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Exact solution of the nonlinear fin problem with exponentially temperature-dependent thermal conductivity and heat transfer coefficient

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Abstract. This article studies a class of fin problems with two nonlinear terms arising from thermal conductivity and convection heat transfer coefficient. A one-dimensional convective straight fin is analysed for exponentially temperature-dependent thermal conductivity and exponentially temperature-dependent heat transfer coefficient. The exact fin temperature excess, heat transfer rate and fin efficiency are obtained and presented graphically. The obtained results show the strong influences of exponent indexes of thermal conductivity and convection transfer coefficient as well as thermogeometric fin parameter on the fin efficiency and heat flow.

Keywords. Exact solution; exponentially temperature-dependent thermal conductivity; exponentially temperature-dependent heat transfer coefficient; fin efficiency; non-linear fin problem; convective fin.

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1. Introduction

Heat transfer is a typical phenomenon in engineering applications and daily life. In heat transfer, various problems are encountered. The heat exchange equipment can transfer heat from a hot fluid to a cold fluid in a certain manner. The study of the influence factors in the heat transfer process and achieving the best heat transfer efficiency are particularly significant. For example, the entropy generation of the micro/nanofluid was investigated in various environments such as magnetic field, thermal radiation etc. [1–3]. Hosseinzadeh *et al* [4] examined the effect of variable Lorentz forces on nanofluid in movable parallel plates. Iram *et al* [5] investigated the effects of temperature and concentration gradient on heat and mass transfer in micropolar fluid. Zangooee *et al* [6] analysed the hydrothermal problem of MHD nanofluid flow between two radiative stretchable rotating disks.

As an indispensable class of heat conduction elements that extend heat transfer surface area, fins enhance thermal transfer efficiency [7]. Fins are widely used in heat-exchange equipments such as heat exchangers, electronic components, semiconductors, solar collectors, etc. [8]. In the past decade, the analysis of fins for enhancing heat transfer has aroused extensive concern of researchers. When the cross-section of a fin is relatively small, the temperature change in the cross-section also is small. Then it only has a significant temperature change along the direction of the length of the fin. In other words, the temperature distribution in the fin is a one-dimensional temperature field along the direction of the length of the fin. A variety of contributions have been made for a one-dimensional fin with temperature-dependent thermal conductivity. An earlier study of temperature-dependent straight fins with internal heat generation may be found in [9], where the governing differential equation is nonlinear. Atouei *et al* [10] solved the heat transfer problem of semispherical convective–radiative porous fins with temperature-dependent properties and heat generation. When neglecting heat source and radiation, convective fins often contain two nonlinear terms, one being temperature-dependent thermal conductivity and the other being temperature-dependent heat transfer coefficient [11]. Their nonlinear dependence on temperature has various forms like constant, linear, power law and so on. Till now, the thermal performance of fins has been studied for constant thermal conductivity and power-law heat transfer coefficient [12,13] and for

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linear thermal conductivity and constant heat transfer coefficient [14–24]. A variety of methods have been formulated to determine the temperature distribution, including the Adomian decomposition method [14–16], the differential transformation method [17,18], the variational iteration method [19,20], the homotopy analysis method [21–23], the double optimal linearisation method [24], etc. The thermal performance of the linear temperature-dependent thermal conductivity and the power-law temperature-dependent thermal transfer coefficient were analysed and resolved [25–28]. Numerical results were obtained by the singular boundary method for a steady-state nonlinear heat conduction problem with temperature-dependent thermal conductivity [29]. The exact solutions of the nonlinear fin problem with the power-law temperature-dependent thermal conductivity and thermal transfer coefficient were investigated [30–32].

In addition to the case of constant, linear and power-law functions, the thermal conductivity and thermal transfer coefficient also have other functions. However, there is little information on the study of a longitudinal fin with exponentially temperature-dependent thermal conductivity and exponentially temperature-dependent thermal transfer coefficient. One-dimensional steady-state heat balance equation has been discussed with exponential function thermal conductivity in [33]. In fact, a more general exponentially temperature-dependent thermal conductivity has been reported in [34]. However, the thermal performance of the nonlinear fin with exponentially temperature-dependent thermal conductivity and thermal transfer coefficient was not studied before, to the best of our knowledge. In particular, no exact solution was reported. This paper aims at deriving an analytic solution for a longitudinal fin with exponentially temperature-dependent thermal conductivity and thermal transfer coefficient. The effects of exponential indexes on the temperature distribution and fin efficiency are examined.

2. Statement of the problem

Consider a convective fin, where the temperature change along the longitudinal direction is significant and the temperature change in the cross-section is negligibly small. Therefore, in the present study, a one-dimensional (1D) fin with cross-sectional area A, length L and perimeter P, is considered, as shown in figure 1. The fin is exposed to a base surface of temperature T_b, and extends into a fluid of temperature T_a. In addition, as heat transfer through the tip end of the fin is relatively small, it is negligible, and so the fin tip is assumed to be insulated. As a result, the boundary conditions of the fin with an adiabatic tip and constant base temperature are stated as follows:

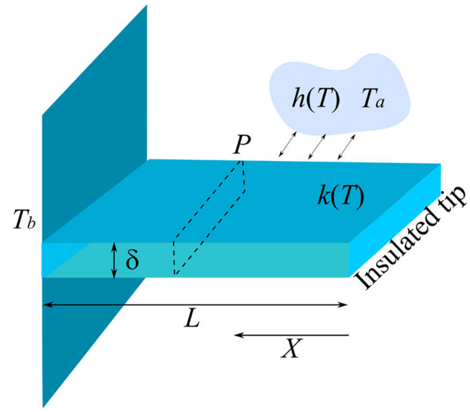

Figure 1. Schematic of a convective fin with rectangular cross-section along with coordinates.

$$\frac{\mathrm{d}T}{\mathrm{d}X} = 0, \quad \text{at } X = 0, \tag{1}$$

$$T = T_b, \text{ at } X = L, \tag{2}$$

where X is the distance from the adiabatic tip. Taking into account that the radiation of heat energy varies with the fourth power of absolute temperature, heat radiation is pronounced and cannot be ignored under high temperature or vacuum insulation [35]. However, in general, when the amount of heat radiated is small enough or negligible compared to convection and heat conduction, we can remove the effect of heat radiation. In the presence of heat radiation, a relevant analysis can be found in [36]. In the absence of surface heat radiation, 1D steady-state heat balance reads as

$$\frac{\mathrm{d}}{\mathrm{d}X}\left[k(T)A\frac{\mathrm{d}T}{\mathrm{d}X}\right] - Ph(T)(T-T_a) = 0, \quad 0 < X < L, \tag{3}$$

where the thermal conductivity $k(T)$ and the convection heat transfer coefficient $h(T)$ at any position X depend on the fin temperature $T(X)$ at the local position. Here, we only consider the situation where they exponentially depend on the temperature change, that is,

$$k(T) = k_0 \mathrm{e}^{\alpha_0 T}, \quad h(T) = h_0 \mathrm{e}^{\beta_0 T}, \tag{4}$$

where k_0 and h_0 are the reference values of thermal conductivity and convection heat transfer coefficient, respectively, α_0 and β_0 are exponential indexes of thermal conductivity and convection heat transfer coefficient, respectively.

For the convenience of later analysis, the following dimensionless variables are introduced:

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad x = \frac{X}{L}, \quad M = L\sqrt{\frac{h_a P}{k_a A}}, \tag{5}$$

and moreover, we denote

$$k(T) = k_a \mathrm{e}^{\alpha\theta}, \quad h(T) = h_a \mathrm{e}^{\beta\theta}, \tag{6}$$

where

$$\alpha = \alpha_0 (T_b - T_a), \quad k_a = k_0 \mathrm{e}^{\alpha_0 T_a}, \tag{7}$$

$$\beta = \beta_0 (T_b - T_a), \quad h_a = h_0 \mathrm{e}^{\beta_0 T_a}. \tag{8}$$

With these variables, the 1D steady-state heat balance equation (3) together with the boundary conditions (1) and (2) becomes

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{e}^{\alpha\theta}\frac{\mathrm{d}\theta}{\mathrm{d}x}\right] - M^2\theta \mathrm{e}^{\beta\theta} = 0, \; 0 < x < 1 \tag{9}$$

subject to

$$\frac{\mathrm{d}\theta}{\mathrm{d}x}(0) = 0, \quad \theta(1) = 1. \tag{10}$$

3. Exact solutions

In order to seek a suitable solution to eq. (9) subject to the boundary conditions (10), let us first discuss it for $\alpha = 0$ and $\alpha \neq 0$, meaning constant thermal conductivity and exponentially temperature-dependent thermal conductivity, respectively. In particular, if $\alpha = 0$ and $\beta = 0$, equivalent to constant thermal conductivity and constant convection heat transfer coefficient, the dimensionless temperature for this classical case has an exact explicit expression [8] as follows:

$$\theta = \frac{\cosh Mx}{\cosh M}. \tag{11}$$

3.1 *Constant thermal conductivity*

In order to make it more convenient to solve this problem, we introduce $u = \mathrm{d}\theta/\mathrm{d}x$ and obtain

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} = u\frac{\mathrm{d}u}{\mathrm{d}\theta}. \tag{12}$$

Thus, the ordinary differential equation (9) can be converted to the following equation:

$$u\,\mathrm{d}u - M^2\theta \mathrm{e}^{\beta\theta}\,\mathrm{d}\theta = 0. \tag{13}$$

When $\beta \neq 0$, we integrate both sides of eq. (13) and obtain

$$\frac{1}{2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 - M^2 \mathrm{e}^{\beta\theta}\left(\frac{\theta}{\beta} - \frac{1}{\beta^2}\right) = C, \tag{14}$$

where C is an integration constant, and we have used $u = \mathrm{d}\theta/\mathrm{d}x$ in the final equation. Obviously, the constant C is related to the temperature at the fin tip, denoted by θ_0. That is, setting $x = 0$ in (14) and remembering $\theta'(0) = 0$, one gets

$$-M^2 \mathrm{e}^{\beta\theta_0}\left(\frac{\theta_0}{\beta} - \frac{1}{\beta^2}\right) = C. \tag{15}$$

With this at hand, eq. (14) is further rewritten as

$$\frac{\mathrm{d}\theta}{\sqrt{\mathrm{e}^{\beta\theta}(\beta\theta - 1) - \mathrm{e}^{\beta\theta_0}(\beta\theta_0 - 1)}} = \frac{\sqrt{2}M}{|\beta|}\mathrm{d}x, \tag{16}$$

where another equation with a negative sign has been removed due to no practical implication.

After integrating both sides of eq. (16), we obtain

$$\frac{\sqrt{2}M}{|\beta|}x = \int_{\theta_0}^{\theta} \frac{\mathrm{d}z}{\sqrt{\mathrm{e}^{\beta z}(\beta z - 1) - \mathrm{e}^{\beta\theta_0}(\beta\theta_0 - 1)}}. \tag{17}$$

Note that the constant θ_0 is still unknown in the above equation. Substituting the remaining boundary condition $\theta(1) = 1$ to eq. (17) leads to the condition that θ_0 must satisfy

$$\frac{\sqrt{2}M}{|\beta|} = \int_{\theta_0}^{1} \frac{\mathrm{d}z}{\sqrt{\mathrm{e}^{\beta z}(\beta z - 1) - \mathrm{e}^{\beta\theta_0}(\beta\theta_0 - 1)}}. \tag{18}$$

Once θ_0 is determined using (18), we put its value into (17) to obtain the temperature excess distribution of a 1D convective fin.

3.2 *Exponentially temperature-dependent thermal conductivity*

Next, we consider the case of exponentially temperature-dependent thermal conductivity. For this case, we first introduce a new unknown function

$$y = \mathrm{e}^{\alpha\theta}. \tag{19}$$

Under the transformation, one rewrites the ordinary differential equation (9) subject to the boundary conditions (10) as follows:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = M^2 y^{\beta/\alpha} \ln y, \tag{20}$$

subject to

$$\frac{\mathrm{d}y}{\mathrm{d}x}(0) = 0, \quad y(1) = \mathrm{e}^{\alpha}. \tag{21}$$

Using the transformation $u = \mathrm{d}y/\mathrm{d}x$, one gets

$$u\,\mathrm{d}u = M^2 y^{\beta/\alpha} \ln y\,\mathrm{d}y. \tag{22}$$

In the following, let us first consider a special case, i.e. $\beta = -\alpha$. For this case, we integrate both sides of the ordinary differential equation (22) to get

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - M^2 \ln^2 y = C, \tag{23}$$

where C is an unknown constant, and we have used $u = \mathrm{d}y/\mathrm{d}x$. The boundary condition $\mathrm{d}y/\mathrm{d}x = 0$ at $x = 0$ allows us to obtain

$$-M^2 \ln^2 y_0 = C. \tag{24}$$

We plug the result (24) into eq. (23) to eliminate the constant C, and the ordinary differential equation (23) can be rewritten as follows:

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = M^2(\ln^2 y - \ln^2 y_0), \tag{25}$$

which, by further substituting $y = \mathrm{e}^{\alpha\theta}$ into (25), is further rewritten as

$$M\,\mathrm{d}x = \frac{\mathrm{e}^{\alpha\theta}\,\mathrm{d}\theta}{\sqrt{\theta^2 - \theta_0^2}}. \tag{26}$$

After integrating with respect to both sides of eq. (26) and recalling the condition $\theta(0) = \theta_0$, one has

$$Mx = \int_{\theta_0}^{\theta} \frac{\mathrm{e}^{\alpha z}\,\mathrm{d}z}{\sqrt{z^2 - \theta_0^2}}, \tag{27}$$

where θ_0 can be determined by $\theta(1) = 1$. To this end, by setting $x = 1$ in the above equation, the undetermined parameter θ_0 should meet the following equation:

$$M = \int_{\theta_0}^{1} \frac{\mathrm{e}^{\alpha z}\,\mathrm{d}z}{\sqrt{z^2 - \theta_0^2}}. \tag{28}$$

Once θ_0 is determined using (28), we put its value into eq. (27) to obtain the temperature excess distribution of a 1D convective fin.

For the remaining case, i.e., $\alpha+\beta \neq 0$, the differential equation (22) can be solved by integrating both sides of the equation, yielding

$$\frac{1}{2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - M^2\alpha^2 y^{(1+\beta/\alpha)}\left[\frac{\ln y}{\alpha(\alpha+\beta)} - \frac{1}{(\alpha+\beta)^2}\right] = C, \tag{29}$$

where C is an unknown constant, and we have used $u = \mathrm{d}y/\mathrm{d}x$.

In a similar manner, eq. (29) can be converted to the following differential equation in the original variables:

$$\frac{\mathrm{e}^{\alpha\theta}\,\mathrm{d}\theta}{\sqrt{\mathrm{e}^{(\alpha+\beta)\theta}\left[(\alpha+\beta)\,\theta-1\right]-\mathrm{e}^{(\alpha+\beta)\theta_0}\left[(\alpha+\beta)\,\theta_0-1\right]}} = \frac{\sqrt{2}M}{|\alpha+\beta|}\mathrm{d}x. \tag{30}$$

Integrating both sides of eq. (30) and imposing the condition $\theta_0 = \theta(0)$ we obtain

$$\frac{\sqrt{2}M}{|\alpha+\beta|}x = \int_{\theta_0}^{\theta} \frac{\mathrm{e}^{\alpha z}\,\mathrm{d}z}{\sqrt{\mathrm{e}^{(\alpha+\beta)z}\left[(\alpha+\beta)\,z-1\right]-\mathrm{e}^{(\alpha+\beta)\theta_0}\left[(\alpha+\beta)\,\theta_0-1\right]}}, \tag{31}$$

where θ_0 is an unknown constant that can be determined by the condition $\theta(1) = 1$, i.e.,

$$\frac{\sqrt{2}M}{|\alpha+\beta|} = \int_{\theta_0}^{1} \frac{\mathrm{e}^{\alpha z}\,\mathrm{d}z}{\sqrt{\mathrm{e}^{(\alpha+\beta)z}\left[(\alpha+\beta)\,z-1\right]-\mathrm{e}^{(\alpha+\beta)\theta_0}\left[(\alpha+\beta)\,\theta_0-1\right]}}. \tag{32}$$

4. Results and discussion

4.1 *Temperature excess*

Based on the above-obtained results, we examine the influence of fin parameters on the dimensionless temperature excess distribution. Figure 2 presents a comparison of the dimensionless temperature excess value θ_0 at the fin tip ($x = 0$) for different heat transfer properties. For arbitrary α or β, the dimensionless temperature excess value θ_0 at the fin tip, as seen in figure 2, gradually decreases with the thermogeometric parameter M increasing. It implies that the increase of β decreases the dimensionless temperature excess value θ_0 for given parameters M and α.

Once θ_0 is determined for given material parameters, the dimensionless temperature excess at an arbitrary position in the direction of the length can be evaluated and the numerical results are presented in figure 3

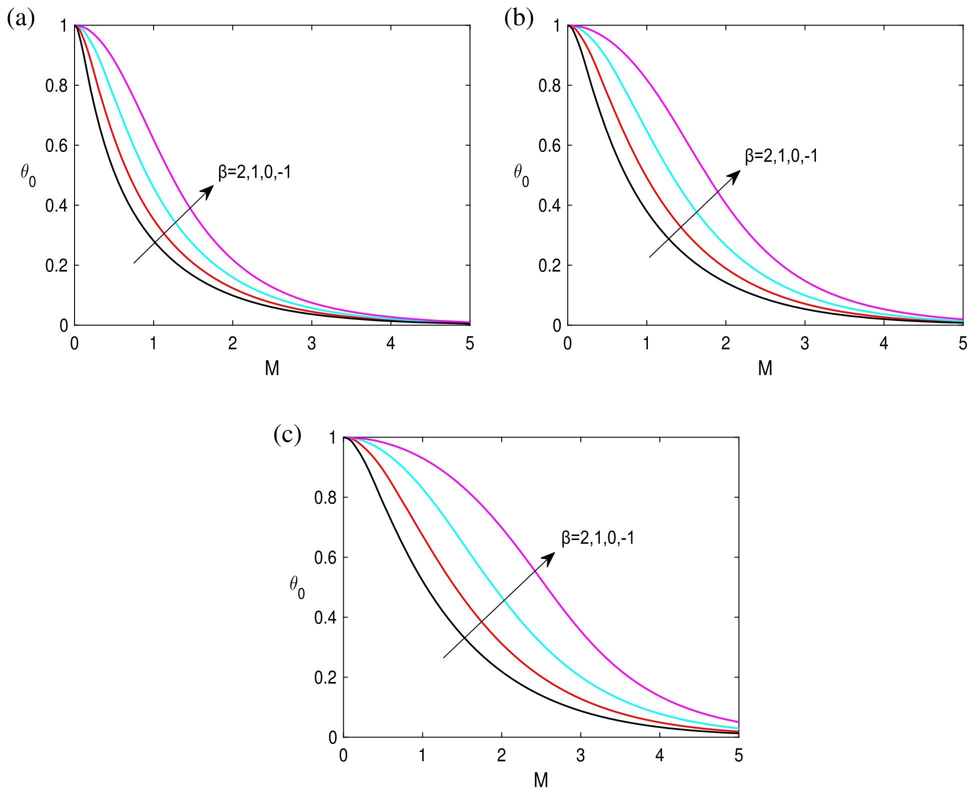

Figure 2. The dimensionless temperature value θ_0 at the fin tip for different values of β. (**a**) $\alpha = -1$, (**b**) $\alpha = 0$ and (**c**) $\alpha = 1$.

for different values of α and β when $M = 1.6$. The dimensionless temperature excess $\theta(x)$ decreases as the position is away from the base or closer to the tip, and the rate of temperature reduction obviously becomes slower as it gets closer to the fin tip. The reason is that as the fin temperature decreases, the temperature difference between the fin and the surrounding environment decreases. Moreover, figure 3 shows that as β decreases or α increases, the temperature excess increases and the temperature gradient decreases. From the physical viewpoint, with the increase of α, the heat conduction coefficient of the fin increases and the thermal resistance in the fin decreases. Therefore, even for a small temperature gradient, the fin will generate a large heat flow. With the decrease of β, the thermal resistance between the fin and the external environment increases, and so the heat transfer from the fin surface to the surrounding environment decreases. Similar results were obtained in [37].

Figures 4 and 5 illustrate the effect of thermogeometric parameter M on the variation of dimensionless temperature excess $\theta(x)$ with the dimensionless length for different parameters β when $\alpha = -1$ and 1, respectively. Obviously, the larger is the value of M, the less is the temperature excess θ and the larger is the temperature gradient, irrespective of the values of α and β. The effect of β in the convection heat transfer coefficient on the temperature excess is shown in figures 4 and 5 for different values of M and α. From figures 4 and 5, it is viewed that the trend of these curves is similar to that for linear temperature-dependent thermal conductivity [38].

4.2 *Heat transfer rate*

Ignoring the heat generation of the fin and heat radiation, the heat transfer rate can be obtained by evaluating the heat loss from the fin surface to the surrounding fluid [8,35],

$$q = k(T)A\left.\frac{\mathrm{d}T}{\mathrm{d}X}\right|_{X=L} \tag{33}$$

or in dimensionless form:

$$\frac{qL}{k_a A(T_b - T_a)} = \mathrm{e}^{\alpha}\theta'(1), \tag{34}$$

where the temperature gradient $\theta'(1)$ at the fin base is readily evaluated once the temperature excess is determined. That is, with the help of (16), (26) and (30) one has

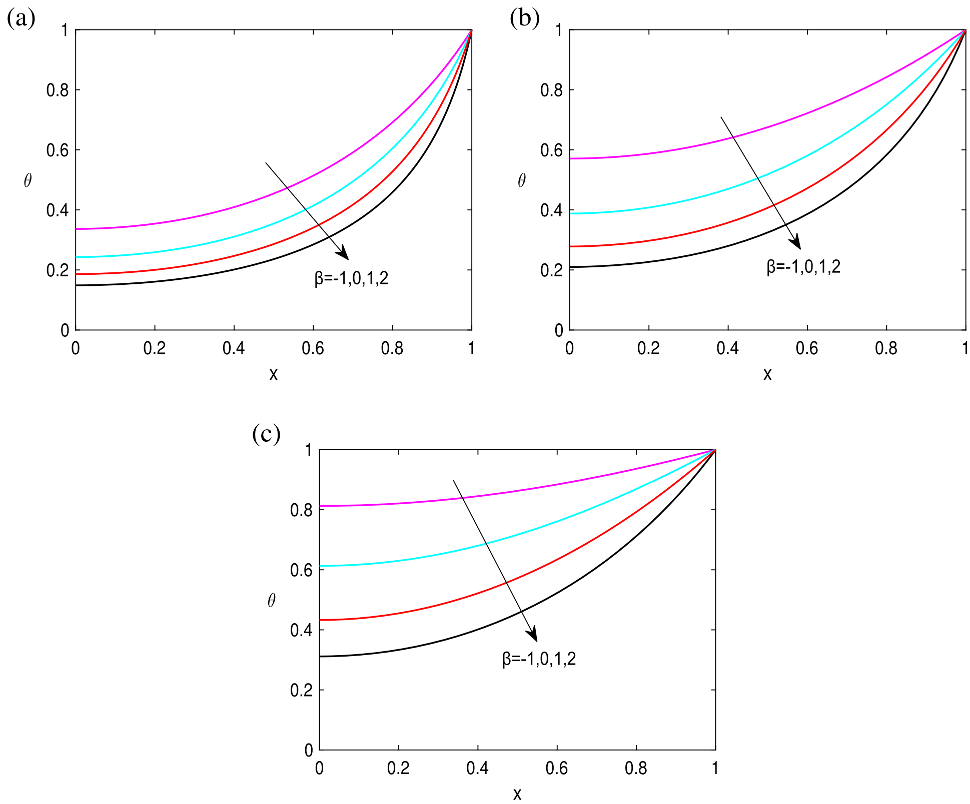

Figure 3. The dimensionless temperature excess θ for $M = 1.6$ and different values of β. (**a**) $\alpha = -1$, (**b**) $\alpha = 0$ and (**c**) $\alpha = 1$.

$$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \begin{cases} \dfrac{M}{\mathrm{e}^{\alpha\theta}\,|\alpha+\beta|}\sqrt{2\left\{\mathrm{e}^{(\alpha+\beta)\theta}\left[(\alpha+\beta)\,\theta-1\right]-\mathrm{e}^{(\alpha+\beta)\theta_0}\left[(\alpha+\beta)\,\theta_0-1\right]\right\}}, & \alpha+\beta\neq 0 \\ \dfrac{M}{\mathrm{e}^{\alpha\theta}}\sqrt{\theta^2-\theta_0^2}, & \alpha+\beta=0. \end{cases} \tag{35}$$

Putting (35) into (34) leads to the following equation:

$$\frac{qL}{k_a A(T_b-T_a)} = \begin{cases} \dfrac{M}{|\alpha+\beta|}\sqrt{2\left\{\mathrm{e}^{(\alpha+\beta)}\left[(\alpha+\beta)-1\right]-\mathrm{e}^{(\alpha+\beta)\theta_0}\left[(\alpha+\beta)\,\theta_0-1\right]\right\}}, & \alpha+\beta\neq 0, \\ M\sqrt{1-\theta_0^2}, & \alpha+\beta=0. \end{cases} \tag{36}$$

For convenience, the dimensionless heat transfer rate may be defined alternatively by [35],

$$Q = \frac{q}{\sqrt{h_a P k_a A}\,(T_b-T_a)} \tag{37}$$

which gives

$$Q = \begin{cases} \dfrac{1}{|\alpha+\beta|}\sqrt{2\left\{\mathrm{e}^{\alpha+\beta}\,(\alpha+\beta-1)-\mathrm{e}^{(\alpha+\beta)\theta_0}\left[(\alpha+\beta)\,\theta_0-1\right]\right\}}, & \alpha+\beta\neq 0, \\ \sqrt{1-\theta_0^2}, & \alpha+\beta=0. \end{cases} \tag{38}$$

In particular, for the special case $a = \beta = 0$, if denoting $M = mL$, $m = \sqrt{h_a P/k_a A}$, from (11) we obtain an explicit expression for Q as

$$Q = \tanh(mL). \tag{39}$$

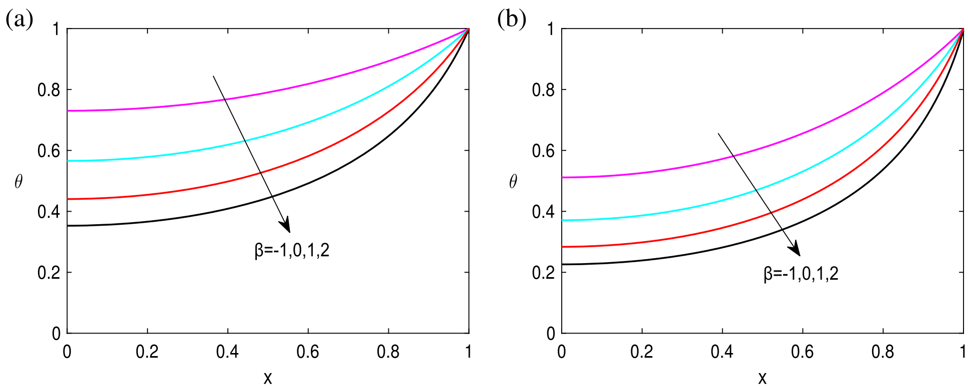

Figure 4. The dimensionless temperature excess θ for $\alpha = -1$ and different values of β. (**a**) $M = 0.8$ and (**b**) $M = 1.2$.

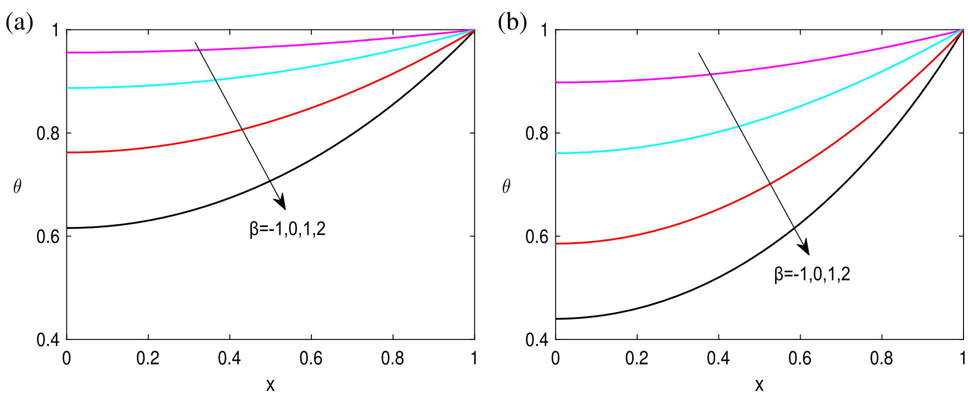

Figure 5. The dimensionless temperature excess θ for $\alpha = 1$ and different values of β. (**a**) $M = 0.8$ and (**b**) $M = 1.2$.

Figure 6 shows the variation of the dimensionless heat transfer rate Q with M for a convective longitudinal fin with different values of α and β. From figure 6, we see that for any α and β, the dimensionless heat transfer rate Q rapidly increases from the origin with M rising and then progressively tends to a steady value. It is obvious that if the thermal resistance of the fin base position $X = L$ remains unchanged (corresponding to the constant parameter α), the increase of β and M causes the heat loss from the fin surface to increase, and then the heat flow through the fin base ($X = L$) increases, which makes the temperature profile steeper. When β and M remain unchanged, the increase of α decreases the heat resistance of the fin base position ($X = L$), and the heat flow through the base position of the fin ($X = L$) increases. Similar conclusions were obtained in [39].

In particular, as $M = mL$ reaches a certain value which strongly depends on α and β, further increase of M has few influences on Q. For example, for $a = \beta = 0$, corresponding to constant conductivity and constant convection transfer coefficient, the exact expression gives $\tanh(mL) > 0.99$ or $mL > 2.65$, which implies that a convective fin of length $L = 2.65/m$ may approximately be taken as an infinitely long fin. More precisely, it is sufficient to choose a convective fin of length $2.65/m$, and Q may reach 99% compared to an infinitely long convective fin. This means that the fin has an optimal length, an excessively long fin waste material and increases the cost without doing any good as the heat emitted from the fin to the external environment is very marginal. Obviously, for other values of α and β, the optimal length has a slight change. It is pointed out that for nonvanishing α and β values, an exact solution was derived before, but not in explicit form. Therefore, for simplicity, we select the value of Q for $M = 5$ as the final steady value and compute a critical thermogeometric parameter M_0 such that $Q(M_0)/Q(5) = 99\%$. Evaluated results are listed in table 1. From table 1, for $a = \beta = 0$, it is seen that $M_0 = 2.642$ is nearly the same as $M_0 = 2.65$. The former reaches 99% relative to $Q(5)$, while the latter reaches 99% relative to $Q(\infty)$. In other words, it is reasonable to assume $Q(5)$ as the final steady value. Under different parameter conditions, the optimal length of the fin is different. Obviously, with the

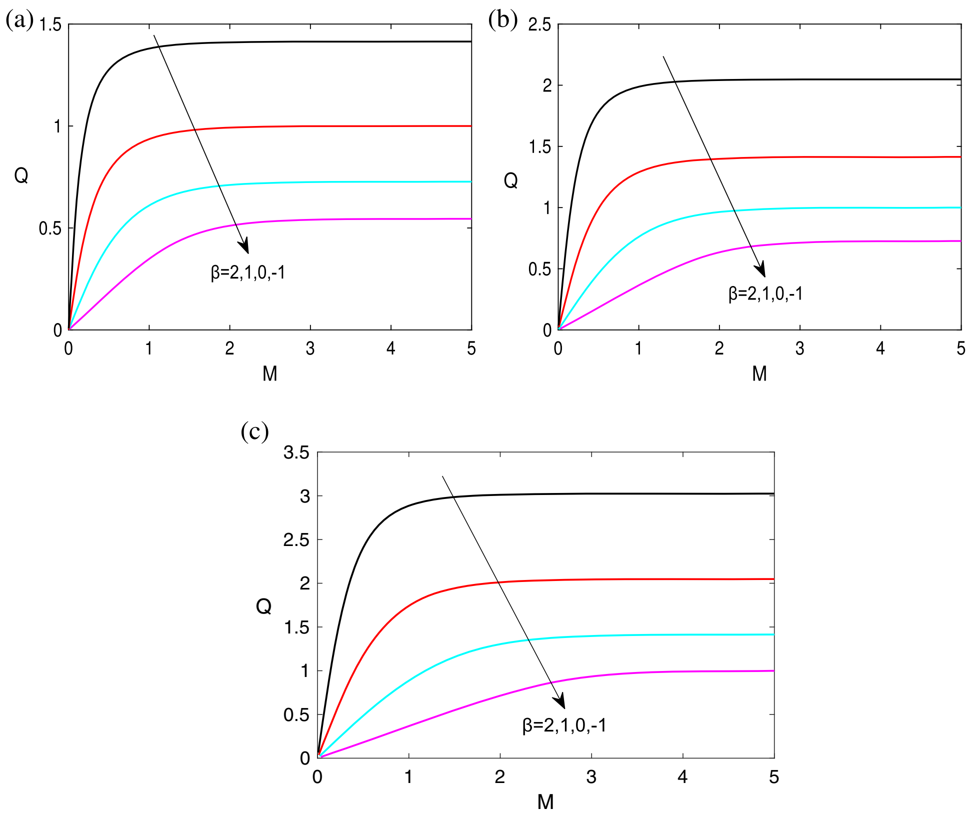

Figure 6. The dimensionless fin heat transfer rate against M for different values of β. (**a**) $\alpha = -1$, (**b**) $\alpha = 0$ and (**c**) $\alpha = 1$.

Table 1. The critical thermogeometric parameter M_0 such that $Q(M_0)/Q(5) = 99\%$.

α	$\beta = -1$	$\beta = 0$	$\beta = 1$	$\beta = 2$
-1	2.913	2.385	1.869	1.360
0	3.302	2.642	1.987	1.431
1	3.905	3.055	2.270	1.604

increase of α and the decrease of β, the optimal length of the fin increases.

4.3 *Fin efficiency*

The fin efficiency is the ratio of the actual heat transferred from the fin surface to the surrounding fluid to the amount of heat when the entire fin area takes the base temperature [8]. In fact, the amount of actual heat transferred from the fin surface to the surrounding fluid is equal to the heat passing through the fin base position $X = L$ or $x = 1$; so the fin efficiency can be obtained as follows:

$$\eta = \frac{k(T)A\,(\mathrm{d}T/\mathrm{d}X)|_{X=L}}{Ph(T_b)(T_b - T_a)L} = \frac{\mathrm{e}^{\alpha-\beta}\theta'(1)}{M^2}. \tag{40}$$

Consequently, one acquires the fin efficiency as follows:

$$\eta = \frac{Q}{M\mathrm{e}^{\beta}}. \tag{41}$$

The value of the fin efficiency directly reflects the performance of the heat exchanger, and a greater fin efficiency value reflects better fin performance. Figure 7 shows the influence of M on the fin efficiency for different values of α and β. Obviously, the fin efficiency decreases as M or β increases. From figure 7, the fin efficiency is 100% for an ideal state with $M = 0$, where the temperature of each place on the fin is equal to the temperature of the base position. When $\beta = -1, \alpha = 1$, the efficiency of the fin is equal to unity or extremely close to unity for the range of M from 0 to 1.6. The

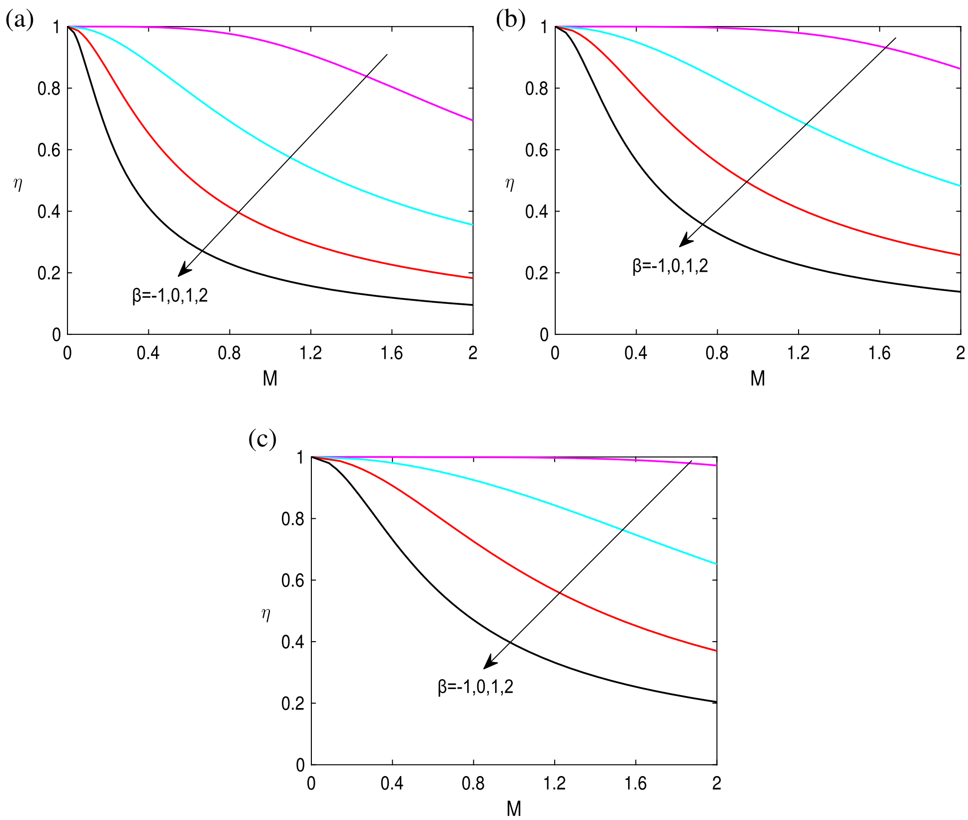

Figure 7. The fin efficiency against M for different values of β. (**a**) $\alpha = -1$, (**b**) $\alpha = 0$ and (**c**) $\alpha = 1$.

influence of M on the thermal efficiency is very small or ignored.

5. Conclusions

In this paper, the temperature excess distribution, the heat transfer rate and the fin efficiency were analytically determined for a convective straight fin with exponentially temperature-dependent thermal conductivity and exponentially temperature-dependent convection heat transfer coefficient. An exact solution was derived in an implicit form. The influence of the parameters on the temperature excess, the heat transfer rate and the fin efficiency was presented graphically and discussed. The obtained results indicate that if α becomes larger, or β and M become less, there is greater temperature excess along the fin length and larger fin efficiency. The dependence of the fin heat transfer rate on α, β and M is also elucidated.

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