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Electric charge quantisation in 331 models with exotic charges

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Abstract. The extensions of the Standard Model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge group are known as 331 models. Different properties such as the fermion assignment and the electric charges of the exotic spectrum, that define a particular 331 model, are fixed by a β parameter. In this article, we study the electric charge quantisation in two versions of the 331 models, set by the conditions $\beta = 1/(3\sqrt{3})$ and $\beta = 0$. In these frameworks, arise exotic particles, for instance, new leptons and gauge bosons with a fractional electric charge. Additionally, depending on the version, quarks with non-standard fractional electric charges or even neutral appear. Considering the definition of electric charge operator as a linear combination of the group generators that annihilates the vacuum, classical constraints from the invariance of the Lagrangian, and gauge and mixed gauge-gravitational anomalies cancellation, the quantisation of the electric charge can be verified in both versions.

Keywords. Electric charge quantisation; 331 models; gauge anomalies cancellation; particles with exotic electric charges.

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1. Introduction

The experimental observation where the electric charge of the particles appears only in quantised units is known as electric charge quantisation. The first efforts to explain this phenomenon included ideas related to higher dimensions [1,2], magnetic monopoles [3] and grand unified theories [4]. However, these approaches have not yet been experimentally verified. In this work, we follow the perspective introduced by [5–7] and [8]. This approach considers two main conditions: (1) classical constraints imposed by the $U(1)_X$ gauge group invariance of the Yukawa Lagrangian and (2) the quantum restrictions arising from the cancellation of the anomalies. In that sense, the above approach for electric charge quantisation will be applied for two versions of the 331 model, set by the conditions $\beta = 1/(3\sqrt{3})$ [9] and $\beta = 0$ [10]. In these versions, new leptons with charge $\pm 2/3\, e$ $(\pm 1/2\, e)$, extra quarks with charges $+1/3\, e, 0\, (\pm 1/6\, e)$, exotic gauge and scalar bosons with charges $\pm 1/3\, e$ and $\pm 2/3\, e$ $(\pm 1/2\, e)$ arise, in addition to a new neutral boson Z'.

Also, particles with fractional electric charges have been proposed by other theoretical models [11,12] and both ATLAS and CMS Collaborations have already performed searches of new heavy lepton-like particles with non-standard electric charges [13,14]. Experimentally, these kinds of particles may be misidentified or unobserved because charged particle identification algorithms generally assume that particles have charges of $\pm 1e$ [15]. The new proposal maintains the special features of the 331 models, such as the relation between the number of fermionic families and the number of colours in QCD. Pires and Ravinez [8] consider the quantisation of the electric charge in a similar fashion but for the minimal 331 model, that is, for $\beta = -\sqrt{3}$. In this work, the authors argue that for models with $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ symmetry, the quantisation of the electric charge is verified when they take into account the three families of fermions, with or without considering the neutrino masses, in contrast to the Standard Model (SM). On the other hand, Dong and Long [16] explain charge quantisation using the general form of the electromagnetic currents under the parity invariance for 331 models with $\beta = -\sqrt{3}$ and $\beta = -1/\sqrt{3}$. Finally, the objective of this study is to verify if the proposed 331 versions satisfy the quantisation of the electric charge.

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2. The models

The general relation for the electric charge operator ($\mathcal{Q}$) in a 331 model is given by

$$\mathcal{Q} = \alpha T_3 + \beta T_8 + \gamma X, \tag{1}$$

where T_3 and T_8 are the diagonal generators of $SU(3)_L$ built as $T_i = \lambda_i/2$ from the Gell–Mann matrices λ_i, with $i = 1, \ldots, 8$ and X is the charge of $U(1)_X$. We consider $\alpha = 1$ in order to properly set the W boson electric charge in the model, as was described in [16].

In addition, the value of β fixes the fermion assignment, the electric charges of the exotic spectrum, and it is used to classify different 331 models [17]. We can write

$$\mathcal{Q} = \text{diag}\left(\pm\frac{1}{2}\left(1+\frac{\beta}{\sqrt{3}}\right), \frac{1}{2}\left(\mp 1 \pm \frac{\beta}{\sqrt{3}}\right), \mp\frac{1}{3}\beta\sqrt{3}\right) + \gamma X I_{3\times 3}, \tag{2}$$

where the upper sign corresponds to the fundamental representation of the Gell–Mann matrices and the lower sign to the conjugate representation. The scalar sector,

$$\eta \sim (\mathbf{1}, \mathbf{3}, X_\eta), \quad \rho \sim (\mathbf{1}, \mathbf{3}, X_\rho), \quad \chi \sim (\mathbf{1}, \mathbf{3}, X_\chi) \tag{3}$$

which develop vacuum expectation value (vev) as

$$\langle \eta^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \quad \langle \rho^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix}, \quad \langle \chi^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ v\chi \end{pmatrix}. \tag{4}$$

With the requirement that the charge operator must annihilate the vacuum, we obtain the following general relations:

$$\gamma = -\frac{1}{2}\left(1+\frac{\beta}{\sqrt{3}}\right)\frac{1}{X_\eta} = -\frac{1}{2}\left(-1+\frac{\beta}{\sqrt{3}}\right)\frac{1}{X_\rho} = \frac{\beta\sqrt{3}}{3}\frac{1}{X_\chi}. \tag{5}$$

Thus, for the particular choice of $\beta = 1/(3\sqrt{3})$, we obtain

$$\gamma = \frac{1}{9X_\chi}, \quad X_\eta = -5X_\chi, \quad X_\rho = 4X_\chi \tag{6}$$

and for $\beta = 0$:

$$\gamma = -\frac{1}{2X_\eta}, \quad X_\rho = -X_\eta, \quad X_\chi = 0. \tag{7}$$

It is straightforward to verify that the scalar fields fulfill the relation

$$X_\rho + X_\eta + X_\chi = 0. \tag{8}$$

In order to cancel anomalies associated with the $SU(3)_L$ gauge group, the leptons and the quark families must be assigned in different $SU(3)_L$ representations. So, for $\beta = 1/(3\sqrt{3})$, the leptons and the third quark family are assigned in triplets, while the first two quark families in antitriplets.

The leptonic sector includes

$$\psi_{iL} = \left(\nu_i, e_i^-, E_i\right)_L^T \sim \left(\mathbf{1}, \mathbf{3}, X_{\ell_i}\right), \quad e_{i_R}^- \sim \left(\mathbf{1}, \mathbf{1}, X_{e_i}\right), \quad E_{i_R} \sim \left(\mathbf{1}, \mathbf{1}, X_{E_i}\right), \tag{9}$$

where $i = 1, 2, 3$.

The two first quark families form $SU(3)_L$ antitriplets

$$Q_{aL} = (d_a, -u_a, D_a)_L^T \sim \left(\mathbf{3}, \mathbf{3}^*, X_{Q_a}\right), \quad u_{a_R} \sim \left(\mathbf{3}, \mathbf{1}, X_{u_a}\right), \quad d_{a_R} \sim \left(\mathbf{3}, \mathbf{1}, X_{d_a}\right), \quad D_{a_R} \sim \left(\mathbf{3}, \mathbf{1}, X_{D_a}\right), \quad a = 1, 2 \tag{10}$$

and the third family is assigned to $SU(3)_L$ triplet:

$$Q_{3L} = (t, b, T)_L^T \sim \left(\mathbf{3}, \mathbf{3}, X_{Q_3}\right), \quad t_R \sim (\mathbf{3}, \mathbf{1}, X_t), \quad b_R \sim (\mathbf{3}, \mathbf{1}, X_b), \quad T_R \sim (\mathbf{3}, \mathbf{1}, X_T). \tag{11}$$

Furthermore, the Yukawa Lagrangian for quarks is

$$-\mathcal{L}_Y^{\text{quarks}} = f_{ab}^u\,\overline{Q_{aL}}\,\rho^*\,u_{bR} + f_{ab}^d\,\overline{Q_{aL}}\,\eta^*\,d_{bR} + f_{ab}^D\,\overline{Q_{aL}}\,\chi^*\,D_{bR} + f^b\,\overline{Q_{3L}}\,\rho\,b_R + f^t\,\overline{Q_{3L}}\,\eta\,t_R + f^T\,\overline{Q_{3L}}\,\chi\,T_R + \text{h.c.} \tag{12}$$

The Yukawa Lagrangian for leptons is

$$-\mathcal{L}_Y^{\text{leptons}} = F_{ij}^e\,\overline{\psi_{iL}}\,\rho\,e_{jR} + F_{ij}^E\,\overline{\psi_{iL}}\,\chi\,E_{jR} + \text{h.c.} \tag{13}$$

On the other hand, for $\beta = 0$ we have leptons and the third quark family in antitriplets, while the first two quark families in triplets. In this case, the leptonic sector of the model is [10]

$$\psi_{iL} = \left(e_i^-, -\nu_i, E_i\right)_L^T \sim \left(\mathbf{1}, \mathbf{3}^*, X_{\ell_i}\right), \quad e_{i_R}^- \sim \left(\mathbf{1}, \mathbf{1}, X_{e_i}\right), \quad E_{i_R} \sim \left(\mathbf{1}, \mathbf{1}, X_{E_i}\right), \tag{14}$$

where $i = 1, 2, 3$.

The two first quark families form the $SU(3)_L$ triplets

$$Q_{aL} = (u_a, d_a, U_a)_L^T \sim \left(\mathbf{3}, \mathbf{3}, X_{Q_a}\right), \quad u_{a_R} \sim \left(\mathbf{3}, \mathbf{1}, X_{u_a}\right), \quad d_{a_R} \sim \left(\mathbf{3}, \mathbf{1}, X_{d_a}\right), \quad U_{a_R} \sim \left(\mathbf{3}, \mathbf{1}, X_{U_a}\right), \quad a = 1, 2 \tag{15}$$

and the third family is assigned to the $SU(3)_L$ anti-triplet:

$$Q_{3L} = (b, -t, T)_L^T \sim \left(\mathbf{3}, \mathbf{3}^*, X_{Q_3}\right), \quad t_R \sim (\mathbf{3}, \mathbf{1}, X_t),$$
$$b_R \sim (\mathbf{3}, \mathbf{1}, X_b), \quad T_R \sim (\mathbf{3}, \mathbf{1}, X_T). \quad (16)$$

Finally, for this version the Yukawa Lagrangian for quarks is

$$\begin{aligned} -\mathcal{L}_Y^{\text{quarks}} &= f_{ia}^u\, \overline{Q_{iL}}\, \eta\, u_{aR} + f_{ia}^d\, \overline{Q_{iL}}\, \rho\, d_{aR} \\ &+ f_{ia}^U\, \overline{Q_{iL}}\, \chi\, U_{aR} + f^a\, \overline{Q_{3L}}\, \eta^*\, d_{aR} \\ &+ f_a^t\, \overline{Q_{3L}}\, \rho^*\, u_{aR} \\ &+ f_a^U\, \overline{Q_{3L}}\, \chi^*\, U_{aR} + \text{h.c.} \end{aligned} \quad (17)$$

with $i = 1, 2$; $a = 1, 2, 3$; $u_{aR} = u_R, c_R, t_R$; $d_{aR} = d_R, s_R, b_R$ and $U_{aR} = U_{1R}, U_{2R}, T_R$. The Yukawa Lagrangian for leptons takes the form

$$-\mathcal{L}_Y^{\text{leptons}} = F_{ij}^e\, \overline{\psi_{iL}}\, \rho\, e_{jR} + F_{ij}^E\, \overline{\psi_{iL}}\, \chi\, E_{jR} + \text{h.c.} \quad (18)$$

Now, in order to obtain different electric charges of the particles in these models, we shall use classical and quantum constraints.

3. Constraints from replicas between families

As the SM particles and the exotic particles in the models present replicas between families, this allows us to reduce the number of hypercharge variables. For $\beta = 1/(3\sqrt{3})$:

$$X_{Q_1} = X_{Q_2} \equiv X_Q \quad (19)$$
$$X_{\ell_1} = X_{\ell_2} = X_{\ell_3} \equiv X_\ell \quad (20)$$
$$X_{u_1} = X_{u_2} = X_t \equiv X_u \quad (21)$$
$$X_{d_1} = X_{d_2} = X_b \equiv X_d \quad (22)$$
$$X_{e_1} = X_{e_2} = X_{e_3} \equiv X_e \quad (23)$$
$$X_{E_1} = X_{E_2} = X_{E_3} \equiv X_E \quad (24)$$
$$X_{D_1} = X_{D_2} \equiv X_D \quad (25)$$

and for $\beta = 0$, we obtain the same conditions, but instead of (25), we have

$$X_{U_1} = X_{U_2} = X_T \equiv X_U. \quad (26)$$

4. Constraints from $U(1)_X$ invariance

From the $U(1)_X$ invariance of the Yukawa Lagrangian, for $\beta = 1/(3\sqrt{3})$ we obtain

$$X_Q = X_d - X_\eta \quad (27)$$
$$X_Q = X_D - X_\chi \quad (28)$$
$$X_Q = X_u - X_\rho \quad (29)$$
$$X_{Q_3} = X_\eta + X_t \quad (30)$$
$$X_{Q_3} = X_\chi + X_T \quad (31)$$
$$X_{Q_3} = X_\rho + X_b \quad (32)$$
$$X_e = X_\ell - X_\rho \quad (33)$$
$$X_E = X_\ell - X_\chi \quad (34)$$

and for $\beta = 0$

$$X_Q = X_u + X_\eta \quad (35)$$
$$X_Q = X_d + X_\rho \quad (36)$$
$$X_Q = X_\chi + X_U \quad (37)$$
$$X_{Q_3} = X_d - X_\eta \quad (38)$$
$$X_{Q_3} = X_u - X_\rho \quad (39)$$
$$X_{Q_3} = X_U - X_\chi. \quad (40)$$

Equations (33) and (34) are still being fulfilled for this case.

5. Constraints from the cancellation of anomalies

As it is known, an anomaly is a symmetry which has been conserved in the classical theory but is broken at the quantum level. In the context of quantum field theory involving chiral fermions, it is important to cancel gauge anomalies in order to obtain a renormalisable theory. In the present paper, we are focussing on the cancellation of gauge and mixed gauge-gravitational anomalies. Thus, the quantum restrictions arising from the cancellation of anomalies imply

$$\left[SU(3)_C\right]^2 U(1)_X \to A_C = 3\sum_q X_{q_L} - \sum_q X_{q_R} = 0,$$

$$\left[SU(3)_L\right]^2 U(1)_X \to A_L = 3\sum_q X_{q_L} + \sum_\ell X_{\ell_L} = 0,$$

$$\begin{aligned} [\text{Grav}]^2\, U(1)_X \to A_G &= 3\sum_{\ell,q}\left[X_{\ell_L} + 3X_{q_L}\right] \\ &- \sum_{\ell,q}\left[X_{\ell_R} + 3X_{q_R}\right] = 0, \end{aligned}$$

$$\begin{aligned} \left[U(1)_X\right]^3 \to A_X &= 3\sum_{\ell,q}\left[X_{\ell_L}^3 + 3X_{q_L}^3\right] \\ &- \sum_{\ell,q}\left[X_{\ell_R}^3 + 3X_{q_R}^3\right] = 0, \end{aligned}$$

where q_L and ℓ_L are the doublets, and q_R and ℓ_R are the singlets, for the SM and exotic fields. Then

$$\begin{aligned} A_C = 3\left\{2X_Q + X_{Q_3}\right\} &- \left(2X_u + 2X_d + 2X_{D,Ux}\right. \\ &\left. + X_t + X_b + X_T\right) = 0 \end{aligned} \quad (41)$$

$$A_L = 3\left\{2X_Q + X_{Q_3}\right\} + 3X_\ell = 0 \quad (42)$$

$$A_G = 3\,(3X_\ell) - \{3X_e + 3X_E\} = 0. \tag{43}$$

From eq. (43), we have

$$A_G = 9X_\ell - 3X_e - 3X_E = 0. \tag{44}$$

For $\beta = 1/(3\sqrt{3})$:
Using eqs (6), (33) and (34) in (44), we obtain

$$X_\ell = -5X_\chi. \tag{45}$$

For $\beta = 0$:
Replacing eqs (7), (33) and (34) in (44), we have

$$X_\ell = X_\eta. \tag{46}$$

6. Results

6.1 *For* $\beta = 1/(3\sqrt{3})$

Subtracting eqs (27) and (32) and using (22) and (8):

$$X_{Q_3} = X_Q - X_\chi. \tag{47}$$

Replacing eqs (45) and (47) in (42), we obtain

$$X_Q = 2X_\chi. \tag{48}$$

Then

$$X_{Q_3} = X_\chi, \quad X_D = 3X_\chi$$
$$X_T = 0, \quad X_d = -3X_\chi, \quad X_u = 6X_\chi$$
$$X_e = -9X_\chi, \quad X_E = -6X_\chi. \tag{49}$$

In this case, the charge operator equation (2) is given by

$$\mathcal{Q} = \mathrm{diag}\,(\pm 5/9, \mp 4/9, \mp 1/9) + \frac{X}{9X_\chi}\mathrm{I}_{3\times 3}. \tag{50}$$

As was previously explained, the upper sign corresponds to the fundamental representation (triplet) of the Gell–Mann matrices and the lower sign to the conjugate representation (antitriplet). For the lepton triplet, we obtain

$$\mathcal{Q}\psi_L = \left(\mathrm{diag}\,(5/9, -4/9, -1/9) - \frac{5}{9}\mathrm{I}_{3\times 3}\right)\psi_L$$
$$\mathcal{Q}\psi_L = \mathrm{diag}\,(0, -1, -2/3)\,\psi_L. \tag{51}$$

Thus, we find the quantisation of the electric charge with the correct electric charges for leptons

$$\mathcal{Q}_{\nu_{e,\mu,\tau}} = 0, \quad \mathcal{Q}_{e,\mu,\tau} = -1, \quad \mathcal{Q}_{E,M,\mathcal{T}} = -2/3. \tag{52}$$

For the quark antitriplet:

$$\mathcal{Q}Q = \mathrm{diag}\,(-1/3, 2/3, 1/3)\,Q \tag{53}$$
$$\mathcal{Q}_{d,s} = -1/3, \quad \mathcal{Q}_{u,c} = 2/3, \quad \mathcal{Q}_{D_1,D_2} = 1/3 \tag{54}$$

and for the quark triplet:

$$\mathcal{Q}Q_3 = \mathrm{diag}\,(2/3, -1/3, 0)\,Q_3 \tag{55}$$
$$\mathcal{Q}_b = -1/3, \quad \mathcal{Q}_t = 2/3, \quad \mathcal{Q}_T = 0. \tag{56}$$

6.2 *For* $\beta = 0$

Subtracting eqs (36) and (38) and using (7):

$$X_{Q_3} = X_Q. \tag{57}$$

Replacing eqs (46) and (57) in (42), we obtain

$$X_Q = -\frac{1}{3}X_\eta. \tag{58}$$

Then

$$X_{Q_3} = -1/3X_\eta, \quad X_{U,T} = -1/3X_\eta$$
$$X_d = 2/3X_\eta, \quad X_u = -4/3X_\eta$$
$$X_e = 2X_\eta, \quad X_E = X_\eta \tag{59}$$

and

$$\mathcal{Q} = \mathrm{diag}\,(\mp 1/2, \mp 1/2, 0) - \frac{X}{2X_\eta}\mathrm{I}_{3\times 3}. \tag{60}$$

For the lepton triplet:

$$\mathcal{Q}\psi_L = \left(\mathrm{diag}\,(-1/2, 1/2, 0) - \frac{1}{2}\mathrm{I}_{3\times 3}\right)\psi_L$$
$$\mathcal{Q}\psi_L = \mathrm{diag}\,(-1, 0, -1/2)\,\psi_L \tag{61}$$

and

$$\mathcal{Q}_{\nu_{e,\mu,\tau}} = 0, \quad \mathcal{Q}_{e,\mu,\tau} = -1, \quad \mathcal{Q}_{E,M,\mathcal{T}} = -1/2. \tag{62}$$

For the quarks triplet:

$$\mathcal{Q}Q_3 = \mathrm{diag}\,(-1/3, 2/3, -1/6)\,Q \tag{63}$$
$$\mathcal{Q}_b = -1/3, \quad \mathcal{Q}_t = 2/3, \quad \mathcal{Q}_T = -1/6. \tag{64}$$

Finally, for the quarks, antitriplet:

$$\mathcal{Q}Q = \mathrm{diag}\,(2/3, -1/3, -1/6)\,Q_3 \tag{65}$$
$$\mathcal{Q}_{u,c} = 2/3, \quad \mathcal{Q}_{d,s} = -1/3, \quad \mathcal{Q}_{U_1,U_2} = -1/6. \tag{66}$$

7. Conclusion

In this work, we have considered two versions of the 331 model, with the particular feature of containing extra leptons with fractional electric charges and non-standard electric charges for the new quarks. By considering constraints from the classical and quantum levels, we have shown, for both $\beta = 1/(3\sqrt{3})$ and $\beta = 0$, that the quantisation of the electric charge can be obtained by using the Yukawa sector and the cancellation of chiral anomalies when the three families are taken together and independent of the neutrino, as happens in the other 331 versions. As can be observed from our procedure, different β values produce different constraint equations as a result of imposing the $U(1)_\chi$ invariance of the Yukawa Lagrangian and the cancellation of anomalies. This is because β fixes the fermion representations in the multiplets of the group. In that sense, in our approach, we think that the extension of the charge quantisation for an arbitrary β is not straightforward.

An analysis using the general form of the electromagnetic currents under parity invariance for arbitrary beta, and the cancellation of chiral anomalies for two specific values of the mentioned parameter, allows us to obtain the quantisation of the electric charge as was shown in ref. [16].

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