Pramana – J. Phys. (2020) 94:71
https://doi.org/10.1007/s12043-020-1931-z

Execution of Fredkin gate by a set of free fermions

F SOBERANES[1] and M ÁVILA[2],*

[1]Technologico de Estudios Superiores de Chalco, Tlapala 56600, Chalco, Edo. Mex., Mexico
[2]Centro Universitario UAEM Valle de Chalco, María Isabel, 56615 Valle de Chalco, Edo. Mex., Mexico
*Corresponding author. E-mail: manvlk@yahoo.com

MS received 8 August 2019; revised 2 December 2019; accepted 24 December 2019

Abstract. It is not a trivial task to answer whether a free fermion-based architecture for a quantum computer can efficiently execute basic gates such as the Fredking gate. We show that a set of free fermions can efficiently execute Fredkin gate.

Keywords. IBM quantum computer; quantum Fredkin gate; free fermions.

PACS Nos 05.30.−d; 03.67.−a

The achievement of IBM quantum computer is due to the employment of free fermion-architecture [1–5]. Bound fermions and their relation to quantum information processing have been studied extensively in the past [6–12]. In particular, entanglement properties have been addressed. Quantum spin chains are excellent tools for studying the crucial aspects of quantum information processing such as entanglement and transference of states because several simple models can be solved analytically. There exist several efficient numerical techniques.

In order to study entanglement, Eisler and Zimboras [13] have introduced an approach based on energy currents in quantum spin chains. On the other hand, it is so clear that an operative quantum hardware such as the IBM quantum computer, whose architecture is based on free fermions, must be able to process primitive operations (gates) in an efficient way. In the present paper, it is shown that a primitive gate such as the Fredkin gate can be executed efficiently by a set of free fermions.

The paper is organised as follows: In §1 the model is introduced, in §2 the numerical results are shown while in §3, conclusions obtained from the results are discussed.

1. The model

In the present paper, the XX spin chain carrying energy currents are relevant. However, such a constraint will not allow the fermions to be free. In what follows we shall account for a set of free fermions.

The XX model for $N = 3$ fermions is defined using the following Hamiltonian:

$$H^{xx} = -S_1^x S_2^x - S_1^y S_2^y - S_2^x S_3^x - S_2^y S_3^y - hS_1^z - hS_2^z - hS_3^z, \quad (1)$$

where $\{S_a^x, S_a^y, S_a^z\}$ are the Pauli matrices and h is the transverse magnetic field. It is assumed that the Hamiltonian of eq. (1) is constrained to carry energy currents and consequently the $N = 3$ fermions of (1) are not free.

In ref. [13] it was shown that by adding an appropriate current term to eq. (1), the system becomes a set of free fermions. That is, by adding to eq. (1) the following term

$$J^E = hS_1^x S_2^y - hS_1^y S_2^x + S_2^z S_1^y S_3^x - S_2^z S_1^x S_3^y + hS_2^x S_3^y - hS_2^y S_3^x, \quad (2)$$

the resulting system is a set of free fermions [13] which are described by the Hamiltonian

$$H^E = H^{xx} - \lambda J^E, \quad (3)$$

where λ is a parameter.

On the other hand, the so-called Fredkin gate operates on three qubits and it is defined by the following transformations:

$|000\rangle \to |000\rangle$

$|010\rangle \to |010\rangle$

$|110\rangle \to |111\rangle$

Published online: 23 April 2020

$$|001\rangle \rightarrow |001\rangle$$
$$|011\rangle \rightarrow |011\rangle$$
$$|101\rangle \rightarrow |101\rangle$$
$$|111\rangle \rightarrow |110\rangle. \quad (4)$$

The more general 3-qubit state at time t is

$$|\psi(t)\rangle = C_0(t)|000\rangle + C_1(t)|010\rangle + C_2(t)|100\rangle + C_3(t)|110\rangle + C_4(t)|001\rangle + C_5(t)|011\rangle + C_7(t)|101\rangle + C_6(t)|111\rangle, \quad (5)$$

where $|C_0(t)|^2+|C_1(t)|^2+|C_2(t)|^2+\cdots+|C_7(t)|^2 = 1$. If we employ the following initial conditions

$$C_0(t=0) = 0$$
$$C_1(t=0) = 0$$
$$C_2(t=0) = 0$$
$$C_3(t=0) = 1$$
$$C_4(t=0) = 0$$
$$C_5(t=0) = 0$$
$$C_6(t=0) = 0$$
$$C_7(t=0) = 0, \quad (6)$$

then we conclude that the Fredkin gate of eq. (4) has been executed after an elapsed time $t = T$ when it is satisfied that

$$|C_0(t=T)|^2 = 0$$
$$|C_1(t=T)|^2 = 0$$
$$|C_2(t=T)|^2 = 0$$
$$|C_3(t=T)|^2 = 0$$
$$|C_4(t=T)|^2 = 0$$
$$|C_5(t=T)|^2 = 0$$
$$|C_6(t=T)|^2 = 1$$
$$|C_7(t=T)|^2 = 0. \quad (7)$$

2. Execution of Fredkin gate

The respective Schrödinger equation associated with the set of free fermions described by the Hamiltonian of eq. (3) is

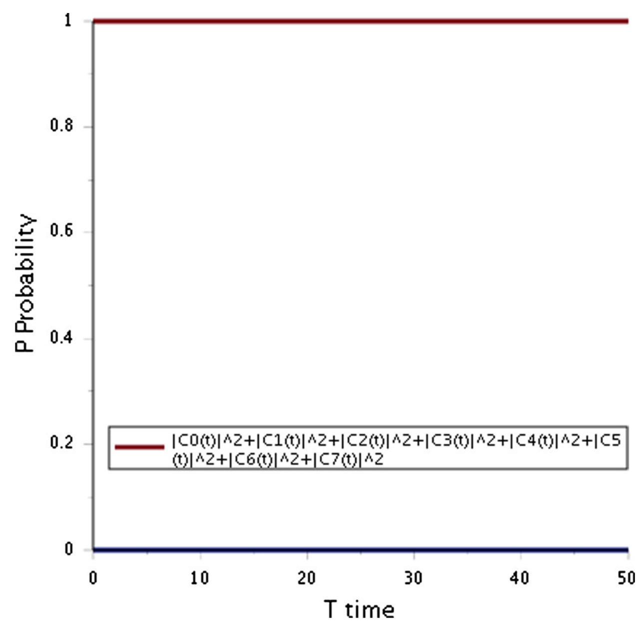

Figure 1. The quantity $|C_0|^2+|C_1|^2+|C_2|^2+\cdots+|C_7|^2$ as a function of time. The coefficients $|C_i(t)|^2$ $(i = 0, 1, \ldots, 7)$ are solutions of eq. (9).

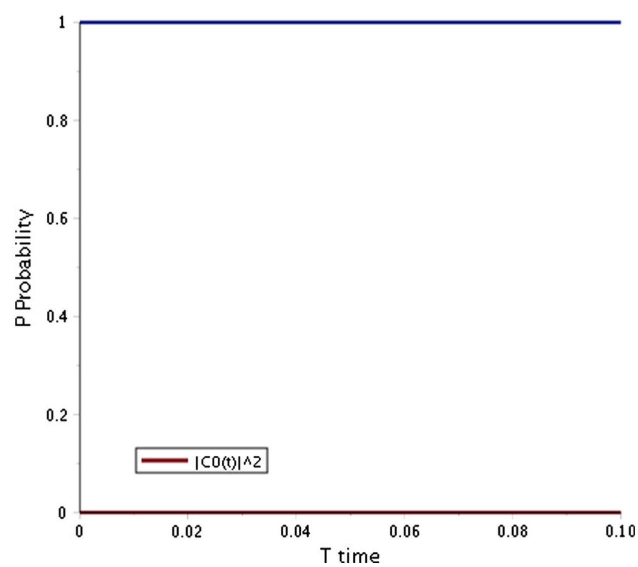

Figure 2. The quantity $|C_0|^2$ of eq. (9) as a function of time.

$$i\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H^E|\psi(t)\rangle. \quad (8)$$

Substitute eqs (1)–(3) and (5) in the above equation and we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}C_0(t) = -i[-3hC_0]$$
$$\frac{\mathrm{d}}{\mathrm{d}t}C_1(t) = -i[-hC_1 - 2C_2 + 2ih\lambda C_2 - 2i\lambda C_4]$$

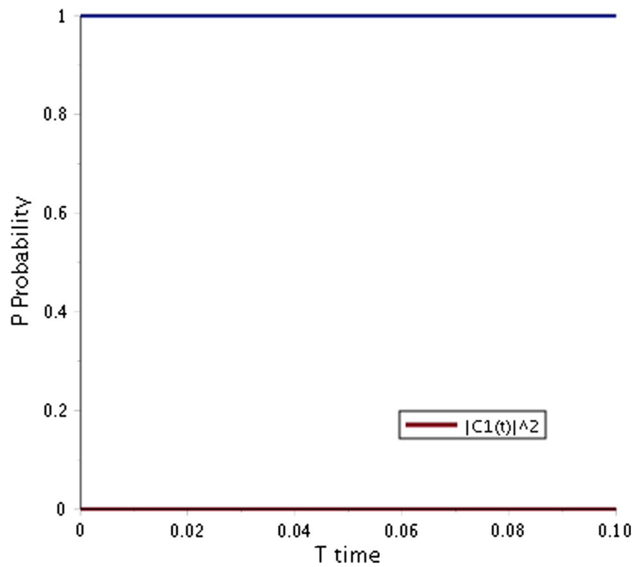

Figure 3. The quantity $|C_1|^2$ of eq. (9) as a function of time.

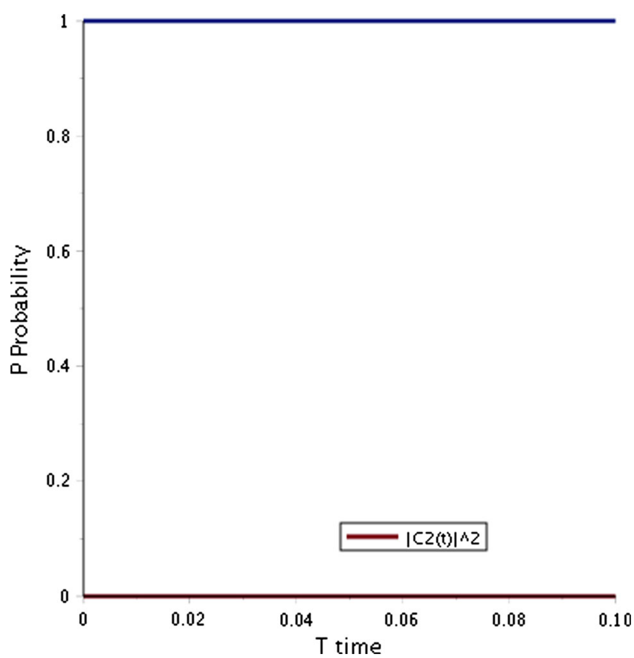

Figure 4. The quantity $|C_2|^2$ of eq. (9) as a function of time.

$$\frac{\mathrm{d}}{\mathrm{d}t}C_2(t) = -i[-2C_1 - 2ih\lambda C_1 - hC_2 - 2C_4 + 2ih\lambda C_4]$$
$$\frac{\mathrm{d}}{\mathrm{d}t}C_3(t) = -i[hC_3 - 2C_5 + 2ih\lambda C_5 + 2i\lambda C_6]$$
$$\frac{\mathrm{d}}{\mathrm{d}t}C_4(t) = -i[-2i\lambda C_1 - 2C_2 + 2ih\lambda C_2 - hC_4]$$
$$\frac{\mathrm{d}}{\mathrm{d}t}C_5(t) = -i[-2ih\lambda C_3 - 2C_3 + hC_5 - 2C_6 + 2ih\lambda C_6]$$

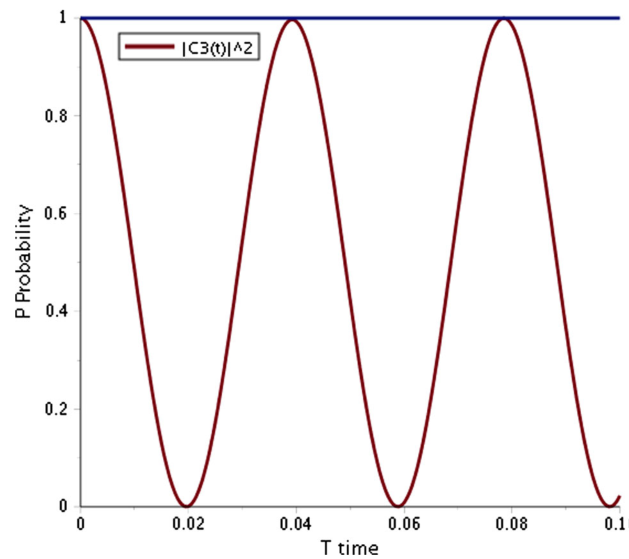

Figure 5. The quantity $|C_3|^2$ of eq. (9) as a function of time.

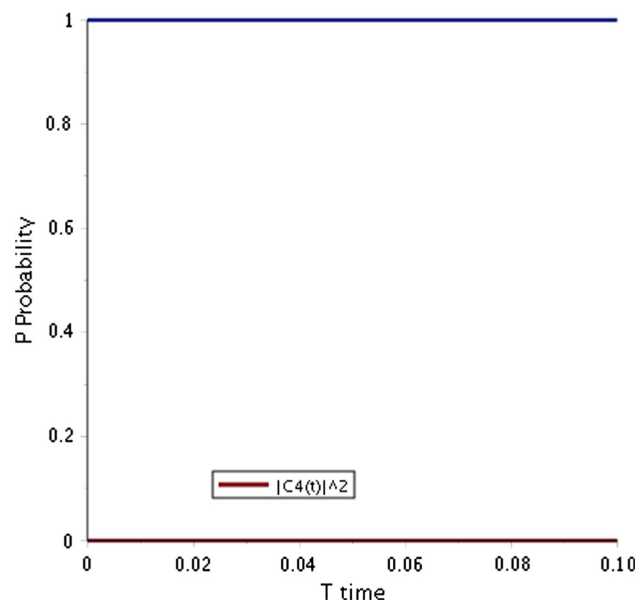

Figure 6. The quantity $|C_4|^2$ of eq. (9) as a function of time.

$$\frac{\mathrm{d}}{\mathrm{d}t}C_6(t) = -i[-2i\lambda C_3 - 2C_5 - 2ih\lambda C_5 + hC_6]$$
$$\frac{\mathrm{d}}{\mathrm{d}t}C_7(t) = -i[-3hC_7]. \tag{9}$$

In the present work we solve the above system of equations in the presence of a weak magnetic field, i.e. $h = 0.01$ T and in the regime of a strong coupling with the current J^E of eq. (2) in eq. (3) by choosing $\lambda = 40$.

In the present section, the system of equations of eq. (9) is solved numerically for the quantities $|C_i(t)|^2$ ($i = 0, 1, \ldots, 7$) as a function of time. In figures 1–8 the obtained results are shown. We observe from figure 1

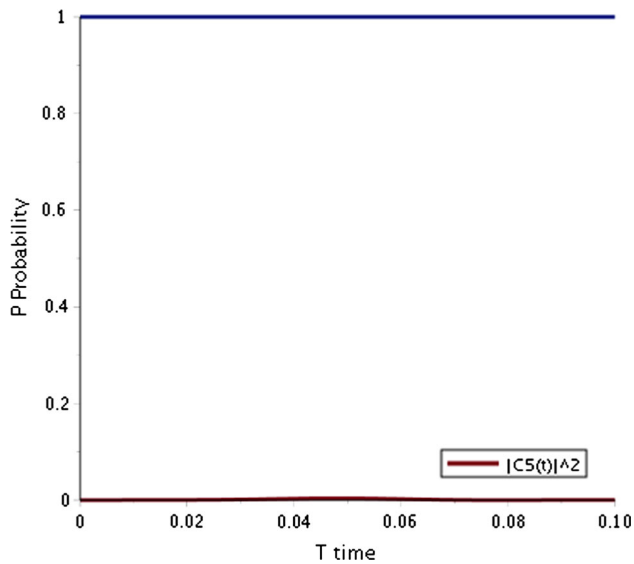

Figure 7. The quantity $|C_5|^2$ of eq. (9) as a function of time.

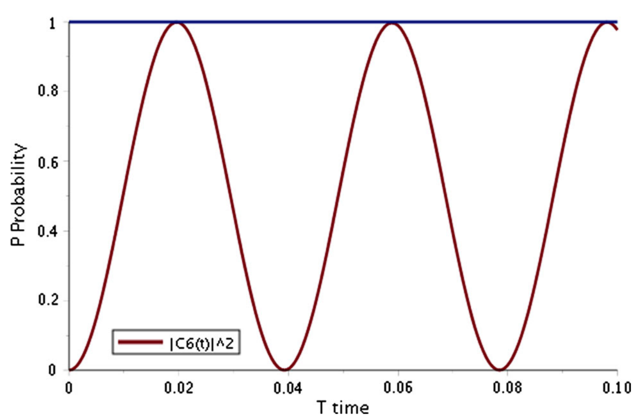

Figure 8. The quantity $|C_6|^2$of eq. (9) as a function of time.

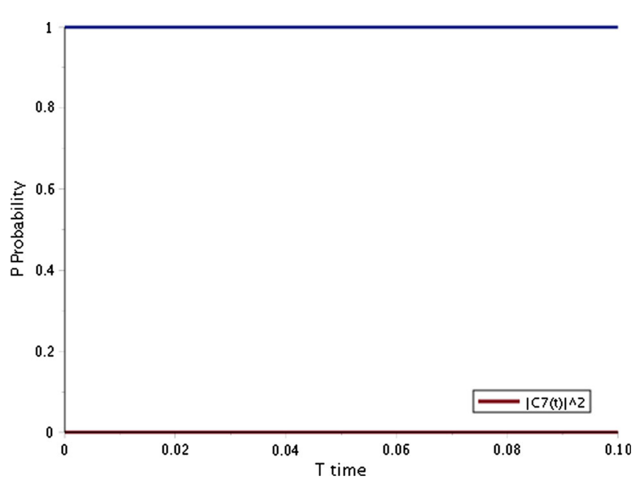

Figure 9. The quantity $|C_7|^2$ of eq. (9) as a function of time.

that the condition of the probability conservation

$$|C_0(t)|^2 + |C_1(t)|^2 + |C_2(t)|^2 + \cdots + |C_7(t)|^2 = 1, \quad (10)$$

is satisfied by the coefficients of eq. (9). Let us observe that the conditions of eq. (7) are also satisfied. That is, from figure 2 it is concluded that $|C_0(t)|^2 = 0$ is satisfied. From figures 3–9 it is also concluded that $|C_1(t)|^2 = 0$, $|C_2(t)|^2 = 0$, $|C_4(t)|^2 = 0$, $|C_5(t)|^2 = 0$ and $|C_7(t)|^2 = 0$. In particular we note that

$$|C_3(t = 0.02)|^2 = 0$$
$$|C_0(t = 0.02)|^2 = 0 = 1 \quad (11)$$

which are in agreement with conditions (6) and (7).

3. Conclusions

We have considered a set of free fermions whose dynamical behaviour is given by the Hamiltonian of eq. (3). With such a Hamiltonian we build the respective Schrödinger equation in terms of the coefficients $C_i(t)$ of eq. (5). The Schrödinger equation for the present set of free fermions is given by a set of coupled first-order differential equations of eq. (9). By solving numerically the Schrödinger equation (9), we first show that the conservation of probability $|C_0(t)|^2 + |C_1(t)|^2 + |C_2(t)|^2 + \cdots + |C_7(t)|^2 = 1$ is satisfied as is evident from figure 1. We then find that the obtained coefficients also satisfy the Fredkin gate conditions of eqs (6) and (7). It is concluded that the Fredkin gate is efficiently executed by a set of free fermions. Finally, we must observe that for the Fredkin gate to be executed by a set of free fermions, the presence of the homogeneous magnetic field h is necessary. Indeed, if $h = 0$ the diagonal contributions of the Schrödinger equation (9) to $C_3(t)$ and $C_6(t)$ would vanish. The above would spoil the figure of control qubit.

References

[1] B Roy, P Goswami and V Juricic, *Phys. Rev. B* **97**, 205117 (2018)

[2] E Mascot, S Cocklin, S Rachel and D K Morr, arXiv:1811.06664

[3] IBM QX, https://quantumexperience.ng.bluemix.net/qx

[4] Rigetti QPU, https://www.rigetti.com

[5] T E O'Brien, P Rozek and A R Akhmerov, *Phys. Rev. Lett.* **120**, 220504 (2018)

[6] A Osterloh, L Amico, G Falci and R Fazio, *Nature (London)* **416**, 608 (2002)

[7] T J Osborne and M A Nielsen, *Phys. Rev. A* **66**, 032110 (2002)

[8] G Vidal, J I Latorre, E Rico and A Kitaev, *Phys. Rev. Lett.* **90**, 227901 (2003)
[9] B Q Jin and V E Korepin, *J. Stat. Phys.* **116**, 79 (2004)
[10] M Fannes, B Haegemann and M Mosonyi, *J. Math. Phys.* **44**, 6005 (2003)
[11] J P Keating and F Mezzadri, *Commun. Math. Phys.* **252**, 543 (2004)
[12] V Popkov and M Salerno, *Phys. Rev. A* **71**, 012301 (2005)
[13] V Eisler and Z Zimboras, *Phys. Rev. A* **71**, 042318 (2005)