xi_i\right)\right], \tag{2.25}$$

and the corresponding $u_M = 2(\ln F_M^*)_x$ are M-lump solutions of (1.1).

Next we investigate 1-lump solutions of (1.1) as an example. The 1-lump solution can be obtained from (2.25),

$$F_1^* = \xi_1\xi_2 + B_{12}, \tag{2.26}$$

where

$$\xi_i = x + p_i y + \frac{5}{3}p_i^2 t, \quad i = 1, 2,$$

$$B_{12} = \frac{6(p_1 + p_2)}{(p_1 - p_2)^2}$$

and

$$p_2 = \overline{p_1}.$$

If we use q_1 and q_2 to denote the real and imaginary parts of p_1 respectively, then we have

$$F_1^* = (x^* + q_1 y^*)^2 + (q_2 y^*)^2 - \frac{3q_1}{q_2^2}, \tag{2.27}$$

where

$$x^* = x - \frac{5}{3}(q_1^2 + q_2^2)t, \tag{2.28}$$

$$y^* = y + \frac{10}{3}q_1 t. \tag{2.29}$$

Then we have

$$u_1 = 2(\ln F_1^*)_x = \frac{4(x^* + q_1 y^*)}{(x^* + q_1 y^*)^2 + (q_2 y^*)^2 - 3q_1/q_2^2}. \tag{2.30}$$

u_1 is integrable if and only if $q_1 < 0$ and $q_2 \neq 0$. Then, F_1^* is a positive quadratic function. More results about it can be found in refs [2–4]. Under these conditions, the rational solution u_1 is a permanent lump solution. This solution decays as $O(x^{-2}, y^{-2})$ when $|x|, |y| \to \infty$ and the moving velocities on two directions are

$$v_x = \frac{5}{3}(q_1^2 + q_2^2) \tag{2.31}$$

and

$$v_y = -\frac{10}{3}q_1. \tag{2.32}$$

In figure 1, the evolution of this solution is shown for a particular choice of parameters q_1 and q_2.

Multiple lump solutions of (1.1) can also be obtained from (2.25). For instance, taking $M = 2$, we have

$$\begin{aligned} F_2^* = {} & \xi_1\xi_2\xi_3\xi_4 + B_{12}\xi_3\xi_4 + B_{13}\xi_2\xi_4 + B_{14}\xi_2\xi_3 \\ & + B_{23}\xi_1\xi_4 + B_{24}\xi_1\xi_3 + B_{34}\xi_1\xi_2 \\ & + B_{12}B_{34} + B_{13}B_{24} + B_{14}B_{23}, \end{aligned} \tag{2.33}$$

where

$$\xi_i = x + p_i y + \frac{5}{3}p_i^2 t, \quad i = 1, 2, 3, 4,$$

$$B_{ij} = \frac{6(p_i + p_j)}{(p_i - p_j)^2}, \quad i, j = 1, 2, 3, 4$$

and

$$p_2 = \overline{p_1}, \quad p_4 = \overline{p_3}.$$

If we set $p_1 = q_1 + iq_2$, $p_3 = q_3 + iq_4$ and $q_1, q_3 < 0$, $q_2q_4 \neq 0$, then F_2^* is a positive function which only contains quartic and quadratic perfect square terms, and we can obtain a rational solution by using the transformation (2.1)

$$u_2 = 2(\ln F_2^*)_x, \tag{2.34}$$

u_2 is a permanent 2-lump solution of (1.1). Under the condition of q_1, q_2, q_3 and q_4 above, u_2 is integrable.

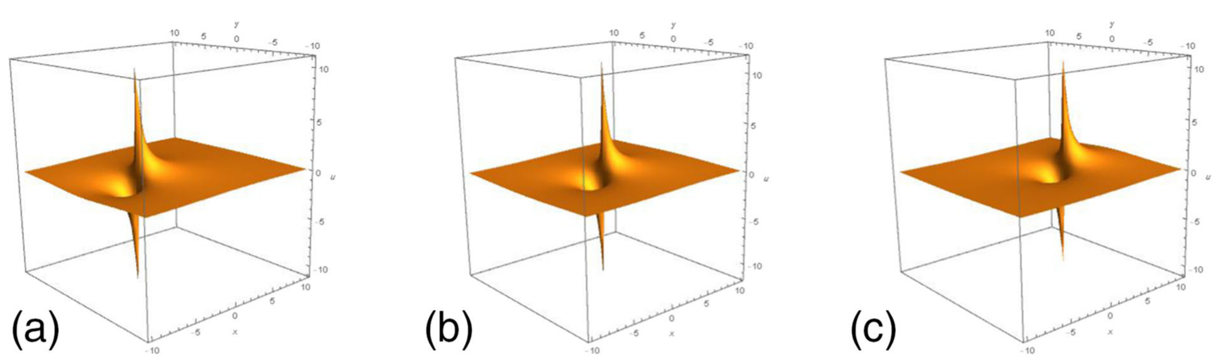

Figure 1. Evolutions of 1-lump solutions when $q_1 = -0.01$ and $q_2 = 1$ at time (**a**) $t = -2$, (**b**) $t = 0$ and (**c**) $t = 2$.

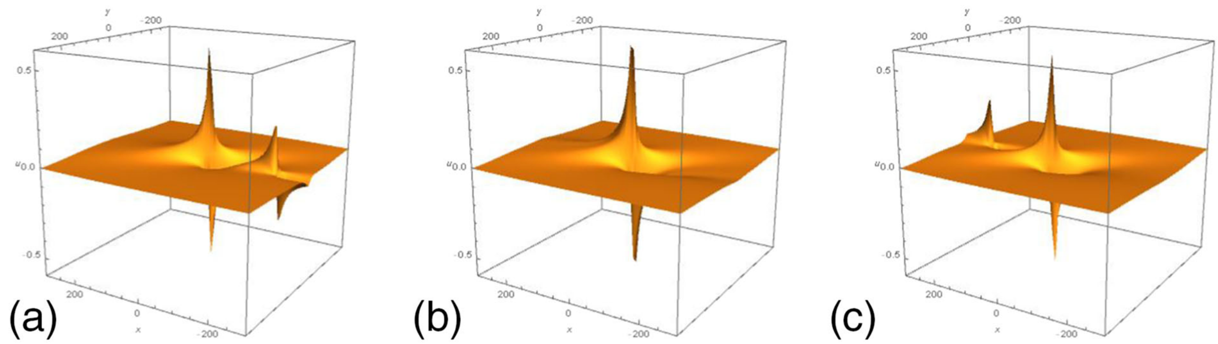

Figure 2. Evolutions of 2-lump solutions when $q_1 = -1$, $q_2 = 1$, $q_3 = -1$ and $q_4 = 5$ at time (**a**) $t = -5$, (**b**) $t = 0$ and (**c**) $t = 5$.

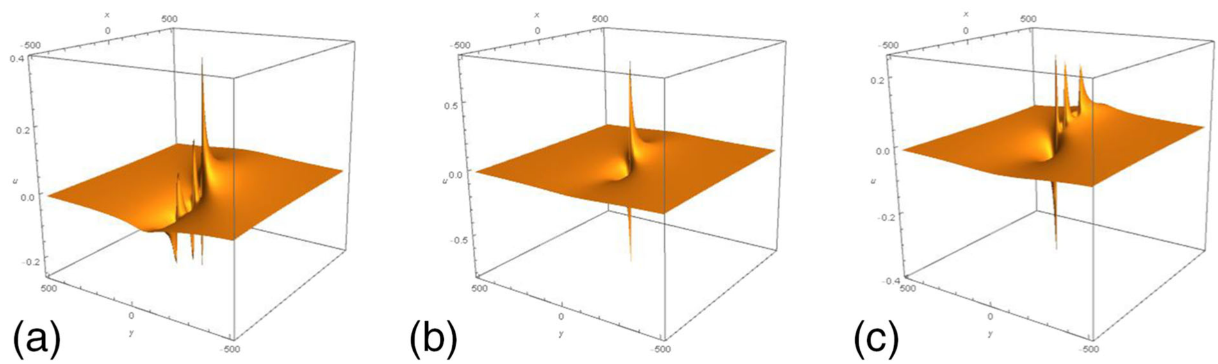

Figure 3. Evolutions of 3-lump solutions when $q_1 = -1$, $q_2 = 1$, $q_3 = -1$, $q_4 = 2$, $q_5 = -1$ and $q_6 = 3$ at time (**a**) $t = -15$, (**b**) $t = 0$ and (**c**) $t = 15$.

In figure 2, the evolution of this solution is shown for a particular choice of parameters q_1, q_2, q_3 and q_4.

We can also obtain the expression of F_3^* by taking $M = 3$ in (2.25). This expression contains 76 terms. So we do not write it out here. When $p_2 = \overline{p_1}$, $p_4 = \overline{p_3}$, $p_6 = \overline{p_5}$, and $q_1, q_3, q_5 < 0$, $q_2q_4q_6 \neq 0$, where $p_1 = q_1 + iq_2$, $p_3 = q_3 + iq_4$, $p_5 = q_5 + iq_6$, we find that F_3^* is a positive function which only contains sextic, quartic and quadratic perfect square terms. Substituting F_3^* into transformation (2.1), we get $u_3 = 2(\ln F_3^*)_x$, which is a permanent integrable 3-lump solution of (1.1). In figure 3, the evolution of this solution is shown for a particular choice of parameters q_1, q_2, q_3, q_4, q_5 and q_6. Similarly, the explicit expressions of all multiple lump solutions of (1.1) can be obtained, and the evolution of these solutions can also be studied. For M-lump solution, there are M lump waves propagating in the same direction at different speeds. Lump waves keep their physical properties after collision.

3. Lump–kink solutions

In this section, the interaction solutions between lumps and kink solitons will be investigated. The interaction solution between 1-lump and 1-kink soliton of eq. (1.1) was studied by Deng *et al* (more details can be seen in [27]). We shall discuss the interactive solution between 1-lump and 2-kink soliton. To this end, we write f in the following form:

$$f_{1,2} = g_1^2 + g_2^2 + k_7h_1 + k_8h_2 + k_9h_1h_2 + c, \qquad (3.1)$$

where

$$g_1 = a_1x + a_2y + a_3t + a_4,$$
$$g_2 = a_5x + a_6y + a_7t + a_8,$$
$$h_1 = \mathrm{e}^{k_1x+k_2y+k_3t},$$
$$h_2 = \mathrm{e}^{k_4x+k_5y+k_6t},$$

a_i $(i = 1, \ldots, 8)$, k_i $(i = 1, \ldots, 9)$ and c are undetermined coefficients. Substituting (3.1) into eq. (2.10) and considering the coefficients of every terms, we obtain a system of 42 nonlinear equations. We use Mathematica to solve it and show the solution as follows:

$$a_2 = \frac{-4a_6^2 + 9a_1^2k_1^4}{12a_1k_1^2},$$
$$a_3 = \frac{5(16a_6^4 - 216a_1^2a_6^2k_1^4 + 81a_1^4k_1^8)}{432a_1^3k_1^4},$$

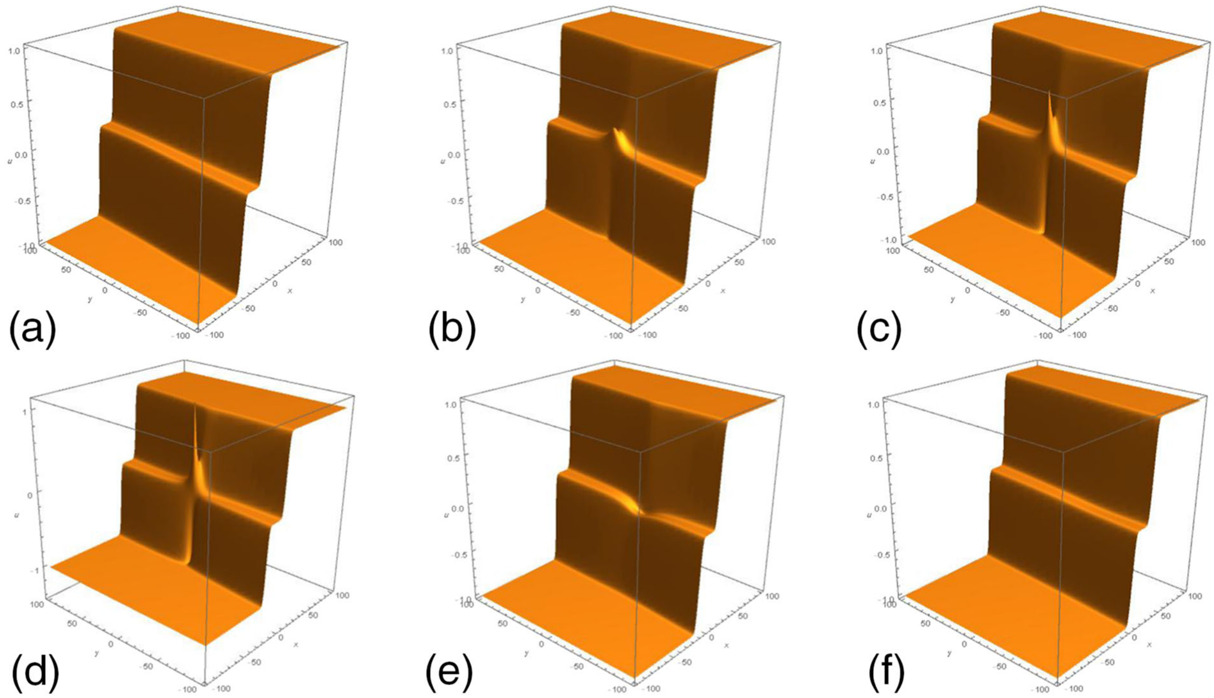

Figure 4. Evolutions of lump–kink solution when $a_1 = a_4 = a_8 = 1$, $a_6 = -1$, $k_1 = k_8 = 1$ and $k_7 = 0.02$ at time (**a**) $t = -50$, (**b**) $t = -5$, (**c**) $t = -1$, (**d**) $t = 0$, (**e**) $t = 15$ and (**f**) $t = 50$.

$$
\begin{aligned}
a_5 &= 0,\\
a_7 &= \frac{5(-4a_6^3 + 9a_1^2 a_6 k_1^4)}{18a_1^2 k_1^2},\\
k_2 &= \frac{-4a_6^2 + 3a_1^2 k_1^4}{12a_1^2 k_1},\\
k_3 &= \frac{5a_6^4}{27a_1^4 k_1^3} - \frac{5a_6^2 k_1}{6a_1^2} + \frac{3k_1^5}{16},\\
k_4 &= -k_1, \quad k_5 = -k_2, \quad k_6 = -k_3,\\
k_9 &= \frac{k_1^2 k_7 k_8}{a_1^2} + \frac{9k_1^6 k_7 k_8}{4a_6^2},\\
c &= -\frac{a_1^2(-4a_6^2 + 9a_1^2 k_1^4)}{4a_6^2 k_1^2},
\end{aligned}
\tag{3.2}
$$

and the coefficients need to satisfy the following conditions:

$$a_1 a_6 k_1 \neq 0, \quad k_7 > 0, \quad k_8 > 0, \quad c > 0. \tag{3.3}$$

It is clear that $f_{1,2}$ is a positive function, and the lump–kink solution of eq. (1.1) can be obtained by using the transformation (2.1)

$$u_{1,2} = 2(\ln f_{1,2})_x.$$

$u_{1,2}$ is integrable because $f_{1,2}$ is positive. It is noted that $u_{1,2}$ degenerates to the interactive solution between one lump and twin stripe solitons in [27] when $k_7 = k_8$. In figure 4, the evolution of $u_{1,2}$ is shown by choosing fixed values for $a_1, a_4, a_6, a_8, k_1, k_7$ and k_8 when time (t) varies. Firstly, we can see that there exists a 2-kink soliton. With the flow of time, a lump wave appears and gradually propagates, and finally it is swallowed by the kink soliton. In other words, the interaction between lump waves and kink solitons is not elastic in this case. Analogously, we can further investigate the interactive solutions between N-kink and M-lump based on the previous M-lump solutions.

4. Conclusion

Lump wave solution of nonlinear partial differential equation is an attractive research direction in mathematics and physics field. It can describe the rogue wave phenomena that often appear in optics, acoustics and many other branches of physics. 1-lump wave solutions have been systematically studied in many existing literatures. In this paper, M-lump solutions of (2 + 1)-dimensional CDGKS equation are constructed by applying the 'long-wave' limit method. Then the related lump–kink solutions are obtained, and the dynamical properties of all these solutions are analysed using some figures. We can see the universality of this method. It can be applied to many other nonlinear evolution equations to get their M-lump solutions. In addition, we can investigate other classes of interactive solutions of lump waves to the nonlinear systems.

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