



Dust-acoustic rogue waves in non-thermal plasmas

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Abstract. The nonlinear propagation of dust-acoustic waves (DAWs) and associated dust-acoustic rogue waves (DARWs), which are governed by the nonlinear Schrödinger equation, is theoretically investigated in a four-component plasma medium containing inertial warm negatively charged dust grains and inertialess non-thermal distributed electrons as well as isothermal positrons and ions. The modulationally stable and unstable parametric regimes of DAWs are numerically studied for the plasma parameters. Furthermore, the effects of temperature ratios of ion-to-electron and ion-to-positron, and the number density of ion and dust grains on the DARWs are investigated. It is observed that physical parameters play very crucial roles in the formation of DARWs. These results may be useful in understanding the electrostatic excitations in dusty plasmas in space and laboratory situations.

Keywords. Nonlinear Schrödinger equation; modulational instability; rogue waves.

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1. Introduction

The research regarding the propagation of nonlinear electrostatic perturbation in a four-component electron-positron-ion-dust (FCEPID) plasma medium (FCEPIDPM) has received great attention among the plasma physicists due to the existence of FCEPID not only in space plasmas, viz., the hot spots on dust rings in the galactic centre [1–4], auroral zone [2], around pulsars [3,5], interstellar medium [6], Milky Way [6], accretion disks near neutron stars [6], Jupiter's magnetosphere [7] but also in laboratory experiments. Many researchers have studied dust-acoustic (DA) waves (DAWs) [3–5], dust-ion-acoustic waves (DIAWs) [1,2] and associated nonlinear structures in four-component dusty plasma. The presence of positron drastically changes the mechanism of the formation and propagation of nonlinear electrostatic structures, viz., shock, solitons [2], supersolitons [1,7], double layers (DLs) [2], etc.

The existence of the non-thermal particles (viz., electrons [1,2,6,7], positrons [1,2], ions, etc.), which rigorously changes the dynamics of the plasma medium and the mechanism of the formation of various

electrostatic pulses in space plasmas has been confirmed by the Viking [8] and Freja satellites [9]. Cairns *et al* [10] investigated the electrostatic solitary structures in a non-thermal plasma, and found that both negative and positive density perturbations can exist in the presence of non-thermal electrons. Paul *et al* [1] showed that the existence of non-thermal electrons and positrons in a FCEPID model supports the solitary waves of both polarities. Banerjee and Maitra [2] studied DIAWs, i.e., solitons and DLs, in a FCEPIDPM having non-thermal electrons and positrons and observed that the positive potential cannot exist after a particular value of non-thermal parameter (α).

The rogue waves (RWs), which appear due to the modulational instability (MI) of the carrier waves, have been observed in different branches of science, viz., oceanography [11], biology, optics [12], finance [13] and plasma physics [14–17], and are also governed by the nonlinear Schrödinger equation (NLSE) [14–17]. Chowdhury *et al* [15] examined the stability conditions for the positron-acoustic waves (PAWs) in a multicomponent plasma medium, and found that the critical wave number (k_c), which indicates the stable and unstable parametric regimes of PAWs, reduces with the increase

in the value of α . Kourakis and Shukla [16] studied the MI of the DIAWs in a three-component plasma medium and showed that the existence of the negative dust grains reduces the value of k_c . Rahman *et al* [17] reported the stable and unstable domains in a dusty plasma medium having non-thermal plasma species, and found that the amplitude of the DARWs increases with non-thermality of the plasma species.

Recently, Saberian *et al* [4] studied the DA solitons and DLs in a FCEPIDPM. Esfandyari-Kalejahi *et al* [5] investigated DA solitary waves (DASWs) in a multicomponent dusty plasma in the presence of non-thermal electrons, and found that the amplitude of the DASWs increases with charge state of the negative dust grains. Jehan *et al* [3] considered an unmagnetised FCEPIDPM having negatively charged massive inertial dust grains, inertialess electrons, positrons and ions for examining DASWs, and observed that the existence of positrons and their temperature can change the sign of the nonlinear coefficient of the governing equation. The present paper is an extension of the work of Jehan *et al* [3] by studying the MI of the DAWs in an unmagnetised FCEPIDPM having non-thermal electrons featuring Cairns' distribution. This paper also examines the nonlinear properties of DARWs.

The manuscript is organised as follows: The basic governing equations of our plasma model are presented in §2. The MI and RWs are presented in §4. Results and discussion are provided in §5. The conclusion is provided in §6.

2. Governing equations

We consider the propagation of DAWs in an unmagnetised collisionless FCEPIDPM consisting of inertial warm negatively charged massive dust grains (mass = m_d ; charge $q_d = -Z_d e$) and inertialess non-thermal Cairns' distributed electrons (mass = m_e ; charge $q_e = -e$) as well as isothermal positrons (mass m_p ; charge $q_p = +e$) and ions (mass m_i ; charge $q_i = +Z_i e$), where Z_d (Z_i) is the number of electrons (protons) residing on negatively (positively) charged massive dust grains (ions). Overall, the charge neutrality condition for our plasma model can be written as

$$n_{e0} + Z_d n_{d0} = n_{p0} + Z_i n_{i0}.$$

Now, the basic set of normalised equations can be written as

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \sigma_1 n_d \frac{\partial n_d}{\partial x} = \frac{\partial \phi}{\partial x}, \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu_e n_e - (1 + \mu_e - \mu_i) n_p - \mu_i n_i + n_d, \quad (3)$$

where n_d is the number density of warm dust grains normalised by its equilibrium value n_{d0} ; u_d is the dust fluid speed normalised by the DA wave speed $C_d = (Z_d k_B T_i / m_d)^{1/2}$ (with T_i being the ion temperature, m_d being the dust grain mass and k_B being the Boltzmann constant); ϕ is the electrostatic wave potential normalised by $k_B T_i / e$ (with e being the magnitude of single electron charge); the time and space variables are normalised by $\omega_{pd}^{-1} = (m_d / 4\pi Z_d^2 e^2 n_{d0})^{1/2}$ and $\lambda_{Dd} = (k_B T_i / 4\pi Z_d n_{d0} e^2)^{1/2}$, respectively; $P_d = P_{d0} (N_d / n_{d0})^\gamma$ (with P_{d0} being the equilibrium adiabatic pressure of the dust and $\gamma = (N + 2) / N$, where N is the degree of freedom and for one-dimensional case, $N = 1$ then $\gamma = 3$); $P_{d0} = n_{d0} k_B T_d$ (with T_d being the temperature of the warm dust grain); and other plasma parameters are: $\sigma_1 = 3T_d / Z_d T_i$, $\mu_e = n_{e0} / Z_d n_{d0}$ and $\mu_i = Z_i n_{i0} / Z_d n_{d0}$. Now, the expression for the number density of non-thermal electrons following the Cairns' distribution [1,10] can be written as

$$n_e = [1 - \beta \sigma_2 \phi + \beta \sigma_2^2 \phi^2] \exp(\sigma_2 \phi), \quad (4)$$

where $\sigma_2 = T_i / T_e$ (T_e being the temperature of the isothermal electron) and $\beta = 4\alpha / (1 + 3\alpha)$ with α being the parameter determining the fast particles present in our plasma model. Now, the expression for the number density of isothermal positrons following the Maxwellian distribution can be written as [3]

$$n_p = \exp(-\sigma_3 \phi), \quad (5)$$

where $\sigma_3 = T_i / T_p$ (T_p being the temperature of the isothermal positrons). Now, the expression for the number density of isothermal ions following the Maxwellian distribution can be written as [3]

$$n_i = \exp(-\phi). \quad (6)$$

Now, by substituting eqs (4)–(6) in eq. (3), and expanding up to third order of ϕ , we get

$$\frac{\partial^2 \phi}{\partial x^2} + 1 = n_d + S_1 \phi + S_2 \phi^2 + S_3 \phi^3 + \dots, \quad (7)$$

where

$$S_1 = (\sigma_2 - \beta \sigma_2 + \sigma_3) \mu_e + (1 - \sigma_3) \mu_i + \sigma_3,$$

$$S_2 = [(\sigma_2^2 - \sigma_3^2) \mu_e + (\sigma_3^2 - 1) \mu_i - \sigma_3^2] / 2,$$

$$S_3 = [(3\beta + 1) \mu_e \sigma_2^3 + \mu_e \sigma_3^3 + (1 - \sigma_3^3) \mu_i + \sigma_3^3] / 6.$$

The terms containing S_1 , S_2 and S_3 in the right-hand side of eq. (7) are the contributions of inertialess electrons,

positrons and ions. We note that eqs (1), (2) and (7) now represent the basis set of normalised equations to describe the nonlinear dynamics of the DAWs, and associated DARWs in an unmagnetised FCEPIDPM under consideration.

3. Derivation of the NLSE

To study the MI of the DAWs, we want to derive the NLSE by employing the reductive perturbation method, and for that case, first we can write the stretched coordinates in the form [14–17]

$$\xi = \epsilon(x - v_g t), \tag{8}$$

$$\tau = \epsilon^2 t, \tag{9}$$

where v_g is the group speed and ϵ is a small parameter. Then we can write the dependent variables as

$$n_d = 1 + \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} n_{dl}^{(m)}(\xi, \tau) \exp[il(kx - \omega t)], \tag{10}$$

$$u_d = \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} u_{dl}^{(m)}(\xi, \tau) \exp[il(kx - \omega t)], \tag{11}$$

$$\phi = \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} \phi_l^{(m)}(\xi, \tau) \exp[il(kx - \omega t)], \tag{12}$$

where k and ω are real variables representing the carrier wave number and frequency, respectively. The derivative operators can be written as [14–17]

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - \epsilon v_g \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau}, \tag{13}$$

$$\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial \xi}. \tag{14}$$

Now, by substituting eqs (8)–(14) into eqs (1), (2) and (7), and collecting the terms containing ϵ , the first-order ($m = 1$ with $l = 1$) reduced equations can be written as

$$n_{d1}^{(1)} = \frac{k^2}{S} \phi_1^{(1)}, \tag{15}$$

$$u_{d1}^{(1)} = \frac{k\omega}{S} \phi_1^{(1)}, \tag{16}$$

$$n_{d1}^{(1)} = -k^2 \phi_1^{(1)} - S_1 \phi_1^{(1)}, \tag{17}$$

where $S = \sigma_1 k^2 - \omega^2$. Hence these relations provide the dispersion relation of DAWs

$$\omega^2 = \sigma_1 k^2 + \frac{k^2}{S_1 + k^2}. \tag{18}$$

The second-order ($m = 2$ with $l = 1$) equations are given by

$$n_{d1}^{(2)} = \frac{k^2}{S} \phi_1^{(2)} - \frac{2ik\omega(v_g k - \omega)}{S^2} \frac{\partial \phi_1^{(1)}}{\partial \xi}, \tag{19}$$

$$u_{d1}^{(2)} = \frac{k\omega}{S} \phi_1^{(2)} - \frac{i(v_g k - \omega)(\omega^2 + \sigma_1 k^2)}{S^2} \frac{\partial \phi_1^{(1)}}{\partial \xi}, \tag{20}$$

with the compatibility condition

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\omega^2 - S^2}{k\omega}. \tag{21}$$

The coefficients of ϵ for $m = 2$ and $l = 2$ provide the second-order harmonic amplitudes which are found to be proportional to $|\phi_1^{(1)}|^2$

$$n_{d2}^{(2)} = S_4 |\phi_1^{(1)}|^2, \tag{22}$$

$$u_{d2}^{(2)} = S_5 |\phi_1^{(1)}|^2, \tag{23}$$

$$\phi_2^{(2)} = S_6 |\phi_1^{(1)}|^2, \tag{24}$$

where

$$S_4 = \frac{2S_6 k^2 S^2 - \sigma_1 k^6 - 3\omega^2 k^4}{2S^3},$$

$$S_5 = \frac{S_4 \omega S^2 - \omega k^4}{k S^2},$$

$$S_6 = \frac{\sigma_1 k^6 + 3\omega^2 k^4 - 2S_2 S^3}{6k^2 S^3}.$$

Now, we consider the expression for $m = 3$ with $l = 0$ and $m = 2$ with $l = 0$, which leads the zeroth harmonic modes. Thus, we obtain

$$n_{d0}^{(2)} = S_7 |\phi_1^{(1)}|^2, \tag{25}$$

$$u_{d0}^{(2)} = S_8 |\phi_1^{(1)}|^2, \tag{26}$$

$$\phi_0^{(2)} = S_9 |\phi_1^{(1)}|^2, \tag{27}$$

where

$$S_7 = \frac{S_9 S^2 - \sigma_1 k^4 - \omega^2 k^2 - 2\omega v_g k^3}{S^2(\sigma_1 - v_g^2)},$$

$$S_8 = \frac{S_7 v_g S^2 - 2\omega k^3}{S^2},$$

$$S_9 = \frac{2\omega v_g k^3 + \sigma_1 k^4 + \omega^2 k^2 - 2S_2 S^2(\sigma_1 - v_g^2)}{S^2(1 - S_1 v_g^2 + \sigma_1 S_1)}.$$

Finally, the third harmonic modes ($m = 3$ and $l = 1$), with the help of (15)–(27), give a set of equations, which can be reduced to the following NLSE:

$$i \frac{\partial \Phi}{\partial \tau} + P \frac{\partial^2 \Phi}{\partial \xi^2} + Q |\Phi|^2 \Phi = 0, \tag{28}$$

where $\Phi = \phi_1^{(1)}$ for simplicity. In eq. (28), the dispersion coefficient P is given as

$$P = \frac{F_1(v_g k - \omega) - S^3}{2\omega k^2 S},$$

where $F_1 = \omega^3 + 3\omega\sigma_1 k^2 - 3v_g k \omega^2 - v_g \sigma_1 k^3$. Also in eq. (28), the nonlinear coefficient Q is given as

$$Q = \frac{2S_2 S^2 (S_9 + S_6) + 3S_3 S^2 - F_2}{2\omega k^2},$$

where

$$F_2 = (k^4 \sigma_1 + \omega^2 k^2)(S_4 + S_7) + 2\omega k^3 (S_5 + S_8).$$

The space and time evolution of the DAWs in an unmagnetised FCEPIDPM are directly governed by the coefficients P and Q , and indirectly governed by different plasma parameters such as α , μ_i , μ_e , σ_1 , σ_2 , σ_3 and k . Thus, these plasma parameters significantly affect the stability conditions of the DAWs in an unmagnetised FCEPIDPM.

4. Modulational instability and rogue waves

The stable and unstable parametric regimes of DAWs are organised by the sign of P and Q of eq. (28) [14–20]. When P and Q have the same sign (i.e., $P/Q > 0$), the evolution of the DAWs amplitude is modulationally unstable in the presence of external perturbations. On the other hand, when P and Q have opposite signs (i.e., $P/Q < 0$), the DAWs are modulationally stable in the presence of external perturbations. The plot of P/Q against k yields stable and unstable parametric regimes of the DAWs. The point, at which the transition of P/Q curve intersects with the k -axis, is known as the threshold or critical wave number k ($= k_c$) [14–20].

The governing equation for highly energetic DARWs in the modulationally unstable parametric regime ($P/Q > 0$) can be written as [21,22]

$$\begin{aligned} \Phi(\xi, \tau) &= \sqrt{\frac{2P}{Q}} \left[\frac{4(1 + 4iP\tau)}{1 + 16P^2\tau^2 + 4\xi^2} - 1 \right] \exp(2iP\tau). \end{aligned} \quad (29)$$

Equation (29) describes that a large amount of wave energy, which causes due to the nonlinear characteristics of the plasma medium, is localised into a comparatively small area in space.

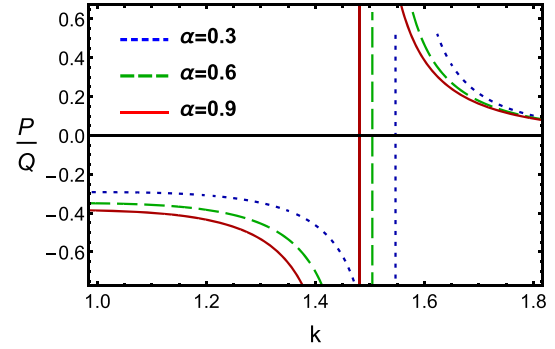


Figure 1. Plot of P/Q vs. k for different values of α when $\mu_e = 0.5$, $\mu_i = 0.7$, $\sigma_1 = 0.003$, $\sigma_2 = 0.4$ and $\sigma_3 = 0.5$.

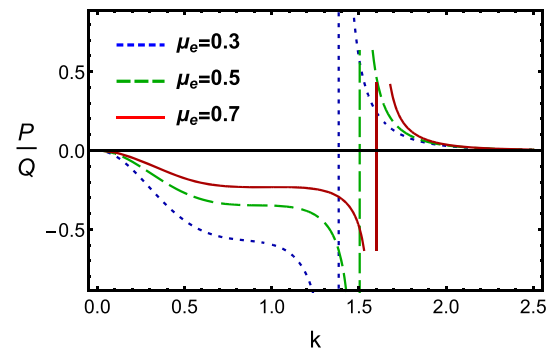


Figure 2. Plot of P/Q vs. k for different values of μ_e when $\alpha = 0.6$, $\mu_i = 0.7$, $\sigma_1 = 0.003$, $\sigma_2 = 0.4$ and $\sigma_3 = 0.5$.

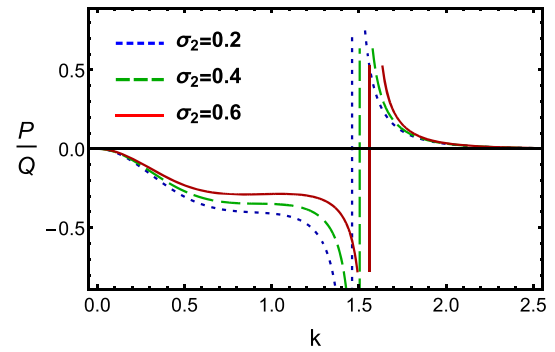


Figure 3. Plot of P/Q vs. k for different values of σ_2 when $\alpha = 0.6$, $\mu_e = 0.5$, $\mu_i = 0.7$, $\sigma_1 = 0.003$ and $\sigma_3 = 0.5$.

5. Results and discussion

We have numerically analysed the stable and unstable parametric regimes of DAWs in figures 1–3. The effects of non-thermality of the electrons in organising the stable and unstable parametric regimes of the DAWs can be seen in figure 1 which indicates that k_c as well as modulationally stable parametric regime of DAWs decrease with the increase in the value of α , and this result is in good agreement with the result of ref. [15]. So, the

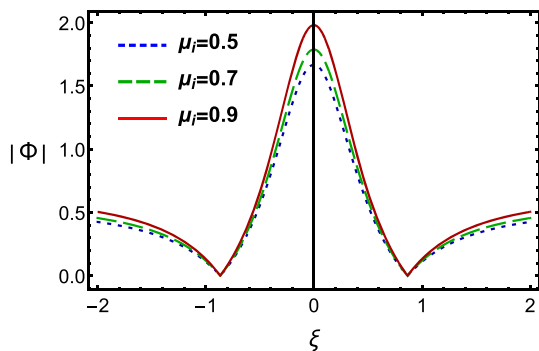


Figure 4. Plot of $|\Phi|$ vs. ξ for different values of μ_i when $k = 1.7$, $\tau = 0$, $\alpha = 0.6$, $\mu_e = 0.5$, $\sigma_1 = 0.003$, $\sigma_2 = 0.4$ and $\sigma_3 = 0.5$.

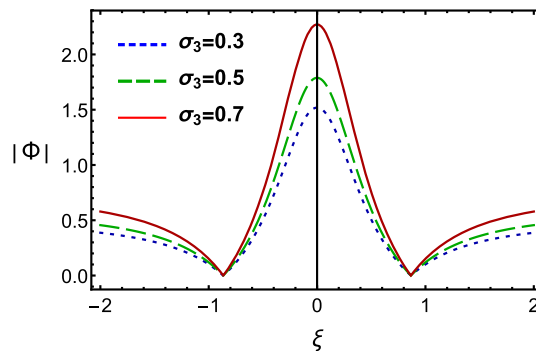


Figure 5. Plot of $|\Phi|$ vs. ξ for different values of σ_3 when $k = 1.7$, $\tau = 0$, $\alpha = 0.6$, $\mu_e = 0.5$, $\mu_i = 0.7$, $\sigma_1 = 0.003$ and $\sigma_2 = 0.4$.

DAWs become modulationally unstable for small values of k for excess non-thermality of the plasma species.

Figure 2 indicates that the presence of non-thermal electrons and negatively charged warm dust grains can significantly modify the stability conditions of DAWs in an unmagnetised FCEPIDPM. It is clear from this figure that (a) when $\mu_e = 0.3, 0.5$ and 0.7 then the corresponding $k_c \equiv 1.4$ (dotted blue curve), $k_c \equiv 1.5$ (dashed green curve) and $k_c \equiv 1.6$ (solid red curve); (b) the critical value increases with the increase in the value of μ_e ; (c) the modulationally stable (unstable) parametric regime of DAWs increases with an increase in the value of electron (dust) number density for a constant value of the charge state of negative dust grains (via μ_e).

The effects of ion to electron temperature (via σ_2) on the stable and unstable parametric regimes can be observed in figure 3, and it is clear from this figure that (a) k_c increases with σ_2 ; (b) the modulationally stable parametric regime increases with ion temperature while decreases with electron temperature. So, the temperature of the electron and the ion plays an opposite role in recognising the modulationally stable and unstable parametric regimes of DAWs in an unmagnetised FCEPIDPM.

We have numerically analysed eq. (29) in figures 4 and 5 to understand the nonlinear property of the FCEPIDPM as well as the mechanism of the formation of DARWs associated with DAWs in the modulationally unstable parametric regime. Figure 4 indicates that (a) the amplitude and width of the DARWs increase with an increase in the value of μ_i ; (b) the nonlinearity, which organises the shape of the DARWs, of the plasma medium increases with an increase in the value of ion number density whereas the nonlinearity of the plasma medium decreases with dust number density when their charge states remain constant. Figure 5 indicates the temperature effects of ion and electron (via σ_3) on the generation of DARWs in FCEPIDPM.

The ion (positron) temperature enhances (suppresses) both amplitude and width of the DARWs associated with DAWs in the modulationally unstable parametric regime.

6. Conclusion

In this study, we have performed a nonlinear analysis of DAWs in an unmagnetised FCEPIDPM consisting of inertial negatively charged warm dust grains and inertialess non-thermal Cairns' distributed electrons as well as isothermal positrons and ions. The evolution of DAWs is governed by the standard NLSE, and the coefficients P and Q of NLSE can recognise the modulationally stable and unstable parametric regimes of DAWs in the presence of external perturbation. It is observed that the DAWs become unstable for small values of k for excess non-thermality (via α) of the plasma species, and the stable (unstable) parametric regimes of the DAWs increases with the increase in the value of electron (dust) number density at a constant value of the dust charge state. The ion (positron) temperature enhances (suppresses) both amplitude and width of the DARWs. Finally, these results may be applicable in understanding the conditions of the MI of DAWs and associated DARWs in astrophysical environments (viz., the hot spots on dust rings in the galactic centre [1–4], auroral zone [2], around pulsars [3,5], interstellar medium [6], Milky Way [6], accretion disks near neutron stars [6], Jupiter's magnetosphere [7], etc.) and laboratory devices.

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