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Systematics of multinucleon transfer in heavy-ion reactions

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Abstract. One-neutron pickup reactions for 52 projectile–target combinations were analysed using a systematics between transfer cross-sections and ground-state Q-values. One-neutron pickup transfer shows a good correlation between reduced transfer cross-sections and ground-state Q-values (Q_{gg}) if one separate the systems into two groups based on their $Z_p Z_t$ product. Also, similar kind of systematics is applied to 2n, 3n and 4n pickup transfer and a good correlation is obtained between reduced transfer cross-sections and Q_{gg} values, where no $Z_p Z_t$ dependence is seen.

Keywords. Multinucleon transfer; reduced transfer cross-sections; neutron separation energy.

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1. Introduction

Development of various spectrometers [1–5] and the availability of many heavy-ion beams from different accelerator facilities have given a new impetus to the field of heavy-ion reactions. Quasielastic processes are the dominant processes in heavy-ion reactions at energies close to the Coulomb barrier [6–8]. One-neutron transfer channel is one of them [6,9–11]. To study two-neutron transfer cross-section enhancement and search for pair transfer effects, multinucleon transfer processes have been investigated [12–19]. Neutron transfer with positive Q-values has been identified as one of the major reasons in the observed enhancement of sub-barrier fusion [20–27].

The role of transfer channels in explaining the observed enhancement of sub-barrier fusion cross-sections has been noticed in the measurements of ^{58}Ni + ^{58,64}Ni by Beckerman *et al* [23]. Timmers *et al* [24] studied the fusion of calcium and zirconium isotopes (^{40}Ca + ^{90,96}Zr) to examine the interplay between neutron transfer and sub-barrier fusion enhancement. A large enhancement was observed in the sub-barrier fusion cross-sections of ^{40}Ca + ^{96}Zr, a system with positive Q-value in neutron transfer, compared to ^{40}Ca + ^{90}Zr, a system which has no positive Q-value in neutron transfer channels. Sargsyan *et al* [28] studied the role of nuclear deformation and neutron transfer in sub-barrier capture processes using the quantum diffusion model and noted that the experimentally observed sub-barrier fusion enhancement is mainly related to the change in the quadruple deformation of the colliding nuclei.

Fragmentation processes and fission of heavy nuclei are extensively used for the synthesis of neutron- and proton-rich isotopes. However, the production cross-sections are very low. In recent years, much effort has been made to produce new neutron-rich heavy nuclei by multinucleon transfer reactions [29–39]. Zagrebaev and Greiner [40] pointed out that multinucleon transfer reactions can be used to produce neutron-rich heavy nuclei. They used a model based on the Langevin-type dynamical equations of motion [40,41], to predict the cross-sections for the production of neutron-rich nuclei. Saiko and Karpov [42] used a modified multidimensional dynamical model of nucleus–nucleus collisions based on the Langevin equations to analyse multinucleon transfer processes in low-energy collisions for spherical and static deformed nuclei. They observed a reasonable agreement between the calculated and the measured energy, angular, charge, and isotopic distributions of reaction products for a number of multinucleon transfer reactions with medium-mass and heavy nuclei. In addition to this dynamical model, to investigate the multinucleon transfer processes theoretically, the semiclassical GRAZING model [43–45] has been used with great success [9]. Furthermore, dinuclear system

(DNS) model [46–48] also was used for describing the transfer reactions. Microscopic approaches, such as the time-dependent Hartree–Fock (TDHF) approach [49–53] and improved quantum molecular dynamics (ImQMD) model have been proposed [54–56] for describing multinucleon transfers. Even though these models were successful in predicting transfer cross-sections, the recently measured heavy ^{204}Hg + ^{198}Pt reaction by Welsh *et al* [29] show significant deviation from these models.

The Q-value dependence of multinucleon transfer reactions has been studied theoretically and experimentally in the last few decades [9–11,57–63]. Alder and Trautmann [60] investigated the dependence of the transfer reactions on the Q-values with the semiclassical theory of transfer reactions. Artukh *et al* [61] measured the differential cross-sections for the formation of various isotopes in the ^{16}O + ^{232}Th reaction. They observed an exponential dependence of the measured yields on the Q-values of the reactions. Abul-Magd *et al* [62] explained this in terms of the statistical theory of nuclear reactions.

van den Berg *et al* [64] successfully explained the observed dependence between transfer cross-sections and ground-state Q-values (Q_{gg}). They also observed that ^{149}Sm(^{58}Ni,^{59}Ni)^{148}Sm reaction deviates from the above systematics in a significant way. Rehm *et al* [65] applied this systematics for one- and two-nucleon transfer reactions induced by medium heavy projectiles with some success. Reisdorf *et al* [66] extended this systematics by including ^{86}Kr-induced reactions on ^{76}Ge, ^{104}Ru and ^{130}Te targets, which they have measured. They could not find a good correlation between reduced transfer cross-sections (σ_{red}) and Q_{gg} even though they separated the systems based on the product of their Z_p and Z_t values.

In this scenario, we have extended the systematics proposed by van den Berg *et al* [64] by considering a large dataset. To obtain a good correlation between σ_{red} and Q_{gg}, we have separated the systems into two groups based on their Z_p and Z_t products. In this work, we have applied this systematics for 1n, 2n, 3n and 4n neutron transfer reactions. The systematics developed in this article is restricted to reactions induced by heavier projectiles with masses $A \geq 16$ and $Z_pZ_t < 4400$. For one-neutron transfer systematics, we have included 52 projectile–target combinations, spanning over a large range of Q-values and we have included 38 projectile–target combinations for two-neutron transfer. Also 10 and 8 projectile–target combinations were included for three- and four-neutron transfer reactions respectively. We discuss 1n transfer systematics in §2, 2n transfer systematics in §3 and 3n and 4n transfer systematics together in §4.

2. Systematics for one-neutron transfer reactions

To explore the effects of Q-value on transfer cross-sections of heavy-ion induced reactions, we have collected angle- and energy-integrated one-neutron transfer cross-sections from literature. The energies of the reactions were chosen to be above the Bass fusion barrier. The angle- and energy-integrated one-neutron transfer cross-sections show scattered distribution with Q_{gg}, which is shown in figure 1. The transfer cross-sections were not corrected for any energy dependence, because the dependence of σ_{tr} on energy is found to be weak near the barrier energies [64].

Reisdorf *et al* [66] introduced a grouping of reactions based on their Z_pZ_t values, where they separated the systems into three groups with $Z_pZ_t < 1200$, $Z_pZ_t \geq 1800$ and those with intermediate values of Z_pZ_t. Such a separation could not show any correlation between σ_{tr} and Q_{gg}. To improve this grouping criterion and to look for a possible correlation between Q_{gg} and σ_{tr}, we have redefined Z_pZ_t into two groups with $Z_pZ_t < 1400$ and $Z_pZ_t \geq 1400$. Such a criterion was found to be reasonable considering that the quasifission [67–73] starts to appear in reactions with $Z_pZ_t = 1000$ or above and plays a dominant role around $Z_pZ_t = 1400$ [74]. Moreover, Z_pZ_t have direct relation with the effective fissility parameter (χ_{eff}) of the systems [75]. In terms of fissility, the grouping criteria become, $\chi_{\text{eff}} < 0.6$ ($Z_pZ_t < 1400$) and $\chi_{\text{eff}} > 0.6$ ($Z_pZ_t \geq 1400$). If one uses effective fissility as a selection parameter, we do not expect any change in the systematics.

In order to quantitatively separate the Q-value effects from the experimental transfer cross-section, we have used reduced transfer cross-section as given in eq. (1) introduced by van den Berg *et al* [64], where B_i and

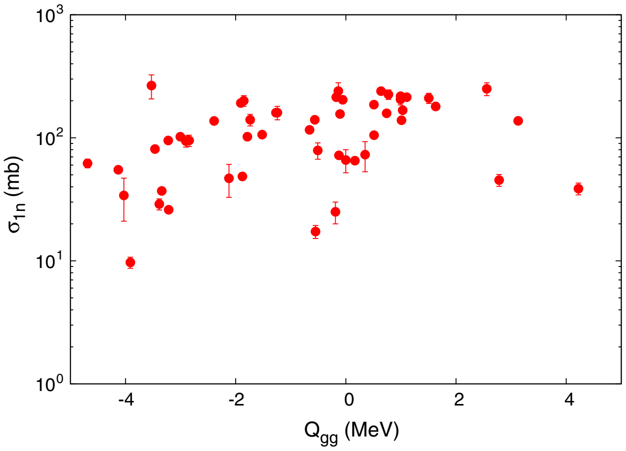

Figure 1. One-neutron pickup reaction cross-sections as a function of Q_{gg}, for the systems listed in table 1.

Table 1. Angle- and energy-integrated 1n transfer cross-sections ($\sigma_{1\mathrm{n}}$(exp)) for systems used in this systematics along with the ground-state Q-values (Q_{gg}), binding energies of the transferred neutron in the entrance (B_i) and the exit channels (B_f) which are taken from NNDC [76]. $\sigma_{1\mathrm{n}}$(syst) and $\sigma_{1\mathrm{n}}$(gr) are the cross-sections obtained from the systematics and GRAZING calculations respectively.

System	Z_pZ_t	Q_{gg} (MeV)	B_i (MeV)	B_f (MeV)	$\sigma_{1\mathrm{n}}$(exp) (mb)	$\sigma_{1\mathrm{n}}$(syst) (mb)	$\sigma_{1\mathrm{n}}$(gr) (mb)	Reference
^{28}Si + ^{62}Ni	392	−2.122	10.60	8.47	46.8±14	44.13	59.2	[77]
^{33}S + ^{90}Zr	640	−0.551	11.97	11.42	17.3±2.1	36.99	30.5	[78]
^{33}S + ^{91}Zr	640	4.223	7.19	11.42	38.6±4.2	38.46	61.6	[78]
^{33}S + ^{92}Zr	640	2.782	8.63	11.42	45.3±5	46.18	67.5	[78]
^{16}O + ^{208}Pb	656	−3.225	7.37	4.14	85	100.98	38.4	[79]
^{18}O + ^{206}Pb	656	−4.131	8.09	3.96	45	65.10	4.0	[80]
^{32}S + ^{93}Nb	656	−0.189	8.83	8.64	25±5	73.09	28.6	[81]
^{32}S + ^{92}Mo	672	−4.029	12.67	8.64	34±13	30.37	12.8	[81]
^{32}S + ^{96}Mo	672	−0.513	9.15	8.64	79±12	67.45	79.6	[82]
^{32}S + ^{98}Mo	672	−0.001	8.64	8.64	66±14	74.85	49.7	[81]
^{32}S + ^{100}Mo	672	0.347	8.29	8.64	73 ±20	80.51	53.6	[81]
^{58}Ni + ^{58}Ni	784	−3.217	12.22	9.00	26±10	25.00	36.7	[8]
^{58}Ni + ^{64}Ni	784	−0.658	9.66	9.00	116±15	60.06	97.3	[8]
^{40}Ca + ^{96}Zr	800	0.513	7.85	8.36	105	88.65	106.7	[83]
^{32}S + ^{144}Sm	992	−1.878	10.52	8.64	48.5±2.5	46.04	36.1	[84]
^{40}Ca + ^{124}Sn	1000	−0.127	8.49	8.36	75	79.23	143.3	[85]
^{48}Ca + ^{124}Sn	1000	−3.343	8.49	5.15	37	67.40	64.2	[86]
^{32}S + ^{166}Er	1088	0.166	8.48	8.64	65.2±3	77.57	55.8	[84]
^{28}Si + ^{208}Pb	1148	1.106	7.37	8.47	214±2	186.65	175.3	[87]
^{86}Kr + ^{76}Ge	1152	−3.912	9.43	5.52	9.7±1	42.52	17.6	[66]
^{37}Cl + ^{208}Pb	1394	−1.260	7.37	6.11	160±0.088	114.18	119.2	[7]
^{58}Ni + ^{112}Sn	1400	−1.789	10.79	9.00	102±5	81.32	86.0	[88]
^{58}Ni + ^{116}Sn	1400	−0.564	9.56	9.00	140±4	120.42	154.9	[88]
^{58}Ni + ^{120}Sn	1400	−0.105	9.10	9.00	156±4	134.29	182.9	[88]
^{58}Ni + ^{124}Sn	1400	0.510	8.49	9.00	186±4	153.31	196.4	[88]
^{64}Ni + ^{112}Sn	1400	−4.690	10.79	6.10	62±5	40.01	56.1	[88]
^{64}Ni + ^{116}Sn	1400	−3.465	9.56	6.10	81±5	79.01	90.8	[88]
^{64}Ni + ^{120}Sn	1400	−3.007	9.10	6.10	102±6	101.06	110.8	[88]
^{64}Ni + ^{124}Sn	1400	−2.391	8.47	6.10	137±7	186.40	127.9	[88]
^{40}Ar + ^{208}Pb	1476	−1.269	7.37	6.10	215	218.27	81.3	[32]
^{86}Kr + ^{104}Ru	1584	−3.385	8.90	5.52	28.9± 3	96.60	50.5	[66]
^{40}Ca + ^{208}Pb	1640	0.995	7.37	8.36	135	196.75	186.8	[89]
^{58}Ni + ^{144}Sm	1736	−1.520	10.52	9.00	105±7	89.46	154.8	[64,90]
^{58}Ni + ^{149}Sm	1736	3.129	5.87	9.00	136±17	192.28	209.3	[64,90]
^{58}Ni + ^{150}Sm	1736	1.013	7.99	9.00	138±25	166.03	222.3	[90]
^{58}Ni + ^{152}Sm	1736	0.742	8.26	9.00	156±17	160.05	167.8	[90]
^{58}Ni + ^{154}Sm	1736	1.032	7.97	9.00	165±10	166.63	216.0	[64,90]
^{46}Ti + ^{208}Pb	1804	1.513	7.37	8.88	210±20	184.12	261.2	[91]
^{48}Ti + ^{208}Pb	1804	0.775	7.37	8.14	225±20	202.63	247.1	[91]
^{50}Ti + ^{208}Pb	1804	−0.995	7.37	6.37	205±20	215.50	210.1	[91]
^{86}Kr + ^{130}Te	1872	−2.904	8.41	5.52	94±10	123.41	35.0	[66]
^{58}Ni + ^{232}Th	2520	2.559	6.44	9.00	250±30	191.13	273.5	[92]
^{58}Ni + ^{208}Pb	2296	1.631	7.37	9.00	178	181.64	258.2	[93]
^{64}Ni + ^{208}Pb	2296	−1.270	7.37	6.10	160	156.77	200.0	[65]
^{76}Se + ^{192}Pt	2652	−1.243	8.66	7.42	160±20	146.78	123.1	[94]
^{76}Se + ^{198}Pt	2652	−0.137	7.56	7.42	240±40	201.21	157.4	[94]
^{82}Se + ^{192}Pt	2652	−2.844	8.66	5.82	95±10	120.37	149.6	[94]
^{82}Se + ^{198}Pt	2652	−1.738	7.56	5.82	140±15	194.50	146.1	[94]

Table 1. *Continued.*

System	Z_pZ_t	Q_{gg} (MeV)	B_i (MeV)	B_f (MeV)	σ_{1n}(exp) (mb)	σ_{1n}(syst) (mb)	σ_{1n}(gr) (mb)	Reference
^{86}Kr + ^{208}Pb	2952	−1.853	7.37	5.52	200	206.33	168.3	[95]
^{90}Zr + ^{208}Pb	3280	−0.174	7.37	7.19	210	213.80	276.2	[83]
^{197}Au + ^{130}Te	4108	−1.907	8.42	6.51	185	155.00	184.0	[31]
^{136}Xe + ^{198}Pt	4212	−3.530	7.56	4.03	252±52	156.18	55.9	[33]

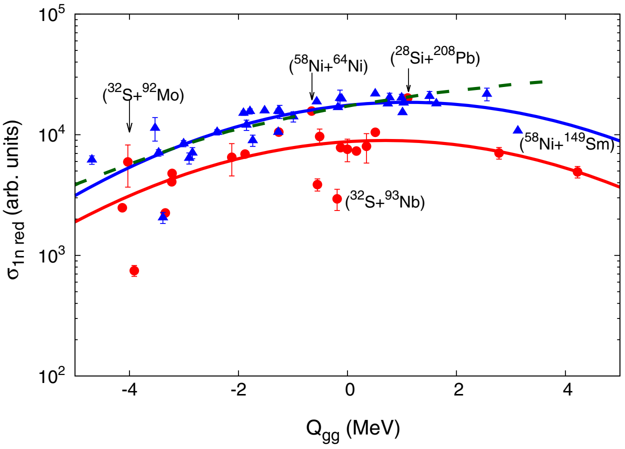

Figure 2. Reduced one-neutron pickup reaction cross-sections as a function of Q_{gg}. Triangles and circles correspond to the reactions with $Z_pZ_t \geq 1400$ and $Z_pZ_t < 1400$ respectively. Solid lines are fit using eq. (2) with different parameters and the dashed line corresponds to the integral of eq. (2), reproduced from van den Berg *et al* [64] (see text).

B_f are the binding energies of the transferred neutron in the entrance and the exit channels. Here x stands for the number of transfer neutrons.

$$\sigma_{\rm red}(xn) = \sigma_{\rm exp}(xn) \times (B_iB_f)^{1.1}. \qquad (1)$$

The reduced cross-sections evaluated by eq. (1) for 52 projectile–target combinations (systems are listed in table 1) are shown as a function of Q_{gg} in figure 2. The general trend, as can be seen in figure 2, of these reduced transfer cross-sections with increasing Q_{gg} can be understood from the Q-matching behaviour [64]. A typical Q window for neutron transfer reactions can be approximated by a Gaussian function given in eq. (2), where a, m and s are fitting coefficients.

$$\sigma_{\rm red}(Q_{gg}) = a * \exp(-(Q_{gg}-m)^2/2s^2). \qquad (2)$$

The solid lines in figure 2 are the results of least square fit to the data using eq. (2). Fitted parameters are listed in table 2. Three reactions, ^{32}S + ^{92}Mo, ^{58}Ni + ^{64}Ni and ^{28}Si + ^{208}Pb (all of them with $Z_pZ_t < 1400$) fall along the curve that fits the systems with $Z_pZ_t \geq 1400$. One can notice that in the reactions ^{32}S + ^{92}Mo and ^{28}Si + ^{208}Pb, while the target nuclei have neutron shell closure, the projectiles do not have any such shell closure. This observed deviation from the systematics may be attributed to the neutron shell closure effect of the target nuclei. The dashed line in figure 2 corresponds to the integral of eq. (2), used by van den Berg *et al* [64] to fit $\sigma_{\rm red}$ (with $m = 0$ and $s = 5.9$). In the case of ^{58}Ni + ^{149}Sm reaction, one of the case having odd-neutron number in the target, was shown to have deviated significantly from van den Berg [64] systematics. They suggested that this deviation is possibly due to the neglect of pairing corrections. However, for the same reaction, $\sigma_{\rm red}$ are seen to be close to our fit. A similar trend can be seen where the observed $\sigma_{\rm red}$ falls far below the systematics in the case of ^{32}S + ^{93}Nb. Considering ^{32}S + ^{93}Nb with odd proton target, we may assume that a similar pairing correction is required, as suggested by van den Berg *et al* [64] in the case of ^{58}Ni + ^{149}Sm reaction. However, more datasets are required for odd proton target nuclei to draw any conclusion. The overall agreement between the systematics and measured experimental cross-sections is very good. In table 1, we list the experimental one-neutron transfer cross-sections (σ_{1n}) collected from literature along with systematics ($\sigma_{\rm syst}$).

For one of the heaviest reactions included in this systematics ^{90}Zr + ^{208}Pb ($Z_pZ_t = 3280$), the measured cross-section for one-neutron pickup reaction is $\sigma_{\rm exp} = 210$ mb and from systematics we obtain, $\sigma_{\rm syst} = 213.9$ mb. Similarly, in the case of ^{91}Zr(^{33}S,^{34}S)^{90}Zr reaction, which has the largest positive Q-value (4.223 MeV) included in this systematics, we see very good agreement with the systematics ($\sigma_{\rm exp} = 38.6 \pm 4.2$ mb and $\sigma_{\rm syst} = 37.0$ mb).

The total transfer cross-sections measured using chemical techniques, for ^{86}Kr + ^{76}Ge, ^{86}Kr + ^{104}Ru, ^{86}Kr + ^{130}Te [66], generally show deviation from the systematics. Also ^{33}S + ^{90}Zr and ^{48}Ca + ^{124}Sn show deviation from the systematics. In systems with $Z_pZ_t > 4000$, for example ^{197}Au + ^{130}Te ($\sigma_{\rm exp} = 177.0$ mb and $\sigma_{\rm syst} = 155.0$ mb) and ^{136}Xe + ^{198}Pt ($\sigma_{\rm exp} = 252 \pm 52$ mb and $\sigma_{\rm syst} = 156.18$ mb) we can see significant deviations. Accordingly, one can say that this

Table 2. Fit parameters obtained with eq. (2) for various transfer systematics.

Transfer/Channel	Parameters		
	a	*m*	*s*
1n ($Z_pZ_t < 1400$)	8962±470	0.69±0.31	3.23±0.33
1n ($Z_pZ_t \geq 1400$)	18570±816	1.08±0.40	3.23±0.33
−1n,1n ($Z_pZ_t \geq 1400$)	18816±718	0.93±0.30	3.33±0.28
2n	6322±385	5.19±0.57	3.88±0.61
3n	6217±1788	32.55±3.53	13.58±0.70
4n	577±239	3.82±2.10	5.57±2.84

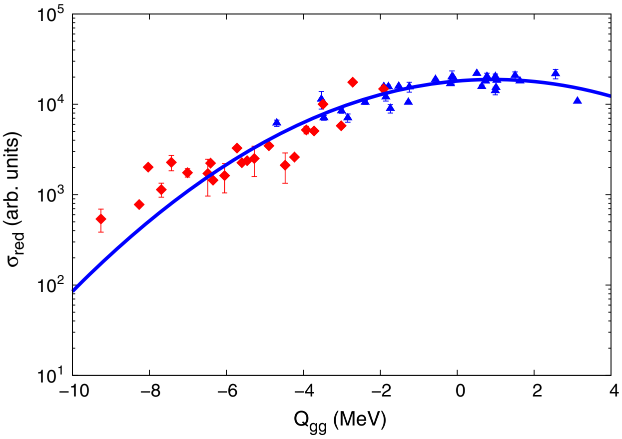

Figure 3. Reduced one-neutron transfer reaction cross-sections ($Z_pZ_t \geq 1400$) as a function of Q_{gg}. Triangles and diamonds correspond to the pickup and stripping reactions respectively. The line is a fit using eq. (2).

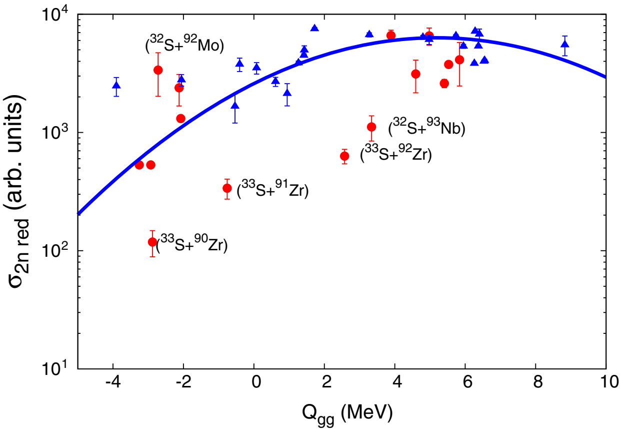

Figure 4. Reduced two-neutron pickup reaction cross-sections as a function of Q_{gg}. Triangles and circles correspond to the reactions with $Z_pZ_t \geq 1400$ and $Z_pZ_t < 1400$ respectively. The line is a fit using eq. (2).

systematics show significant deviation for the systems with $Z_pZ_t > 4000$ ($\chi_{\text{eff}} > 1$).

For a comparison with our systematics, transfer cross-sections calculated using GRAZING [43] are also listed in table 1, as σ_{1n}(gr). For many of these reactions, the calculated cross-sections using GRAZING code show large deviations from the measured cross-sections.

In addition to pickup reactions, in figure 3, we plot one-neutron stripping reactions with $Z_pZ_t \geq 1400$ as a function of Q_{gg}. This plot shows very good correlation between σ_{red} and Q_{gg}. In the case of 2n, 3n and 4n transfers, we have considered only pickup channels and have not included stripping due to the paucity of experimental data.

3. Systematics for two-neutron pickup reactions

There are many attempts to investigate correlated pair transfer effects in multinucleon transfer reactions [12, 16–19,96,97]. von Oertzen *et al* [12] reported experimental two-neutron transfer enhancement due to the pairing interaction between the superfluid tin nuclei (^{120}Sn + ^{112}Sn reaction). In the case of lighter system, ^{18}O + ^{174}Yb reaction, Sahu *et al* [16] observed a large enhancement in the 2n- and 2n-correlated transfer cross-sections. In order to investigate the possible influence of the two valence neutrons, Jha *et al* [17] studied pair transfer enhancement for ^{18,16}O + ^{90}Zr systems. They found that the cross-sections for 2n and 2n-correlated transfer channels are enhanced in the case of ^{18}O + ^{90}Zr compared to ^{16}O + ^{90}Zr system. Recently, Montanari *et al* [18] studied ^{60}Ni + ^{112}Sn reaction in a wide energy range, from the Coulomb barrier to far below barrier energies. Their results agree very well with microscopic calculations which incorporate nucleon–nucleon pairing correlations.

Rehm *et al* [65] obtained two-neutron transfer systematics with 29 reactions induced by various projectiles on ^{208}Pb, Sn and Ni targets. They found that the binding-energy corrected two-neutron cross-sections follow a systematics with Q_{gg} for the reactions with these magic nuclei. We have extended this systematics for 38 projectile–target combinations. To remove binding energy dependence, we defined reduced cross-sections

Table 3. Angle- and energy-integrated 2n transfer cross-sections (σ_{2n}(exp)) for systems used in this systematics along with Q_{gg}, binding energies of the transferred neutron in the entrance (B_i) and the exit channels (B_f) which are taken from NNDC [76]. σ_{2n}(syst) and σ_{2n}(gr) are the cross-sections obtained from the systematics and GRAZING calculations respectively.

System	Z_pZ_t	Q_{gg} (MeV)	B_i (MeV)	B_f (MeV)	σ_{2n}(exp) (mb)	σ_{2n}(syst) (mb)	σ_{2n}(gr) (mb)	Reference
^{28}Si + ^{62}Ni	392	0.667	7.82	10.61	18.5±5.5	24.94	5.49	[77]
^{33}S + ^{90}Zr	640	−2.883	9.32	6.99	1.2±0.3	21.58	0.49	[78]
^{33}S + ^{91}Zr	640	−0.760	11.97	6.99	2.6±0.5	47.56	2.12	[78]
^{33}S + ^{92}Zr	640	2.574	7.19	6.99	8.5±1.2	69.15	3.35	[78]
^{18}O + ^{206}Pb	656	−3.255	6.74	7.61	8.0	6.58	0.32	[80]
^{32}S + ^{93}Nb	656	3.341	7.89	11.42	7.9±1.9	9.79	6.51	[81]
^{32}S + ^{92}Mo	672	−2.720	10.11	11.42	15±6	3.558	1.51	[81]
^{32}S + ^{98}Mo	672	4.595	6.82	11.42	26±8	51.53	9.74	[81]
^{32}S + ^{100}Mo	672	5.839	5.93	11.42	40±14	59.47	11.58	[81]
^{58}Ni + ^{58}Ni	784	−2.077	10.25	11.39	7.0	6.08	1.22	[8]
^{58}Ni + ^{64}Ni	784	3.892	6.84	11.39	55.0±6	49.37	9.97	[8]
^{40}Ca + ^{96}Zr	800	5.531	6.46	11.48	33.0	54.28	15.83	[83]
^{40}Ca + ^{124}Sn	1000	5.408	5.95	11.48	25.0	60.21	21.05	[85]
^{48}Ca + ^{124}Sn	1000	−2.928	5.95	6.36	9.78	12.85	5.61	[86]
^{28}Si + ^{208}Pb	1148	4.977	6.74	10.61	60±10	57.31	39.4	[87]
^{58}Ni + ^{112}Sn	1400	1.431	8.17	11.39	34±3	27.56	7.03	[88]
^{58}Ni + ^{116}Sn	1400	3.278	7.55	11.39	50±2	41.64	25.27	[88]
^{58}Ni + ^{120}Sn	1400	4.799	6.48	11.39	53±2	52.44	30.76	[88]
^{58}Ni + ^{124}Sn	1400	5.952	5.95	11.39	52±2	61.25	40.86	[88]
^{64}Ni + ^{112}Sn	1400	−3.906	8.17	8.95	22±4	3.51	3.3	[88]
^{64}Ni + ^{116}Sn	1400	−2.059	7.55	8.95	27±3	10.52	11.48	[88]
^{64}Ni + ^{120}Sn	1400	−0.538	6.84	8.95	18±5	22.46	13.36	[88]
^{64}Ni + ^{124}Sn	1400	0.614	5.95	8.95	34±3	39.75	17.97	[88]
^{40}Ar + ^{208}Pb	1476	1.420	6.74	9.43	46.7	41.01	7.18	[32]
^{40}Ca + ^{208}Pb	1640	5.737	6.74	11.49	55.0	52.87	41.92	[89]
^{58}Ni + ^{144}Sm	1736	1.265	8.60	11.39	25.00	24.59	22.80	[64,90]
^{58}Ni + ^{149}Sm	1736	6.375	8.14	11.39	36.84	42.82	46.12	[64,90]
^{58}Ni + ^{150}Sm	1736	6.530	5.87	11.39	37.71	60.67	49.62	[90]
^{58}Ni + ^{152}Sm	1736	6.533	5.60	11.39	41.43	62.53	26.17	[90]
^{58}Ni + ^{154}Sm	1736	6.552	5.87	11.39	39.99	59.44	47.60	[64,90]
^{46}Ti + ^{208}Pb	1804	6.402	6.74	11.63	56±6	51.01	78.25	[91]
^{48}Ti + ^{208}Pb	1804	4.976	6.74	10.94	54±5	55.82	57.06	[91]
^{50}Ti + ^{208}Pb	1804	0.075	6.74	7.81	45±5	4.06	54.78	[91]
^{58}Ni + ^{208}Pb	2296	6.281	6.74	11.39	60.55	51.62	70.04	[93]
^{64}Ni + ^{208}Pb	2296	0.944	6.74	8.95	40.0	38.34	44.56	[65]
^{58}Ni + ^{232}Th	2520	8.829	5.12	11.39	63±12	47.16	49.62	[92]
^{90}Zr + ^{208}Pb	3280	1.723	6.74	8.63	86.33	48.85	102.9	[83]
^{197}Au + ^{130}Te	4108	−0.405	6.08	7.58	55.7	4.79	58.5	[31]

as given in eq. (1), with B_i and B_f, the binding energies of the transferred second neutron in the entrance and the exit channels respectively. The reduced two-neutron cross-sections are shown in figure 4, as a function of Q_{gg}, for the systems listed in table 3.

From figure 4, one can see a good correlation between two-neutron transfer cross-sections and Q_{gg} as already seen for one-neutron pickup and stripping. However, in the case of two-neutron transfer, we are not able to separate them into different groups based on their Z_pZ_t values as has been done in the case of one-neutron transfer. Excepting for ^{48}Ca + ^{124}Sn, ^{58}Ni + ^{58}Ni, ^{18}O + ^{206}Pb and ^{64}Ni + ^{120}Sn reactions, many of the reactions with large negative Q-values show deviation from this systematics. As observed in one-neutron pickup systematics, the reduced cross-sections for ^{32}S + ^{93}Nb reaction is lower than that from the systematics. However, ^{58}Ni + ^{149}Sm, in which one-neutron pickup

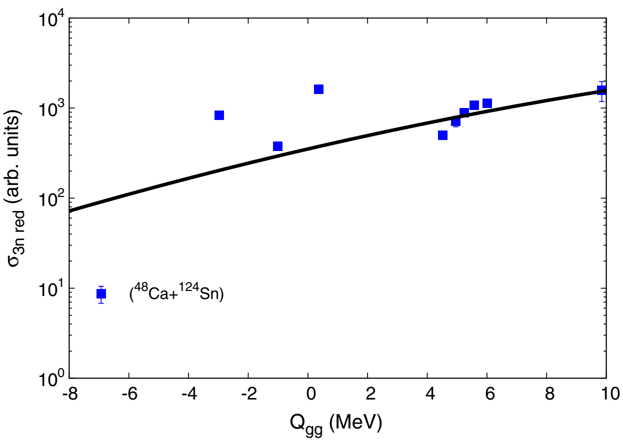

Figure 5. Reduced three-neutron pickup reaction cross-sections as a function of Q_{gg}, for the systems listed in table 4. The line is a fit using eq. (2).

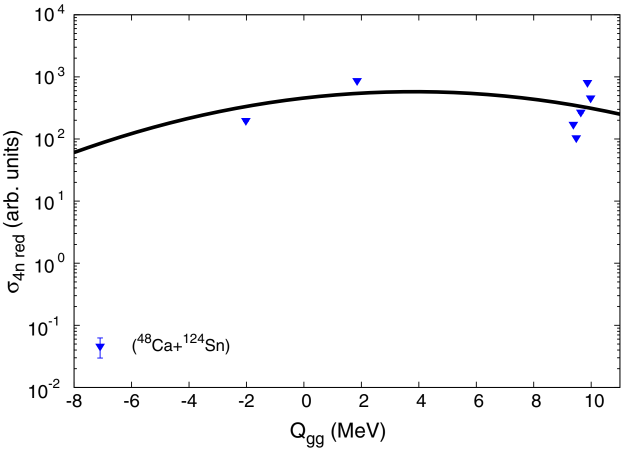

Figure 6. Reduced four-neutron pickup reaction cross-sections as a function of Q_{gg}, for the systems listed in table 5. The line is a fit using eq. (2).

cross-sections showed lower cross-sections than expected from systematics, do not show such a trend in the two-neutron pickup. Furthermore, all the sulphur-induced reactions show lower cross-sections than expected from the systematics and follow a different trend in two-neutron transfer systematics, except for ^{32}S + ^{92}Mo reaction. In table 3, we list the experimental two-neutron transfer cross-sections (σ_{2n}(exp)) collected from literature along with systematics (σ_{2n}(syst)) and cross-sections obtained by GRAZING calculations (σ_{2n}(gr)).

4. Systematics for three- and four-neutron pickup reactions

Similar to two-neutron transfer reactions, we have applied the same systematics for three- and four-neutron pickup reactions with 10 and 8 projectile–target combinations respectively. In both these cases, to remove the binding energy dependence on transfer cross-sections, we have used eq. (1), where B_i and B_f are the neutron separation energy of the transferred third and fourth neutron in the entrance and exit channels of 3n and 4n pickup reactions respectively. The respective reduced transfer cross-sections are shown in figures 5 and 6. In tables 4 and 5 we list the experimental three- and four-neutron transfer cross-sections (σ_{3n} and σ_{4n} respectively) collected from literature along with systematics (σ_{syst}) and cross-sections obtained by GRAZING calculations respectively. For most of these reactions, a good correlation was obtained between σ_{red} and Q_{gg} for 3n and 4n transfer reactions. However, in the case of ^{48}Ca + ^{124}Sn reaction, both 3n and 4n transfers show deviations from the systematics.

Table 4. Angle- and energy-integrated 3n transfer cross-sections (σ_{3n}(exp)) for systems used in this systematics along with Q_{gg}, binding energies of the transferred neutron in the entrance (B_i) and the exit channels (B_f) which are taken from NNDC [76]. σ_{3n}(syst) and σ_{3n}(gr) are the cross-sections obtained from the systematics and GRAZING calculations respectively.

System	Z_pZ_t	Q_{gg} (MeV)	B_i (MeV)	B_f (MeV)	σ_{3n}(exp) (mb)	σ_{3n}(syst) (mb)	σ_{3n}(gr) (mb)	Reference
^{40}Ca + ^{96}Zr	800	5.246	8.22	7.93	9	8.07	1.49	[83]
^{40}Ca + ^{124}Sn	1000	4.526	8.82	7.93	4.7	7.86	2.49	[85]
^{48}Ca + ^{124}Sn	1000	−6.929	8.82	4.81	0.14	1.43	0.20	[86]
^{58}Ni + ^{124}Sn	1400	4.956	8.82	7.82	6.5±3	7.05	7.44	[98]
^{40}Ar + ^{208}Pb	1476	−1.008	8.09	5.66	5.6	4.22	0.117	[32]
^{40}Ca + ^{208}Pb	1640	5.584	8.09	7.93	11.07	8.65	4.81	[89]
^{58}Ni + ^{208}Pb	2296	6.015	8.09	7.82	11.88	9.46	14.68	[93]
^{58}Ni + ^{232}Th	2520	9.854	6.79	7.82	20±5	18.76	7.23	[92]
^{90}Zr + ^{208}Pb	3280	0.371	8.09	6.73	20	8.50	27.67	[83]
^{197}Au + ^{130}Te	4108	−2.971	8.78	6.22	10.26	2.45	11.0	[31]

Table 5. Angle- and energy-integrated 4n transfer cross-sections (σ_{4n}(exp)) for systems used in this systematics along with Q_{gg}, binding energies of the transferred neutron in the entrance (B_i) and the exit channels (B_f) which are taken from NNDC [76]. σ_{4n}(syst) and σ_{4n}(gr) are the cross-sections obtained from the systematics and GRAZING calculations respectively.

System	Z_pZ_t	Q_{gg} (MeV)	B_i (MeV)	B_f (MeV)	σ_{4n}(exp) (mb)	σ_{4n}(syst) (mb)	σ_{4n}(gr) (mb)	Reference
^{40}Ca + ^{96}Zr	800	9.642	6.73	11.13	2.35	2.93	0.23	[83]
^{40}Ca + ^{124}Sn	1000	9.486	6.17	11.13	1	3.38	0.41	[85]
^{48}Ca + ^{124}Sn	1000	−7.094	6.17	6.01	0.00089	1.57	0.01	[86]
^{58}Ni + ^{124}Sn	1400	9.382	6.17	10.60	1.74±1.25	3.56	1.41	[98]
^{40}Ca + ^{208}Pb	1640	9.984	6.73	11.13	3.98	2.68	1.57	[89]
^{58}Ni + ^{208}Pb	2296	9.879	6.73	10.60	7.46	2.95	5.44	[93]
^{90}Zr + ^{208}Pb	3280	1.858	6.73	8.22	10.58	6.44	14.18	[83]
^{197}Au + ^{130}Te	4108	−2.027	6.29	7.23	2.99	4.85	4.25	[31]

5. Conclusion

We have analysed and developed a systematics for the transfer cross-sections, with projectiles in the mass range $A = 16$–197 and with target nuclei $A = 58$–232. The developed systematics show a good correlation between σ_{red} and Q_{gg}. One can use these systematics to predict cross-sections even for heavier systems ($Z_pZ_t < 4000$) which are experimentally more difficult to measure, where the nuclear spectroscopic studies are of high importance.

One-neutron pickup transfer reactions show a good correlation between σ_{red} and Q_{gg} if one separates the systems into two groups based on their Z_pZ_t product. The reactions with target nuclei having odd nucleon number, ^{58}Ni + ^{149}Sm and ^{32}S + ^{93}Nb, show deviations from one-neutron pickup systematics. Moreover, this systematics starts to deviate for systems with $Z_pZ_t > 4000$ ($\chi_{\text{eff}} > 1$).

In the two-neutron transfer systematics, reactions with large negative Q-values deviate from the systematics. However, the result for ^{48}Ca + ^{124}Sn, ^{58}Ni + ^{58}Ni, ^{18}O + ^{206}Pb and ^{64}Ni + ^{120}Sn systems agree with systematics. All the sulphur-induced reactions show lower cross-sections than expected from the systematics and follow a different trend in two-neutron transfer systematics, except for ^{32}S + ^{92}Mo reaction. In the case of ^{48}Ca + ^{124}Sn reaction, 1n transfer deviates from systematics, while 2n transfer is in agreement with systematics. These systems have proton shell closure in both the projectile and the target nuclei. Usually, collision between the proton shell closed nuclei (open neutron shell) are used to study the pair transfer effects [12,13,15]. However, the role of nucleon–nucleon correlation, as expressed by the transfer of pairs, is not fully understood yet and need further investigations [9].

Similar to two-neutron systematics, three- and four-neutron systematics also show good agreement with the experimental cross-sections. ^{48}Ca + ^{124}Sn reaction shows similar deviation from systematics for 3n and 4n transfers as observed in the one-neutron transfer systematics.

In conclusion, transfer systematics show very good correlation with Q_{gg} values if one uses reduced cross-sections, for one-neutron to four-neutron transfers. Further studies are needed to extend this systematics for proton pickup/stripping reactions.

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