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Joint remote state preparation of an arbitrary eight-qubit cluster-type state

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Abstract. In this paper, we put forward a scheme to realise joint remote state preparation (JRSP) of an arbitrary eight-qubit cluster-type state with two non-maximally entangled Greenberger–Horne–Zeilinger (GHZ) states in a recursive manner. The senders begin by helping the remote receiver to construct one intermediate state which is related to the target state closely. Then, the receiver introduces auxiliary qubits and applies appropriate local operations to obtain the target eight-qubit cluster-type state. It is shown that one new GHZ channel can be distributed among three participants with a certain probability if the initial attempt fails. Moreover, compared with the previous protocols, in our scheme both quantum resources and classical communications are considerably reduced.

Keywords. Joint remote state preparation; Greenberger–Horne–Zeilinger state; cluster-type state; recursive.

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1. Introduction

Since Bennett *et al* [1] first proposed the concept of quantum teleportation, quantum information processing has been greatly developed in recent years. After that, great interest was aroused in quantum information transmission, such as controlled teleportation [2], quantum cloning [3,4], quantum state sharing [5,6], quantum secure direct communication [7,8], etc. Furthermore, a new method called remote state preparation (RSP) has been presented by Lo [9] and Pati [10]. Compared with quantum teleportation, RSP requires less classical communication cost and entanglement cost. Due to these unique advantages and features, a variety of RSP protocols have been widely proposed theoretically and experimentally [11–24]. For example, Dai *et al* [12] proposed a novel scheme for remote preparation of a two-qubit entangled state via partially entangled states. Then, Wang *et al* [14] presented a protocol for remotely preparing a four-particle cluster state via two partially entangled Greenberger–Horne–Zeilinger state (GHZ) states. Recently, Wei *et al* [16] have introduced a novel scheme for remote preparation of an arbitrary multi-qubit state with maximally entangled two-qubit states.

It might be unreliable to let one person to hold all information, especially when it is significant and sensitive. To deal with this situation, another new type of protocol known as joint remote state preparation (JRSP) was proposed. Since Xia *et al* [25] put forward the concept of JRSP, there have been numerous studies on joint remote preparation of various quantum states [26–43]. Liu *et al* [26] introduced a novel scheme for JRSP of arbitrary two- and three-qubit entangled states with two and three GHZ states, respectively. Moreover, Long *et al* [27] investigated a scheme for multiparty JRSP of an arbitrary GHZ state via positive operator-valued measurement (POVM), which consisted of multiple senders and one receiver. Furthermore, Zhang *et al* [28] put forward a new JRSP scheme, which involved multiple senders and multiple receivers. Recently, Wang *et al* [29] investigated a protocol for joint remote preparing multi-qubit states, which was applicable to the situation where various types of noise acted on the transmission system.

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In addition, as one important entangled resource in quantum information, cluster states have been widely used due to their maximum connectivity and reliable security. Owing to these advantages, cluster states can be efficiently applied to quantum information processing tasks [44–49], such as quantum teleportation [44,45], JRSP [46], dense coding [47], quantum state splitting [48], etc. Since they are harder to be destroyed by local operations, cluster states have been proved to be superior to GHZ states and W states [50]. Therefore, JRSP of cluster states has attracted much attention where various entangled states have been utilised as quantum channels. For instance, Zhan *et al* [30] proposed two JRSP schemes of four-qubit cluster states via six EPR pairs and two six-qubit entangled states, respectively. Further, An *et al* [33] reinvestigated and improved Zhan's protocols. In their protocol, only three (not six as in [30]) EPR pairs were utilised as the quantum channel. Lately, to enhance success probability, Wang *et al* [34] presented a protocol for deterministic JRSP of a four-qubit cluster state via two GHZ states. Their scheme could achieve 100% success probability. However, quantum channels used in the aforementioned protocols were maximally entangled states. Generally, under normal circumstances, it is impossible to maintain maximally entangled state at one's disposal due to the inevitable environmental noise. Taking this into consideration, Hou [37] came up with a scheme for JRSP of four-particle cluster states based on two-qubit partially entangled states and then extended it to multi-party scenario. Recently, Choudhury and Samanta [38] proposed a scheme to remotely prepare an arbitrary six-qubit cluster state with one Bell state and two GHZ states as quantum channels.

Note that in most of the JRSP schemes where non-maximally entangled channels are used, if JRSP fails, the channel will be completely destroyed leading to the loss of precious channel resources. In this sense, it is significant to explore a recursive scheme to enhance channel resource efficiency. In addition, although several schemes of remote preparation of cluster states have been investigated, JRSP of eight-qubit cluster-type state has never been explored. In this article, we work for JRSP of arbitrary eight-qubit cluster-type states. We also show that even when non-maximally entangled channels are utilised in JRSP, it is possible to reuse quantum resource even if the initial attempt fails.

This paper is organised as follows. In §2, a new scheme for JRSP of arbitrary eight-qubit cluster-type states is introduced. In §3, we analyse the efficiency of our protocol. Finally, a concise conclusion is given in §4.

2. JRSP of arbitrary eight-qubit cluster-type states

For simplicity, we suppose there are three participants: two senders (Alice and Bob) and one receiver (Charlie). Both Alice and Bob are willing to help the remote receiver Charlie to prepare an eight-qubit cluster-type state, as described below:

$$|C_8\rangle = a_0 e^{i\theta_0}|00000000\rangle + a_1 e^{i\theta_1}|00001111\rangle + a_2 e^{i\theta_2}|11110000\rangle - a_3 e^{i\theta_3}|11111111\rangle. \quad (1)$$

Here $\theta_0 = 0, \theta_k \in [0, 2\pi)(k = 1, 2, 3)$. The real coefficients a_0, a_1, a_2, a_3 satisfy the normalisation condition $\sum_{k=0}^{3} a_k^2 = 1$. In this scheme, all information of state (1) is distributed as follows. Alice knows the amplitude information a_0, a_1, a_2, a_3. The phase information θ_k $(k = 0, 1, 2, 3)$ is split in two parts ϕ_k^A and ϕ_k^B which satisfy the relationship $(\phi_k^A, \phi_k^B) \in R, \phi_k^A + \phi_k^B = \theta_k$. In detail, Alice knows $\{\phi_0^A, \phi_1^A, \phi_2^A, \phi_3^A\}$ while Bob knows $\{\phi_0^B, \phi_1^B, \phi_2^B, \phi_3^B\}$. In this case, neither Alice nor Bob can separately help Charlie to reconstruct the target state. Charlie does not have any information about the prepared state.

Assume Alice, Bob and Charlie share two non-maximally entangled three-qubit GHZ states as the quantum channel at the very beginning,

$$|\varphi\rangle_{A_1B_1C_1} = (\alpha_0|000\rangle + \beta_0|111\rangle)_{A_1B_1C_1}, \quad (2)$$

$$|\varphi\rangle_{A_2B_2C_2} = (\alpha_1|000\rangle + \beta_1|111\rangle)_{A_2B_2C_2}. \quad (3)$$

Here all real coefficients satisfy the relationships $|\alpha_t|^2 + |\beta_t|^2 = 1$ and $|\alpha_t| < |\beta_t|$ $(t = 0, 1)$. Alice is in possession of qubits A_1 and A_2. Bob possesses qubits B_1 and B_2 and Charlie possesses qubits C_1 and C_2, as shown in figure 1.

Hence, the combined state of the whole system can be written as

$$\begin{aligned}
|\varphi\rangle_{A_1B_1C_1A_2B_2C_2} &= (\alpha_0|000\rangle + \beta_0|111\rangle)_{A_1B_1C_1} \\
&\quad \otimes (\alpha_1|000\rangle + \beta_1|111\rangle)_{A_2B_2C_2} \\
&= (\alpha_0\alpha_1|000000\rangle + \alpha_0\beta_1|000111\rangle \\
&\quad + \beta_0\alpha_1|111000\rangle + \beta_0\beta_1|111111\rangle)_{A_1B_1C_1A_2B_2C_2} \\
&= (b_0|000000\rangle + b_1|000111\rangle \\
&\quad + b_2|111000\rangle + b_3|111111\rangle)_{A_1B_1C_1A_2B_2C_2}. \quad (4)
\end{aligned}$$

Here, the coefficients $b_0 = \alpha_0\alpha_1, b_1 = \alpha_0\beta_1, b_2 = \beta_0\alpha_1, b_3 = \beta_0\beta_1$ and $b_m = b_0$.

In order to help Charlie to remotely obtain the desired eight-qubit entangled cluster state described in eq. (1),

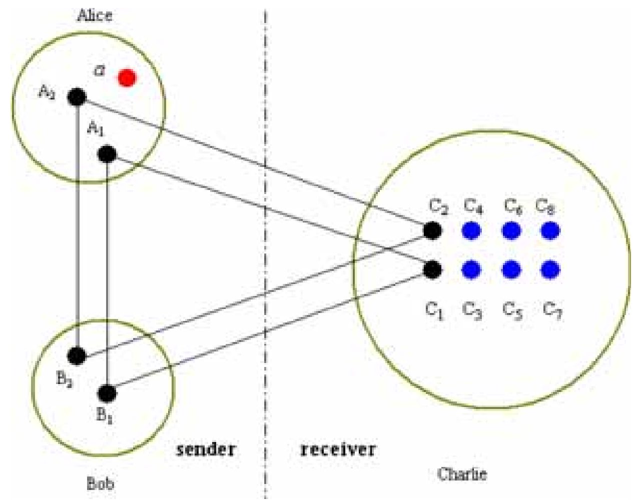

Figure 1. Schematic diagram of our scheme. Here black dots denote entangled particles, the red dot represents the auxiliary particle introduced by Alice, blue dots represent the auxiliary particles introduced by Charlie and bold black lines stand for quantum channels.

Alice and Bob cooperate in the following steps, as shown in figure 2.

Step 1. Alice introduces an auxiliary qubit T with the state $|0\rangle_T$ and performs amplitude modulation by unitary operation $U_{A_1A_2T}$, given by

$$U_{A_1A_2T}=\begin{bmatrix} U_0 & 0 & 0 & 0\\ 0 & U_1 & 0 & 0\\ 0 & 0 & U_2 & 0\\ 0 & 0 & 0 & U_3 \end{bmatrix} \tag{5}$$

with

$$U_i=\begin{bmatrix} \dfrac{a_ib_m}{b_i} & -\sqrt{1-\left(\dfrac{a_ib_m}{b_i}\right)^2}\\ \sqrt{1-\left(\dfrac{a_ib_m}{b_i}\right)^2} & \dfrac{a_ib_m}{b_i} \end{bmatrix}$$

$$(i=0,1,2,3)$$

Now the state of the whole system becomes

$$|\varphi\rangle_{A_1B_1C_1A_2B_2C_2T}$$
$$=b_m(a_0|000000\rangle+a_1|000111\rangle$$
$$+a_2|111000\rangle+a_3|111111\rangle)_{A_1B_1C_1A_2B_2C_2}|0\rangle_T$$
$$+\Big(\sqrt{b_0^2-(a_0b_m)^2}|000000\rangle$$
$$+\sqrt{b_1^2-(a_1b_m)^2}|000111\rangle$$

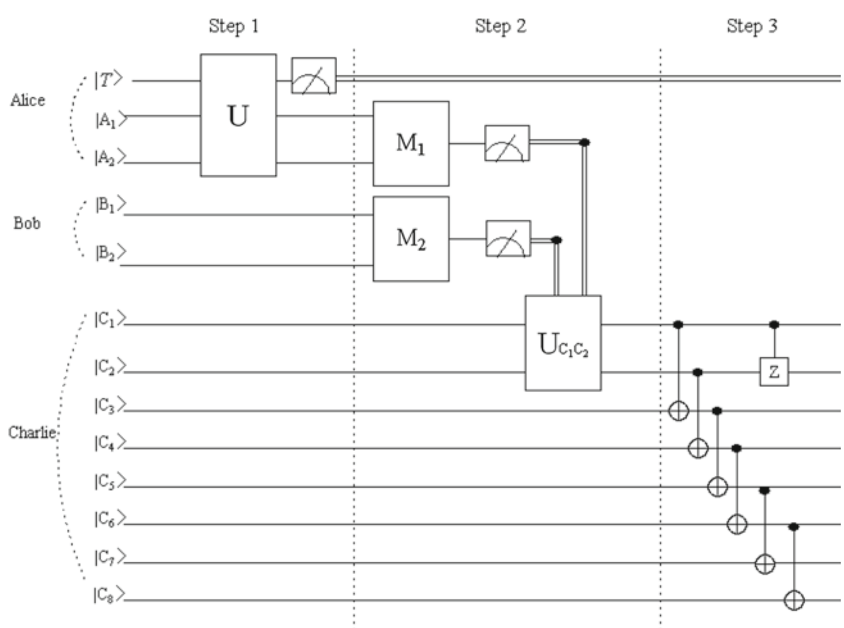

Figure 2. Proposed quantum circuit for JRSP of an arbitrary eight-qubit cluster-type state. U denotes the unitary operation performed by Alice for amplitude modulation, M_1 and M_2 represent the projection measurement performed by Alice and Bob respectively, to realise phase modulation. According to Alice and Bob's measurement results, $U_{C_1C_2}$ denotes the unitary operation performed by Charlie to obtain two-qubit intermediate state.

$$+\sqrt{b_2^2-(a_2b_m)^2}|111000\rangle$$
$$\left.+\sqrt{b_3^2-(a_3b_m)^2}|111111\rangle\right)_{A_1B_1C_1A_2B_2C_2}|1\rangle_T. \quad (6)$$

Next, Alice makes a single qubit measurement on her auxiliary particle T with basis $\{|0\rangle, |1\rangle\}$. Obviously, if the result of the measurement is $|0\rangle_T$, the above state reduces to

$$|\varphi\rangle_{A_1B_1C_1A_2B_2C_2} = (a_0|000000\rangle + a_1|000111\rangle + a_2|111000\rangle + a_3|111111\rangle)_{A_1B_1C_1A_2B_2C_2}. \quad (7)$$

The success probability can be calculated as $|b_m|^2$.

Step 2. Alice performs a two-qubit projective measurement M_1 on her qubits A_1 and A_2 under a set of mutually orthogonal basis vectors given by

$$|\mu_{00}^A\rangle = \frac{1}{2}(|00\rangle + e^{-i\phi_1^A}|01\rangle + e^{-i\phi_2^A}|10\rangle + e^{-i\phi_3^A}|11\rangle)_{A_1A_2},$$
$$|\mu_{01}^A\rangle = \frac{1}{2}(|00\rangle - e^{-i\phi_1^A}|01\rangle + e^{-i\phi_2^A}|10\rangle - e^{-i\phi_3^A}|11\rangle)_{A_1A_2},$$
$$|\mu_{10}^A\rangle = \frac{1}{2}(|00\rangle + e^{-i\phi_1^A}|01\rangle - e^{-i\phi_2^A}|10\rangle - e^{-i\phi_3^A}|11\rangle)_{A_1A_2},$$
$$|\mu_{11}^A\rangle = \frac{1}{2}(|00\rangle - e^{-i\phi_1^A}|01\rangle - e^{-i\phi_2^A}|10\rangle + e^{-i\phi_3^A}|11\rangle)_{A_1A_2}. \quad (8)$$

Meanwhile, Bob carries out another two-qubit projective measurement M_2 on his two qubits B_1 and B_2 with the following set of mutually orthogonal basis vectors:

$$|\nu_{00}^B\rangle = \frac{1}{2}(|00\rangle + e^{-i\phi_1^B}|01\rangle + e^{-i\phi_2^B}|10\rangle + e^{-i\phi_3^B}|11\rangle)_{B_1B_2},$$
$$|\nu_{01}^B\rangle = \frac{1}{2}(|00\rangle - e^{-i\phi_1^B}|01\rangle + e^{-i\phi_2^B}|10\rangle - e^{-i\phi_3^B}|11\rangle)_{B_1B_2},$$
$$|\nu_{10}^B\rangle = \frac{1}{2}(|00\rangle + e^{-i\phi_1^B}|01\rangle - e^{-i\phi_2^B}|10\rangle - e^{-i\phi_3^B}|11\rangle)_{B_1B_2},$$
$$|\nu_{11}^B\rangle = \frac{1}{2}(|00\rangle - e^{-i\phi_1^B}|01\rangle - e^{-i\phi_2^B}|10\rangle + e^{-i\phi_3^B}|11\rangle)_{B_1B_2}. \quad (9)$$

Note that in terms of mutually orthogonal bases above, the joint state in eq. (7) can be rewritten in the following form:

$$\begin{aligned}
|\varphi\rangle_{A_1B_1C_1A_2B_2C_2}
&= \frac{1}{4}|\mu_{00}^A\rangle_{A_1A_2}(|\nu_{00}^B\rangle_{B_1B_2}(a_0|00\rangle \\
&+ a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&+ a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2} \\
&+ |\nu_{01}^B\rangle_{B_1B_2}(a_0|00\rangle - a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&+ a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2} \\
&+ |\nu_{10}^B\rangle_{B_1B_2}(a_0|00\rangle + a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&- a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2} \\
&+ |\nu_{11}^B\rangle_{B_1B_2}(a_0|00\rangle - a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&- a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}) \\
&+ \frac{1}{4}|\mu_{01}^A\rangle_{A_1A_2}(|\nu_{00}^B\rangle_{B_1B_2}(a_0|00\rangle \\
&- a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&+ a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2} \\
&+ |\nu_{01}^B\rangle_{B_1B_2}(a_0|00\rangle + a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&+ a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2} \\
&+ |\nu_{10}^B\rangle_{B_1B_2}(a_0|00\rangle - a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&- a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2} \\
&+ |\nu_{11}^B\rangle_{B_1B_2}(a_0|00\rangle + a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle \\
&- a_2e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}) \\
&+ \frac{1}{4}|\mu_{10}^A\rangle_{A_1A_2}(|\nu_{00}^B\rangle_{B_1B_2}(a_0|00\rangle \\
&+ a_1e^{i(\phi_1^A+\phi_1^B)}|01\rangle
\end{aligned}$$

$$- a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}$$
$$+ |\nu_{01}^B\rangle_{B_1B_2}(a_0|00\rangle - a_1 e^{i(\phi_1^A+\phi_1^B)}|01\rangle$$
$$- a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}$$
$$+ |\nu_{10}^B\rangle_{B_1B_2}(a_0|00\rangle + a_1 e^{i(\phi_1^A+\phi_1^B)}|01\rangle$$
$$+ a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}$$
$$+ |\nu_{11}^B\rangle_{B_1B_2}(a_0|00\rangle - a_1 e^{i(\phi_1^A+\phi_1^B)}|01\rangle$$
$$+ a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2})$$
$$+ \frac{1}{4}|\mu_{11}^A\rangle_{A_1A_2}(|\nu_{00}^B\rangle_{B_1B_2}(a_0|00\rangle$$
$$- a_1 e^{i(\phi_1^A+\phi_1^B)}|01\rangle$$
$$- a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}$$
$$+ |\nu_{01}^B\rangle_{B_1B_2}(a_0|00\rangle + a_1 e^{i(\phi_1^A+\phi_1^B)}|01\rangle$$
$$- a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}$$
$$+ |\nu_{10}^B\rangle_{B_1B_2}(a_0|00\rangle - a_1 e^{i(\phi_1^A+\phi_1^B)}|01\rangle$$
$$+ a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle - a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}$$
$$+ |\nu_{11}^B\rangle_{B_1B_2}(a_0|00\rangle + a_1 e^{i(\phi_1^A+\phi_1^B)}|01\rangle$$
$$+ a_2 e^{i(\phi_2^A+\phi_2^B)}|10\rangle + a_3 e^{i(\phi_3^A+\phi_3^B)}|11\rangle)_{C_1C_2}). \tag{10}$$

There are four possible measurement outcomes $|\mu_{m_1m_2}^A\rangle_{A_1A_2}$ and $|\nu_{n_1n_2}^B\rangle_{B_1B_2}$ $(m_1, m_2, n_1, n_2 \in \{0, 1\})$ obtained by Alice and Bob, respectively. Both Alice and Bob broadcast their respective measurement results to the receiver Charlie via classic communication. According to the measurement results, Charlie performs the corresponding unitary operation $U_{C_1C_2}$ to obtain the two-qubit intermediate state.

The detailed relationship between the measurement outcomes $|\mu_{m_1m_2}^A\rangle_{A_1A_2}$ and $|\nu_{n_1n_2}^B\rangle_{B_1B_2}$ and the unitary operation $U_{C_1C_2}$ is provided in table 1. For example, if the measurement outcomes of Alice and Bob are $|\mu_{01}^A\rangle_{A_1A_2}$ and $|\nu_{00}^B\rangle_{B_1B_2}$ respectively, the state of particles C_1 and C_2 becomes

$$|\varphi\rangle_{C_1C_2} = (a_0|00\rangle - a_1 e^{i(\varphi_1^A+\varphi_1^B)}|01\rangle$$
$$+ a_2 e^{i(\varphi_2^A+\varphi_2^B)}|10\rangle - a_3 e^{i(\varphi_3^A+\varphi_3^B)}|11\rangle)_{C_1C_2}. \tag{11}$$

Table 1. The relationship between the measurement outcomes of Alice and Bob and the unitary operations performed by Charlie.

Measurement results	Unitary operation $U_{C_1C_2}$
$\|\mu_{00}^A\rangle_{A_1A_2}, \|\nu_{00}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes I_{C_2}$
$\|\mu_{00}^A\rangle_{A_1A_2}, \|\nu_{01}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes Z_{C_2}$
$\|\mu_{00}^A\rangle_{A_1A_2}, \|\nu_{10}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes I_{C_2}$
$\|\mu_{00}^A\rangle_{A_1A_2}, \|\nu_{11}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes Z_{C_2}$
$\|\mu_{01}^A\rangle_{A_1A_2}, \|\nu_{00}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes Z_{C_2}$
$\|\mu_{01}^A\rangle_{A_1A_2}, \|\nu_{01}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes I_{C_2}$
$\|\mu_{01}^A\rangle_{A_1A_2}, \|\nu_{10}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes Z_{C_2}$
$\|\mu_{01}^A\rangle_{A_1A_2}, \|\nu_{11}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes I_{C_2}$
$\|\mu_{10}^A\rangle_{A_1A_2}, \|\nu_{00}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes I_{C_2}$
$\|\mu_{10}^A\rangle_{A_1A_2}, \|\nu_{01}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes Z_{C_2}$
$\|\mu_{10}^A\rangle_{A_1A_2}, \|\nu_{10}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes I_{C_2}$
$\|\mu_{10}^A\rangle_{A_1A_2}, \|\mu_{11}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes Z_{C_2}$
$\|\mu_{11}^A\rangle_{A_1A_2}, \|\nu_{00}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes Z_{C_2}$
$\|\mu_{11}^A\rangle_{A_1A_2}, \|\nu_{01}^B\rangle_{B_1B_2}$	$Z_{C_1} \otimes I_{C_2}$
$\|\mu_{11}^A\rangle_{A_1A_2}, \|\nu_{10}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes Z_{C_2}$
$\|\mu_{11}^A\rangle_{A_1A_2}, \|\nu_{11}^B\rangle_{B_1B_2}$	$I_{C_1} \otimes I_{C_2}$

When Charlie receives the measurement results, he performs unitary operation $I_{C_1} \otimes Z_{C_2}$ onto his particles C_1 and C_2.

In this way, an intermediate two-qubit state containing all information of the target eight-qubit cluster-type state is obtained by Charlie, as shown now:

$$|\varphi\rangle_{C_1C_2} = (a_0|00\rangle + a_1 e^{i\theta_1}|01\rangle$$
$$+ a_2 e^{i\theta_2}|10\rangle + a_3 e^{i\theta_3}|11\rangle)_{C_1C_2}. \tag{12}$$

Step 3. In order to obtain the desired eight-qubit cluster-type state in eq. (1), Charlie should introduce six auxiliary qubits with the initial state $|000000\rangle_{C_3C_4C_5C_6C_7C_8}$. Then he applies several CNOT gates on the selective qubits to obtain the target eight-qubit cluster-type state:

$$|\varphi^{(1)}\rangle_{C_1C_2C_3C_4C_5C_6C_7C_8}$$
$$= (\mathrm{CNOT}_{C_6C_8}\mathrm{CNOT}_{C_5C_7}\mathrm{CNOT}_{C_4C_6}\mathrm{CNOT}_{C_3C_5}$$
$$\mathrm{CNOT}_{C_2C_4}\mathrm{CNOT}_{C_1C_3})$$
$$|\varphi\rangle_{C_1C_2}|000000\rangle_{C_3C_4C_5C_6C_7C_8}$$
$$= (a_0|00000000\rangle + a_1 e^{i\theta_1}|00001111\rangle$$
$$+ a_2 e^{i\theta_2}|11110000\rangle$$
$$+ a_3 e^{i\theta_3}|11111111\rangle)_{C_1C_3C_5C_7C_2C_4C_6C_8}, \tag{13}$$

where

$$\mathrm{CNOT} = |0\rangle\langle 0| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|) + |1\rangle\langle 1| \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|). \quad (14)$$

Finally, he applies a CZ gate operation on qubits C_1 and C_2. The state in eq. (13) becomes

$$\begin{aligned} &|\varphi^{(2)}\rangle_{C_1C_3C_5C_7C_2C_4C_6C_8} \\ &\quad = \mathrm{CZ}_{C_1C_2}(a_0|00000000\rangle \\ &\qquad + a_1 e^{i\theta_1}|00001111\rangle + a_2 e^{i\theta_2}|11110000\rangle \\ &\qquad + a_3 e^{i\theta_3}|11111111\rangle)_{C_1C_3C_5C_7C_2C_4C_6C_8} \\ &\quad = (a_0|00000000\rangle + a_1 e^{i\theta_1}|00001111\rangle \\ &\qquad + a_2 e^{i\theta_2}|11110000\rangle \\ &\qquad - a_3 e^{i\theta_3}|11111111\rangle)_{C_1C_3C_5C_7C_2C_4C_6C_8}, \end{aligned} \quad (15)$$

where

$$\mathrm{CZ} = |0\rangle\langle 0| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|) + |1\rangle\langle 1| \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|). \quad (16)$$

Now he obtains the target eight-qubit cluster-type state.

Here we make some notes about the above procedure. If the measurement result is $|1\rangle_T$ in Step 1, the state in eq. (4) will collapse into

$$\begin{aligned} &|\varphi\rangle_{A_1B_1C_1A_2B_2C_2} \\ &\quad = \Big(\sqrt{b_0^2 - (a_0 b_m)^2}|000000\rangle \\ &\qquad + \sqrt{b_1^2 - (a_1 b_m)^2}|000111\rangle \\ &\qquad + \sqrt{b_2^2 - (a_2 b_m)^2}|111000\rangle \\ &\qquad + \sqrt{b_3^2 - (a_3 b_m)^2}|111111\rangle\Big)_{A_1B_1C_1A_2B_2C_2}. \end{aligned} \quad (17)$$

It is observed that once qubits A_1, B_1 and C_1 are measured with the basis $\{|0\rangle, |1\rangle\}$ and the measurement outcome is $|111\rangle_{A_1B_1C_1}$, the state in eq. (17) will collapse to the following state:

$$|\varphi\rangle_{A_2B_2C_2} = \Big(\sqrt{b_2^2 - (a_2 b_m)^2}|000\rangle + \sqrt{b_3^2 - (a_3 b_m)^2}|111\rangle\Big)_{A_2B_2C_2}. \quad (18)$$

It shows that one new GHZ channel is to be shared by Alice, Bob and Charlie. Therefore, it is possible for three participants to continue performing JRSP if another GHZ channel is introduced among them.

3. Performance efficiency

In the JRSP scheme, quantum resource consumption, classical information consumption and the prepared target state are usually used to evaluate the efficiency of the protocol. Recently, Choudhury and Samanta [38] defined the intrinsic efficiency of the JRSP protocol as $\eta = q_p/(q_c + x_t)$, where q_p is the number of particles in the prepared state, q_c is the number of qubits consumed and x_t represents the number of classical bits consumed. Here we also adopt their index to evaluate the intrinsic efficiency (η) of our scheme.

We first discuss the usage of classical information in our schemes. It is obvious that total classical information cost is four bits in our proposed scheme. In Step 2 of our protocol, Alice and Bob respectively perform two-qubit projective measurement on their qubits and then tell their measurement outcomes to the receiver.

Then, we discuss the consumption of quantum resources in our scheme. Here, the consumption of quantum resources consists of the number of qubits used in quantum channels and the number of auxiliary qubits. In detail, we first use two non-maximally entangled three-qubit GHZ states. Then, Alice should introduce one auxiliary qubit to implement the amplitude modulation operation. In addition, in Step 3 Charlie should introduce six auxiliary qubits to obtain the desired state. Therefore, the total quantum resource consumption is 13 qubits.

To sum up, the intrinsic efficiency of our protocol can be calculated as $\eta = 8/17$. In addition, according to the method in ref. [38], we can also calculate the efficiency of Zhan's scheme [30], Wang's scheme [34] and Choudhury's scheme [38], as shown in table 2.

From table 2, it can be found that our scheme has several advantages. First, the cluster-type state we prepare is eight-qubit cluster-type state, while only four-qubit cluster-type state is prepared in schemes given in [30,34]. Next, we notice that previous schemes are not optimal in terms of the classical information resource utilisation while our scheme only requires 4-bit classical resource. Additionally, an important difference between these strategies and ours lies in the fact that in our scheme once amplitude modulation fails, the collapsed channel can be reused by three participants with a certain probability, which enhances the efficiency of the valuable quantum resource. The analysis clearly shows that our scheme is better and more economical than other schemes [30,34,38].

Table 2. Comparison of efficiency among several JRSP protocols.

Schemes	Quantum channel	Quantum resource consumption	Classical resource consumption	Prepared quantum state	Intrinsic efficiency (η)
Scheme [30]	Bell	12	8	Four-qubit cluster state	1/5
Scheme [34]	GHZ	8	6	Four-qubit cluster state	2/7
Scheme [38]	GHZ-Bell	11	11	Six-qubit cluster state	3/11
Ours	GHZ	13	4	Eight-qubit cluster state	8/17

4. Discussion and conclusion

In summary, we present a novel JRSP protocol of arbitrary eight-qubit cluster-type state with two non-maximally entangled GHZ states as the quantum channel. In our scheme, the sender Alice should pre-adjust the initial channel with an auxiliary qubit to achieve amplitude modulation by local unitary operations. Then, both senders Alice and Bob perform two-qubit projective measurements on their own qubits, respectively. Finally, the receiver can obtain the desired state by performing several operations. Obviously, compared with previous schemes, our scheme has the following merits: First, the sender Alice can modulate the channel in a recursive way. If the first attempt fails, one new GHZ state can be distributed between Alice, Bob and Charlie with a certain probability. Second, our scheme requires minimum classical resources and demonstrates a higher intrinsic efficiency. Third, we employ non-maximally entangled three-qubit GHZ states as the shared quantum resource in advance, which manifests strong quantum correlations. Based on these advantages, we hope our scheme can stimulate more investigations on JRSP protocols in quantum information processing.

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