Pramana – J. Phys. (2020) 94:60
https://doi.org/10.1007/s12043-020-1927-8

A generalised approach to calculate various transport observables for a linear array of series and parallel quantum dots

SUSHILA DEVI[1,*], P K AHLUWALIA[1] and SHYAM CHAND[2]

[1]Department of Physics, H.P. University, Shimla 171 005, India
[2]University Institute of Information Technology, H.P. University, Shimla 171 005, India
*Corresponding author. E-mail: sushilabhatt45@gmail.com

MS received 3 March 2019; revised 11 December 2019; accepted 20 December 2019

Abstract. A systematic generalised approach to find transport observables for a linear array of different quantum dot (QD) systems has been discussed, using non-equilibrium Green function (NEGF) formalism, in the presence of on-dot Coulomb interaction and inter-dot tunnelling. The equation of motion (EOM) method has been used to derive expressions for Green functions (GFs) within the simplest mean-field approximation to tackle the Coulomb correlation term. Starting from the mathematical structures of GFs for single, double and triple quantum dot systems, the expressions for GFs and transport observables have been generalised for the quantum dot systems containing N number of quantum dots in series as well as parallel linear array of dots. Further, the formulae so obtained have been used for numerical calculations of transmission probability and the I–V characteristics of linear arrays of quantum dots in series as well as parallel configuration containing up to three dots. The results show that, with the increase in number of dots in the scattering region, transmission probability and electron current decrease in series case, while both quantities increase in parallel configuration of dots. The inter-dot tunnelling leads to the splitting of transmission peaks in double QD system in series case whereas, it induces Fano effect in triple QD system in parallel configuration.

Keywords. Quantum transport; quantum dots; linear array of dots; non-equilibrium Green function; Landauer–Buttiker formula.

PACS Nos **73.23.Hk; 73.21.La; 73.63.Kv; 73.40.Gk**

1. Introduction

With the advancement in the nanoscale fabrication technology, it has become possible to apply confinement techniques to fabricate nanostructures having less than three degrees of freedom of motion for electrons [1]. These low-dimensional structures, namely, quantum wells (2-D system), quantum wires (1-D system) and quantum dots (0-D system), have already attracted tremendous research activities during the past three decades. The quantum dots (QDs) [2,3], having been regarded as artificial atoms and the smallest solid-state nanostructures, are the most promising candidates for their applications in nanoelectronics [4–6], quantum information processing [7–12], quantum optics and photonics [13–17], medical application [18–20] etc. The electron transport through QDs is also one of the most exciting fields in modern transport theory and has been explored extensively during the past three decades [21–25]. The Kondo effect [26–29], Coulomb blockade [30,31], Fano effect [32–34], Dicke effect [35,36], Josephson tunnelling [37–39] etc. are some of the interesting phenomena associated with electron transport through QDs, which can be observed under specific conditions imposed on QD systems for respective phenomenon. Interestingly, all the parameters of QDs like shape, size, level spacing, spin–orbit interaction [40], coupling with environment etc. are fully tuneable, as a result of which, QD systems give an advantage of studying these novel phenomena in a variety of ways under controllable conditions.

A typical QD system, designed to study quantum transport, consists of two macroscopic electron reservoirs called leads having band-like spectrum and a central scattering region containing one or more QDs [41] having discrete energy levels due to confinement effect. Depending on the number of QDs in the central scattering region, it is possible to fabricate

Published online: 31 March 2020

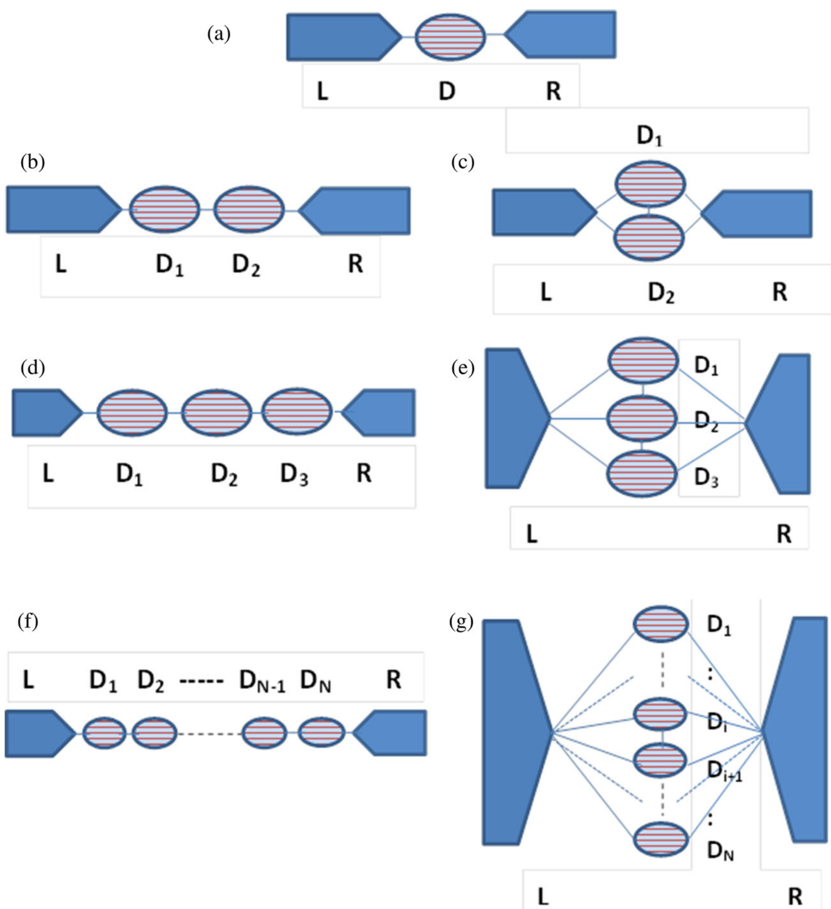

Figure 1. Schematic diagrams of different quantum dot systems. (**a**) Single QD system, (**b**) coupled QD system in series, (**c**) coupled QD system in parallel, (**d**) triple QD system in series, (**e**) triple QD system in parallel, (**f**) N-QD system in series and (**g**) N-QD system in parallel. Here L(R) represent left(right) leads and D_i is the ith dot.

single QD system, coupled QD systems (series and parallel), linear triple QD systems (series and parallel) and a linear array of N-QD systems (series and parallel) (figure 1). Experimental realisations of such systems, electron transport through them and their optical properties, have been explored in recent past in many works [42–49].

Many theoretical transport formalisms, like master equation [20], slave-boson approach [28], renormalisation group method [50] etc. are available for studying quantum transport through QDs. But, non-equilibrium Green function (NEGF) approach [51–55] is the best formalism to successfully explore quantum transport through QDs [56–59] and has contributed immensely during the past two decades in the theoretical study of transport through single and double QD systems [60–68]. It has been observed that, the analytical calculations of expressions for GFs and other transport observables are simplest in single QD system. However, significant difficulties are encountered when one tries to get these expressions for double QD systems. As the number of dots in the scattering region increases, these calculations become increasingly complicated. Therefore, only limited theoretical works are available for transport through triple and higher array of dots.

Further, the application of QD arrays is expected in the nanoelectronic devices, quantum computing [7,11,12] and quantum dot solar cells [69,70]. Hence, it becomes important to know the electronic and transport properties of QD systems with larger number of dots. Although there are a few theoretical studies which have attempted to explore transport through QD arrays [46,71,72], these studies are limited to non-interacting case and to the best of our knowledge, none of the theoretical works takes Coulomb correlation into consideration, while studying electron transport through QD systems containing three or more than three QDs in the scattering region.

Recent development on the experimental part to fabricate linear arrays of QDs, with high precision for measurement of different parameters, has been very encouraging [73–75] and are found to be necessary developments towards the physical implementation of elements of quantum computing. Volk *et al* [73] made the sequential arrangement of eight quantum dots forming a 'Qubyte' register by adding QDs by $n + 1$ method and studied the flow and tunnelling rate of electron on each QD. The scalable approach to physically transport a single electron through a linear 1D array of quantum dots was experimentally demonstrated by Mills *et al* [74] which offers an important path for experimental work on transport through 1D array with larger number of dots. Ito *et al* [75] fabricated a semiconductor quintuple quantum dot (5QD) or series coupled five QDs with a concept relevant for further increasing the number of QDs. These experimental works provide an important step towards realising controllable large-scale multiple QD systems and offer playground for quantum computation and simulation.

Motivated by this, we present a generalised approach to calculate various transport observables, on the basis of generalised expressions for GFs for different QD systems in Coulomb blockade regime. To this effect, we express the analytically calculated GFs of dots in single, double and triple QD systems in the form of matrices. Then, on the basis of the trend being followed, we generalise the final forms of GFs for longer linear chain of QDs in series as well as in parallel configuration of dots. Hence, out of this generalisation we are able to write final forms of GFs for any linear array of N-QD ($N = 1, 2, 3, \ldots, N$) in series as well as in parallel configuration, in the presence of on-dot Coulomb correlation. This generalisation further leads to generalised expressions for transport observables like tunnelling current and transmission probability. The expressions so obtained have been used to numerically calculate the transmission probability and the I–V characteristics of linear arrays of QDs in series as well as in parallel configuration containing up to three dots in the scattering region. As, with the advancement in the nanofabrication techniques, linear arrays of QDs are currently available for experimental examination, the results presented in the present work would be an important theoretical study to demonstrate the electron transport phenomena through the linear array of multiple dots and may further help understand transport through complex molecules [76] etc.

Rest of the paper is organised as follows: §2 deals with the setting of model Hamiltonian for linear QD systems in series and parallel configurations. In §3, the calculations for GFs for different QD systems have been presented and in §4 various transport observables have been derived. Section 5 contains the numerical results and a brief summary of conclusions is presented in §6.

2. Model Hamiltonians

Over the years, various energies and interactions in QD systems have been modelled successfully using model [77] by suitably modifying its original form. The shapes of model Hamiltonians for different QD systems, written in second quantisation form, are very well known and can be found in many theoretical studies done for various QD systems [56–62,65–68]. In the following part we write the total Hamiltonian for different QD systems.

To write the total Hamiltonian, we take leads and each dot to be different systems, which interact with each other via hybridisation terms.

2.1 *Hamiltonian for the single QD system*

For a single QD system, the Hamiltonian has the following form:

$$\begin{aligned} H = {} & \sum_{k\sigma} \epsilon_k^{\mathrm{L}} a_{k\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} \epsilon_p^{\mathrm{R}} b_{p\sigma}^{\dagger} b_{p\sigma} \\ & + \sum_{\sigma} \epsilon c_{\sigma}^{\dagger} c_{\sigma} + U n_{\uparrow} n_{\downarrow} \\ & + \left(\sum_{k\sigma} V_k^{\mathrm{L}} c_{\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} V_p^{\mathrm{R}} c_{\sigma}^{\dagger} b_{p\sigma} + \mathrm{h.c.} \right), \end{aligned} \quad (1)$$

where $\epsilon_{k(P)}^{\mathrm{L(R)}}$ are the kinetic energies of non-interacting free electrons in the left(right) leads having band-like spectrum, $a_{k\sigma}^{\dagger}(a_{k\sigma})$ and $b_{p\sigma}^{\dagger}(b_{p\sigma})$ are the electron creation(annihilation) operators for the left(right) leads. However, the confined electrons in QD are having discrete energy levels such that ϵ is the energy of electrons at Fermi level, $c_{\sigma}^{\dagger}(c_{\sigma})$ are creation(annihilation) operators of electrons on QD, U is the intra-dot Coulomb interaction energy of electrons on QD and $n_{\sigma} = c_{\sigma}^{\dagger} c_{\sigma}$ is the number operator for electron occupation. Terms in the parenthesis of eq. (1) represent hopping of electrons between QD and leads, with $V_{k(p)}^{\mathrm{L(R)}}$ as the coupling potential of the left(right) barrier with dot.

Now, on the analogy of single QD system, the form of Hamiltonian for QD system with more than one QD between the leads can easily be visualised. If there is a linear array of N QDs, then the form of Hamiltonian will depend on whether the dots are connected in series or parallel configuration between the leads. From the topology of dots given in figure 1, it is clear that, in series case, only the first and the last dot will be connected to the left and the right leads respectively, while all QDs will be connected to both the leads in parallel configuration. To generalise the form of Hamiltonian, we modify eq. (1) for double and triple QD systems and then extend the analogy for N-QD systems.

2.2 *Hamiltonian for the double QD system*

Now on the analogy of single QD system and topology of double dots in figures 1b and 1c, the Hamiltonian for the double QD system has the following forms:

(a) *Series configuration*

$$H = \sum_{k\sigma} \epsilon_k^{\mathrm{L}} a_{k\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} \epsilon_p^{\mathrm{R}} b_{p\sigma}^{\dagger} b_{p\sigma} + \sum_{i\sigma} \epsilon_i c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{i} U_i n_{i\uparrow} n_{i\downarrow} + \left(\sum_{k\sigma} V_k^{\mathrm{L}} c_{1\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} V_p^{\mathrm{R}} c_{2\sigma}^{\dagger} b_{p\sigma} + \sum_{\sigma} V_d c_{1\sigma}^{\dagger} c_{2\sigma} + \mathrm{h.c.} \right). \quad (2)$$

(b) *Parallel configuration*

$$H = \sum_{k\sigma} \epsilon_k^{\mathrm{L}} a_{k\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} \epsilon_p^{\mathrm{R}} b_{p\sigma}^{\dagger} b_{p\sigma} + \sum_{i\sigma} \epsilon_i c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{i} U_i n_{i\uparrow} n_{i\downarrow} + \left(\sum_{k\sigma i} V_{ki}^{\mathrm{L}} c_{i\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma i} V_{pi}^{\mathrm{R}} c_{i\sigma}^{\dagger} b_{p\sigma} + \sum_{\sigma} V_d c_{1\sigma}^{\dagger} c_{2\sigma} + \mathrm{h.c.} \right). \quad (3)$$

2.3 *Hamiltonian for the linear triple QD system*

In the case of triple QD systems shown in figures 1d and 1e, the model Hamiltonian can be written in the following forms:

(a) *Series configuration*

$$H = \sum_{k\sigma} \epsilon_k^{\mathrm{L}} a_{k\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} \epsilon_p^{\mathrm{R}} b_{p\sigma}^{\dagger} b_{p\sigma} + \sum_{i\sigma} \epsilon_i c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{i} U_i n_{i\uparrow} n_{i\downarrow} + \left[\sum_{k\sigma} V_k^{\mathrm{L}} c_{1\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} V_p^{\mathrm{R}} c_{3\sigma}^{\dagger} b_{p\sigma} + \sum_{\sigma} V_d (c_{1\sigma}^{\dagger} c_{2\sigma} + c_{2\sigma}^{\dagger} c_{3\sigma}) + \mathrm{h.c.} \right]. \quad (4)$$

(b) *Parallel configuration*

$$H = \sum_{k\sigma} \epsilon_k^{\mathrm{L}} a_{k\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} \epsilon_p^{\mathrm{R}} b_{p\sigma}^{\dagger} b_{p\sigma} + \sum_{i\sigma} \epsilon_i c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{i} U_i n_{i\uparrow} n_{i\downarrow} + \left[\sum_{k\sigma i} V_{ki}^{\mathrm{L}} c_{i\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma i} V_{pi}^{\mathrm{R}} c_{i\sigma}^{\dagger} b_{p\sigma} + \sum_{\sigma} V_d (c_{1\sigma}^{\dagger} c_{2\sigma} + c_{2\sigma}^{\dagger} c_{3\sigma}) + \mathrm{h.c.} \right]. \quad (5)$$

2.4 *Hamiltonian for the linear array of N-QD system*

Proceeding in a similar manner, we can generalise the expressions for the model Hamiltonian for a linear array of N-QD systems, as follows:

(a) *Series configuration*

$$H = \sum_{k\sigma} \epsilon_k^{\mathrm{L}} a_{k\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} \epsilon_p^{\mathrm{R}} b_{p\sigma}^{\dagger} b_{p\sigma} + \sum_{i\sigma} \epsilon_i c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{i} U_i n_{i\uparrow} n_{i\downarrow} + \left[\sum_{k\sigma} V_k^{\mathrm{L}} c_{1\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} V_p^{\mathrm{R}} c_{n\sigma}^{\dagger} b_{p\sigma} + \sum_{\sigma} V_d (c_{1\sigma}^{\dagger} c_{2\sigma} + c_{2\sigma}^{\dagger} c_{3\sigma} + \cdots + c_{n-2\sigma}^{\dagger} c_{n-1\sigma} + c_{n-1\sigma}^{\dagger} c_{n\sigma}) + \mathrm{h.c.} \right]. \quad (6)$$

(b) *Parallel configuration*

$$H = \sum_{k\sigma} \epsilon_k^{\mathrm{L}} a_{k\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma} \epsilon_p^{\mathrm{R}} b_{p\sigma}^{\dagger} b_{p\sigma} + \sum_{i\sigma} \epsilon_i c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{i} U_i n_{i\uparrow} n_{i\downarrow} + \left[\sum_{k\sigma i} V_{ki}^{\mathrm{L}} c_{i\sigma}^{\dagger} a_{k\sigma} + \sum_{p\sigma i} V_{pi}^{\mathrm{R}} c_{i\sigma}^{\dagger} b_{p\sigma} + \sum_{\sigma} V_d (c_{1\sigma}^{\dagger} c_{2\sigma} + c_{2\sigma}^{\dagger} c_{3\sigma} + \cdots + c_{n-2\sigma}^{\dagger} c_{n-1\sigma} + c_{n-1\sigma}^{\dagger} c_{n\sigma}) + \mathrm{h.c.} \right], \quad (7)$$

where ϵ_i is the energy of the discrete energy level on the ith dot, $c_{i\sigma}^{\dagger}$ $(c_{i\sigma})$ are creation(annihilation) operators of electrons on the dots, U_i is the intra-dot Coulomb interaction energy of electrons on the ith dot and

$n_{i\sigma} = c_{i\sigma}^{\dagger} c_{i\sigma}$ is the number operators for electron occupation. Here, $V_{k(p)i}$L(R) are the coupling potentials of the left(right) barriers with the ith dot and is assumed to be a tunnelling matrix. In all the cases, V_d represents inter-dot tunnelling.

3. Generalisation of Green functions

The quantum transport through any QD system is typically a non-equilibrium problem. Hence, over the period of many years, NEGF approach has been the most successful theoretical tool to explore the electron transport phenomenon in QD systems [56,57,61,65,66]. In NEGF formalism, three types of Green functions (GFs) are of particular importance. These three types of GFs are: retarded GF (G^r), advanced GF (G^a) and lesser GF ($G^<$). The basic definitions and properties of these GFs can be found in many works [25,60,62], which deal with transport through the QD systems. In the present case, these GFs have been derived using the equation of motion (EOM) method [25,56]. The EOM for GFs generates an endless chain of higher-order GFs [60]. To truncate these higher-order GFs, we employ the simplest mean-field approximations [39] to tackle Coulomb interaction term, thereby all the GFs of the dots get closed to their final forms.

The aim of this paper is to generalise the final forms of GFs for different QD systems with any number of dots in the scattering region. With the help of the expressions for GFs of the dots in single, coupled and triple QD system, a generalisation of dot GFs for longer linear chains of QDs can be carried out for series as well as parallel configurations of dots.

3.1 *For the single QD system*

If we write the EOM for GFs of the dot for single QD system using Hamiltonian written in eq. (1) and make the simplest mean-field approximation (where GF is of the type $\langle\langle n_\sigma C_\sigma, c_\sigma^\dagger\rangle\rangle \approx n_\sigma \langle\langle C_\sigma, c_\sigma^\dagger\rangle\rangle$), then the final form of the retarded GF can be closed to the following form:

$$G^r = \langle\langle c_\sigma, c_\sigma^\dagger\rangle\rangle^r = \frac{1}{\omega - \epsilon - U n_\sigma + \frac{i}{2}\Gamma}. \quad (8)$$

This can be expressed in matrix form as

$$\mathbf{G}^{\mathrm{r}} = \left[\omega - \epsilon - U n_\sigma + \frac{i}{2}\Gamma\right]^{-1}. \quad (9)$$

3.2 *For the coupled QD system*

(a) *Series configuration*
If we write the EOM for GFs of the dots in series configuration of double QD system using Hamiltonian written in eq. (2) and make the simplest mean-field approximation, then the final forms of the retarded GFs are found to be close to the following forms:

$$G_{11}^r = \langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = \frac{\omega - \varepsilon_2 - U n_{2-\sigma} + \frac{i}{2}\Gamma_{22}^{\mathrm{R}}}{\left(\omega - \epsilon_1 - U n_{1-\sigma} + \frac{i}{2}\Gamma_{11}^{\mathrm{L}}\right)\left(\omega - \epsilon_2 - U n_{2-\sigma} + \frac{i}{2}\Gamma_{22}^{\mathrm{R}}\right) - |V_d|^2} \quad (10)$$

$$G_{22}^r = \langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = \frac{\omega - \varepsilon_1 - U n_{1-\sigma} + \frac{i}{2}\Gamma_{11}^{\mathrm{L}}}{\left(\omega - \epsilon_1 - U n_{1-\sigma} + \frac{i}{2}\Gamma_{11}^{\mathrm{L}}\right)\left(\omega - \epsilon_2 - U n_{2-\sigma} + \frac{i}{2}\Gamma_{22}^{\mathrm{R}}\right) - |V_d|^2} \quad (11)$$

$$G_{21}^r = \langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = \frac{V_d^*}{\left(\omega - \epsilon_1 - U n_{1-\sigma} + \frac{i}{2}\Gamma_{11}^{\mathrm{L}}\right)\left(\omega - \epsilon_2 - U n_{2-\sigma} + \frac{i}{2}\Gamma_{22}^{\mathrm{R}}\right) - |V_d|^2} \quad (12)$$

$$G_{12}^r = \langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = \frac{V_d}{\left(\omega - \epsilon_1 - U n_{1-\sigma} + \frac{i}{2}\Gamma_{11}^{\mathrm{L}}\right)\left(\omega - \epsilon_2 - U n_{2-\sigma} + \frac{i}{2}\Gamma_{22}^{\mathrm{R}}\right) - |V_d|^2}. \quad (13)$$

The above GFs can nicely be expressed in a matrix form as follows:

$$\mathbf{G}^r = \begin{bmatrix} G_{11}^r & G_{12}^r \\ G_{21}^r & G_{22}^r \end{bmatrix} = \begin{bmatrix} \omega - \epsilon_1 - U n_{1-\sigma} + \frac{i}{2}\Gamma_{11}^{\mathrm{L}} & -V_d \\ -V_d^* & \omega - \epsilon_2 - U n_{2-\sigma} + \frac{i}{2}\Gamma_{22}^{\mathrm{R}} \end{bmatrix}^{-1}. \quad (14)$$

(b) *Parallel configuration*

Now, using Hamiltonian of eq. (3), the GFs for parallel double QD are closed to the following final forms:

$$G^r_{11} = \langle\langle c_{1\sigma}, c^\dagger_{1\sigma}\rangle\rangle^r = \frac{\omega - \varepsilon_2 - Un_{2-\sigma} + \frac{i}{2}\Gamma_{22}}{(\omega - \epsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma_{11})(\omega - \epsilon_2 - Un_{2-\sigma} + \frac{i}{2}\Gamma_{22}) - (-V_d + \frac{i}{2}\Gamma_{12})(-V_d^* + \frac{i}{2}\Gamma_{21})} \tag{15}$$

$$G^r_{22} = \langle\langle c_{2\sigma}, c^\dagger_{2\sigma}\rangle\rangle^r = \frac{\omega - \varepsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma_{11}}{(\omega - \epsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma_{11})(\omega - \epsilon_2 - Un_{2-\sigma} + \frac{i}{2}\Gamma_{22}) - (-V_d + \frac{i}{2}\Gamma_{12})(-V_d^* + \frac{i}{2}\Gamma_{21})} \tag{16}$$

$$G^r_{21} = \langle\langle c_{2\sigma}, c^\dagger_{1\sigma}\rangle\rangle^r = \frac{V_d^* - \frac{i}{2}\Gamma_{21}}{(\omega - \epsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma_{11})(\omega - \epsilon_2 - Un_{2-\sigma} + \frac{i}{2}\Gamma_{22}) - (-V_d + \frac{i}{2}\Gamma_{12})(-V_d^* + \frac{i}{2}\Gamma_{21})} \tag{17}$$

$$G^r_{12} = \langle\langle c_{1\sigma}, c^\dagger_{2\sigma}\rangle\rangle^r = \frac{V_d - \frac{i}{2}\Gamma_{12}}{(\omega - \epsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma_{11})(\omega - \epsilon_2 - Un_{2-\sigma} + \frac{i}{2}\Gamma_{22}) - (-V_d + \frac{i}{2}\Gamma_{12})(-V_d^* + \frac{i}{2}\Gamma_{21})}. \tag{18}$$

Clearly, these GFs can be written in matrix form as

$$G^r = \begin{bmatrix} G^r_{11} & G^r_{12} \\ G^r_{21} & G^r_{22} \end{bmatrix} = \begin{bmatrix} \omega - \epsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma_{11} & -V_d + \frac{i}{2}\Gamma_{12} \\ -V_d^* + \frac{i}{2}\Gamma_{21} & \omega - \epsilon_2 - Un_{2-\sigma} + \frac{i}{2}\Gamma_{22} \end{bmatrix}^{-1}. \tag{19}$$

It is worth mentioning here that, another configuration for double QD system in parallel topology, in which only one dot is attached to lead while the other one is side-coupled with this dot often called as T-shaped geometry [64,78–80], is also of importance. Though, it will not be possible to get GFs of dots for T-shaped configuration through the results of generalisation process, yet, GFs of dots for this configuration can simply be obtained as a special case of parallel DQD when $\Gamma_{22} = \Gamma_{12} = \Gamma_{21} = 0$ in eq. (20), as the lead-dot coupling $V_{k2}^{\mathrm{L(R)}} = 0$ in the corresponding Hamiltonian [eq. (3)].

3.3 *For the triple QD system*

Now, after following similar procedure as in coupled QD case, we find that the corresponding matrices for various GFs in triple QD systems have the following forms:

(a) *Series configuration*

$$\mathbf{G^r} = \begin{bmatrix} \omega - \epsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma^{\mathrm{L}}_{11} & -V_{12} & 0 \\ -V_{12}^* & \omega - \epsilon_2 - Un_{2-\sigma} & -V_{23} \\ 0 & -V_{23}^* & \omega - \epsilon_3 - Un_{3-\sigma} + \frac{i}{2}\Gamma^{\mathrm{R}}_{33} \end{bmatrix}^{-1}. \tag{20}$$

(b) *Parallel configuration*

$$\mathbf{G}^r = \begin{bmatrix} \omega - \epsilon_1 - Un_{1-\sigma} + \frac{i}{2}\Gamma_{11} & -V_{12} + \frac{i}{2}\Gamma_{12} & \frac{i}{2}\Gamma_{13} \\ -V_{12}^* + \frac{i}{2}\Gamma_{21} & \omega - \epsilon_2 - Un_{2-\sigma} + \frac{i}{2}\Gamma_{22} & -V_{23} + \frac{i}{2}\Gamma_{23} \\ \frac{i}{2}\Gamma_{31} & -V_{23}^* + \frac{i}{2}\Gamma_{32} & \omega - \epsilon_3 - Un_{3-\sigma} + \frac{i}{2}\Gamma_{33} \end{bmatrix}^{-1}. \tag{21}$$

Detailed calculations of GFs for various quantum dots are shown in Appendix A. At this stage, one can easily visualise the scope for the generalisation of expressions for GFs in series as well as in parallel configurations, as a systematic trend is being set with increase in the order of matrix *vis-à-vis* topology of QDs in a given system. Now, each element in a given matrix can directly be predicted with the consideration of dot topology and its possible coupling with environment (other dots/leads).

Here we find that the analogy can be extended to any number of dots in the linear array.

3.4 *For N-QD system*

By looking at the trend being followed to write these GFs in QD systems containing one, two and three dots in the scattering region, one can easily generalise the form of GFs for N-QD system, as follows:

(a) *Series configuration*

$$\mathbf{G}^r = \begin{bmatrix} \omega-\epsilon_1-Un_{1-\sigma}+\frac{i}{2}\Gamma^{\mathrm{L}}_{11} & -V_{12} & \cdots & 0 & 0 \\ -V^*_{12} & \omega-\epsilon_2-Un_{2-\sigma} & & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & \omega-\epsilon_{N-1}-Un_{N-1-\sigma} & -V_{N-1N} \\ 0 & 0 & & -V^*_{N-1N} & \omega-\epsilon_N-Un_{N-\sigma}+\frac{i}{2}\Gamma^{\mathrm{R}}_{NN} \end{bmatrix}^{-1}. \tag{22}$$

(b) *Parallel configuration*

$$\mathbf{G}^r = \begin{bmatrix} \omega-\epsilon_1-Un_{1-\sigma}+\frac{i}{2}\Gamma_{11} & -V_{12}+\frac{i}{2}\Gamma_{12} & \cdots & \frac{i}{2}\Gamma_{1N-1} & \frac{i}{2}\Gamma_{1N} \\ -V^*_{12}+\frac{i}{2}\Gamma_{21} & \omega-\epsilon_2-Un_{2-\sigma}+\frac{i}{2}\Gamma_{22} & & \frac{i}{2}\Gamma_{2N-1} & \frac{i}{2}\Gamma_{2N} \\ \vdots & & \ddots & & \vdots \\ \frac{i}{2}\Gamma_{N-11} & \frac{i}{2}\Gamma_{N-12} & \cdots & \omega-\epsilon_{N-1}-Un_{N-1-\sigma} & -V_{N-1N}+\frac{i}{2}\Gamma_{N-1N} \\ \frac{i}{2}\Gamma_{N1} & \frac{i}{2}\Gamma_{N2} & & -V^*_{NN-1}+\frac{i}{2}\Gamma_{NN-1} & \omega-\epsilon_N-Un_{N-\sigma}+\frac{i}{2}\Gamma_{NN} \end{bmatrix}^{-1}. \tag{23}$$

In all the above cases, $\Gamma_{ij} = \Gamma^{\mathrm{L}}_{ij} + \Gamma^{\mathrm{R}}_{ij}$ and for an $N \times N$ matrix, the lead-dot coupling matrices can be written as

(a) *Series configuration*

$$\Gamma^{\mathrm{L}} = \begin{bmatrix} \Gamma^{\mathrm{L}}_{11} & 0 & \cdots & 0 & 0 \\ 0 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

and

$$\Gamma^{\mathrm{R}} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 0 & 0 \\ 0 & 0 & \cdots & 0 & \Gamma^{\mathrm{R}}_{NN} \end{bmatrix}.$$

(b) *Parallel configuration*

$$\Gamma^{\mathrm{L(R)}} = \begin{bmatrix} \Gamma^{\mathrm{L(R)}}_{11} & \Gamma^{\mathrm{L(R)}}_{12} & \cdots & \Gamma^{\mathrm{L(R)}}_{1N-1} & \Gamma^{\mathrm{L(R)}}_{1N} \\ \Gamma^{\mathrm{L(R)}}_{21} & \Gamma^{\mathrm{L(R)}}_{22} & & \Gamma^{\mathrm{L(R)}}_{2N-1} & \Gamma^{\mathrm{L(R)}}_{2N} \\ \vdots & & \ddots & & \vdots \\ \Gamma^{\mathrm{L(R)}}_{N-11} & \Gamma^{\mathrm{L(R)}}_{N-12} & \cdots & \Gamma^{\mathrm{L(R)}}_{N-1N-1} & \Gamma^{\mathrm{L(R)}}_{N-1N} \\ \Gamma^{\mathrm{L(R)}}_{N1} & \Gamma^{\mathrm{L(R)}}_{N2} & & \Gamma^{\mathrm{L(R)}}_{NN-1} & \Gamma^{\mathrm{L(R)}}_{NN} \end{bmatrix}.$$

Here,

$$\Gamma^{\mathrm{L(R)}}_{ij} = \sum_{k(p)}^{\mathrm{L(R)}} V^{\mathrm{L(R)}*}_{k(p)i(j)} V^{\mathrm{L(R)}}_{k(p)i(j)} g^r_{k(p)} = 2\pi \sum_{k(p)}^{\mathrm{L(R)}} V^{\mathrm{L(R)}*}_{k(p)} V^{\mathrm{L(R)}}_{k(p)} \delta\left(\omega - \varepsilon^{\mathrm{L(R)}}_{k(p)}\right)$$

are different possible lead-dot coupling matrix elements.

The expressions for the advanced GFs can also be generalised in a similar manner. However, final forms of these GFs are not shown but it can be seen that these GFs are the complex conjugates of the corresponding retarded GFs. Further, the expressions for lesser GFs can be obtained in terms of retarded and advanced GFs [53, 58]. Hence in this way, we are able to generalise the expressions for all the GFs of linear array of QDs with any number of dots.

4. Expression for the tunnelling current

To write the generalised expression for the current, we follow Meir and Wingreen technique [57,65], with the help of which we get the following Landauer–Buttiker formula [65]:

$$I = \frac{e}{h}\sum_{\sigma}\int (f_{\mathrm{L}} - f_{\mathrm{R}})\mathrm{Tr}\big(\mathbf{G}^{a}\mathbf{\Gamma}^{\mathrm{R}}\mathbf{G}^{r}\mathbf{\Gamma}^{\mathrm{L}}\big)\mathrm{d}\omega$$
$$= \frac{e}{h}\sum_{\sigma}\int (f_{\mathrm{L}} - f_{\mathrm{R}})T(\omega)\mathrm{d}\omega, \quad (24)$$

where $T(\omega) = \mathrm{Tr}(\mathbf{G}^{a}\mathbf{\Gamma}^{\mathrm{R}}\mathbf{G}^{r}\mathbf{\Gamma}^{\mathrm{L}})$ is the transmission coefficient and $f_{\mathrm{L(R)}}$ are the Fermi distribution functions for the left(right) leads. As an example, derivation of the expression for total current for the double QD system in parallel configuration are given in Appendix B. Hence, to get the expressions for the current and the transmission coefficient for a given linear array of QDs, one can substitute the corresponding GF matrices in the above expressions.

5. Numerical results

Equation (21) is the famous Landauer–Buttiker formula for the current and is a well-established relation for the current through mesoscopic conductor. In this article, it has been written for different QD systems. Here the matrix product involves four matrices and their shapes have been generalised for N-QD system in series as well as in parallel configurations of N dots, on the basis of the trend being followed in the expressions of GFs in single, double and triple QD systems in series and parallel configurations. Though some of the previous studies also have given the shapes of GFs for coupled and triple QD systems, here we have done the same in the presence of Coulomb interaction term also.

The significance of this theoretical work can be acknowledged for the simple reason that, it presents a systematic approach to generalise the expressions for GFs for any QD system in series and parallel configurations. In particular, when the number of dots in the scattering region is three or more, it is always cumbersome to get the expressions for GFs. But, the expressions for GFs in the present study can serve as 'ready reference' if anybody wishes to explore the transport properties of any number of QDs in series as well as in parallel configurations.

To assess the worth of the present generalisation, we have used the generalised expressions of GFs for the numerical calculations of transmission probability and I–V characteristics, for the linear configurations of N dots in scattering region up to $N = 3$, in series as well as in parallel configurations. For simplicity, we have assumed that the dots are symmetrically coupled with leads. Experimentally, it is observed that on-dot Coulomb repulsion U ($U_1 = U_2 = U_3$) is a dominant energy parameter. Hence, all energy parameters can be expressed in units of U. Further, we have taken the three dots with the same energy, i.e. $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$ and same inter-dot coupling V_d. In addition to this, we also consider a non-magnetic case, for which $n_{i\sigma} = n_{i-\sigma} = n = \frac{1}{2}$, the average occupation number for a level on QD. These parameters can also be calculated self-consistently.

Figure 2 shows the variation of transmission coefficient (unnormalised) as a function of energy, for series (a) and parallel (b) configuration for single ($N = 1$), double ($N = 2$) and triple ($N = 3$) QD systems. In series case, it is revealed that the height of transmission peak decreases with increase in the number of dots in the scattering region, while in parallel case height of the peak increases with increase in the number of dots. The explanation for this behaviour can clearly be understood from the simple reason that there is only single transmission channel/path available in series configuration of dots and path length for electron flow tends to increase with the increase in the number of dots placed in series, leading to a decrease in transmission probability as the number of dots increases. But, in parallel case, the number of transmission channels increases with increase in the number of dots, resulting in an increase in electron transmission probability.

In parallel configuration, a notable feature in transmission peaks appears for triple QD system, where inter-dot coupling-induced splitting of peaks occurs, resulting in a Lorenzian-type peak and a Fano shape-type peak existing around $\omega = \varepsilon + Un$. The appearance of Fano peaks shows the Fano effect, which in this case occurs due to the quantum interference phenomena between discrete levels of dots and energy continuum in the leads. In double QD systems, the Fano effect has been observed in asymmetric parallel configuration only [34] and it disappears when dots are arranged in symmetric parallel configuration [67] in the absence of flux. The result in this work is also the same as Fano effect is not appearing in transmission spectrum for symmetric parallel double ($N = 2$) QD system. Fano effect in symmetric parallel double QD can appear only in the presence of flux [68]. But interestingly, the present work reports the appearance of Fano effect in triple ($N = 3$) QD system even in symmetric parallel configuration and in the absence of flux. Further, it is worth noticing that Fano peaks disappear when inter-dot tunnelling (V_d) is removed (blue line) and only Lorenzian peaks appear in transmission probability (figure 3). Hence, it can be concluded that, inter-dot tunnelling is responsible to induce

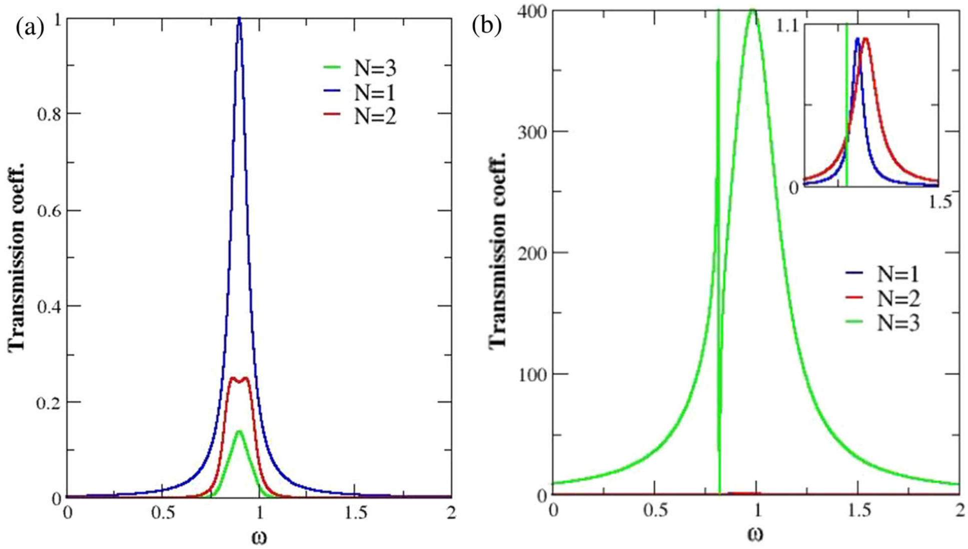

Figure 2. Graph showing the variation of transmission coefficient vs. energy for (**a**) series and (**b**) parallel configurations for N dots connected to leads with $\varepsilon = 0.4U$, $\Gamma = 0.1U$, $V_d = 0.06U$.

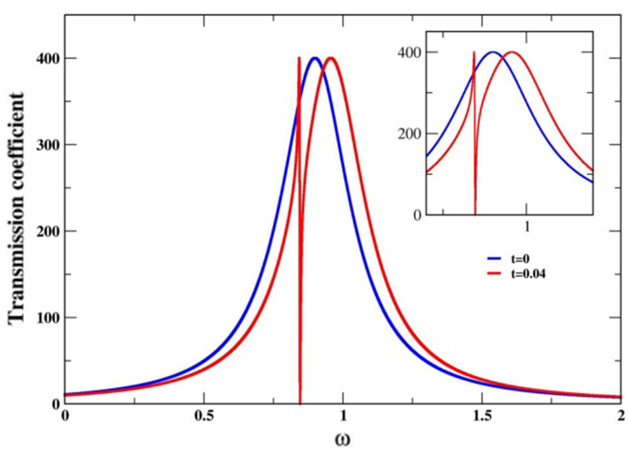

Figure 3. Transmission coefficient vs. energy graph for triple quantum dot at $\varepsilon = 0.4U$, $\Gamma = 0.1U$, $V_d = t = 0$ (blue line) and $V_d = t = 0.04U$ (red line).

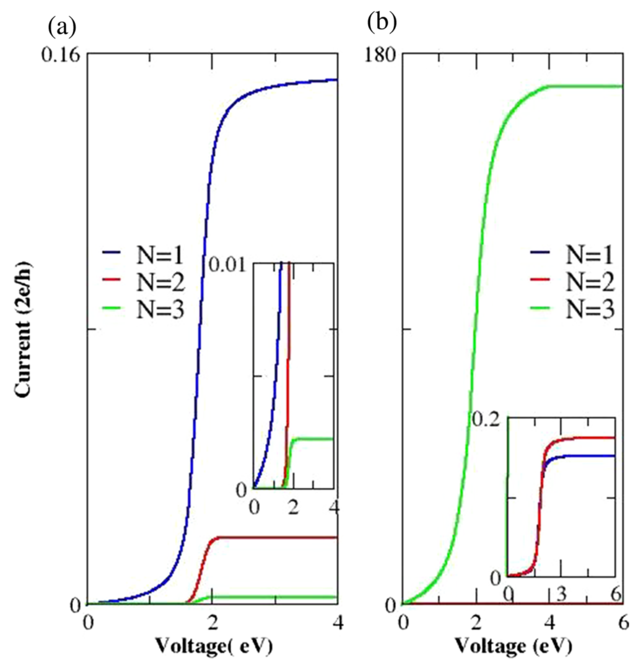

Figure 4. Graph showing the variation of I–V for (**a**) series and (**b**) parallel configurations for N dots connected to leads with $\varepsilon = 0.4U$, $\Gamma = 0.1U$, $V_d = 0.06U$.

Fano effect in triple QD system even in symmetric configurations.

It is interesting to note that there is a splitting of transmission peak for double QD system ($N = 2$) in series case and splitting is caused by inter-dot coupling. This has been reported in some of the theoretical works earlier also [61]. In the present work, with the given parameters, no splitting is found in transmission peak for triple QD system ($N = 3$) in series configurations.

The appearance of Fano resonance in parallel triple QD system has also been obtained in conductance spectrum. Figure 4 shows the variation of I–V characteristics for different number of dots in (a) series and in (b) parallel linear arrangement of dots. The decrease(increase) in current with increase in the number of dots in series(parallel) cases is as per the expectations and the results of the previous theoretical works related to such system, which can be understood on the basis of the same explanation as given for transmission probability behaviour.

Further, we have also presented the calculations of current through the QD systems with single, double and triple QDs in series (figure 4a) as well as in parallel configuration (figure 4b) of QDs between the leads. It is interesting to note that there is splitting of transmission peak for the double QD system ($N = 2$) in series case and splitting is caused by inter-dot coupling. This has been reported in some of the theoretical works earlier also [61]. In the present work with the given parameters, no splitting is found in transmission peak for triple QD ($N = 3$) in series configurations.

6. Conclusions

In the present work, we have made a generalisation of formulae for various transport observables after deriving the generalised expressions for GFs for N number of QDs in series and parallel linear arrays connected to leads. On the basis of these generalised expressions, we calculated transmission probability and I–V characteristics numerically, for these linear arrays containing up to $N = 3$ dots in scattering regions. On the basis of the results obtained, the following conclusions can be drawn:

- With mean-field approximation, the GFs for linear array of dots in series as well as parallel configurations of QD systems can easily be generalised for any number of dots, leading to generalised expressions for current and transmission coefficient.
- The height of transmission peaks and electron current decrease with increase in the number of dots in the scattering region in series case, while in parallel case height of the peaks and amount of current increase with increase in the number of dots.
- The inter-dot tunnelling leads to the splitting of transmission peaks in double QD system in series case whereas, it induces Fano effect in triple QD system in parallel configuration.
- The expressions for GFs obtained in the present work can serve as 'ready reference' to study the transport properties of different linear array of QDs in series as well as in parallel configurations in Coulomb blockade regime.

Appendix A

A.1 Equations of motion for Green's functions of dots

(a) *For the single quantum dot*

$$\omega\langle\langle c_\sigma, c_\sigma^\dagger\rangle\rangle^r = 1 + \varepsilon\langle\langle c_\sigma, c_\sigma^\dagger\rangle\rangle^r + U\langle\langle n_{-\sigma}c_\sigma, c_\sigma^\dagger\rangle\rangle^r + V_k^{\mathrm{L}}\langle\langle a_{k\sigma}, c_\sigma^\dagger\rangle\rangle^r + V_p^{\mathrm{R}}\langle\langle b_{p\sigma}, c_\sigma^\dagger\rangle\rangle^r \tag{A.1}$$

Further,

$$\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_k^{\mathrm{L}*} g_k^r \langle\langle c_\sigma, c_\sigma^\dagger\rangle\rangle^r$$

and

$$\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_p^{\mathrm{R}*} g_k^r \langle\langle c_\sigma, c_\sigma^\dagger\rangle\rangle^r.$$

(b) *For the double quantum dot in series*

$$(\omega-\epsilon_1)\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma}n_{1-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_k^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \tag{A.2}$$

$$(\omega-\epsilon_2)\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = 1 + V_d^*\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma}n_{2-\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_p^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \tag{A.3}$$

$$(\omega-\epsilon_2)\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma}n_{2-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_p^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \tag{A.4}$$

$$(\omega-\epsilon_1)\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = V_d\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma}n_{1-\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_k^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r. \tag{A.5}$$

Here

$$\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_{k1}^{\mathrm{L}*} g_k^r \langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r$$

and

$$\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_{p2}^{\mathrm{R}*} g_k^r \langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r.$$

(c) *For the double quantum dot in parallel*

$$(\omega-\epsilon_1)\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma}n_{1-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{k1}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{p1}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \tag{A.6}$$

$$(\omega-\epsilon_1-n_1U_1)\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{k1}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{p1}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \tag{A.7}$$

$$\left(\omega-\epsilon_1-n_1U_1-i\frac{\Gamma_{11}}{2}\right)\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + i\frac{\Gamma_{12}}{2}\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \tag{A.8}$$

$$\left(\omega-\epsilon_2-n_2U_2-i\frac{\Gamma_{22}}{2}\right)\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_d\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + i\frac{\Gamma_{21}}{2}\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \tag{A.9}$$

$$\left(\omega-\epsilon_2-n_2U_2-i\frac{\Gamma_{22}}{2}\right)\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + i\frac{\Gamma_{21}}{2}\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \tag{A.10}$$

$$(\omega - \epsilon_1 - n_1 U_1)\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{k1}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_{p1}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \quad (A.11)$$

$$\left(\omega - \epsilon_1 - n_1 U_1 - i\frac{\Gamma_{11}}{2}\right)\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + i\frac{\Gamma_{12}}{2}\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r. \quad (A.12)$$

The Green's functions $\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r$ and $\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r$ arise due to the tunnelling of electrons between dots and leads. The EOM for these gives the following Dyson equations:

$$\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_{k1}^{\mathrm{L}*} g_k^{\mathrm{r}}\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{k2}^{\mathrm{L}*} g_k^r\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r$$

$$\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_{p1}^{\mathrm{R}*} g_k^r\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{p2}^{\mathrm{R}*} g_k^r\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r.$$

(d) *For the triple quantum dots in series*

$$(\omega - \epsilon_1)\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma} n_{1-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_k^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \quad (A.13)$$

$$(\omega - \epsilon_2)\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = 1 + V_d^*\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_d\langle\langle c_{3\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma} n_{2-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \quad (A.14)$$

$$(\omega - \epsilon_3)\langle\langle c_{3\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r = 1 + V_d^*\langle\langle c_{2\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + U_3\langle\langle c_{3\sigma} n_{3-\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_p^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r \quad (A.15)$$

$$(\omega - \epsilon_1)\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = V_d\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma} n_{1-\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_k^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \quad (A.16)$$

$$(\omega - \epsilon_1)\langle\langle c_{1\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r = V_d\langle\langle c_{2\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma} n_{1-\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_k^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r \quad (A.17)$$

$$(\omega - \epsilon_2)\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_d\langle\langle c_{3\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma} n_{2-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \quad (A.18)$$

$$(\omega - \epsilon_2)\langle\langle c_{2\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r = V_{\mathrm{d}}^*\langle\langle c_{1\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_d\langle\langle c_{3\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma} n_{2-\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r \quad (A.19)$$

$$(\omega - \epsilon_3)\langle\langle c_{3\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_3\langle\langle c_{3\sigma} n_{3-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_p^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \quad (A.20)$$

$$(\omega - \epsilon_3)\langle\langle c_{3\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_3\langle\langle c_{3\sigma} n_{3-\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_p^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r. \quad (A.21)$$

The Dyson equations for lead-dot GFs in series combination are

$$\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_{k1}^{\mathrm{L}*} g_k^r\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r$$

and

$$\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_{p3}^{\mathrm{R}*} g_k^{\mathrm{r}}\langle\langle c_{3\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r.$$

(e) *For the triple quantum dots in parallel*

$$(\omega - \epsilon_1)\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma} n_{1-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{k1}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{p1}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \quad (A.22)$$

$$(\omega - \epsilon_2)\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = 1 + V_d^*\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_d\langle\langle c_{3\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_{k2}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma} n_{2-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{p2}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \quad (A.23)$$

$$(\omega - \epsilon_3)\langle\langle c_{3\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r = 1 + V_d^*\langle\langle c_{2\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + U_3\langle\langle c_{3\sigma} n_{3-\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_{k3}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_{p3}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r \quad (A.24)$$

$$(\omega - \epsilon_1)\langle\langle c_{1\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = V_{\mathrm{d}}\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma} n_{1-\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_{k1}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_{p1}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \quad (A.25)$$

$$(\omega - \epsilon_2)\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_d\langle\langle c_{3\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma} n_{2-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{p2}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{k2}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \quad (A.26)$$

$$(\omega - \epsilon_3)\langle\langle c_{3\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + U_3\langle\langle c_{3\sigma} n_{3-\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{k3}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + V_{p3}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r \quad (A.27)$$

$$(\omega - \epsilon_1)\langle\langle c_{1\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r = V_d\langle\langle c_{2\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + U_1\langle\langle c_{1\sigma} n_{1-\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_{k1}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_{p1}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r \quad (A.28)$$

$$(\omega - \epsilon_2)\langle\langle c_{2\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{1\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_d\langle\langle c_{3\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + U_2\langle\langle c_{2\sigma} n_{2-\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_{p2}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r + V_{k2}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{3\sigma}^\dagger\rangle\rangle^r \quad (A.29)$$

$$(\omega - \epsilon_3)\langle\langle c_{3\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r = V_d^*\langle\langle c_{2\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + U_3\langle\langle c_{3\sigma} n_{3-\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_{k3}^{\mathrm{L}}\langle\langle a_{k\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r + V_{p3}^{\mathrm{R}}\langle\langle b_{p\sigma}, c_{2\sigma}^\dagger\rangle\rangle^r \quad (A.30)$$

$$\left(\omega - \epsilon_1 - n_1 U_1 - i\frac{\Gamma_{11}}{2}\right)\langle\langle c_{1\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r = 1 + V_d\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r + i\frac{\Gamma_{12}}{2}\langle\langle c_{2\sigma}, c_{1\sigma}^\dagger\rangle\rangle^r$$

$$+i\frac{\Gamma_{13}}{2}\langle\langle c_{3\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r \tag{A.31}$$

$$\left(\omega - \epsilon_2 - n_2U_2 - i\frac{\Gamma_{22}}{2}\right)\langle\langle c_{2\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r$$
$$= V_d\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r + i\frac{\Gamma_{21}}{2}\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r \tag{A.32}$$

$$\left(\omega - \epsilon_3 - n_3U_3 - i\frac{\Gamma_{33}}{2}\right)\langle\langle c_{3\sigma}, c^{\dagger}_{3\sigma}\rangle\rangle^r$$
$$= 1 + V_d\langle\langle c_{2\sigma}, c^{\dagger}_{3\sigma}\rangle\rangle^r + i\frac{\Gamma_{32}}{2}\langle\langle c_{2\sigma}, c^{\dagger}_{3\sigma}\rangle\rangle^r. \tag{A.33}$$

The EOM for lead-dot coupling GFs are given by following Dyson equations:

$$\begin{aligned}\langle\langle a_{k\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r = {} & V^{\mathrm{L}*}_{k1} g^r_k\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r \\ & + V^{\mathrm{L}*}_{k2} g^r_k\langle\langle c_{2\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r \\ & + V^{\mathrm{L}*}_{k3} g^r_k\langle\langle c_{3\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r \\ \langle\langle b_{p\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r = {} & V^{\mathrm{R}*}_{p1} g^r_k\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r \\ & + V^{\mathrm{R}*}_{p2} g^r_k\langle\langle c_{2\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r \\ & + V^{\mathrm{R}*}_{p3} g^r_k\langle\langle c_{3\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r.\end{aligned}$$

The GFs of QDs in various configurations can be obtained by using the simplest mean-field approximation, such that GFs of the type $\langle\langle c_{i\sigma}n_{i-\sigma}, c^{\dagger}_{j\sigma}\rangle\rangle^r \approx n_{i-\sigma}\langle\langle c_{i\sigma}, c^{\dagger}_{j\sigma}\rangle\rangle^r$ and hierarchy of GFs in the above EOMs get closed to the final forms of the respective GFs.

Appendix B

B.1 Current

To discuss the method for deriving relation for the total current, we take the case of parallel double quantum dot system. The expression for the current due to the left lead is

$$I_{\mathrm{L}} = \frac{e}{h}\sum_{ki\sigma}\int\left(V^{\mathrm{L}}_{ki}\langle\langle a_{k\sigma}, c^{\dagger}_{i\sigma}\rangle\rangle^<_{\omega} - V^{\mathrm{L}*}_{ki}\langle\langle c_{k\sigma}, a^{\dagger}_{i\sigma}\rangle\rangle^<_{\omega}\right)\mathrm{d}\omega \quad (i = 1, 2). \tag{B.1}$$

Similarly, the current due to the right lead can be written as

$$I_{\mathrm{R}} = \frac{e}{h}\sum_{pi\sigma}\int\left(V^{\mathrm{R}}_{pi}\langle\langle b_{p\sigma}, c^{\dagger}_{i\sigma}\rangle\rangle^<_{\omega} - V^{\mathrm{R}*}_{pi}\langle\langle c_{k\sigma}, b^{\dagger}_{i\sigma}\rangle\rangle^<_{\omega}\right)\mathrm{d}\omega \quad (i = 1, 2). \tag{B.2}$$

The Dyson equations for the GFs appearing in these expressions are

$$\begin{aligned}&\langle\langle a_{k\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^<_{\omega} \\ &= V^{\mathrm{L}*}_{k1} g^r_k\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^<_{\omega} + V^{\mathrm{L}*}_{k1} g^<_k\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^a_{\omega} \\ &\quad + V^{\mathrm{L}*}_{k2} g^r_k\langle\langle c_{2\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^<_{\omega} + V^{\mathrm{L}*}_{k2} g^<_k\langle\langle c_{2\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^a_{\omega} \\ &\langle\langle a_{k\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^<_{\omega} \\ &= V^{\mathrm{L}*}_{k2} g^r_k\langle\langle c_{2\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^<_{\omega} + V^{\mathrm{L}*}_{k2} g^<_k\langle\langle c_{2\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^a_{\omega} \\ &\quad + V^{\mathrm{L}*}_{k1} g^r_k\langle\langle c_{1\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^<_{\omega} + V^{\mathrm{L}*}_{k1} g^<_k\langle\langle c_{1\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^a_{\omega} \\ &\langle\langle c_{1\sigma}, a^{\dagger}_{k\sigma}\rangle\rangle^<_{\omega} \\ &= V^{\mathrm{L}}_{k1} g^<_k\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r_{\omega} + V^{\mathrm{L}}_{k1} g^a_k\langle\langle c_{1\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^<_{\omega} \\ &\quad + V^{\mathrm{L}}_{k2} g^<_k\langle\langle c_{2\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^r_{\omega} + V^{\mathrm{L}}_{k2} g^a_k\langle\langle c_{2\sigma}, c^{\dagger}_{1\sigma}\rangle\rangle^<_{\omega} \\ &\langle\langle c_{2\sigma}, a^{\dagger}_{k\sigma}\rangle\rangle^<_{\omega} \\ &= V^{\mathrm{L}}_{k2} g^<_k\langle\langle c_{2\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^r_{\omega} + V^{\mathrm{L}}_{k2} g^a_k\langle\langle c_{2\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^<_{\omega} \\ &\quad + V^{\mathrm{L}}_{k1} g^<_k\langle\langle c_{1\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^r_{\omega} + V^{\mathrm{L}}_{k1} g^a_k\langle\langle c_{1\sigma}, c^{\dagger}_{2\sigma}\rangle\rangle^<_{\omega}\end{aligned}$$

$$I_{\mathrm{L}} = \frac{e}{h}\sum_{\sigma}\int i\mathrm{Tr}\left[\Gamma^{\mathrm{L}}G^< + f_{\mathrm{L}}\Gamma^{\mathrm{L}}(G^r - G^a)\right]\mathrm{d}\omega \tag{B.3}$$

$$I_{\mathrm{R}} = \frac{e}{h}\sum_{\sigma}\int i\mathrm{Tr}\left[\Gamma^{\mathrm{R}}G^< + f_{\mathrm{R}}\Gamma^{\mathrm{R}}(G^r - G^a)\right]\mathrm{d}\omega. \tag{B.4}$$

Further,

$$I = I_{\mathrm{L}} = -I_{\mathrm{R}} = \frac{I_{\mathrm{L}} - \mathrm{I}_{\mathrm{R}}}{2}.$$

The total current in the couple quantum dots is

$$I = \frac{ie}{2h}\sum_{\sigma}\int \mathrm{Tr}\left[\left(\Gamma^{\mathrm{L}} - \Gamma^{\mathrm{R}}\right)G^< + (\Gamma^{\mathrm{L}}f_{\mathrm{L}} - \Gamma^{\mathrm{R}}f_{\mathrm{R}})(G^r - G^a)\right]\mathrm{d}\omega. \tag{B.5}$$

For the proportionate case

$$\Gamma^{\mathrm{L}} = \lambda\Gamma^{\mathrm{R}} \quad \text{and} \quad x = \frac{1}{1+\lambda}.$$

The current can be written as $I = xI_{\mathrm{L}} - (1 - x)I_{\mathrm{R}}$. Hence,

$$I = \frac{ie}{h}\sum_{\sigma}\int (f_{\mathrm{L}} - f_{\mathrm{R}}) \times \mathrm{Tr}\left[\left(\frac{\Gamma^{\mathrm{L}}\Gamma^{\mathrm{R}}}{\Gamma^{\mathrm{L}} - \Gamma^{\mathrm{R}}}\right)(G^r - G^a)\right]\mathrm{d}\omega. \tag{B.6}$$

Also, by making use of the identity $G^r - G^a = -i\left(\Gamma^{\mathrm{L}} - \Gamma^{\mathrm{R}}\right)G^r G^a$, the expression for the total current leads to the following well-known Landauer–Buttiker current formula:

$$I = \frac{e}{h}\sum_{\sigma}\int (f_{\mathrm{L}} - f_{\mathrm{R}})\mathrm{Tr}\left(G^a\Gamma^{\mathrm{R}}G^r\Gamma^{\mathrm{L}}\right)\mathrm{d}\omega$$
$$= \frac{e}{h}\sum_{\sigma}\int (f_{\mathrm{L}} - f_{\mathrm{R}})T(\omega)\mathrm{d}\omega. \quad (\mathrm{B.7})$$

References

[1] P Harrison and Alex Valavanis, *Quantum wells, wires and dots: Theoretical and computational physics of semiconductor nanostructures* (John Wiley & Sons, 2016)
[2] L Kouwenhoven and C M Marcus, *Phys. World* **11**, 35 (1998)
[3] D Bimberg, M Grundmann and N N Ledentsov, *Quantum dot heterostructures* (Wiley, New York, 1999)
[4] K Goser, P Glosekotter and V Dienstuhl, *Nano electronics & nano systems* (Springer, Berlin, 2004)
[5] George W Hanson, *Fundamentals of nanoelectronics* (Pearson, London, 2011)
[6] P S Peercy, *Nature* **406(6799)**, 1023 (2000)
[7] A Steane, *Rep. Prog. Phys.* **61**, 117 (1998)
[8] D Deutsch, *Proc. R. Soc. Lond. A* **400**, 97 (1985)
[9] D Loss and D P DiVincenzo, *Phys. Rev. A* **57**, 120 (1998)
[10] D D Awschalom, D Loss and N Samarth, *Semiconductor spintronics and quantum computation* (Springer, Berlin, Heidelberg, 2002)
[11] J M Elzerman *et al*, *Nature* **430**, 431 (2004)
[12] R Hanson, L P Kouwenhoven, J R Petta, S Tarucha, L M K Vandersypen, *Rev. Mod. Phys.* **79**, 1217 (2007)
[13] Peter Lodahl, Sahand Mahmoodian and Søren Stobbe, *Rev. Mod. Phys.* **87**, 347 (2015)
[14] S Stobbe, J Johansen, P T Kristensen, J M Hvam and P Lodahl, *Phys. Rev. B* **80**, 155307 (2009)
[15] Q Wang, S Stobbe and P Lodahl, *Phys. Rev. Lett.* **107**, 167404 (2011)
[16] J L O'Brien, A Furusawa and J Vuckovic, *Nature Photon.* **3**, 687 (2009)
[17] R Khordad and B Mirhosseini, *Pramana – J. Phys.* **85**, 723 (2015)
[18] Ankhi Maiti and Sagarika Bhattacharyya, *Int. J. Chem. Chem. Eng.* **3**, 37 (2013)
[19] X Michalet *et al*, *Science* **307**, 5709 (2005)
[20] H M Azzazy, M M Mansour and S C Kazmierczak, *Clinical Biochem.* **40**, 917 (2007)
[21] W G van der Wiel, S De Franceschi, J M Elzerman, T Fujisawa, S Tarucha and L P Kouwenhoven, *Rev. Mod. Phys.* **75**, 101 (2003)
[22] W Izumida, O Sakai and S Suzuki, *J. Phys. Soc. Jpn* **70**, 1045 (2001)
[23] G Chen, G Klimeck and S Dutta, *Phys. Rev. B* **50**, 8035 (1994)
[24] C A Stafford and S Das Sarma, *Phys. Rev. Lett.* **72**, 3590 (1994)
[25] Shyam Chand, G Rajput, K C Sharma and P K Ahluwalia, *Pramana – J. Phys.* **72**, 887 (2009)
[26] W Izumida and O Sakai, *Phys. Rev. B* **62**, 10260 (2000)
[27] T Aonu and M Eto, *Phys. Rev. B* **63**, 125327 (2001)
[28] R Aguado and D C Langreth, *Phys. Rev. Lett.* **85**, 1946 (2000)
[29] D Goldhaber-Gordon *et al*, *Phys. Rev. Lett.* **81**, 5225 (1998)
[30] H Grabert and M H Devoret, *Single charge tunnelling* (Plenum Press, New York, 1992)
[31] M A Kastner, *Rev. Mod. Phys.* **64**, 849 (1992)
[32] K Kobayashi, H Aikawa, S Katusmoto and Y Iye, *Phys. Rev. Lett.* **88**, 1 (2002)
[33] P A Orellana, M L Ladron de Guevera and F Claro, *Phys. Rev. B* **70**, 233315 (2004)
[34] M L Ladron de Guevara, F Carlo and P A Orellana, *Phys. Rev. B* **67**, 195335 (2003)
[35] T Vorrath and T Brandes, *Phys. Rev. B* **68**, 035309 (2003)
[36] E Vernek, P A Orellana and S E Ulloa, *Phys. Rev. B* **82**, 165304 (2010)
[37] J D Pillet, C H L Quay, P Morfin, C Bena, A Levy Yeyati and P Joyez, *Nat. Phys.* **6**, 965 (2010)
[38] S De Franceschi, L Kouwenhoven, C Schonenberger and W Wernsdorfer, *Nat. Nanotechnol.* **5**, 703 (2010)
[39] G Rajput, R Kumar and Ajay, *Superlatt. Microstruct.* **73**, 193 (2014)
[40] D A Eremin, E N Grishanov, I Y Popov and A A Boitsev, *Pramana – J. Phys.* **92**: 95 (2019)
[41] G Klimeck, G Chen and S Datta, *Phys. Rev. B* **50**, 2316 (1994)
[42] K Wrześniewski and I Weymann, *Phys. Rev. B* **92**, 045407 (2015)
[43] J Cai and G D Mahan, *Phys. Rev. B* **78**, 035115 (2008)
[44] B H Teng, H K Sy, Z C Wang,Y Q Sun and H C Yang, *Phys. Rev. B* **75**, 012105 (2007)
[45] W Z Shangguan, T C A Yeung, Y B Yu and C H Kam, *Phys. Rev. B* **63**, 235323 (2001)
[46] Y Han, W-J Gong, H-M Wang and A Du, *J. Appl. Phys.* **112**, 123701 (2012)
[47] F R Braakman, P Barthelemy, C Reich, W Wegscheider and L M K Vandersypen, *Nature Nanotechnol.* **67**, 2 (2013)
[48] W Wang, J Liao, J Sun and N Gu, *Superlatt. Microstruct.* **44**, 721 (2008)
[49] V P Kunets, M Rebello Sousa Dias, T Rembert, M E Ware, Yu I Mazur, V Lopez-Richard, H A Mantooth, G E Marques and G J Salamo, *J. Appl. Phys.* **113**, 183709 (2013)
[50] K G Wilson, *Rev. Mod. Phys.* **47**, 773 (1975)
[51] L V Keldysh, *Zh. Eksp. Teor. Fiz.* **47**, 1515 (1964); [*Sov. Phys. JETP* **20**, 1018 (1965)]
[52] L P Kadanoff and G Baym, *Quantum statistical mechanics* (Benjamin, New York, 1962)

[53] C Caroli, R Combescot, P Nozieres and D Saint-James, *J. Phys. C* **4**, 916 (1971)
[54] G D Mahan, *Many particle physics* (Plenum Press, New York, 2000)
[55] D N Zubarev, *Sov. Phys. Usp.* **3**, 320 (1960); H Haug and A P Jauho, *Quantum kinetics in transport and optics of semiconductors* (Springer, New York, 2008)
[56] C Lacroix, *J. Phys. F* **11**, 2389 (1981)
[57] A P Jouho, N S Wingreen and Y Meir, *Phys. Rev. B* **50**, 5528 (1994)
[58] Y Meir and N S Wingreen, *Phys. Rev. Lett.* **68**, 2512 (1992)
[59] N S Wingreen and Y Meir, *Phys. Rev. B* **49**, 11040 (1994)
[60] C Niu, L Liu and T Lin, *Phys. Rev. B* **51**, 5130 (1995)
[61] S Lamba and S K Joshi, *Phys. Rev. B* **62**, 1580 (2000)
[62] J Q You and H Z Zheng, *Phys. Rev. B* **60**, 13314 (1999)
[63] A S Adourian, C Livermore and R M Westervelt, *Appl. Phys. Lett.* **75**, 424 (1999)
[64] L G Mourokh, N J M Horing and A Y Smirnov, *Phys. Rev. B* **66**, 085332 (2002)
[65] S Chand, R K Moudgil and P K Ahluwalia, *Physica B* **405**, 239 (2010)
[66] S Devi, B B Brogi, P K Ahluwalia and S Chand, *Physica B* **539**, 111 (2018)
[67] G Rajput, S Chand, P K Ahluwalia and K C Sharma, *Physica B* **405**, 4323 (2010)
[68] B B Brogi, S Chand and P K Ahluwalia, *Physica B* **461**, 110 (2015)
[69] A J Nozik, *Physica E* **14**, 115 (2002)
[70] P V Kamat, *J. Phys. Chem. C* **111**, 2834 (2007)
[71] H Li, T Lü and P Sun, *Phys. Lett. A* **343**, 403 (2005)
[72] Z-T Jiang and C-C Zhong, *Chin. Phys. B* **25**, 67302 (2016)
[73] C Volk *et al*, *npj Quantum Information* (2019)
[74] A R Mills, D M Zajac, M J Gullans, F J Schupp, T M Hazard and J R Petta, *Nat. Commun.* **10** (2019)
[75] Takumi Ito *et al*, *Sci. Rep.* **6**, 39113 (2016)
[76] R Vohra and R S Sawhney, *Pramana – J. Phys.* **90**: 58 (2018)
[77] P W Anderson, *Phys. Rev.* **124**, 41 (1961)
[78] Tae-Suk Kim and S Hershfield, *Phys. Rev. B* **63**, 245326 (2001)
[79] X Q Wang, G Y Yi and W J Gung, *Superlatt. Microstruct.* **109**, 366 (2017)
[80] D Boese, W Hofstetter and H Schoeller, *Phys. Rev. B* **66**, 125315 (2002)