Pramana – J. Phys. (2020) 94:51
https://doi.org/10.1007/s12043-020-1914-0

Zakharov–Kuznetsov–Burgers equation in a magnetised non-extensive electron–positron–ion plasma

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MS received 3 June 2019; revised 6 November 2019; accepted 9 December 2019

Abstract. In this paper, we have studied the three-dimensional (3D) electron-acoustic waves (EAWs) in a three-component complex plasma containing q-non-extensive distributed hot electrons and positrons. The propagation characteristics of the 3D electron-acoustic (EA) shock waves under the influence of magnetic field have been studied. Our present plasma model supports the negative potential shocks. Combined action of dissipation (η), non-extensivity (q), concentration of positrons (β), temperature ratio of cold electrons to positrons (σ) and magnetic field (ω_c) on the EA shock waves has been studied in detail and the findings obtained here will be beneficial in future astrophysical investigations.

Keywords. Electron-acoustic shock waves; Zakharov–Kuznetsov–Burgers equation; dissipative medium.

PACS Nos **52.30.Ex; 52.27.Ny; 52.35.Sb**

1. Introduction

The experimental data obtained from spacecraft missions communicate the existence of non-Maxwellian distribution functions in space plasma environments. The shape of such a function appears to have extended tail, posturing particles with energy higher than the average thermal ones. The important non-Maxwellian functions for plasma particles which have been investigated until now include Kappa distribution, cairns, q-non-extensive distributions etc. In the previous decades, it is proved that systems with long-range interactions and long-time memory cannot be fully explained with the conventional Boltzmann–Gibbs (BG) equation. Therefore a need was recognised to develop new statistics. Consequently, non-extensive statistics based on the derivation of Boltzmann–Gibbs–Shannon (BGS) statistics materialised for studying systems with long-range interactions and long-term memory [1,2]. During the development phase, it was established that the q-equilibrium distribution function follows the power-law structure. One of the possible q-distribution function exhibiting electron non-extensive behaviour is proposed as [3]

$$f_e(v) = C_q\left[1-(q-1)\left(\frac{m_e v^2}{2K_\mathrm{B}T_e}-\frac{e\phi}{K_\mathrm{B}T_e}\right)\right]^{1/(q-1)},$$

where the constant of normalisation is

$$C_q = n_{e0}\frac{\Gamma(1/(1-q))}{\Gamma\big(1/(1-q)-(1/2)\big)}\sqrt{\frac{m_e(1-q)}{2\pi K_\mathrm{B}T_e}}$$

for $-1 < q < 1$. The significance, applications and development of such distribution are given in refs [4–8]. In addition to electrons and ions, positrons are also major constituents of astrophysical and space plasma. Electron–positron–ion (e–p–i) plasma exists in different regions of magnetosphere, auroral region, in the geotail as observed by the FAST, where high-energy electrons and positrons have been observed [9–13]. This motivates the need to investigate the plasma waves in e–p–i plasma. Annihilation process is the other very important difference to distinguish between e–p–i plasma from standard plasma. In some astrophysical situations, the analysis of the corresponding annihilation spectrum allows for the estimate of the conditions in the environment where the annihilation takes place. It can be shown [14] that usually the pair plasma will last

Published online: 06 March 2020

sufficiently long for the collective interaction to take place; otherwise a process of creation of pairs is required to balance the annihilation rate. This is the situation in laboratory environments as well, where, in fact, the annihilation is not of much importance; the annihilation time turns out to be of the order of 1 s.

Numerous powerful strategies have been built up to study electron-acoustic shock waves (EASWs). One of the popular method used for describing small but finite-amplitude solitary waves is the KdV model. The one-dimensional geometry is not sufficient to explain the complete picture of all solitary waves formed in nature. In 1974, Zakharov and Kuznetsov [15] proposed a model to study the three-dimensional (3D) problems of solitary waves in different plasma systems. This Zakharov–Kuznetsov (ZK) equation is a partial differential equation which describes the behaviour of nonlinear wave motion in magnetised plasma. We examined some previous work aimed at solving ZK equation. Mace and Hellberg [16] have studied the nonlinear electron-acoustic (EA) solitons in magnetised plasma with two-electron population and found that only rarefactive solitons are supported by the given plasma model. They further discussed the critical values of plasma parameters at which negative potential EA waves changed to positive potential solitons. Furthermore, various researchers [17–22] have used the Zakharov-Kuznetsov (ZK) equation to study the nonlinear dynamics of solitary structures in different plasma environments. Moslem and Sabry [23] derived the Zakharov–Kuznetsov–Burgers (ZKB) equation for dust ion-acoustic waves propagating in dissipative magnetised plasma. They obtained the exact solution of the ZKB equation using the extended tanh function method. Shalaby *et al* [25] studied the 3D electron-acoustic waves (EAWs) in a magnetised plasma featuring nonthermal distribution of hot electrons. The ZK equation was derived from reductive perturbation method. Effects of density and temperature ratio of hot to cold electrons were discussed on the soliton structures. Only negative potential EA solitary waves were found to exist. Saini *et al* [24] showed the existence of ion-acoustic waves with two-polarity potential in magnetised plasma. From the solution of ZK equation, they found the influence of different plasma parameters on the existence domain of solitary waves. Ata ur Rahman *et al* [26] investigated the 3D electrostatic solitary waves in relativistic degenerate magnetoplasma. The ZK equation was derived to study the dependence of physical behaviour of ion-acoustic waves on plasma number density and direction cosines. Authors applied their results to explain the pulsating white dwarfs.

In all the aforementioned investigations, the effect of positrons was not taken into account. It seems that more efforts and studies are required to study plasma waves in e–p–i plasma. A few years back, Mushtaq and Shah [27] derived and analysed ZK equation to examine the regions of ion-acoustic waves in relativistic magnetoplasma with positrons. Electrostatic structures in e–p–i plasma under the influence of magnetic field and superthermality were investigated by Williams and Kourakis [28]. It is found that strong superthermality and positron concentration suppressed the amplitude and width of solitary waves. From the above literature review, it is clear that the study of ZKB equation with electrons and positrons following q non-extensive distribution in magnetised dissipative e–p–i plasma is not reported. The pursuit of this research is motivated by recent progress in the area. Therefore, in this work, reductive perturbation is utilised to get ZKB equation in magnetised e–p–i plasma. Its analytical solution is then presented which is used to study the effect of various plasma parameters on the nonlinear propagation of these waves.

This paper is organised as follows: Section 2 is devoted to the derivation of the ZKB equation using Tsallis distributed electrons and positrons. Section 3 deals with the solution of the ZKB equation. Section 4 is devoted to the detailed analysis of the numerical results and finally conclusion is presented in §5.

2. Formulation of the problem

The plasma with two-electron populations is known to exist frequently in space, where EA wave may play a significant role. In the present model, magnetised plasma containing two-electron population, i.e. hot and cold electrons with positrons, has been considered. Of these electron populations, cold component is indicated by subscript c, the hot one by subscript h and positrons by subscript b. The fluid equations governing the dynamics of this can be written as

$$\frac{\partial n_c}{\partial t} + \nabla \cdot (n_c \mathbf{u_c}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u_c}}{\partial t} + (\mathbf{u_c} \cdot \nabla)\mathbf{u_c} + \frac{5\alpha(1+\alpha)^{2/3}}{3\theta} n_c^{-1/3} \nabla n_c - \alpha \nabla \phi = \eta \nabla^2 \mathbf{u_c} + \omega_c \mathbf{u_c} \times \mathbf{e_z}, \tag{2}$$

$$\nabla^2 \phi = \frac{1}{\alpha} n_c + n_h - \left(\frac{1+\alpha+\beta}{\alpha}\right) - \frac{\beta}{\alpha} n_p, \tag{3}$$

where

$$\alpha = \frac{n_{ho}}{n_{co}}, \quad \theta = \frac{T_h}{T_c} \tag{4}$$

$$\beta = \frac{n_{bo}}{n_{co}}, \quad \sigma = \frac{T_h}{T_b}. \tag{5}$$

Cold electron fluid velocity (u_c) is normalised by $C_e = (K_\mathrm{B}T_h/\alpha m)^{1/2}$ and ϕ is the electrostatic wave potential normalised by $K_\mathrm{B}T_h/e$.

The differential form of eqs (1)–(3) can be written as

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_{cx}) + \frac{\partial}{\partial y}(n_c u_{cy}) + \frac{\partial}{\partial z}(n_c u_{cz}) = 0, \tag{6}$$

$$\frac{\partial u_{cx}}{\partial t} + u_{cx}\frac{\partial u_{cx}}{\partial x} + u_{cy}\frac{\partial u_{cx}}{\partial y} + u_{cz}\frac{\partial u_{cx}}{\partial z} + \frac{5\alpha(1+\alpha)^{2/3}}{3\theta} n_c^{-1/3}\frac{\partial n_c}{\partial x} - \alpha\frac{\partial \phi}{\partial x} = \eta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u_{cx} + \omega_c u_{cy}, \tag{7}$$

$$\frac{\partial u_{cy}}{\partial t} + u_{cx}\frac{\partial u_{cy}}{\partial x} + u_{cy}\frac{\partial u_{cy}}{\partial y} + u_{cz}\frac{\partial u_{cz}}{\partial z} + \frac{5\alpha(1+\alpha)^{2/3}}{3\theta} n_c^{-1/3}\frac{\partial n_c}{\partial y} - \alpha\frac{\partial \phi}{\partial y} = \eta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u_{cy} - \omega_c u_{cx}, \tag{8}$$

$$\frac{\partial u_{cz}}{\partial t} + u_{cx}\frac{\partial u_{cz}}{\partial x} + u_{cy}\frac{\partial u_{cz}}{\partial y} + u_{cz}\frac{\partial u_{cz}}{\partial z} + \frac{5\alpha(1+\alpha)^{2/3}}{3\theta} n_c^{-1/3}\frac{\partial n_c}{\partial z} - \alpha\frac{\partial \phi}{\partial z} = \eta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u_{cz}, \tag{9}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{\alpha}n_c + n_h - \left(\frac{1+\alpha+\beta}{\alpha}\right) - \frac{\beta}{\alpha}n_p, \tag{10}$$

where $\alpha = n_{ho}/n_{co}$, $\beta = n_{po}/n_{co}$, $\theta = T_h/T_c$ and $\sigma = T_p/T_c$. n_c and u_c are normalised by their equilibrium values n_{c0} and $C_e = (K_\mathrm{B}T_h/\alpha m)^{1/2}$. The spatial and temporal variables are normalised by hot electron Debye length $\lambda_\mathrm{D} = (k_\mathrm{B}T_h/4\pi n_{h0}e^2)^{1/2}$ and inverse of cold electron plasma frequency $\omega_{pc}^{-1} = (m/4\pi n_{h0}e^2)^{1/2}$. ϕ represents the electrostatic wave potential normalised by $K_\mathrm{B}T_h/e$ and $\lambda_\mathrm{D}^2\omega_{pc}$ and kinematic viscosity η normalised by $\lambda_\mathrm{D}^2\omega_{pc}$. Following the model of Amour and Tribeche [29], the non-extensive distribution for the hot electrons is taken as

$$n_h = [1 + (q-1)\phi]^{(q+1)/(2(q-1))}, \tag{11}$$

where q is the strength of the non-extensivity. When $q \to 1$, eq. (11) reduces to the well-known Maxwell–Boltzmann distribution. After expansion, eq. (11) becomes

$$n_h = 1 + \frac{q+1}{2}\phi + \frac{(q+1)(3-q)}{8}\phi^2 + \cdots. \tag{12}$$

Similarly, the non-extensive distribution of positron is taken as

$$n_p = [1 - \sigma(q-1)\phi]^{(q+1)/(2(q-1))}.$$

Expanding n_p, we get

$$n_p = 1 - \sigma\left(\frac{q+1}{2}\right)\phi + \sigma^2\frac{(q+1)(3-q)}{8}\phi^2 + \cdots. \tag{13}$$

Using these expansions in Poisson equation, we get

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\alpha}n_c + a_1\phi + a_2\phi^2 - \frac{1}{\alpha}, \tag{14}$$

where

$$a_1 = \left(1 + \frac{\beta\sigma}{\alpha}\right)\frac{q+1}{2},$$

$$a_2 = \left(1 - \frac{\beta\sigma^2}{\alpha}\right)\frac{(q+1)(3-q)}{8}. \tag{15}$$

In order to derive the ZKB equation, reductive perturbation technique is used in which space and time coordinates are stretched as $\tau = \epsilon^{3/2}t$, $\zeta = \epsilon^{1/2}x$, $\chi = \epsilon^{1/2}y$ and $\xi = \epsilon^{1/2}(z - \lambda t)$ where λ is the wave velocity. Following this, the dependent variables n_c, n_b, u_c, u_b and ϕ are expanded as

$$n_c = 1 + \epsilon n_{c1} + \epsilon^2 n_{c2} + \cdots,$$

$$u_{cx} = \epsilon^{3/2}u_{cx1} + \epsilon^2 u_{cx2} + \cdots,$$

$$u_{cy} = \epsilon^{3/2}u_{cy1} + \epsilon^2 u_{cy2} + \cdots,$$

$$u_{cz} = \epsilon u_{cz1} + \epsilon^2 u_{cz2} + \cdots,$$

$$\phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots. \tag{16}$$

Since we have considered the weak damping situation, assume that $\eta \approx \epsilon^{1/2}\eta_0$. Substituting (16) into (6), (9) and (14) and collect the coefficients of $\epsilon^{3/2}$ as

$$n_{1c} = -\alpha a_1\phi_1 \tag{17}$$

$$u_{1c} = -\alpha a_1\lambda\phi_1 \tag{18}$$

$$\lambda^2 = \frac{5\alpha(1+\alpha)^{2/3}}{3\theta} + \frac{1}{a_1}. \tag{19}$$

Equation (19) represents the phase velocity of the shock waves. It is observed that it depends on non-extensivity, obliqueness, density and temperature ratio of two population of electrons. Now equate the lowest orders of ϵ on both sides of x and y components of momentum equation, and we get

$$\frac{3\alpha(1+\alpha)^2}{\theta}\frac{\partial n_1}{\partial \zeta} = \alpha\frac{\partial \phi_1}{\partial \zeta} + \omega_c u_{1y} \quad (20)$$

$$\frac{3\alpha(1+\alpha)^2}{\theta}\frac{\partial n_1}{\partial \chi} = \alpha\frac{\partial \phi_1}{\partial \chi} - \omega_c u_{1x} \quad (21)$$

and

$$\lambda\frac{\partial u_{1x}}{\partial \xi} + \omega_c u_{2y} = 0 \quad (22)$$

$$\lambda\frac{\partial u_{1y}}{\partial \xi} - \omega_c u_{2x} = 0. \quad (23)$$

By collecting the higher-order terms of ϵ on both sides of (6), (9) and (14), we obtain

$$-\lambda\frac{\partial n_2}{\partial \xi} + \frac{\partial u_{2x}}{\partial \zeta} + \frac{\partial u_{2y}}{\partial \chi} + \frac{\partial u_{2z}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{1c}u_{1c}) + \frac{\partial n_1}{\partial \tau} = 0 \quad (24)$$

$$-\lambda\frac{\partial u_{2z}}{\partial \xi} + u_{1z}\frac{\partial u_{1z}}{\partial \xi} + \frac{\partial u_{1z}}{\partial \tau} + \frac{5\alpha(1+\alpha)^{2/3}}{3\theta}\frac{\partial n_2}{\partial \xi} - \frac{5\alpha(1+\alpha)^{2/3}}{9\theta}n_1\frac{\partial n_1}{\partial \xi} = \alpha\frac{\partial \phi_2}{\partial \xi} + \eta_0\left(\frac{\partial^2}{\partial \zeta^2} + \frac{\partial^2}{\partial \chi^2} + \frac{\partial^2}{\partial \xi^2}\right)u_{1z} \quad (25)$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \frac{1}{\alpha}n_{2c} + a_1\phi_2 + a_2(\phi_1)^2. \quad (26)$$

Now, using eqs (17)–(23), the ZKB equation is obtained in Cartesian coordinates as

$$\frac{\partial \phi}{\partial \tau} + A\phi\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} + C\frac{\partial}{\partial \zeta^2}\left(\frac{\partial^2}{\partial \zeta^2} + \frac{\partial^2}{\partial \chi^2}\right)\phi = D\left(\frac{\partial^2}{\partial \zeta^2} + \frac{\partial^2}{\partial \chi^2} + \frac{\partial^2}{\partial \xi^2}\right)\phi, \quad (27)$$

where $\phi_1 = \phi$. In eq. (27), the quantity A is the nonlinearity coefficient whereas B, C and D are the coefficients of dispersion and dissipation respectively, defined as

$$A = -B\left(2a_2 + \alpha\left(3\lambda^2 + \frac{3\alpha}{\theta}\right)a_1^3\right), \quad (28)$$

$$B = \frac{1}{2a_1^2\lambda},$$

$$C = -B\left(1 + \frac{\lambda^4 a_1^2}{\omega^2}\right) \quad \text{and} \quad D = \frac{\eta_0}{2}. \quad (29)$$

Here, it is worth mentioning that only the last term of eq. (27) contained the parameter η_0 (kinematic viscosity). For $\eta_0 \to 0$, eq. (27) leads to the formation of solitary waves but the presence of dissipative term leads to the formation of shock waves. Thus, one can say that the dissipative coefficient η_0 plays an important role in the characterisation of the nonlinear wave structures.

3. Solution of the ZKB equation

To derive the solution of eq. (27), consider the transformation

$$r = (l_x\zeta + l_y\chi + l_z\xi - U\tau), \quad (30)$$

where l_x, l_y and l_z are the direction cosines along ζ, χ and ξ respectively. Using (30) in eq. (27), we obtain the following differential equation:

$$-U\frac{d\phi}{dr} + Al\phi\frac{d\phi}{dr} + lFs^2\frac{d^3\phi}{\partial r^3} = Ds\frac{d^2\phi}{dr^2}, \quad (31)$$

where

$$F = Bl_z^2 + C(l_x^2 + l_y^2).$$

Now, we we shall employ the tanh (tangent hyperbolic) method (Malfliet [30], Kourakis *et al* [31]) to obtain the exact solution of eq. (30). Introducing the new variable $Y = \tanh(\rho r)$ to eq. (31), we get

$$-u\frac{d\phi}{dY} + Al\phi\frac{d\phi}{dY} + Fl\rho^2\frac{d}{dY} \times\left[(1-Y^2)\frac{d}{dY}\left((1-Y^2)\frac{d\phi}{dY}\right)\right] - C\rho\frac{d}{dY}\left((1-Y^2)\frac{d\phi}{dY}\right) = 0. \quad (32)$$

Let us assume that the solution of eq. (32) is in the form of

$$\phi(Y) = f_0 + f_1(Y) + f_2(Y)^2. \quad (33)$$

Using eq. (33) into (32), we get a set of algebraic equations. After equating the coefficients of different powers of Y, we obtain

$$f_0 = \frac{u}{A} + \frac{3D^2}{25AFl_z^2}, \quad f_1 = -\frac{6D^2}{25AFlz^2},$$

$$f_2 = -\frac{3D^2}{25AFl_z^2}, \quad s = \frac{D}{10Fl_z}$$

and

$$u = \frac{6D^2}{25AFl_z}.$$

Substitute all these values in eq. (33), we get

$$\phi = \frac{3D^2}{25FAl_z^2}[1 - \tanh^2 r + 2(1 - \tanh r)]$$

or

$$\phi = \phi_m\left(1 - \frac{1}{4}\left[1 + \tanh\left(\frac{r}{\delta}\right)\right]^2\right), \tag{34}$$

where $\phi_m = (12D^2/25AFl^2)$ and $\delta(= \rho^{-1}) = 10Fl/D$ are respectively the amplitude and width of the shock.

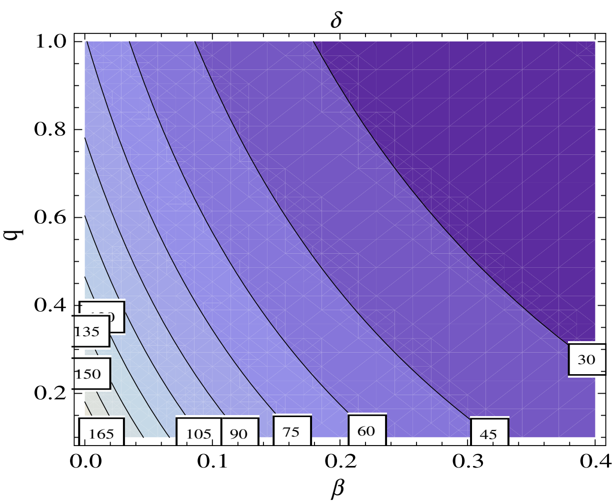

Figure 1. Contour plot of δ with q and β at $\alpha = 2$ and $\theta = 200$.

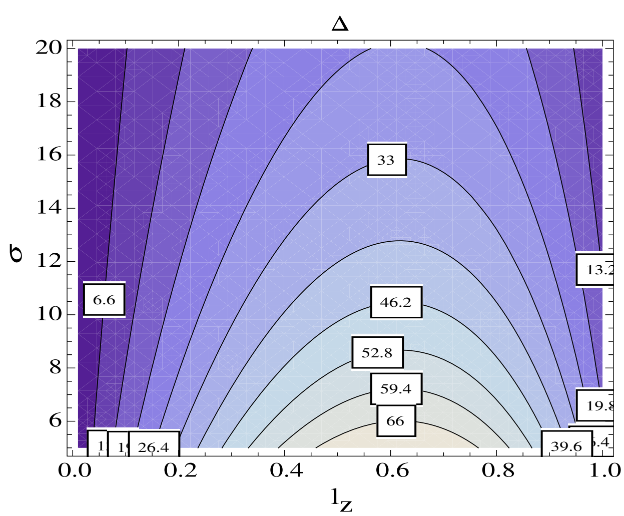

Figure 2. Contour plot of δ with l_z and σ at $\alpha = 2$ and $\theta = 200$.

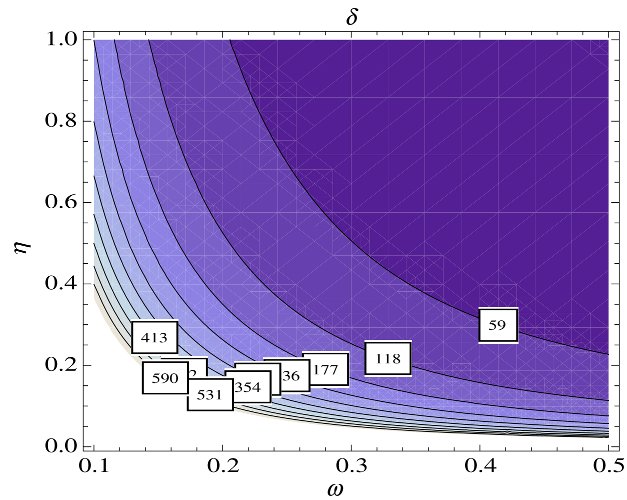

Figure 3. Contour plot of δ with η_0 and ω_c at $\alpha = 2$ and $\theta = 200$ and $q = 0.4$.

4. Parametric analysis of electron-acoustic shock waves

Numerical analysis is carried out to study the 3D EA shock waves under the impact of magnetic field along with q-non-extensive positrons. We have numerically examined the effects of various physical plasma parameters: non-extensive parameter (q), positron concentration (β), relative positron temperature (σ), ω_c and η_0 on the propagation profile of EA shock waves. For the typical numerical analysis, the experimental parametric values are $\theta = 200$, $\beta = 0.2$, $\sigma = 10$, $\eta_0 = 0.4$, $l_z = 0.7$ and $\omega_c = 0.4$. In our work, nonlinear coefficient $A < 0$, and therefore only rarefactive solitons are possible. The contour plot of shock wave width (δ) against q, β, σ, ω_c and η_0 are shown in figures 1–3. It is obvious from figures 1–3 that δ decreases with increase in q, β, η, σ and ω_c. It means that the deviation of particle distribution from Maxwellian equilibrium leads to a narrower width. However, for $\omega_c < 0.23$, the decrease in width is much greater than for $\omega_c > 0.23$. This indicates that for large values of magnetic field, δ does not suffer significant changes. Furthermore, for $l_z < 0.58$, width increases with l_z whereas it becomes narrower for $l_z > 0.58$. The dependence of shock potential (ϕ) on different values of various plasma parameters are illustrated in figures 4–9. Figure 4 gives the spatial variation of r for different values of q (= 0.2 (solid curve), 0.4 (dashed curve) and 0.8 (dotted curve)) at $\theta = 200$, $\beta = 0.2$, $\sigma = 10$, $\eta_0 = 0.4$, $l_z = 0.7$ and $\omega_c = 0.4$. It is remarked that an increase in q (i.e. decrease in non-extensivity) leads to the increase in the amplitude of the EA shock waves. It is stressed that an increase in q suppresses the nonlinearity leading to the enhancement in

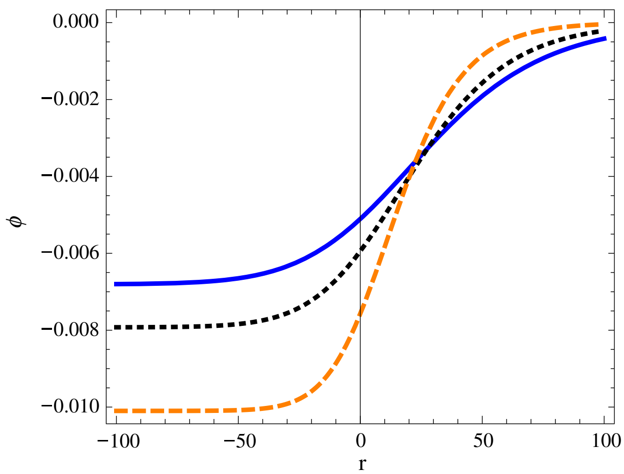

Figure 4. Variation of ϕ with r for different values of q. $q = 0.2$ (solid curve), 0.4 (dotted curve), 0.8 (dashed curve) at $\alpha = 2$ and $\beta = 0.2$.

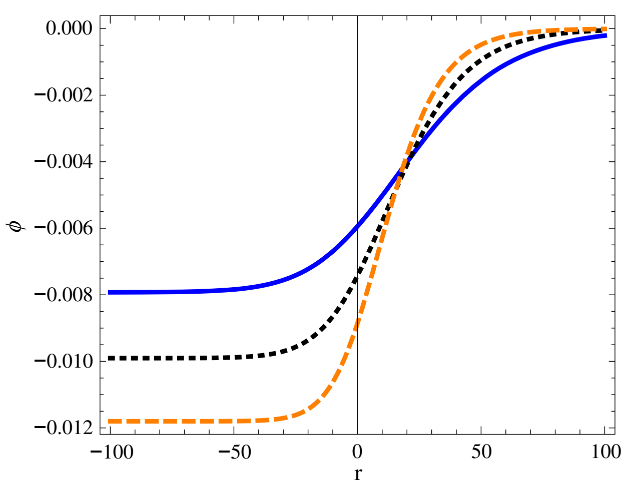

Figure 5. Variation of ϕ with r for different values of β. $\beta = 0.2$ (solid curve), 0.3 (dotted curve), 0.4 (dashed curve) at $\alpha = 2$ and $q = 0.4$.

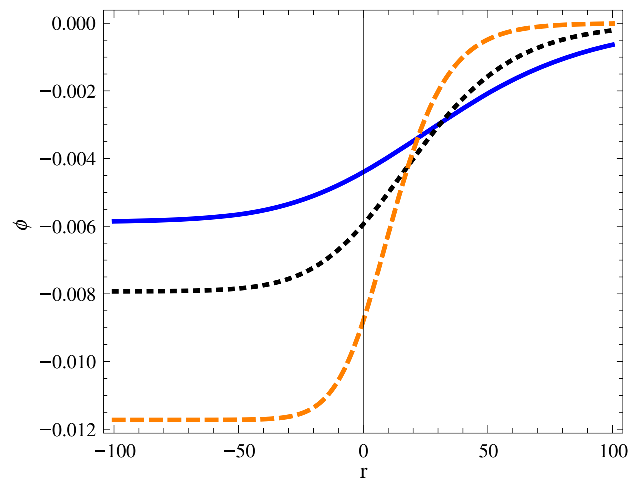

Figure 6. Variation of ϕ with r for different values of σ. $\sigma = 5$ (solid curve), 10 (dotted curve), 20 (dashed curve) at $\alpha = 2$ and $q = 0.4$.

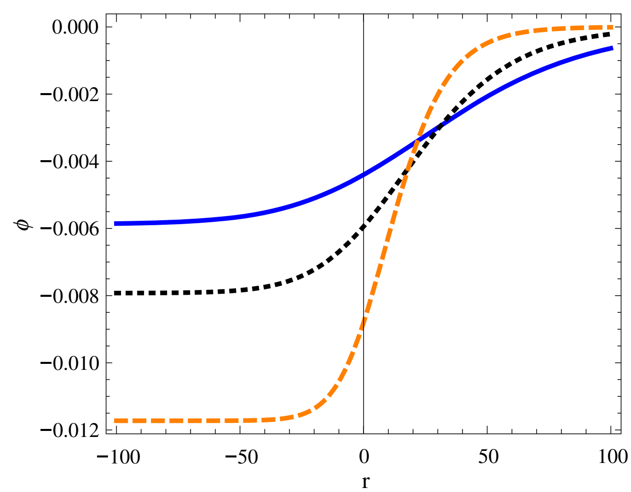

Figure 7. Variation of ϕ with r for different values of ω_c. $\omega_c = 0.2$ (solid curve), 0.3 (dotted curve), 0.4 (dashed curve) at $\alpha = 2$ and $q = 0.4$.

the amplitude of the EA shock waves. Figures 5 and 6 show that the amplitude of EA shock waves increases for higher β and θ. It means that positron temperature and concentration enhance the shock amplitude.

The Lorentz force on a moving charged particle in the presence of magnetic field acts in a direction perpendicular to both the direction of magnetic field and to the direction in which charge particle is moving. It changes the shape of the particle trajectory to a helix. In the present investigation, the effect of magnetic field on the properties of three-dimensional EA shock waves in the e–p–i plasma comes into play via dispersion coefficient (C). The influence of the variation of the magnetic field strength as well as obliqueness of EA shock waves with the direction of ambient magnetic field on the properties of shock structures is presented in figures 7 and 8. To study the effectiveness of ω_c, the graph is plotted between ϕ and ξ by giving variations in magnetic field parameter ω_c (= 0.2, 0.3, 0.4) with $\theta = 200$, $\beta = 0.2$, $\sigma = 10$, $\eta_0 = 0.4$, $l_z = 0.7$ and $q = 3$. In figure 7, the dependence of wave potential on the magnetic field has been shown for the same set of parameters as shown in the previous figure. It can be seen that with the increase in strength of the magnetic field, the amplitude of the shock increases in magneto e–p–i plasma. With increase in l_z, the amplitude as well as width are enhanced for negative potential shocks (as seen in figure 8). The physical explanation is that the strength of the magnetic field affects the dispersion properties of plasma by making it

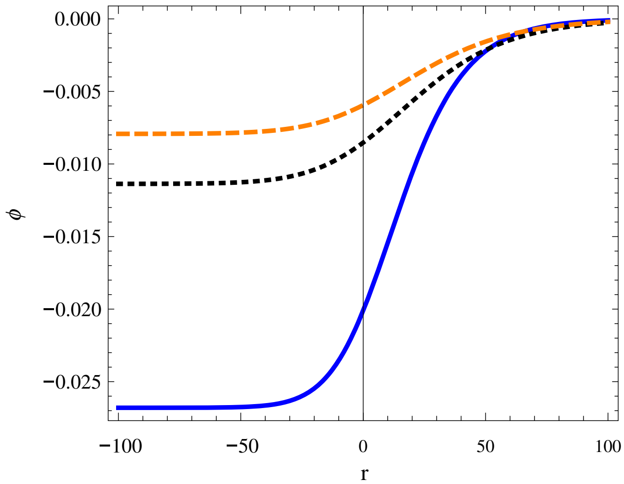

Figure 8. Variation of ϕ with r for different values of l_z. $l_z = 0.3$ (solid curve), 0.5 (dotted curve), 0.7 (dashed curve) at $\alpha = 2$ and $q = 0.4$.

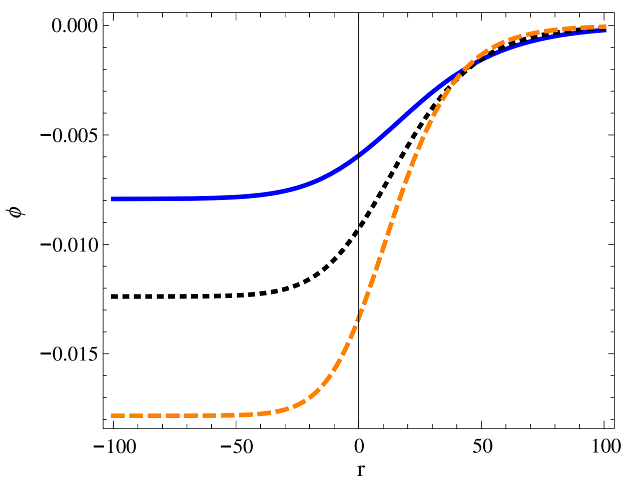

Figure 9. Variation of ϕ with r for different values of η_0. $\eta_0 = 0.3$ (solid curve), 0.5 (dotted curve), 0.7 (dashed curve) at $\alpha = 2$ and $q = 0.4$.

difficult to disperse the plasma in a direction perpendicular to the magnetic field. It is also inferred that more obliquely propagating EA waves give rise to longer and wider shocks.

As C contains viscosity term η_0, it becomes important to study the effect of η_0 on the shock structures. As C is directly proportional to the amplitude and inversely proportional to width, the amplitude increases whereas width of shocks shrinks with increase in the value of viscosity. The impact of η on the shock potential is depicted in figure 9. It can be seen in this figure that as we increase η_0 $(= 0.3, 0.5, 0.7)$, solitary pulses of increased amplitude develops. This means that dissipation term leads to steeper and narrower shock waves.

5. Conclusion

We have presented an investigation of 3D EA shock waves in magnetised plasma carrying positrons along with non-extensive hot electrons. By employing the reductive perturbation technique, we have derived a ZKB equation to highlight the effects of q, β, σ, ω_c and η_0 on the propagation profile of EA shock waves in the present case. In our work, nonlinear coefficient $A < 0$, and therefore only rarefactive solitons are possible. Summarising, we have seen that

(1) Width of the shock wave decreases with increase in β, η_0, q, σ and ω_c. Furthermore, for $l_z < 0.58$, width increases with l_z whereas it becomes narrower for $l_z > 0.58$.
(2) Potential of the shock wave increases as one departs away from non-extensive distribution.
(3) With increase in obliqueness l_z, the amplitude as well as width are enhanced for negative potential shocks.
(4) As the dissipation coefficient contains η_0, it becomes important to study the effect of kinematic viscosity term on the shock structures. Since C is directly proportional to the amplitude and inversely proportional to the width, amplitude increases whereas width of shocks shrinks with increase in the value of viscosity. This means that dissipation term leads to steeper and narrower shock waves.

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